Origami^5

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AN A K PETERS BOOK

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Fifth International Meeting of Origami Science, Mathematics, and Education

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Cover Illustrations: See "A Systematic Approach to Twirl Design," Figure 15 (p. 120), and "Reconstructing David Huffman's Legacy in Curved-Crease Folding," Figure 14 (p. 49).

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Preface

Origami^5 follows in the large footprints of four volumes,[1] each documenting work presented at an extraordinary series of meetings that have explored the connections between origami, mathematics, science, technology, education, and other academic fields. The idea for these meetings originated with Professor Humiaki Huzita, who organized the First International Meeting of Origami Science and Technology, held December 6–7, 1989, at the Casa di Ludovico Ariosto in Ferrara, Italy. Five years later, under the leadership of Professor Koryo Miura, the Second International Meeting of Origami Science and Scientific Origami took place November 29–December 2, 1994, at Seian University of Art and Design, Otsu, Shiga, Japan. This meeting officially expanded beyond origami mathematics and science to include origami design, origami in education, and the history of origami. The third meeting, held at Asilomar, Pacific Grove, California, March 9–11, 2001, was organized by Professor Thomas Hull; titled The Third International Meeting of Origami Science, Mathematics and Education, it became known by

[1] [Huzita 89] *Proceedings of the First International Meeting of Origami Science and Technology*, edited by Humiaki Huzita. Padova, Italy: Dipartimento di Fisica Galileo Galilei dell'Universita degli studi di Padova, 1989.

[Miura et al. 97] *Origami Science and Art: Proceedings of the Second International Meeting of Origami Science and Scientific Origami*, edited by Koryo Miura, Tomoko Fuse, Toshizaku Kawaski, and Jun Maekawa. Otsu, Shiga, Japan: Seian University of Art and Design, 1997.

[Hull 02] *Origami*^{3}*: Third International Meeting of Origami Science, Mathematics and Education*, edited by Thomas Hull. Natick, MA: A K Peters, 2002.

[Lang 09] *Origami*^{4}*: Fourth International Meeting of Origami Science, Mathematics and Education*, edited by Robert J. Lang. Natick, MA: A K Peters, 2009.

its acronym, 3OSME, which inspired the format (and name) for subsequent meetings. The fourth meeting, 4OSME, organized by Robert J. Lang, took place September 6–10, 2006, at the California Institute of Technology in Pasadena, California. Most recently, the fifth such meeting, 5OSME, co-organized by Eileen Tan, Benjamin Tan, and Patsy Wang-Iverson, was held on July 13–17, 2010, at the Singapore Management University in Singapore. The majority of papers in this book are based on presentations from this meeting (with a few post-meeting contributions as well).

Origami^5 follows the precedent set by the second meeting and continued at 3OSME and 4OSME to expand the interdisciplinary connections to the world of origami. This book begins with a section on origami history, art, and design. It is followed by sections on origami in education, origami science, engineering, and technology, and it culminates with a section on origami mathematics—the pairing that inspired the first such meeting. The scope of the collected papers is broad; within this one volume, one can find historical information, artists' descriptions of their processes, various perspectives and approaches to the use of origami in education, mathematical tools for origami design, applications of folding in engineering and technology, and original and cutting-edge research on the mathematical underpinnings of the field.

We begin with a section on *origami history, art, and design*, in which the first two papers contrast Eastern and Western aspects of origami. Koshiro Hatori revisits the history of origami and identifies its origin in both the West and the East. Koichi Tateishi examines differences between current Japanese and western origami practices and attributes them to cross-linguistic differences between the two cultures. Then Arnold Tubis and Crystal Mills discuss the surprising role of origami in the creation of the original American flag by Betsy Ross. Erik Demaine et al. integrate history and design in their examination of the work of curved-crease folding pioneer David Huffman and, for the first time, unlock his secrets and reconstruct several of his most famous works.

Origami artists follow many different paths to create their art, often with a mathematical bent, always with a strong emphasis on aesthetic considerations. In a series of papers on art and design, several authors present the reader with eclectic but engaging papers that describe their approaches to the creation of origami art, all with a strong mathematical or technological flavor. Cheng Chit Leong describes curved folding and ties together aesthetical and mathematical considerations. Christine Edison explores the properties of twisted curved corrugations. Andrew Hudson presents a novel algorithm that can be generalized for construction of origami figures from regular polygons based on their rotational symmetry. Miyuki Kawamura and Hiroyuki Moriwaki straddle design and technology with their new technique for illuminating origami models. Faye Goldman demonstrates how an idea can be adapted for use with new material, leading to esthetically pleasing polyhedral models. Krystyna Burczyk and Wojciech Burczyk discuss their systematic approach to using twirls and spirals with differently shaped papers to produce an almost limitless array of new creations. Building upon his work presented at 4OSME, Matthew Gardiner describes advances he has made with oribotics with the aid of technology and inspiration from surprising sources.

The second section focuses on *origami in education.* Education is a shared global enterprise, with increasing numbers of countries using international assessments, such as *TIMSS: Trends in International Mathematics and Science Study* and *PISA: Programme for International Student Assessment*, in reading, mathematics, and science as benchmarks for measuring progress in educating their students. The chapters in this education section offer insight into educational systems in different countries and cultures through the lens of origami. Miri Golan shows how the teaching of origami is aligned with the van Hiele theory of teaching geometry. Maria Lluïsa Fiol et al. rediscover Froebel's recognition of the value of origami in the education of young children and describe the care and time necessary to educate and prepare teachers in the use of origami to stimulate student learning and creativity. Christine Edison presents cases of the effectiveness of origami in re-engaging students with school in an economically deprived environment.

We then turn to an examination of origami's effectiveness and descriptions of specific ways that origami may be used in education. Norma Boakes studies whether constructing origami models can enhance college students' spatial skills. James Morrow and Charlene Morrow offer a detailed approach to learning origami through reverse engineering of models. Sue Pope and Tung Ken Lam present ways in which novices can be introduced to origami and describe how origami can be used to teach challenging mathematics concepts and to avoid common misconceptions. Michael Winckler et al. describe the use of origami to teach geometry to grade eight students in a gymnasium. Shi-Pui Kwan closes out the education section with a discussion of his use of origami in high school geometry.

Origami science, engineering, and technology are the focus of the next group of papers. It is exciting to see the development of what is considered by many as an art form have more direct impact on society through applications to science and technology. This section includes methods for patterning origami mazes and cylinders, origami used as engineered structures, computer aided origami, and fantastical origami-inspired devices.

The section starts with two papers on rigid origami: first, a design technique for rigid folding using thick panels by Tomohiro Tachi, followed by a kinematic analysis of patterned cylinders from Kunfeng Wang and Yan Chen. We then see how pattern folding techniques can be used for their mechanical structural characteristics in papers from Jiayao Ma and Zhong You, Mark Schenk and Simon Guest, and Yves Klett and Klaus Drechsler. These papers demonstrate how origami can be useful for designing structures such as bumpers in cars and airplane fuselages. Steven Gray et al. describe software for aiding origami construction of a self-folding origami robot, while Naoya Tsurutaet al. and Hugo Akitaya et al. describe systems and algorithms for automated diagramming of origami instruction. Finally, Noy Bassiket al. and Kaori Kuribayashi-Shigetomi and Shoji Takeuchi show self-folding micro origami structures and bio-compatible origami structures that could lead to bio-implantable devices.

The *mathematics of origami* has always involved a creative pairing between "origami-math"—the fundamental laws underlying mathematics—and "computational origami"—the mathematics specific to the origami design problem. The final section, devoted to origami mathematics, starts off with a series of papers on mathematical design algorithms beginning with Jun Maekawa's analysis of the design of knots folded from strips of paper. Nadia Benbernou et al. provide patterns for folding arbitrary 3D shapes composed of cubes; Herng Yi Cheng describes how to construct arbitrary biplanar three-dimensional solids; Jun Mitani gives an algorithm for polyhedral and/or curved rotationally symmetric solids.

Then we have a group of papers giving design algorithms for generally planar structures. Erik Demaine et al. present a recipe for construction of arbitrary mazes. Robert J. Lang and Alex Bateman show how a large class of twist tessellations can be designed, solving a longstanding problem in the process, and Lang gives a prescription for designing woven tessellations, introducing a new law of "flat-unfoldability" along the way.

We then shift focus to the mathematical underpinnings of origami design. Tomohiro Tachi and Erik Demaine give a surprising explanation for the utility of highly symmetric crease patterns in the design world based on the number fields of the points constructed thereby, and this leads into a series of works that describe geometric constructions using origami. Robert Orndorff introduces a new construction of square roots, while Emma Frigerio and Kazuo Haga describe two families of geometric construction based on earlier work by Haga, one of the pioneers of origami constructions.

Continuing the theme of geometric constructions, we have several studies of axiomatic origami, i.e., the foundations of the geometric constructions presented earlier in this section. Eulàlia Tramuns creates formal measures of the number of steps required in origami constructions and compares them to other geometric construction tools. Toshikazu Kawasaki and Roger Alperin examine origami constructions in other spaces: spherical and hyperbolic, respectively. We then move to the area of flat-foldability: Hidefumi Kawasaki presents a new proof of a flat-foldability result. Ryuhei Uehara gives new results on configuration counting in the one-dimensional stamp folding problem, followed by Thomas Hull and Eric Chang's work establishing bounds on the number of crease assignments at a single vertex. The section closes with a proof by Erik Demaine et al. that one of the fundamental steps in circle/river packing based origami design is NP-hard.

There is an accompanying website to *Origami*^{5}, http://www.origami-usa.org/origami5, that provides additional resources and links related to the papers.

Acknowledgments

This book, *Origami*^{5}, is the result of a great deal of work by many people that began with the original 5OSME meeting. We editors wish to thank the program committee members who reviewed the abstracts accepted for presentation at the meeting: Philip Chapman-Bell, Martin Demaine, Christine Edison, Emma Frigerio, Thomas Hull, Cheng Chit Leong, Neil Sloane, Tomohiro Tachi, Benjamin Tan, Eileen Tan, Koichi Tateishi, and Seng Kai Wong.

We also thank the individuals working behind the scenes to evaluate the presentations and to review and improve the submitted papers: Roger Alperin, Marshall Bern, Jin-Yi Cai, Martin Demaine, Ken Fan, Emma Frigerio, Simon Guest, Barry Hayes, Thomas Hull, Jason Ku, Daniel Kwan, David Lister, Perla Myers, Jun Mitani, Charlene Morrow, Joseph O'Rourke, Aviv Ovadya, Ryda Rose, Tomohiro Tachi, Arnold Tubis, Ryuhei Uehara, Tamara Veenstra, and those who chose to remain anonymous.

The 5OSME meeting was made possible through the collaborative efforts of many organizations/institutions and people. Partners included the Gabriella and Paul Rosenbaum Foundation, Origami-USA, Principals Academy, and Singapore Management University. Additionally, several sponsors contributed to making the meeting a reality through the sponsorship of student and teacher registrants: BioSym (Biosystems and Micromechanics), Censam (Center for Environmental Sensing and Modeling), Marshall Cavendish, the SMART Centre (Singapore-MIT Alliance for Research and Technology).

In addition to institutional recognition, we must thank the individuals within the institutions who were responsible for the generous contributions: Rohan Abeyaratne, SMART Centre Director; Duriya Aziz, Marshall Cavendish; George Barbastathis, SMART Centre; Madge Goldman, Gabriella and Paul Rosenbaum Foundation; Roger Kamm, BioSyM IRG PI; Les Norford, CENSAM IRG PI; and Kirpal Singh and Sumathi Nair, Wee Kim Wee Centre, SMU.

We thank Cheng Chit Leong for procuring the SMU site for the meeting, Kirpal Singh for offering SMU as the venue, and Sumathi Nair for all

her work and arrangements. We also thank two most important people, without whom the meeting in Singapore would not have occurred: Eileen Tan and Benjamin Tan.

Most of the papers in this book grew out of the meeting presentations, but these papers are original to this book; the authors wrote, re-rewrote, responded to reviews and editing, and in many cases worked through numerous revision cycles. Thanks to their hard work, you have before you, a collection that represents the leading edge of academic origami developments. Without their work, this book would not exist, and so we thank the authors of these papers most of all.

We hope you will enjoy *Origami*^{5}.

Patsy Wang-Iverson
Robert J. Lang
Mark Yim

March, 2011

Part I

Origami History, Art, and Design

History of Origami in the East and the West before Interfusion

Koshiro Hatori

1 Introduction

Origami used to be, and still sometimes is, said to originate in the second century in China. This conjecture was first stated, as far as I know, by Lillian Oppenheimer when she wrote a foreword for Isao Honda's book *How to Make Origami* [Honda 59]. It assumes two things: paper was invented in the second century in China, and origami started just after the invention of paper. I am going to argue that both assumptions are unacceptable.

2 Origin of Origami: Many Misunderstandings and Some Suppositions

When Oppenheimer wrote the foreword to *How to Make Origami*, it was widely believed that paper was invented by the Chinese eunuch Cai Lun (also alphabetized as T'sai Lun) in 105 CE. However, much older paper has been unearthed from some tombs of the Western Han Dynasty (206 BCE–8 CE). The oldest piece, which was discovered at Fan Ma Tan in 1986, is estimated to have been made in the middle of the second century BCE [Komiya 01].

Moreover, recent studies show that high-quality bark paper, called *amate* in Meso-America, *kapa* in Hawaii, and *tapa* in Southeast Asia, dates back to 5000 BCE [Sakamoto 09]. Such ancient paper was so sophisticated that some even had watermarks. The beaten bark paper has a texture that is close to *washi* or Japanese handmade paper. In fact, it has been made from the mulberry tree in Southeast Asia. Even though this bark paper is sometimes regarded as cloth rather than paper, it may well be the origin of *washi* [Sakamoto 08].

The ancient bark paper had such high quality that I believe it could be folded. Then, one may ask whether origami dates back to 7,000 years ago, and my answer would be "no." When one examines the origin of origami, the question should be, in my opinion, about how origami has emerged and developed instead of who folded paper first. It is likely that ancient people folded paper, but such paper folding would have no relationship with today's origami. We cannot trace the history of origami more than a few hundred years.

Some say origami started in the Heian period (794–1185) in Japan. One of the stories they refer to is an anecdote of Abe no Seimei, the most famous *onmyoji* (specialist of a Japanese traditional spiritual cosmology) of the tenth century. The story says that he took a piece of paper and turned it into a real heron to search for his most formidable rival Ashiya Doman.

According to Masao Okamura, however, this tale of Seimei has nothing to do with origami. Some books say he tied a piece of paper to make a knot, some say he cut paper in a shape of a bird, some say he drew a heron on paper, but no book says he folded paper. Okamura's extensive studies revealed that there is no evidence of origami in the Heian period [Okamura 99].

Others mention the *shide*, cut and folded zigzag paper strips used in Shinto rituals, as an example of ancient paper folding. They were originally, however, pieces of cloth offered to the gods. Although it is possible that the *shide* came to be made of paper in the Heian period, I do not see any connection between the *shide* and today's origami.

The origin of origami in Japan is thought to be ceremonial wrappers as represented by *noshi*. *Noshi* was originally a form of folded wrappers for *noshi-awabi*, or stripped and dried abalone meat, although today it is just attached or printed on wrapping paper as a token of good fortune. Another example is a pair of paper butterflies known as *ocho* and *mecho* (Figure 1). They are, in fact, wrappers for sake bottles, although today they are just attached onto the neck of the bottle and used mainly in wedding ceremonies. Some say such wrappers date back to the Heian period, for which I have never seen any evidence.

The *samurai* warriors of the Edo period (1603–1868) were supposed to fold wrapping paper in a specific way according to what was inside when they sent a gift. It is part of the etiquette of the *samurai* class, which was carried down from generation to generation in some houses, most notably

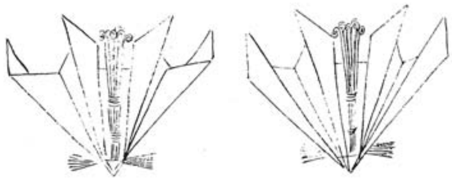

Figure 1. *Ocho* and *mecho* illustrated in the nineteenth century.

Ogasawara, Ise, and Kira. According to Ise Sadatake, who wrote a book of the ceremonial origami *Tsutsumi-no Ki* in 1764, such paper folding was established in the Muromachi period (1333–1573) [Araki 03].

In contrast, the origin of Western origami is thought to be baptismal certificates folded in a "double blintz," that is, folding all the four corners of a square to the center and repeating the same folds on the smaller square. According to Ann Herring, the baptismal certificate of Friedrich Froebel, whom we shall meet in the next section, was also folded [Herring 99].

This custom of folding baptismal certificates seems to have been popular in Central Europe in the seventeenth and eighteenth centuries (Figure 2). Herring suggested that such paper folding may have started before the Protestant Reformation [Herring 99]. So, origami in the West probably dates back to the sixteenth century.

David Lister has noticed that the crease pattern of baptismal certificates is closely similar to the design of old European astrological horoscopes. According to Vicente Palacios, such an "astrological square" was introduced into Spain in the twelfth century [Lister 97]. I must, however, point out that there is no evidence that horoscopes in either Spain or Germany were folded.

Comparing Japanese wrappers with European baptismal certificates, we can observe the difference in the folding styles of the East and the West. The crease lines of *samurai* wrappers run in arbitrary angles, whereas those of baptismal certificates are limited to square grids and diagonals.

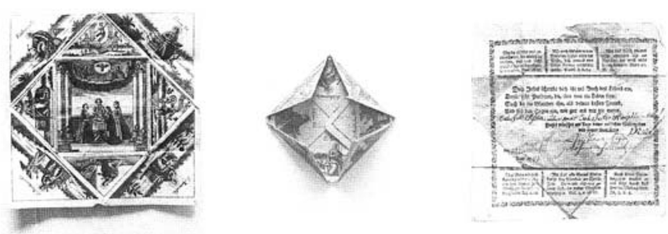

Figure 2. German baptismal certificate from the eighteenth century.

As we shall see in the following section, the difference between Eastern and Western folding stayed with almost all of the origami models until the second half of the nineteenth century, when Japan opened its border and started to exchange cultures with Europe. This fact suggests that the two traditions of origami in Japan and Europe emerged and developed independently of each other.

3 The East and the West: Different Styles, Different Traditions

The most typical European origami model is perhaps the little bird called *pajarita* in Spain and *cocotte* in France. Although its origin is rather vague, I suspect that pajarita must have existed in the late eighteenth century, because horses and riders (Figure 3) in the collection of the German National Museum, which were made around the time of the War of the Sixth Coalition (1812–1814) [Kono 58], appear to have derived from it.

Another popular model in Europe is the boat. Vicente Palacios argued that the boat is recorded in an edition of *Tractatus de Sphaera Mundi* published in Venice in 1490 [Lister 97]. This book was written by Johannes de Sacrobosco (John of Holywood) in the thirteenth century and reprinted more than 60 times through the middle of the seventeenth century. Even though the illustrated boats look like folded models, I would refrain from making judgment until we have more evidence of origami before the nineteenth century.

Also well known is the hat, which John Tenniel has depicted in artwork for Lewis Carroll's *Through the Looking Glass* published in 1872 [Carroll 03]. Although both the boat and the hat are made from rectangular sheets of paper, most of the European traditional models are made from square sheets. Those models are well documented in the context of the Froebelian education system.

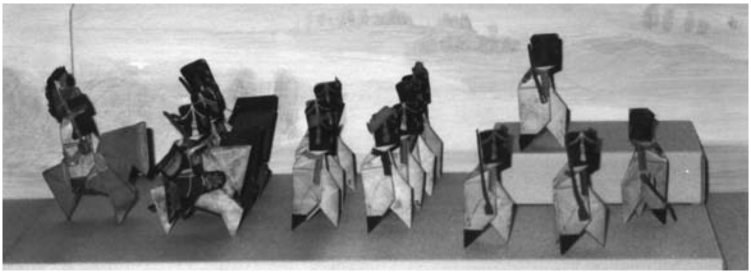

Figure 3. Horses and riders from the early nineteenth century.

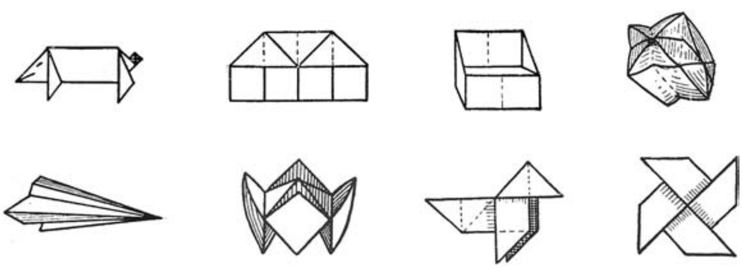

Figure 4. Examples of European traditional models.

Friedrich Wilhelm August Froebel was a German educator who founded the first modern kindergarten in 1837. His education system contained a set of toys called "Gifts" and a set of plays called "Occupations," and one of the most important Occupations was origami. Maria Kraus-Boelté recorded nearly one hundred origami models in her book *The Kindergarten Guide* [Kraus-Boelté and Kraus 82].

Many of the European origami models contained in Kraus-Boelté's book are not included in contemporary Japanese records. The pig, house, sofa (also known as piano or organ), balloon (waterbomb), arrow (paper plane), salt cellar (cootie catcher), bird (*pajarita* or *cocotte*), and windmill shown in Figure 4 were all born in Europe and imported into Japan along with the kindergarten system.

The balloon may be the same model as the "paper prison" mentioned in John Webster's play *The Duchess of Malfi*, which premiered in 1614. That may be the oldest reference to European origami, but again, I would suspend my conclusion for now. Moreover, I do not consider just a few unconvincing references scattered over two centuries enough to prove the existence of origami.

Looking through the European models, one can easily notice that most of them have only creases that are either square grids or diagonals. This is true even in the Chinese junk and the gondola, which are similar to the Japanese *takara-bune*, or treasure ship (Figure 5). The treasure ship has a pointier bow that is folded with sharp-angled creases. This difference is, in my opinion, so critical that I am sure the Chinese junk and *takara-bune* developed independently on the opposite sides of the world.

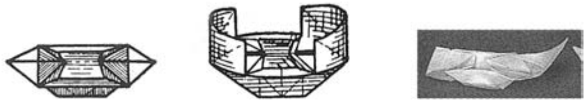

Figure 5. Chinese junk, gondola, and *takara-bune*(left to right).

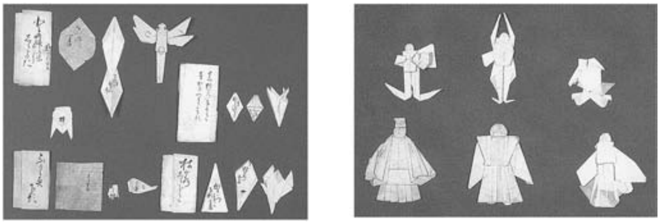

Figure 6. Examples of Japanese traditional models from the early nineteenth century.

The origami history researcher Satoshi Takagi one day bought a box containing many origami pieces. They are considered to have been folded by several persons in the house of Moriwaki from the middle eighteenth century through the nineteenth century. The older pieces are ceremonial wrappers, including *ocho* and *mecho*, and the newer ones are the traditional models we know well, such as the *orizuru* (crane) and *yakko-san* (servant) [Takagi 99].

Most interesting among these pieces are those estimated to have been made in the early nineteenth century (Figure 6), around the same time as the horses and riders were made in Germany. The pieces were made of sheets in different shapes with many cuts, many of which were painted. These are indeed the characteristics of Japanese origami. In contrast, the models in Europe were mostly made of square sheets without cuts.

Many of the Moriwaki models closely resemble those illustrated in contemporary books such as *Kayara-gusa* and *Chushingura Orikata*, but they are slightly different. The variations suggest that many people at that time were making the models, with or without looking at books. In fact, several versions of the *Chushingura Orikata* booklet were published throughout the nineteenth century.

Kayara-gusa was compiled by Adachi Kazuyuki, about whom we know virtually nothing. He copied numerous books to make his own encyclopedia, and completed more than two hundred volumes in 1845 without giving a title to the whole. He reproduced at least three origami books in volumes 27 and 28. We call the origami-related part of this book *Kayara-gusa* because the volumes from 21 to 30 are so titled [Okamura 94].

Kayara-gusa contains 15 patterns of wrappers, diagrams of 25 models, and colored crease patterns for 6 figures [Brossman and Brossman 61]. It also includes two paragraphs, one saying he did not diagram *sembazuru* (connected cranes), boat, *kago* (Japanese palanquin), lotus flower, *sanpo* (tray with stand), box, *komoso* (also known as *komuso*, Zen Buddhist monk), thread holder, or *kabuto* (helmet) because everyone knew how to fold them [Okamura 94]. These models are still popular in today's origami world. It should be noted that most of them were unique to Japan.

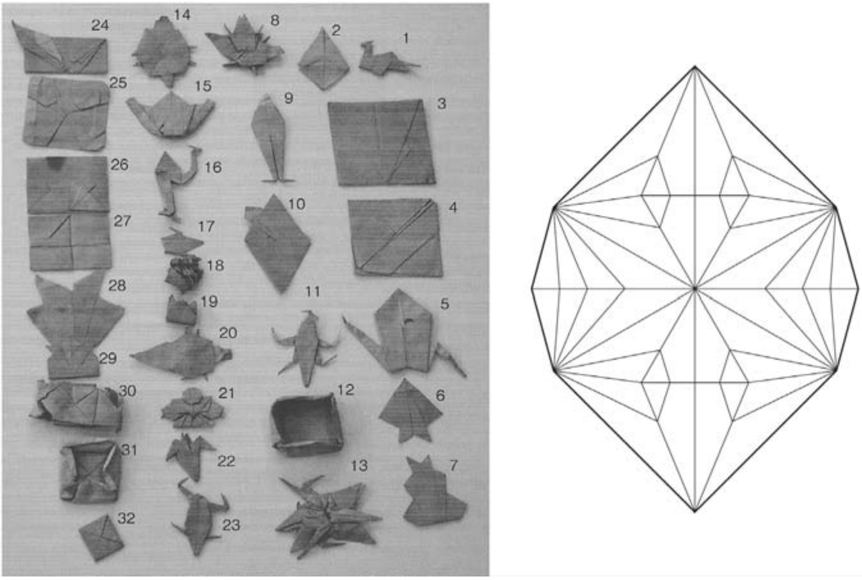

Figure 7. More examples of Japanese traditional models (left) and the crease pattern for the tortoise (right).

The other paragraph says that there were origami models of a peacock, praying mantis, *fukura-suzume* (sparrow ruffling feathers), blowfish (also known as catfish), and *kitsune-no yomeiri* (marriage of fox). They were, and still are, not well-known models, and Adachi himself did not know how to fold them [Okamura 94]. They have been, as it were, lost traditional models. Therefore, it would be surprising that all of the models are contained in the recently discovered origami collection of the Kanchazan Museum (Figure 7, left).

Kuzuhara Koto, a blind *koto* (Japanese stringed musical instrument) teacher born in 1812, made most of the origami pieces in the collection. They include some ceremonial wrappers; some connected cranes similar to those illustrated in *Sembazuru Orikata* published in 1797; many models recorded in *Kayara-gusa*; *tamate-bako* or treasure box, which is perhaps the oldest modular origami model, also depicted in *Ranma Zushiki* in 1734; and even models that have not been found in any other sources, such as the tortoise with the point-splitting crease pattern (Figure 7, right) and the box with the twist fold [Okamura 08].

All of the 66 pieces are unfortunately squashed flat to the extent that some cannot be recognized. Still, the collection is a comprehensive showcase of the diversity of Japanese traditional origami. Although some were made with the same square-grid pattern as the European traditional models, many Japanese models were highly complicated with advanced folding techniques as well as many cuts.

4 Conclusion

When comparing hundreds of traditional models recorded in the eighteenth and nineteenth century, one may be astonished to realize that only a few models were common to Europe and Japan at that time. Moreover, one can notice some differences even between models that appear to be shared between the East and the West.

Not only did the repertoires have little overlap, the folding styles also differed completely between the East and West. The Japanese origami models before the middle of the nineteenth century were made of sheets in various shapes: squares, rectangles, hexagons, octagons, and even many eccentric shapes. They were also folded with many cuts as well as with sophisticated folding techniques, and often were painted. Their European counterparts were made mainly from squares, sometimes from rectangles, and had few cuts. In addition, their crease lines were mostly limited to square grids and diagonals.

The difference has its root in the origin of origami—ceremonial wrappers of the fourteenth century in Japan and baptismal certificates of the sixteenth century in Europe. The crease lines for the wrappers run at different angles, whereas the folds in the baptismal certificates were the double blintz. This fact strongly suggests that Japanese and European origami arose and evolved independently.

In the first years of the Meiji Restoration, in the 1860s and 1870s, the European education system was introduced and adopted in Japan. As a result, European origami was imported to Japan as a part of the kindergarten curriculum. In addition, as people traveled internationally, Japanese origami spread over the Western world. The state of origami as we know it today has been developed as a consequence of such a cultural exchange. Thus, origami has never been a "Japanese" art.

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Deictic Properties of Origami Technical Terms and Translatability: Cross-Linguistic Differences between English and Japanese

Koichi Tateishi

1 Introduction

The hardship that diagram translators and interpreters often have between Japanese and English, and also with other European languages, is due to deictic differences between the languages, resulting in some terms without proper translations. For example, the English direction, "Fold corner to corner and unfold" uses two expressions in Japanese: "Corner-OBJECT corner-*e* fold" and "Corner-OBJECT corner-*ni* fold," where both *e* and *ni* mean *to* in English. When unfolding is the next step, more people tend to use *ni* than *e*. The only difference between *ni* and *e* is that *e* implies gone forever and *ni* indicates staying only tentatively. English does not make this distinction.

There are two possible explanations for this difference:

1. Japanese viewpoints stop at where the paper has been, and then wait and see what happens to the paper, whereas English-speaking people's viewpoints move while they are *making actions* to paper, according to where they are doing so (of course, with *fixed reference points*);

2. English lacks such a semantic distinction and English-speaking people do not care for it. I take the former view, that Japanese diagrams refer only to *similarity*.

The reason that I do not take the second view, at least as the primary reason, is that the second view is a type of strict linguistic determinism, as strongly argued by Sapir and Whorf [Sapir 83]. They often cite an example of a language called Pirahã, an Amazonian language. Pirahã people do not have numeral terms except for *few, a few, many.* Some scholars, including Everett [Everett 05], often cite these as *one, two, many* for propagandistic purposes, which is wrong; the languages' numerical distinction is basically (almost) none, a few, versus many (e.g., [Pinker 07]), and, when asked in Pirahã, they cannot distinguish numerically between three fish and four fish. Everett and his followers argue that words determine concepts and thoughts [Everett 05]. However, Pirahã men (but not women) speak nearly perfect Portuguese for trade with other tribes, and, in such a situation, they understand numbers because they have to understand money and goods to be exchanged. This, as Pinker and Chomsky (although quite indirectly) point out [Pinker 07, Chomsky 95], shows that the meaning does not directly affect thoughts. In the "language" of thoughts, we do have the ability to distinguish between minute differences. However, in real language (a "meta-" language of the thought language), the distinction is categorized in different ways according to the conventions of linguistic communities, as Saussure and Chomsky suggest (the difference between Saussure and Chomsky is whether they take such conventions as social or biological) [Saussure 83, Chomsky 95]. Thus, I would say that the *-e/-ni* fact accords with the linguistic determinism, but only indirectly, because it does not affect our thoughts, which is why English readers can understand the distinctions when explained.

This paper discusses such viewpoint differences in verbal instructions in origami diagrams in Japanese and English. After semantic considerations, the linguistic and/or psychological causes and effects of the differences are considered. Origami is taken for granted in Japanese society outside the world of folders, as easily shown by the following facts: first, the oldest published origami book, *Hiden Senbazuru Orikata*, was published in Japan; second, laymen generally consider origami something Japanese; finally, Japanese frequently answer "origami" when they are asked, "Name whatever you think is a feature of Japanese culture"—along with kimono, tea ceremony, flower arrangement, and so on.[1] This perception has made it hard to construct rigid methods and devices for teaching or telling how to fold, and maybe results in the folders' attitudes toward origami as artwork. In particular, the differences alter creators' psychological attitudes. Because they have not studied anything taken for granted, discrepancies

[1] In this sense, I do not consider my claiming origami being taken for granted an "attitude," but as fact. We do see tons of origami books, from children's to adults', in regional bookstores, and, as a college professor, I encounter students planning to study abroad who are ashamed of not being able even to fold a crane. These factors all show that origami is something presupposed to be a part of Japanese culture.

are created between those who *know* origami and those who do not.[2] Thus, Japanese creators tend to be satisfied with creating only crease patterns, not caring for diagrams. Paper becoming something else *is* a model for Japanese, especially for younger creators of complex models.

2 Previous Studies on Origami Terms

Attempts have been made with regard to fixing the technical terms of origami worldwide, mostly from the Western world. On the Origami-L email list for folders, such a proposal or two (or more) have been made. Lang presents a long list of origami technical terms in the appendix of his seminal book on the mathematical foundations of origami design [Lang 03]. However, such attempts have hardly been made in Japan. The only thing that can be seen in Japan is a list of technical terms with diagrams on the front page of origami books. Given that there are no records of it so far, it seems that Japanese have not paid attention to fixing the terms, or, even if they have, they could not find an effective way of doing so,

Interestingly, though, the Japanese have always shown interest in base forms, i.e., patterns on which various models can be based. Uchiyama Kosho, for example, lists patterns, but not technical terms, on the front pages of his famous epoch-making book [Uchiyama 62], and his father, Uchiyama Mitsuhiro, is quite famous for the patterns themselves, as shown in his *Tatou* book [Yanagi 88]. Moreover, one of the very first books on origami in Japan and in the world, *Hiden Senbazuru Orikata* [Akisato 97], basically shows only patterns. With regard to origami in particular, the Japanese seem to be disinterested in already established methods of folding per se, even in a book format where instruction occupies a very important part. Where have all the steps gone?

I have noted this issue previously [Tateishi 09], pointing out that, as diagrams and verbal instructions help each other, there are cases where too much verbal instruction can cause confusion for readers. I also pointed out that verbal instructions below the diagrams are only for security, so readers are not left alone in a sea of geometrical diagrams. Komatsu presents a significant article on how diagrams can be developed to avoid confusion

[2] All the facts that I will show appear to demonstrate that the Japanese are not good at instructions of origami, but that is not my point. Actually, what I am claiming in this paper is that Japanese folders today are not good at making truly effective origami instructions and diagrams, and I am searching for the reasons. I am not suggesting that people such as Akira Yoshizawa [Yoshizawa 96, Yoshizawa 99] and Kosho Uchiyama [Uchiyama 62] were bad in this respect, because their choices of diagrams, words, and diagrams were neatly done, given the poverty of methods for showing origami models in publication at that time. I am only speaking of recent folders and nonfolders in this paper, although I do not deny that these old masters influenced recent folders.

[Komatsu 03]. However, we both are interested in how the diagrams, not the terms, can be developed.

In this paper, I will add another aspect of the issue: Japanese origami terms are made vague in the first place. Whereas instructions written in English are very referential, those in Japanese are mostly either figurative or metaphorical. For English-speaking people, instructions below diagrams are clearly written instructive advice. Readers will read diagrams and see enough information on how to reproduce the models.[3] However, for the Japanese, instructions are only for help. This paper points the readers to some possible linguistic and pedagogical backgrounds of such differences. Finally, the paper contends that the vagueness in the Japanese origami world may lead to the recent shortage of diagrams of those complex models created by young Japanese origamists.

One may speculate that the competition on notational tools by Kawai, Uchiyama, Yoshizawa, and others, in the 1950s and 1960s might have contributed to the confusion in Japanese diagrams [Kawai 70, Uchiyama 62, Yoshizawa 96, Yoshizawa 99]. This possibility is, of course, one of the factors; however, the fact that this has continued even after the birth of complex origami models, initially perhaps triggered by Maekawa [Maekawa 83] is what I am trying to point out. Maekawa regained the status of crease patterns through the aid of Kunihiko Kasahara [Kasahara 96], and the origami instructions then made a shift in Japan. In the Western world, Peter Engel played a comparable role in reestablishing crease patterns in origami [Engel 89]. Even so, the diagrams had not changed, despite the efforts of people, including Makoto Yamaguchi and Kasahara, whose instructions are truly referential and clear [Yamaguchi 04,Kasahara 96]. This discrepancy *is* what I would like to point out and for which I seek reasons in this paper.

3 Theoretical Backgrounds

Even though I do not blindly accept the Sapir-Whorf hypothesis [Sapir 83], which says that language determines thoughts (and not vice versa), it cannot be denied that the way people speak somehow controls the way they think, exemplified in the frequently cited allegory of Japanese not speaking logically because their language is not logical. Suzuki, for example, points out that various words in Japanese have different ranges of meanings as

[3]Of course, I do not say that everyone can fold the models by this method. If there is an attitude on this point, it is perhaps my scientific attitude as a Chomskyan-theoretic linguist, which always sets an "ideal listener/speaker." I do not deny the existence of performance-dependent differences of individuals, but I will continue to assume ideal creators/authors throughout this paper, as this is the attitude linguists in my field must take.

compared with other languages, including English [Suzuki 73]. The word *lip*, unlike its correspondent *kuchibiru*, can have a mustache on it, for example. As language is a reflection of its cultural background, there is always a problem of translatability. It is the *mouth* (*kuchi*) that has a mustache on it in Japanese.

It is not clear whether the Sapir-Whorf hypothesis or its developments thereof have been accepted in the linguistic analysis of semantics in recent periods. Generative linguistics (e.g., [Chomsky 95]) has never been clear about it. Generativists have put semantics outside their scope, which is genetically endowed inside human beings. Cognitive semantics (e.g., [Lakoff 87]) is not clear on the issue either, because, even though proponents of cognitive semantics quote Whorf [Sapir 83], language for them is a matter of categorization and learning, behind which thoughts and inferences lie. However, we cannot deny that vagueness in verbal expressions is often caused by cross-linguistic differences of categorization of entities, materials, states, and events around us, although, as a linguist, I do not take it as the aforementioned strict linguistic determinism. Language may affect our thought, but not directly. Categorization of senses may affect our cognitive behavior, but this is outside of language; i.e., if categorization of senses by a word is clear-cut, thought will not seek for disambiguation, but if not, we often have to do so—this is not a necessity, on the contrary.

One example that shows the vagueness of Japanese with regard to origami is the verb *oru* (fold). *Oru* can be translated in various ways into English: *fold, bend, twist, break,* and so on. The only common feature of those various *oru*s is that it must be applied to a thin and/or flat base. In contrast, the English verb *fold* is not that ambiguous, and most variations of its meaning pertain to putting a layer onto something else. I will show later that the Japanese verbal noun *ori*, a nominal form of *oru*, can be used even for a case that by no means is folding for English-speaking people, and it is *this* vagueness that causes Japanese origami terms to become vague.

However, given the context of origami folding, *oru*/*ori* is, in most cases, interpreted as folding paper, with no necessity of disambiguation, which shows that the relationship between language and thought is at best indirect. What is at issue is that this disambiguation often “leaks.”

4 Maze of (Un)Translatability

This section introduces various cases that illustrate the difficulties in translation between English and Japanese. As this covers most of the terms in the world of origami, the readers can see how difficult it is to translate one series of diagrams into another language. The terms differ in their degree of (un)translatability; the following sections show them classified into four distinct categories.

4.1 Translatable but Too Generic

Yama-ori and *tani-ori* are the only cases I could find for which direct translation is possible: *valley fold* and *mountain fold*, respectively. These terms have two notable features. First, the term could not possibly be a synthetic compound word in either of the two languages.[4] A synthetic compound word is a word composed of two words, the first of which is semantically the grammatical object of the second. For example, the word *origami creator* is a kind of synthetic compound word, as it refers to one who creates origami (models). In that sense, nobody can fold a mountain or a valley in English, nor in Japanese either. *Tani-o oru* (fold a valley) is simply meaningless. Thus, the meaning of these compound words can be interpreted only figuratively or metaphorically. "Fold like a valley/mountain" is perhaps a proper interpretation, which by itself is rather vague and allows for various interpretations.

This leads us to the second feature of these terms. Even though we find explanations of valley and mountain folds in the front or back pages of origami books, we hardly ever find them in actual diagrams. Rather, there are more cases of "fold and unfold," "fold corner-to-corner," "fold edge-to-edge," "fold in half," and so on. This means that, even though valley and mountain folds are the most popular folds in the world of origami, we seldom use the terms for them in actual instructions. The fact that they indicate the direction of folding does not refute my claim, because, in most such cases, the direction is the only issue that must be disambiguated, which perhaps is why the two terms are so widespread. This is also why the terms are often useful in oral instructions in origami classes, too, but this is off the point because I am speaking of the issue of translatability. The terms should rather be taken as "icons" of directions of folding, whose functions are mostly absorbed into crease lines on diagrams.[5]

4.2 Translatable but with Significant Differences

Next, two types of very popular folding techniques are taken up: inside and outside reverse folds and sinks. Reverse folds, inside and outside, have one

[4] A *compound word* in linguistics, which is my field, means two (or more) words joined to form a unit that functions as a word [Fromkin et al. 07]. They need not be joined without a space between them nor joined by a hyphen. In this sense, the word *mountain* in *mountain fold* is not an adjectival use of a noun, because, if this is the case, we should have such forms as *completely mountain fold, mountain enough fold*, and so on, in which some other word(s) modify the "adjective."

[5] In this sense, they are good examples of vague origami terms well disambiguated, both in Japanese and in English.

feature in common: when it is done, it makes a reverse fold—one partially turns a layer over a folded edge. This is why the two folding techniques, inside and outside, are classified into one category, the reverse fold. Actually, in diagrams written in English, one does see many instructions with "outside reverse fold" or "inside reverse fold" used as a compound verb, perhaps because what the instruction means is strict and unambiguous.

In Japanese, however, the two folding techniques, which do the same action in different directions, are not categorized together; they are linguistically completely distinct. *Naka-wari ori* (inside break-in *ori*) is the term for inside reverse fold, and *kabuse ori* (cover *ori*) is the term for outside reverse fold. *Wari* in Japanese usually means completely dividing a piece into two, which *naka-wari ori* does not. In addition, *kabuse* in Japanese usually means covering and hiding something completely, which *kabuse ori* does not, either. Not only do the Japanese terms miss a very important common feature of the two folding techniques, but they also do not give us unambiguous instructions. *Naka-wari ori* means "fold like breaking pieces" and *kabuse-ori* means "fold like covering," other cases of figurative terms, unlike English.

In addition, the reverse folds and *nakawari-* or *kabuse-ori* have other significant differences. In English diagrams, reverse folds do not necessarily mean only folding by reversing the sides of the sheet halfway. Sometimes, reversing completely can be called reverse folds. However, at least for many Japanese authors, *nakawari-* or *kabuse-ori* refers only to those folds whose outputs look like a beak, like the origami crane's beak. Then, what do the Japanese call those reverse folds that reverse the flap 180 degrees? They say, "Fold like *kabuse-ori*," or "Put the flap inside." These complete reversing processes are what *nakawari* and *kabuse* mean literally, but the Japanese use the terms only for something else. The two terms are already only figurative in Japanese.

The case of *sink* is more serious in Japanese; the English word *sink* means what it does: sink the tip inside layers along the crease lines. Because the original meaning of the instruction is clear enough, we can understand what *unsink* and *spread-sink* mean (*spread-sink* also *does* sink the tip onto the flat surface). What does Japanese use for *sink*? The most frequent case I see is to use the English word *sink*, no translation. That is, Japanese does not have a word for *sink*. The Japanese occasionally use the term *shizume-ori* (sink fold), but this term is problematic for two reasons. First, sinking does not usually create any new crease line, so that it is *tatami* (fold along the crease lines), not *ori* (dividing a layer into two by a crease line), with the meaning of *ori* being figurative again. Second, because the meaning of *ori* is figurative, the meaning of *shizume* also must be (if *sink* is *ori*, then the focus must be on the crease lines created, which do not exist; with a nonexistent focus, one cannot make any action of sinking, strictly speaking). *Shizume-ori* means "do something like sinking by doing something like creating new crease lines."

4.3 Directly Untranslatable Cases

Unlike the clear distinctions in English between *pleat* and *crimp* [Lang 03], Japanese does not usually distinguish between the two, and both are called *dan-ori* (step *ori*). As a result, *dan-ori* is not usually a series of steps; it is again a very figurative and vague expression.

Japanese, however, do use the term *jabara-ori* (bellows *ori*) for box-pleating or pleating through the whole edge of the sheet (again, the result of *jabara-ori* is not bellows). This term corresponds to either of Lang's pleat or crimp, depending on how the surface layer is used, making the direct translation between English and Japanese impossible.

The English *swivel* and Japanese *hikiyose-ori* (pull-onto *ori*) differ in what part in the action is the focus. *Swivel* focuses on the first motion on the layer and on the pivot on which another layer rotates, but *hikiyose-ori* focuses on the second layer pulled up onto the initially folded layer. In terms of an action of *hikiyose* (pulling onto), Japanese *hikiyose-ori* establishes the desired result only indirectly. All of these points, again, make the translation between the two languages extremely difficult. When the English-speaking say, "swivel fold," we often have to make an explanation such as, "fold a layer and then flatten the raised-up layer onto it."

4.4 Lost in Translation

Although it is a special case, the term *Elias-stretch*, named after Neal Elias [Lang 03, p. 464], does not have a Japanese translation, even though the term used for pinching and stretching the pleat-folded layers into a long flap is a frequently used technique in current complex models. When the English-speaking say, "Elias-stretch," we have to divide the process into ten or more steps of pinching the layers and folding the pleat inside. Of course, this is a truly special term named after the individual who introduced the technique. However, English managed to name it, and Japanese did not.

5 Referentiality/Deictic versus Similarity: The Role of a Japanese Verbal Noun *Ori*

This paper thus far has described cases for which translation between English and Japanese is either difficult or impossible. It is surprising that these cover almost all terms of origami. I hypothesize that the difference lies in Japanese terms being ambiguous or vague and in English terms having a reference- or action-based nature. Take, for example, the Elias-stretch described above: even though it is a term for experts, when advanced folders look at the crease patterns and see the instruction "Elias-stretch," they know exactly what they should do.

The point can be extended to the word *sink*, discussed in Section 4.2. "Sinking" presupposes the existence of the pre-creased lines, and the action to be completed is to sink the tip of the corner into clusters of layers along those lines. This is exactly what Japanese folders do not have words to express. *Shizume-ori* is just a simile of what *sink* does: "(Make actions like) fold(ing) like sinking the tip." English origami terms are referentially and deictically unambiguous, given appropriate contexts in diagrams (or even without it); however, when we see diagrams in Japanese, there are still details to fix and struggling with diagrams.

Why are Japanese terms so indirect and figurative? The author hypothesizes that the secret lies in the term *ori* itself. Unlike English nouns such as *sink*, *pleat*, *crimp*, *fold*, and *reverse* that can also be verbs, Japanese *ori* is a noun, derived from the verb *oru* (to fold). The fact that Japanese origami terms can never be verbs can be shown by the lack of expressions like *Shizume-ori-nasai!* (Do sink!), the imperative form, or *shizume-otta* (Sank), the past tense form. This means that terms such as *tani-ori, yama-ori, kabuse-ori, nakawari-ori, dan-ori, jabara-ori, shizume-ori*, and so on are all compound nouns. It is well known, even in English, that already-established compound nouns are not interpreted synthetically but only figuratively. For example, a *blackboard* is never black these days, a *flatfoot* may put on high shoes, and a *jumping bean* may not be jumping now. The same situation applies to Japanese origami terms: they are only figuratively interpreted. Being figurative, Japanese origami terms cannot point the readers to the fixed reference points, so they rely on diagrams and/or crease patterns. It is the combination of diagrams and verbal or written words that have the readers follow the instructions. No over-the-phone teaching is possible with Japanese origami terms.

Such vagueness and figurative nature are seen in other origami terms, too. For example, the meaning of the Japanese term *kado* (literally, corner) is ambiguous between *corner* and *flap*, and *fuchi* (literally, edge) between *edge* and *layer*. Here again, Japanese terms do not have fixed points as reference.

Ori is a very convenient term because it can be attached to anything and create a new folding method. However, because of its convenience and its figurative nature, terms with *ori* are not that stable and easily become obsolete. For example, the term *fukuro-ori* (bag *ori*) means "open and squash" and actually is very convenient to express this series of folding actions. However, it is hardly used these days, and instead, a rather lengthy expression such as *uchigawa-o hirai-te tubusu-youni oru* (fold so that the output will look like it is opened inside and squashed) is used.

Similarly, *kannon-ori* (fold edges to the center line), *kaben-ori* (petal fold), and *zabuton-ori* (blintz fold) are rarely used these days.

In sum, English origami terms refer to actions so that readers know what to do immediately, whereas Japanese origami terms are often vague and/or figurative, or ad hoc, so that, even though they are convenient, they cannot be used as fixed technical terms.

6 Why Don't the Japanese Use Verbs?

Then, the natural question is, why do the Japanese not use compound verbs instead of nouns for origami terms? The reason lies in the linguistic system of Japanese.

First, unlike English, Japanese is a head-final language, which means that the core term of the phrase always comes at the end. For example, in a phrase *origami-o oru* "fold origami," which is about action, the term of the action, the verb, comes at the end. That is why, when English-speaking people say "Elias-stretch," the Japanese have to say:

> [[[[[*kado-o*] *tsubusu*]-*you-ni*] [[[*dan-o*] *tsuman*]-*de*]] [[[*naga-i*] *kado-o*] *tsukuru*]]
>
> (literally, corner-OBJECT-squash-DATIVE pleat-OBJECT pinch-and long-PRESENT corner-OBJECT make),

where the brackets show grammatical meaning units). In this expression, all the disambiguating elements of the phrase and the phrases therein are all at the end in the form of grammatical particles. This means that when one expresses the same concepts, it necessarily takes more words in Japanese than in English to do so.

Then, why not "Elias-stretch"? Here, the restriction on Japanese compound verbs is relevant. Japanese does not make much use of compound verbs in the first place, except for those ending with aspectual verbs like *-hajimeru* (start –ing), *-owaru* (finish –ing), *-tsuzukeru* (continue –ing), *-teiru* (be –ing), and so on. With other common verbs, Japanese has only fixed forms like *naname-yomu* (literally, slant-read meaning scan through). Why? Because Japanese has a denominal verbal suffix *-suru* (do). In Japanese, *-suru* can attach to virtually all nouns and make meaningful verbs. For example, *pasokon* (personal computer) can create with this suffix a compound verb *pasokon-suru* (use a personal computer) and similarly *doroboo* (thief) can create *doroboo-suru* (do robbery). As these two examples show, the semantic relations between a noun and a compound verb with *suru* are completely arbitrary. Of course, one can create words, such as *taniori-suru, nakawariori-suru*, and so on. One can even make *sink-suru.* Because of this very convenient term, the Japanese do not have to rely on compound verbs or any verb at all, but can just make compound nouns for origami. This fact, in turn, leaves Japanese origami terms vague, as discussed above, because compound nouns universally tend to be vague.

I have pointed out that, even though written instructions appear below almost all American diagrams, Japanese diagrams have written instructions only where necessary (for about 60% of all diagrams) [Tateishi 09].[6] Instructions below diagrams are not actually instructions; the real instructions are on the diagrams, with separate written instructions only providing security for readers in the sea of diagrams. I believe this explanation is still correct. Japanese reluctance to use words below diagrams most probably originates from the vagueness of Japanese origami terms, which in turn makes them rely on diagrams or crease patterns.

7 Further Considerations

Even though the Japanese linguistic structure—its morphological structure in particular—prevents Japanese diagrams from being truly explicit and explanatory, the Japanese do have a way around this problem; for example, they could use explanations such as, "Find the crease line just below the top corner, and find also the clusters of layers around the top corner. Now, you slightly open them and push the corner inside. The result must lock the clusters of layers," in place of "Closed sink-*suru*." Why do they not care for it?[7]

The main explanation for this lack of enthusiasm is that the Japanese these days cannot fold.[8] In recent years, Japanese kindergartens and elementary schools hardly use origami in their art and/or math classes. There are several possible reasons: (1) teachers and parents cannot fold origami well these days, perhaps not even a crane; (2) children raised by such origami-ignorant people cannot fold, either. In my semester-based origami class in college, students are certainly interested in origami, but when I ask one or two students to explain and/or teach how to fold a crane, they do not have words to explain it, even in Japanese. Only about half of the students can fold a crane properly. If a crane, the most famous Japanese origami model, produces such disastrous results, outcomes with other models cannot be better; most students cannot fold a helmet, a waterbomb, twin boats, or other popular Japanese models.

[6] I counted origami convention books, because there are individual differences in every country. Satoshi Kamiya, for example, uses words below most diagrams he draws (e.g., [Kamiya 05]).

[7] Actually, Akira Yoshizawa cared for such matters in many of his books. His instructions have been very clear both graphically and verbally [Yoshizawa 96, Yoshizawa 99].

[8] Here, I am speaking of the general public, not of those who attend origami conventions.

I have been teaching origami and its history for five years to students at a women's college, with about 200 students in total. At the beginning of the semester, I give a questionnaire to students to assess their "origami history." Out of 200 students, only 5 said that they had learned something using origami in their school days, no one recalled learning origami in kindergarten, 15 learned origami from their relatives, 158 did not own an origami book, and 126 said the course offered the first opportunity to fold an origami model. Competing activities, such as watching TV, playing video games, and so on, might have contributed to their ignorance of folding origami models. It should be noted that male complex origami folders in Japan are mostly game and/or anime obsessed, which may contribute to male/female differences. Unless they are interested in something tricky and puzzle-like, Japanese children do not even think of folding, and educational efforts in this direction, which is very important to develop their spatial-perceptual faculty, are not emphasized in Japan.

For the Japanese, origami had long been, and still is, taken for granted too much, even though it has practically been ignored in educational institutions today. I often hear the following claims: "Origami books are too difficult to read," "It is impossible to decipher series of diagrams with virtually no instruction at all," and so on. From these statements, I deduce that origami books in Japan, except for some truly good ones, are not for teaching how to fold. Because origami is taken for granted in Japanese society, schools do not even think of teaching folding; this oversight must be corrected. In Japan, origami books mostly seem to be for telling how creators have invented new art forms.[9]

Because there is virtually no origami-based education in Japanese society today, even origami enthusiasts are not taught anything about origami. They somehow manage to decipher ways to fold from hard-to-read diagram books. Because they are not taught, brand-new creators are not generally interested in teaching. They may teach a class, but most of them just hand out a sheet or two with diagrams and/or crease patterns, and say, "Raise your hand if you have questions," which in no sense is teaching. Perhaps, their interest is only in showing their brand-new models, but not sharing them with anybody in the sense of "reproduction," which sharing origami models often means. They therefore do not and cannot work as a resource for the development of the world of folding.

In 1999, when Origami Tanteidan changed its name to JOAS (Japan Origami Academic Society), *The Origami Tanteidan Magazine* started a series called "Crease Pattern Challenge," which shows only crease patterns

[9] Of course, from the creators' viewpoints, origami books publicize their own origami artwork. Creators decide what to put in their own books and how to tell readers what they are (and quite often how to explain how to fold them). From the readers' viewpoints, the authors only state how all these work.

and completed models, and readers are supposed to fold the models by deciphering the complex crease patterns. This, in the end, became reinvention, with the different name of "base forms." From the varieties of crease patterns shown in this series, new folders discover their own "base forms," even though it may be just 40×40 box pleats, and create their new models from them. Because crease patterns cannot be "taught," the new creators are not interested in teaching. The result will be a shortage of good origami model resources in the form of well-written diagrams, in which new generations of creator/folders are not necessarily interested.[10] Even though it may be a recent occurrence, it actually may be a reincarnation of old Japanese creators' reliance on "base forms." This approach has been prominent with relatively older generation complex model folders, but it is increasingly becoming true among younger generations.

Take, for example, a comparison of authors of Crease Pattern Challenges and the centerfold diagrams in *The Origami-Tanteidan Magazine*. After the academic year 2005 (starting from April 2005), there have been 33 issues of *The Origami Tanteidan Magazine* with 32 Crease Pattern Challenges and 36 centerfold diagrams. Among Crease Pattern Challenges, 13 crease patterns (CPs) are created by folders younger than Satoshi Kamiya (the reason why the author chose Satoshi Kamiya is only arbitrary, but the fact that he was featured in the now historic origami magazine *Oru* from Sojusha may justify that—he is the youngest of the "older generation") as opposed to 13 CPs by the older generation and 6 CPs by non-Japanese folders. However, in the centerfold diagrams, only 4 sets of diagrams are contributed by the younger generation, 28 by the older generations, and 4 by non-Japanese folders. I will not even try to draw statistical conclusions from these numbers, but these facts may show that younger creaters do have models, but they do not try to put them in diagram-form instructions. The complexity of their models is, of course, relevant. The more complex the model, the harder it is to draw sequential diagrams. I would like to emphasize the fact that it is harder to see models created by younger generations, except in exhibitions of origami artwork. Origami, on the one hand, is an established form of art, but the world of origami, on the other hand, needs to publicize its works to develop new prospective artists, because most of those who enter the world of origami start by mimicking their predecessors' works. Note that their models are not like those by Eric Joisel [Joisel 10], whose creations are unique.

As origami is taken too much for granted in Japan, creators have no idea of how to start folding from scratch, and they attribute their models to

[10] I do not mean that the Crease Pattern Challenge is harmful. It just shows a dichotomy of origami enthusiasts: those who can be nurtured by self-oriented learning and those who must be taught. I personally am interested in the effects of the spread of crease pattern disposition of models exported from Japan into the Western world these days, which, of course, takes time to investigate.

base forms and/or crease patterns. There has been no successful attempt to reconsider this cross-generational imprinting, so the Japanese cannot think of words for diagrams.

English, however, happens to be well suited as an instructional language. In addition to its relatively fixed word order, it has fewer inflectional systems than other Western languages—due to its historical contact with French, Latin, and Greek and to the United Kingdom's past colonialism—making it an across-the-board *lingua franca* of the world. (In other words, it is easier to learn than other western languages, grammar-wise.) In contrast, Japanese relies on coining new words when one has to give instruction, as in the *-ori* case discussed in Section 5. Furthermore, all the grammatical morphemes as modals, tense, aspects, voice, and so on, are stuck at the end of the sentence as a part of a single verb/auxiliary, which makes it harder to decipher instructions, even for the Japanese. This, in addition to the attitudinal points identified in this section, may have made the Japanese not rely on written instructions.

8 Conclusion

The Japanese cannot think of unambiguous terms for origami partly due to their traditional attitudes toward origami and partly due to the influences of the Japanese language's linguistic structure. With base forms and/or crease patterns as an escape hatch, the Japanese did not, and do not, seriously think about terms for instruction. The situation was different in the Western world because enthusiasts had to teach each other in the most effective ways. For creator-folders, such a situation is harmless. For novices in Japan, however, origami is now a form of art with no good instructive media, with a few exceptions, and they are hesitant to jump into the world of the rich varieties of art forms. I strongly feel that the teaching methods for origami as well as its terms must be seriously studied.

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Betsy Ross Revisited: General Fold and One-Cut Regular and Star Polygons

Arnold Tubis and Crystal Elaine Mills

1 Introduction

The five-pointed star of the American flag has been linked to a meeting in Philadelphia in May or June, 1776, between Betsy Ross and a committee headed by George Washington. Ross advocated the use of a five-pointed star pentagon (*pentagram*) instead of a six-pointed star in the flag, and demonstrated how easily a five-pointed star could be made by a fold and one-cut technique. Although historically controversial, this incident has been widely cited in testimonials, articles, books, and on the Internet, and mentioned in introductions to modern fold and one-cut algorithms (e.g., [Demaine and Demaine 04, Demaine and O'Rourke 07]).

2 Historical Sources for the Story

There are several recent compilations of, and commentary on, the historical bases for the Betsy Ross flag story [Timmins and Yarrington 83, Harker 05, Miller 10, Independence Hall 10]. The following summary is largely based on these sources.

Elizabeth Griscom Ross Ashburn Claypoole (1752–1836) (known as Betsy to family and friends, and thrice married and widowed) was for many years an upholsterer and flag maker in Philadelphia. Documents do indeed exist showing that she made a flag for the Navy during the Revolutionary War. An account of her meeting with then Colonel George Washington, Robert Morris, and Colonel George Ross in 1776 (for which no known official documentation exists) was first publicly delivered in a March 1870 speech by one of her grandsons, William B. Canby (1825–1890), before The Historical Society of Pennsylvania. This speech was followed shortly afterward by affidavits concerning the meeting by one of Betsy's daughters, Rachel Fletcher (1789–1823), in July 1871; one of her granddaughters, Sophia B. Hildebrandt (1806–1891), in May 1870; and one of her nieces, Margaret Donaldson Boggs (1776–1876), in June 1870. Harker points out another written source for the flag story: a letter in 1903 from Rachel Albright (a granddaughter of Betsy) to a friend, Nellie E. Chaffee, whose daughters were interested in Betsy's life story [Harker 05]. The letter is archived in the American Flag House and Betsy Ross Memorial in Philadelphia. Collectively, the Canby speech, the three affidavits, and Rachel Albright's letter state that the meeting occurred in the year 1776, shortly before the signing of the Declaration of Independence, and that Betsy suggested changes in the flag design initially proposed by Washington. Boggs, Canby, and Albright all explicitly refer to Betsy's fold and one-cut five-pointed star. Fletcher's affidavit states that Betsy proposed a 4×3 rectangular rather than a square flag shape and an arrangement for the stars in lines or in some adopted form as a circle, or a star. The full texts of the Canby speech and the three affidavits are available online [Independence Hall 10]. Demaine and Demaine [Demaine and Demaine 04] and Demaine and O'Rourke [Demaine and O'Rourke 07] reference an article [Wilcox 73] in the July 1873 issue of *Harper's New Monthly Magazine* containing the Betsy Ross story described above, with the year of the meeting given as 1777 instead of 1776. Wilcox was probably influenced by the date (June 14, 1777) of the so-called Flag Resolution of the Continental Congress. Harker offers a critical discussion of the significance of this resolution [Harker 05]. It is not evident from the article that the author was aware of the Canby speech and the three affidavits.

Harker has compiled a considerable body of pre-1870 evidence in support of the 1776 events as described by Canby, Fletcher, Hildebrand, Boggs, and Albright [Harker 05]. His basic assertion is that Betsy Ross sewed one or more flags for Washington that featured thirteen five-pointed stars arranged in a circle, and that the Army carried these flags at the Battles of Trenton (December 1776) and Princeton (January 1777). The key pieces of evidence that Harker presents for these assertions are the following:

1. Roberts cites an article in the 1909 publication, *The Journal of American History* by Mrs. Katherine (Wright) Bennett (1854–1944) that recounts the life of Rebecca Prescott Sherman (1743–1813), the sec-

ond wife of Roger Sherman of Connecticut (1721–1793), signer of the Declaration of Independence [Roberts 04]. The article states that Rebecca heard from Roger about a flag, ordered by George Washington, being made and that she visited Betsy Ross and actually assisted her in sewing stars *on the very first flag of the young nation.*

2. In a February 29, 1896, interview in the *Harrisburg Telegraph*, retired General Edward C. Williams (1820–1900) described his experiences in the war with Mexico. His story is confirmed in all specific details by a report of Major William Brindle, Commanding Second Brigade, Volunteer Division, prepared September 15, 1847, the day after the taking of the Mexican Fortress of Chapultepec on the outskirts of Mexico City. Major Brindle reported that after the surrender of Mexican General Bravo, then Captain Williams *ascended to the top of* [the Fortress] *with the first flag made by Betsy Ross, of Philadelphia, which was presented to Washington before the Battle of Trenton, during the Revolution of 1776, which Captain Williams had obtained from the State Library in Harrisburg.* General Williams was born in Philadelphia in 1820 and lived there until 1838. He must have been familiar with the story of Betsy Ross and the American flag at about the time of the fiftieth anniversary of the signing of the Declaration of Independence and the American Revolution.

3. A 1784 painting by Charles Wilson Peale (1741–1827) titled *Washington at the Battle of Princeton, Jan. 3, 1777*, shows Washington having epaulets with five-pointed stars on his shoulders and a flag in the background with five-pointed stars, presumably arranged in a circle. Peale participated in the Battles of Trenton and Princeton, and was known for the painstaking attention to detail in his paintings.

4. A portrait (1832) of Betsy Ross by the famed artist Samuel L. Waldo (1783–1861) suggests that Betsy was considered a person of significance in Philadelphia.

5. A painting (1851) by the artist Ellie Wheeler (1816–1896) shows Betsy Ross and three men, with Ross having a flag on her lap with five-pointed stars in a circle. Wheeler grew up in the neighborhood where Betsy lived and was 20 years old when Betsy died in 1836. It is thus reasonable to assume that the story of the "committee" and Betsy's role in flag designing/making was common knowledge in Philadelphia.

6. A painting (Germany, 1851) by the artist Emanuel Leutze (1816–1868) entitled *Washington Crossing the Delaware* features a flag with a circle of five-pointed stars. Leutze also lived in Philadelphia in the 1820s and 1830s.

7. In a draft of three proposed frescoes for the Ladies Waiting Room (1856) by an architectural draftsman for the consideration of Constantino Brunidi (an Italian artist in the payroll of the Capital Building), one fresco shows a woman (Betsy Ross?) presenting a flag to three men (the committee?), with more uniformed men in the background.

8. The main focus of the remainder of this paper is the Pattern for Stars artifact [Harker 05, Timmins and Yarrington 83]. (See Figure 1.) At a 1963 luncheon meeting of the Women's Committee of the Philadelphia Flag Day Association, Reeves Wetherill of the Society of Free Quakers presented a sample folded five-pointed paper star pattern (a folded 5×8 piece of paper with a partial cut that shows how to obtain a pentagram). We will refer to it as the *Pattern for Stars artifact.* Wetherill explained that the artifact came from an old safe, which his father, Abel Wetherill, had caused to be opened in 1922. Other contents of the safe included a pistol and an old deed signed by John Penn (1729–1795), colonial governor of Pennsylvania. Reeves and Abel Wetherill were descendants of Samuel Wetherill (1736–1816), one of the founders of the Society of Free Quakers.[1] John and Elizabeth (Ross) Claypoole became members of the society in 1785. Harker reports that, according to the present-day clerk of the society, the safe did *not* belong to Samuel Wetherill, but to a Wetherill of a later generation, but that it was clear that the artifact had been in possession of someone associated with the society since the pattern was created [Harker 05]. There are four lines of writing on the corner of the paper in lead pencil as follows:

   H.C. Wilson
   Betsey Ross
   Pattern for
   Stars

   Betsy is spelled as "Betsey" and the "H" appears to overwrite a "W." "Wilson" might possibly refer to Clarissa Claypoole Wilson (1785–1864), Betsy's daughter, who was widowed in 1812 and moved from Baltimore to live with Betsy in Philadelphia.

Although the identity of the person(s) who made the artifact and penciled in the four lines will probably never be determined, *the artifact was apparently considered important enough to be stored for safekeeping along with other items of historical significance.* Photographs of the artifact are found in two publications [Harker 05, Timmins and Yarrington 83]. For

[1] The Society of Free Quakers continues to exist as a philanthropic organization.

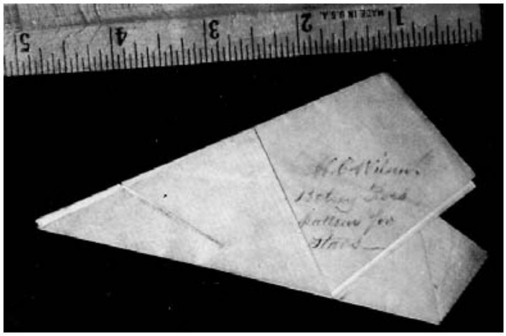

Figure 1. Pattern for Stars artifact with the four-line penciled inscription, Society of Free Quakers, Philadelphia (undated). The partial cut slants downward from the lower portion of the top edge. Photograph by John Balderston Harker, used with permission.

many years, the artifact was on display at the Society of Free Quakers Meeting House in Philadelphia.

3 Replicating the Pattern for Stars Artifact

Figure 2 presents fully land-marked steps for folding a $5'' \times 8''$ piece of paper (the same size paper used in the Pattern for Stars artifact in Figure 1) and obtaining a pentagram with a single cut, just as Betsy Ross supposedly demonstrated in 1776. It is not evident from photos of the Pattern for Stars artifact which, if any, folding landmarks were actually used.

An exact procedure for a fold and one-cut pentagram requires the division of a straight angle into five angles of measure $180°/5 = 36°$. This construction can, of course, be done by folding a golden-ratio triangle (e.g., [Row 05]). However, the required folding is hard to implement accurately with ordinary paper. We therefore use instead, in Steps 3 through 10, the first stage of the Fujimoto iterative method for division of a length or angle into an odd number of equal portions (e.g., [Huzita and Fujimoto 97; Hull 06, pp. 15–26]). This procedure is sufficiently accurate for most practical purposes. For the idealized case of infinitely thin and flexible paper, it gives the division of a straight angle of measure $180°$ into three angles of measure $35.78° = (180° - \arctan(0.75))/4$ and two of measure $36.32° = (180° - 3 \cdot 35.78°)/2$. All other folding steps are theoretically exact. Very thin paper is required if the folding of all layers in Step 11 is to be done accurately. If the paper is too thick, the crease lines of the regular pentagon shape in Step 12 should be made with the paper unfolded.

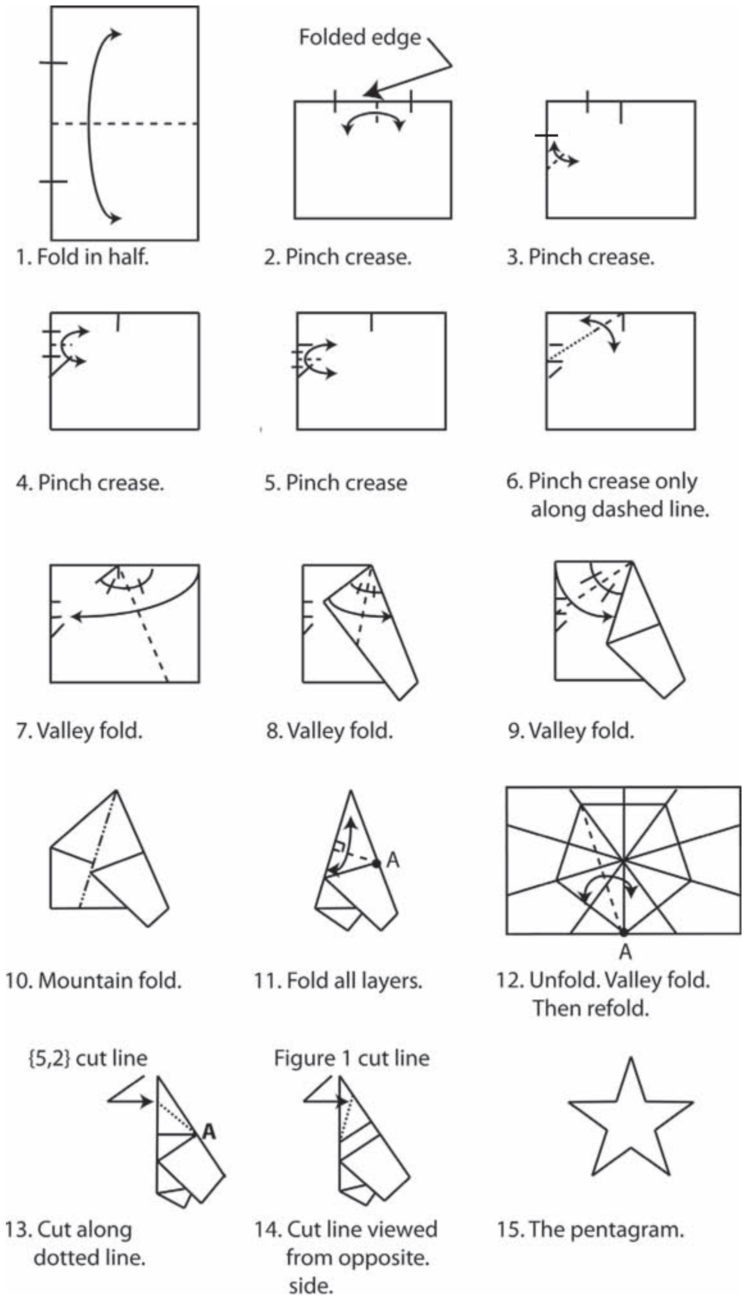

Figure 2. Folding and one cutting the pentagram. The partial cut line in the Pattern for Stars artifact (Figure 1) overlays the cut line shown in Step 14.

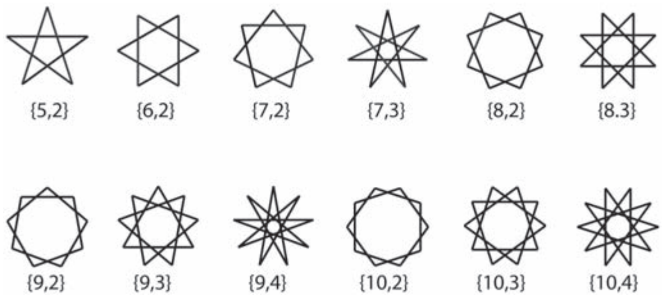

Figure 3. Star polygons, $\{p, x\}$, for $p = 5, 6, 7, \ldots, 10$. The corresponding regular polygons are obtained by joining the neighboring vertices with straight lines.

4 Generalizing the Betsy Ross Method to Fold and One-Cut Any Regular or Star Polygon

A straightforward generalization of the method used to obtain the Betsy Ross Pattern for Stars artifact gives *any* arbitrary regular polygon and its associated star polygon(s). Tubis and Mills previously reported a similar general procedure [Tubis and Mills 09]. It should be noted that the problem of determining the relevant folding for a fold and one-cut polygon (or group of polygons) starting with an *unmarked* piece of paper with no initial creases is different from that solved by Demaine and O'Rourke [Demaine and O'Rourke 07]. In the latter, the polygonal shapes are assumed to be initially inscribed on the paper, and then a general algorithm is derived for folding the paper so that *all* of the polygonal lines are superimposed on top of one another.

A regular polygon is one in which all of the side lengths and interior angles are equal. The sides and bisector lines of the interior angles of the polygon divide a p-sided polygon into p congruent isosceles triangle sections with vertex angle $(180/p)°$. The (regular) star polygon $\{p, x\}$ is a p-pointed star defined by line segments starting at each vertex of the regular p-sided polygon and ending at another vertex, with $x - 1$ vertices in between (e.g., [Coxeter 73, pp. 93–94; Caglayan 08]). It is easily seen that $2 \leq x < p/2$. Thus the $\{5, 2\}$ star polygon (pentagram) is the one with the fewest number of points. Star polygons for $p = 5$–10 are shown in Figure 3.

The first part of the general procedure is to fold the paper in half lengthwise, then to form a triangular section with an internal angle of measure $180°/p$ and with the vertex at the center of the folded edge and one ray along the folded edge, and finally to use this section as a template to fold the piece as in Steps 10 and 11 of Figure 2. Theoretically, this can be done exactly by folding for $p = 3$ through 10 (e.g., [Geretschläger 08]).

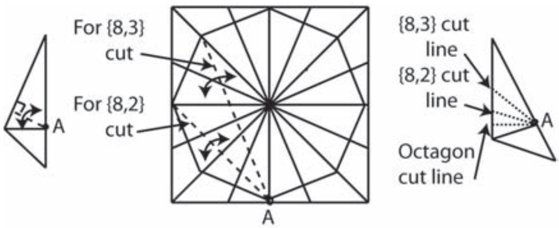

Figure 4. Correspondents of Steps 11–13 of Figure 2 for the case of a regular ($p = 8$) octagon and its star octagons, $\{8, 2\}$ and $\{8, 3\}$, from a starting square.

The method is particularly simple and well known for $p = 3$, 4, 6, and 8. A practical approximate method for $p = 5$ has already been given in Section 3, and may be applied to the case of $p = 10$ by angle bisection. For $p = 9$, one can use a folding procedure to trisect an angle of measure 60°. For $p = 7$, one can use a methodology similar to that for $p = 5$ by noting that arctan(0.5) (= 26.56°) is fairly close to 180°/7 (= 25.71°), and again using an early stage of the Fujimoto procedure to achieve a better approximation to 180°/7. Also, a square is perfectly adequate as the starting paper shape instead of the 8×5 shape used in Section 3.

To make the general method clear, the correspondents of Steps 11–13 of Figure 2 for the case of a regular ($p = 8$) octagon and its two star octagons, $\{8, 2\}$ and $\{8, 3\}$, from a starting square are given in Figure 4.

5 Discussion

Although a considerable body of circumstantial evidence, such as that outlined in this paper, exists today in support of the Betsy Ross flag story, historians will probably never be able to definitively resolve the various controversies and issues (concerning, e.g., the true provenance of the Pattern for Stars artifact) that still surround it. Nevertheless, the story encompasses an extremely interesting confluence of history, origami, and mathematics (geometry and trigonometry) and, as such, provides the basis for many great educational opportunities in mathematics and history classrooms.

Acknowledgment. We wish to thank John Balderston Harker, a fifth generation descendant of Betsy Ross, for sharing with us his extensive research on her life and the history of the American flag, for encouraging us to write this paper, and for providing us with photos of the Pattern for Stars artifact—a possible direct link to his famous ancestor.

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Reconstructing David Huffman's Legacy in Curved-Crease Folding

Erik D. Demaine, Martin L. Demaine, and Duks Koschitz

1 Introduction

David Huffman's curved-crease models (see Figure 1) are elegant, beautiful, and illustrative of Huffman's fascination with curved creases. Huffman's death in 1999 left us without his deep understanding, but his many models and notes provide a glimpse into his thinking. This chapter presents reconstructions of some of David Huffman's curved crease patterns and models, aiming to recover his insight and uncover the mathematical beauty underlying the artistic beauty. These initial reconstructions represent the beginning of an ongoing project with the Huffman family to study and document David Huffman's work in folding.

The first known reference of curved-crease folding is the work of a Bauhaus student in a course by Josef Albers in 1927–1928 [Wingler 69]. This model has creases in concentric circles and a hole in the center. Since the 1930s, Irene Schawinsky, Thoki Yenn, and Kunihiko Kasahara have built similar models with variations on the pleats and the size of the hole [Demaine and Demaine 08]. Other intricate curved-crease origami sculpture was designed by Ronald Resch in the 1970s. From the 1970s to the 1990s, Huffman created hundreds of models, which represent the majority of the work done in this field [Wertheim 04]. Huffman inspired further work on curved creases [Fuchs and Tabachnikov 99] and research on finding the nearest proper folding that approximates a three-dimensional scanned physical model [Kilian et al. 08].

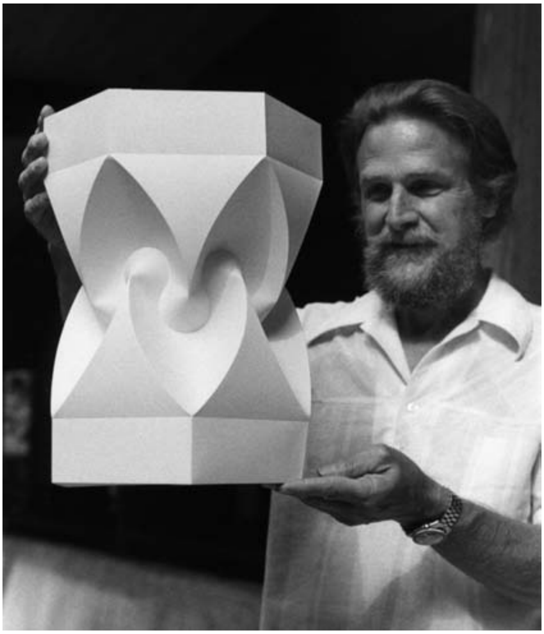

Figure 1. David Huffman and his "Hexagonal Column with Cusps." (Photograph courtesy of University of California, Santa Cruz.)

David Huffman was simultaneously studying the mathematics and the art of curved-crease origami. He analyzed the local mathematical behavior of curved creases in his paper "Curvature and Creases: A Primer on Paper" [Huffman 76] and made sculptures to further study this special kind of folding. Our goal is to better understand the behavior of curved creases demonstrated in his models, given the lack of mathematical and algorithmic tools for designing curved-crease origami.

2 Approach

We are experimenting with both physical models and computer models to reconstruct Huffman's work. We analyze Huffman's designs by carefully studying photographs, measuring his models, and studying features that occur frequently in his designs. Based on personal communication with David Huffman in 1998, we assume that almost all of Huffman's creases are conic section (quadratic) curves. His work spans many areas of mathematical origami; here we select a few designs that use several quadratic curves within a single design or combine curved and straight creases.

Figure 2. Huffman used a spring-loaded ball burnisher similar to this one.

2.1 Folding Methods

The Huffman family provided us with an opportunity to see Huffman's estate, which they manage, and to see a variety of working models and crease patterns. We studied his techniques and work methods by looking at his drawing tools, templates, and model-making equipment. Huffman transferred his crease patterns onto sheets of white, matte PVC ("vinyl") that is 0.01″ thick using French curves. He then traced the creases with a spring-loaded ball burnisher similar to the one shown in Figure 2 to precrease the material. He slowly bent the material to the maximal angle without kinking the uncreased areas. The careful and time-consuming folding technique attests to his incredible patience and love for craft when producing his art.

2.2 Reconstruction Methods

As part of this reconstruction effort, we have decided to stay close to Huffman's way of making models. We deviate from his manual drawing methods in order to create digital files of the crease patterns. The reconstructions are drawn with computer-aided design (CAD) software, Rhinoceros 3D, which allows the use of quadratic curves. Most designs are drawn in two dimensions, folded, and then visually analyzed. Some designs have been recreated virtually in three dimensions and then made into physical models. We made paper versions before producing the final versions from the same material Huffman used in the 1970s.

We use an industrial vinyl cutter/printer to precrease the patterns, which requires format translation to Adobe Illustrator. The *i*-Cut *i*-XL-24M flatbed cutter and router has a 65″ by 120″ vacuum plate and 1″ cutting depth. The machine is furnished with two heads and can crease and cut at the same time. The drop-in tool slots allow the use of a special tool, called a creasing wheel, that gently pushes down onto a surface. The precreased vinyl then needs to be folded into its final shape by hand. Most of the reconstructions are close to the size of the originals. We decided to enlarge some examples to ensure better results, as tolerances become less of an issue.

3 Reconstructions

Huffman created a wide variety of models, most of which were never shown. The approximate reconstructions selected for this paper are grouped into categories that highlight some of the observations we have made during our study. Of course, aesthetic qualities and beauty are big factors, and it is impossible to know Huffman's exact motivations for each design.

We believe that David Huffman was interested in studying vertices of various degrees and that he made some of his models to show how they can be used. He studied vertices that are exploded and separated by polygons. Tessellations represent a large portion of his work, but we include only some that use curved creases. Huffman further studied foldings that have the characteristic of describing a volume. The crease patterns we show in Figures 3 through 15 have tags that identify circles (ci), ellipses (el), and parabolas (pa).

3.1 Degree-1 and -2 Vertices

The model in Figure 3 shows two mountain creases ending within the area of the paper. These vertices are located on the major axis of the ellipse of

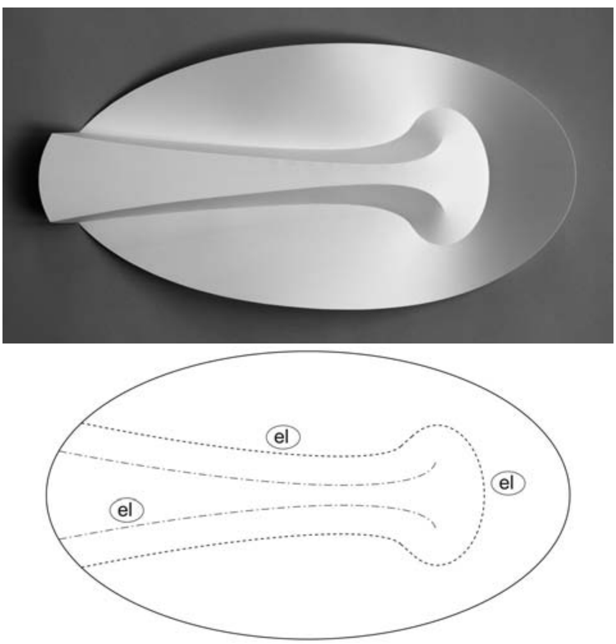

Figure 3. Huffman design using ellipses with two degree-1 vertices.

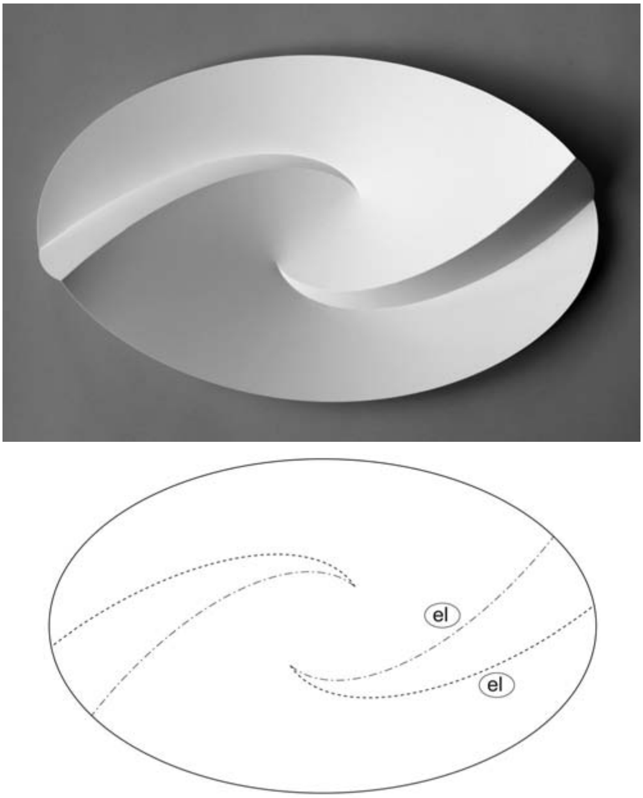

Figure 4. Huffman design using ellipses with two degree-2 vertices.

the valley crease, and the rulings around each degree-1 vertex describe a cone surface. The valley crease is drawn by splicing ellipses together. This design was published on the website of *Grafica Obscura* [Haeberli 96].

The model shown in Figure 4 [Haeberli 96] uses two degree-2 vertices and is drawn using ellipses. There is a ruling line between the two points, but no crease is necessary. The cut-out shape is an ellipse and the mountain and valley assignments alternate radially.

3.2 Inflated Vertices

We observe that Huffman was studying crease patterns with "exploded" vertices of varying degrees. The next series of models shows vertices that are exploded or inflated into flat polygons. Figure 5 shows a noninflated degree-4 vertex with creases that have pairwise common tangents. The creases are all circular arcs and alternate mountain and valley.

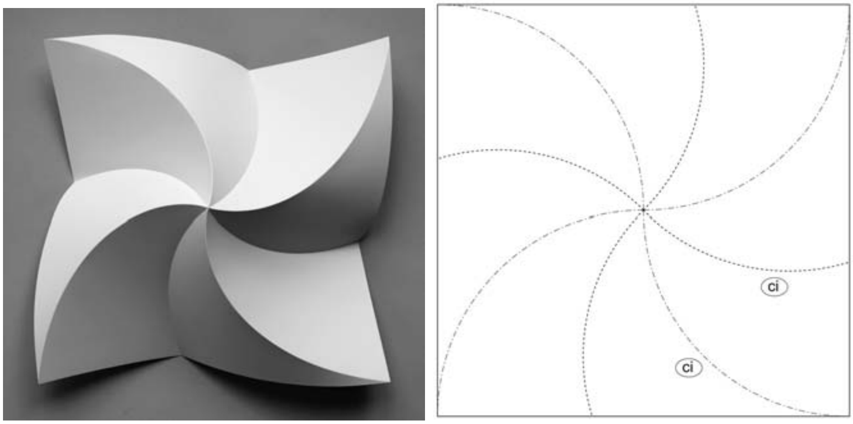

Figure 5. Huffman design using circles (unexploded vertex).

Figure 6 shows a degree-4 vertex inflated into two degree-3 vertices. The connecting element in this case is a straight line. The crease pattern uses ellipses that result in more dramatic curvature changes than in Figure 5.

In Figure 7, a degree-4 vertex has been inflated into four degree-3 vertices. The connecting flat square rotates very little when the design is folded into shape. This example is made of ellipses and is featured on *Grafica Obscura* [Haeberli 96].

Figure 8 shows a crease pattern that displays structural properties similar to the so-called "flashers." Simon Guest studied these shapes in terms

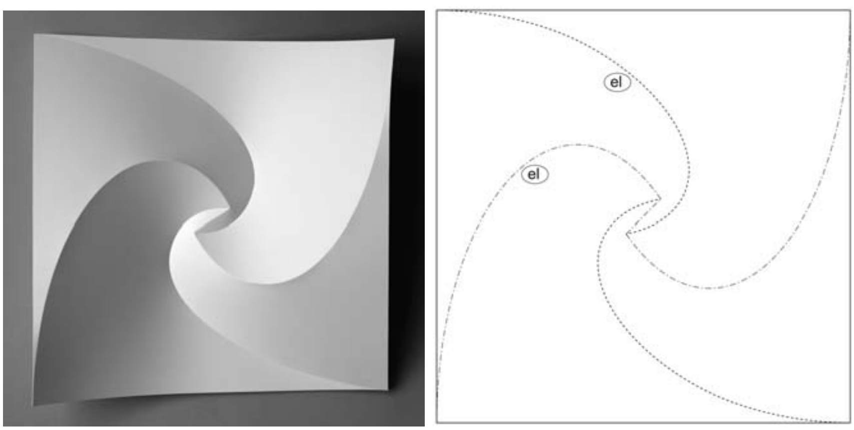

Figure 6. Huffman design using ellipses and a line (exploded vertex).

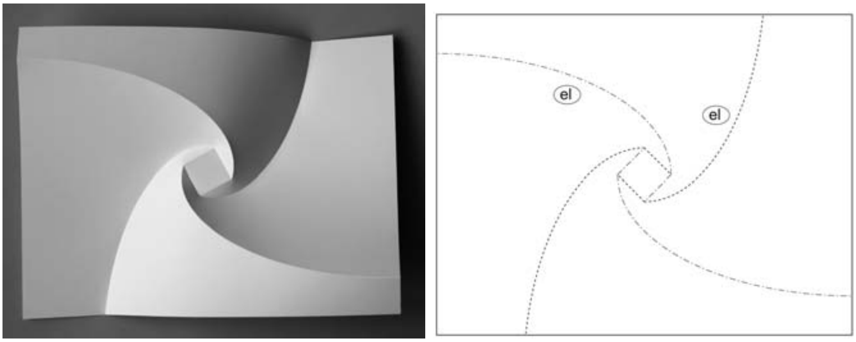

Figure 7. Huffman design using ellipses and a square (exploded vertex).

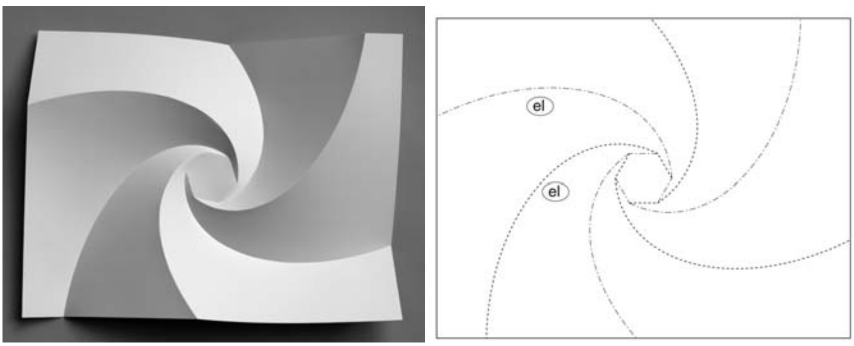

Figure 8. Huffman design using ellipses and a hexagon (exploded vertex).

of how they can curl up into themselves [Guest and Pellegrino 92], and Huffman made several of them. This design is made of ellipses that converge in a degree-6 vertex, which was inflated into six degree-3 vertices. The connecting element is a flat hexagon that rotates very little in comparison to Guest's shapes, and we believe that Huffman was not studying their kinetic behavior, but rather the inflated vertices.

3.3 Tessellations

Figure 9 shows a tessellation with reflectionally and 180° rotationally symmetric tiles. The crease pattern uses circles, and mountain and valley assignments alternate from row to row.

Huffman called the tessellation in Figure 10 "Arches" and used parabolas or ellipses that are connected at their focal points. The resulting arches have parallel rulings. The model shown here was constructed of paper and uses parabolas.

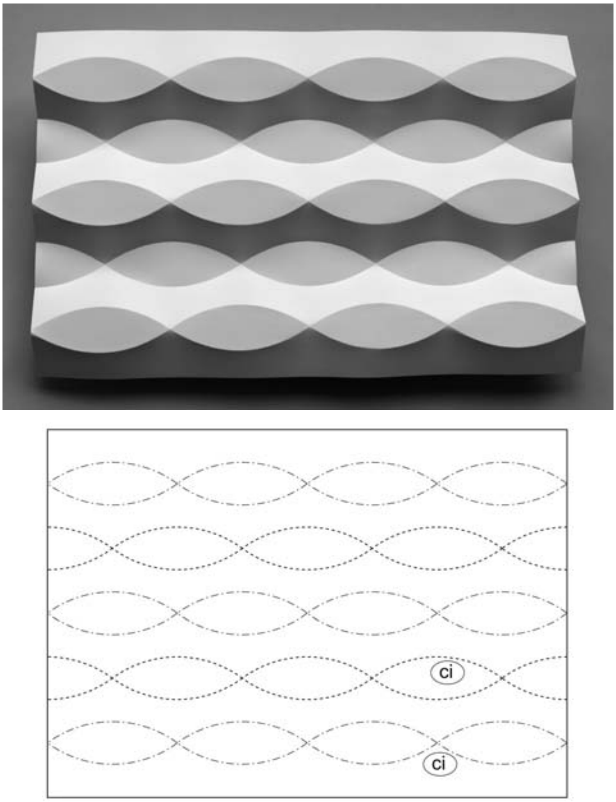

Figure 9. Huffman design using circles.

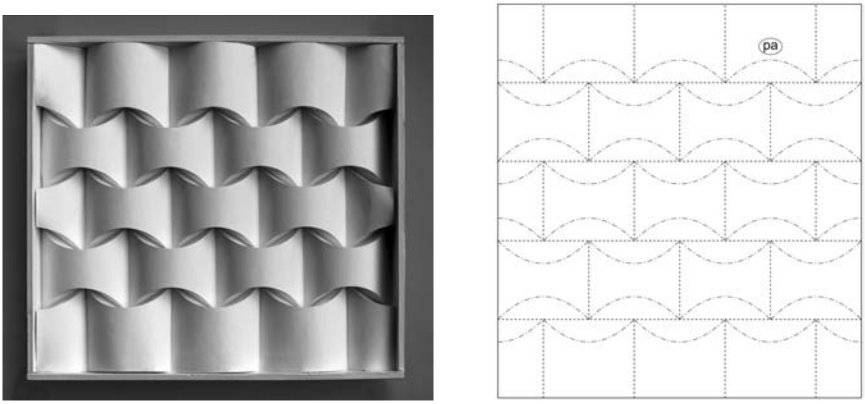

Figure 10. Huffman's "Arches" design using parabolas and lines. (See Color Plate I.)

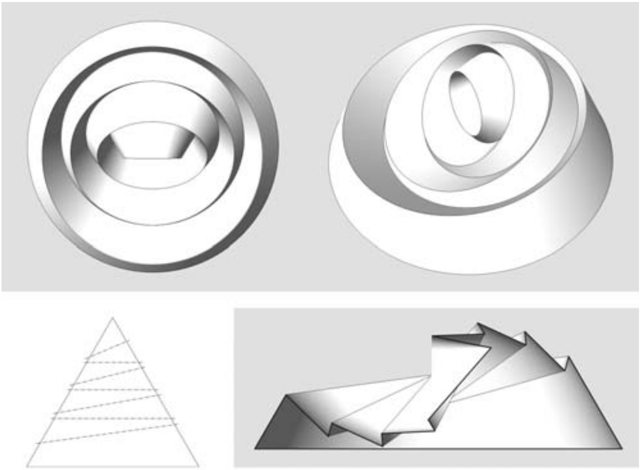

Figure 11. "Cone Reflected 7 Times," top and perspective views (top); section through entire cone and mirrored design (bottom).

3.4 Cones

Figure 11 shows Huffman's "Cone Reflected 7 Times" reconstructed as a digital three-dimensional model. A similar shape was designed by Ron Resch in 1969, which he called "Yellow Folded Cones: Kissing" [Resch 71], where he truncated a cone twice and mirrored it.

Here eight truncated cones are mirrored in an alternating way. The perpendicular cuts to the main axis of the cone result in a circular arc in the crease pattern similar to a design by Hiroshi Ogawa [Ogawa 71]. In the rotated cuts, increasing angles go up from the bottom. The truncated cones need to be unrolled to construct the curves for the crease pattern. These curves are a rare example of Huffman using non-quadratic curves; see Figure 12.

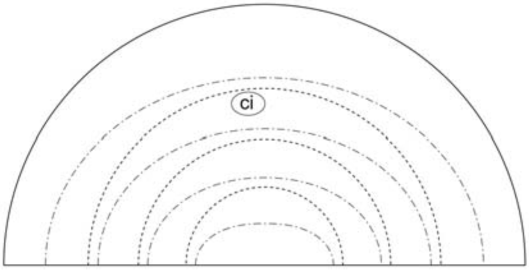

Figure 12. Crease pattern design using circles and non-quadratic curves.

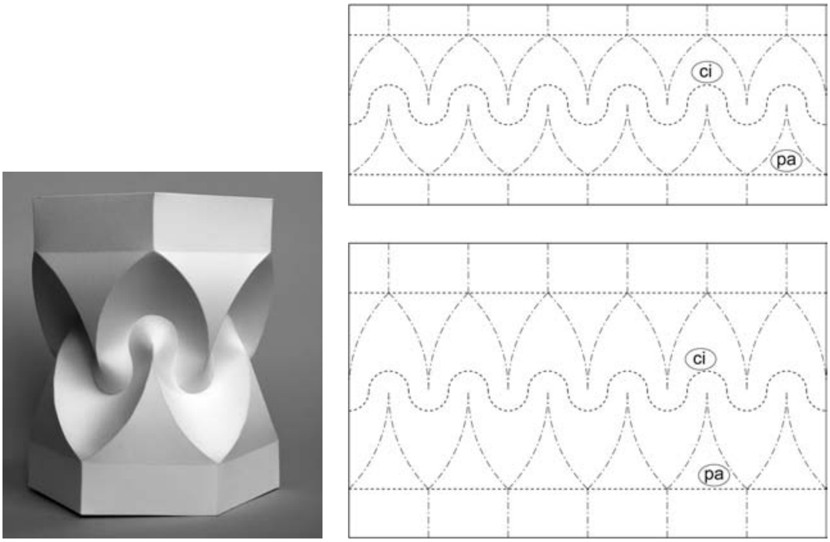

Figure 13. “Hexagonal Column with Cusps” design using circles and lines. The photograph corresponds to the upper crease pattern.

3.5 Complex Shapes

Huffman’s “Hexagonal Column with Cusps,” shown in Figure 13, is remarkable as two sides of the paper meet and create a continuous shape. This stunning and aesthetically very well received example of Huffman’s designs has been reconstructed by Saadya Sternberg [Sternberg 09] and Robert Lang [Lang 10].

This crease pattern combines half-circles, parabolas, and straight lines. Huffman made many versions of this model with different proportions and sometimes even repeated the entire shape twice in a single crease pattern. Figure 13 shows the crease pattern for two differently proportioned versions. The model shown here was made out of paper.

Huffman’s “4-Lobed Cloverleaf Design” is shown in Figure 14. It is symmetric along two axes and comprises lines and ellipses. The inner square is folded such that the triangular faces touch one another—a common way to hide material in straight-crease origami, but a rare characteristic among Huffman’s designs.

The model in Figure 15, named “One Column,” was recreated here from white Zanders elephant hide paper. Huffman joined parabolas together to create two wavy extrusions at different scales. This model is particularly striking. It starts with a very simple crease pattern yet creates dramatic features from folding those creases very tightly.

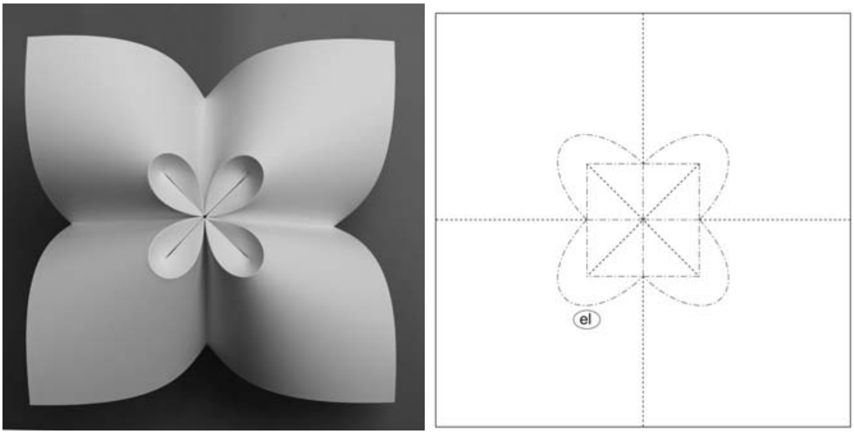

Figure 14. Huffman's "4-Lobed Cloverleaf Design" using ellipses and lines.

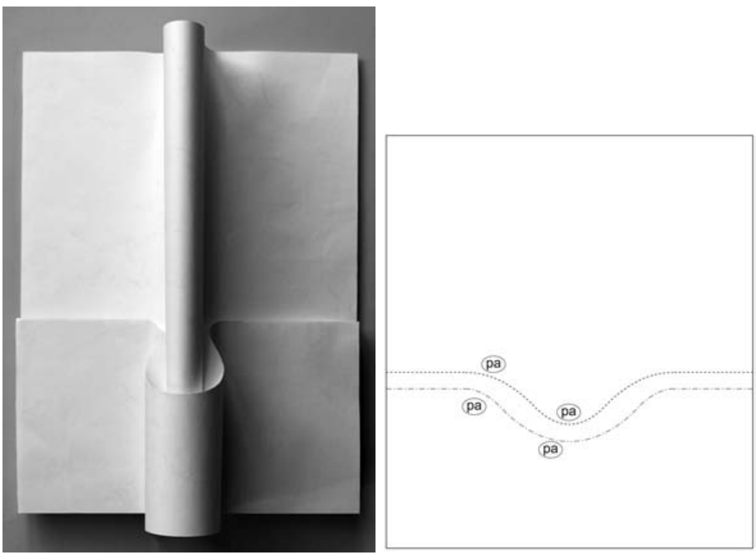

Figure 15. Huffman's "One Column" design using parabolas and lines.

4 Conclusion

Our goal is to expose to the world David Huffman's beautiful artwork and the underlying mathematics that he used to create it. We believe that much can be learned from reconstructing and analyzing his final models, which is the focus of this paper. This reconstruction project is ongoing, with many more models to be studied. We also believe that there is much to be learned from Huffman's notes, sketches and working models, a study which is just beginning.

Ultimately, we aim to develop a theory for how David Huffman designed his curved-crease foldings, to enable future origami artists and mathematicians to build upon his knowledge and expertise. We regret not being able to develop this theory in direct communication with David, but we are confident that the legacy he left behind will enable a fruitful collaboration.

Acknowledgments. We thank the Huffman family—in particular, Elise, Linda, and Marilyn Huffman, and Jeff Grubb—for their ongoing collaboration on this project and for their kind hospitality.

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