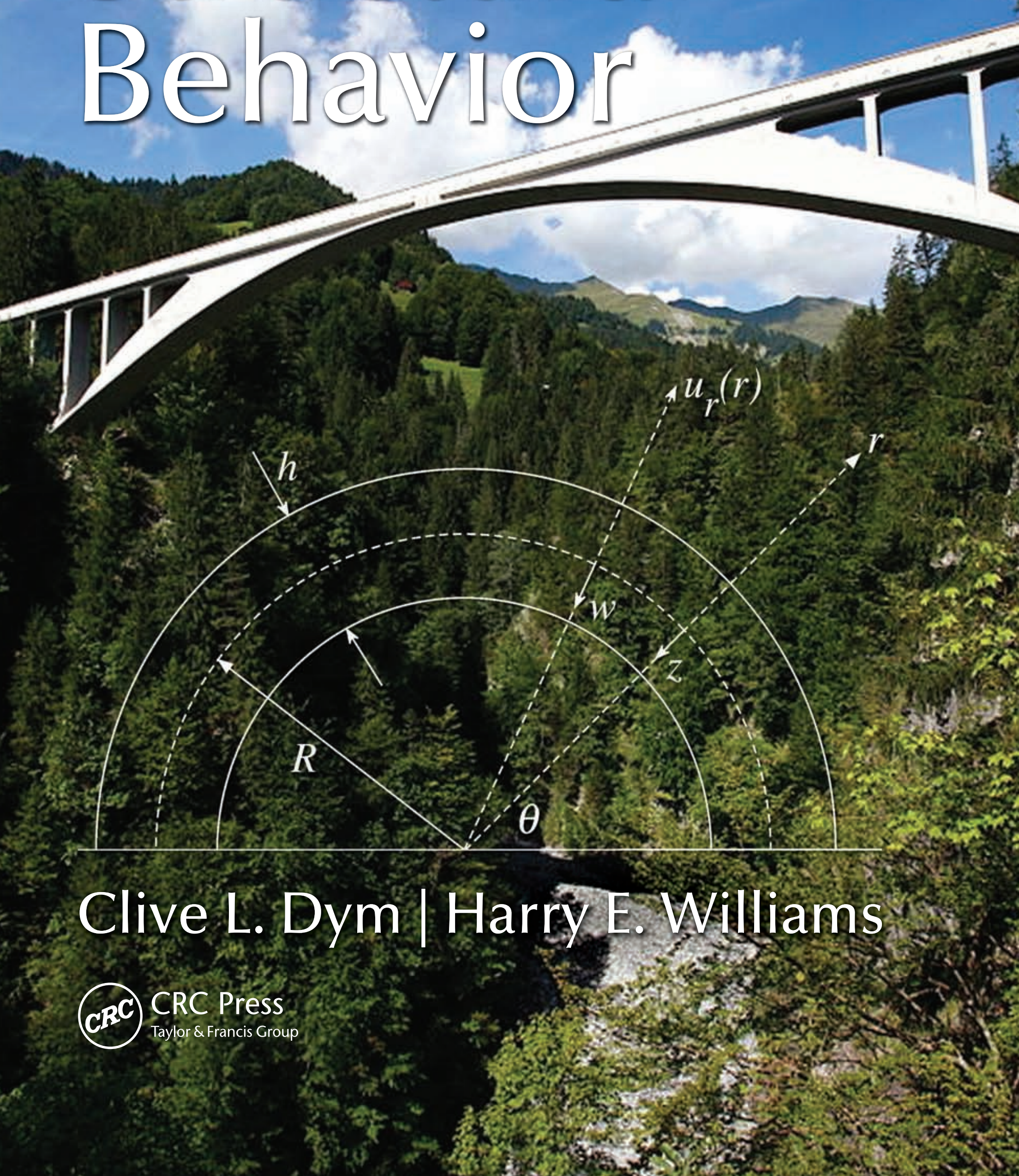
Analytical Estimates of Structural Behavior



Clive L. Dym | Harry E. Williams

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In the spirit of "standing on the shoulders of giants," we dedicate this book to our mentors (and the institutions at which we began to learn)

Anthony E. Armenàkas (Cooper Union)

Joseph Kempner (Brooklyn Polytechnic Institute)

Nicholas J. Hoff (Stanford University)

and

Richard M. Hermes (Santa Clara University)

Julian D. Cole (California Institute of Technology)

George W. Housner (California Institute of Technology)

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Preface

We intend this book to explicitly return the notion of modeling to the analysis of structures by presenting an integrated approach to modeling and estimating structural behavior. The advent of computer-based approaches to structural analysis and design over the last 50 years has only accentuated the need for structural engineers to recognize that we are dealing with *models* of structures, rather than with the actual structures. Further, as tempting as it is to run innumerable computer simulations, closed-form *estimates* can be effectively used to guide and to check numerical results, as well as to confirm in clear terms physical insights and intuitions. What is truly remarkable is that the way of thinking about structures and their models that we propose is rooted in classic elementary elasticity: It depends less on advanced mathematical techniques and far more on thinking about the dimensions and magnitudes of the underlying physics.

A second reason for this book is our concern with traditional classroom approaches to structural analysis. Most introductory textbooks on structural analysis convey the subject as a collection of seemingly unrelated tools available to handle a set of relatively specific problem types. A major divide on the problem-type axis is the distinction drawn between structures that are statically determinate and those that are not. While this also logically conforms to a presentation in an order that reflects the respective degree of difficulty of application, it is often not seen by students as a coherent view of the discipline. Perhaps reflecting a long-standing split in professional affiliations, the classical approaches to structural analysis are often presented as a field entirely distinct from its logical underpinnings in mechanics, especially applied mechanics.

Finally, as noted before, the advent of the computer and its ubiquitous use in the classroom and in the design office has led structural engineering faculty to include elementary computer programs within a shrinking structural curriculum. Thus, students seem to spend more time and effort generating numbers, with less time and effort spent on understanding what meaning—if any—to attach to the numbers that are generated with these programs. This tendency has only strengthened as computers have become still more powerful. Still more unfortunate is that this approach emphasizes another growing dissonance in the education of engineering students: *Problems* in structural behavior and response continue to be formulated largely in mathematical terms, while *solutions* are increasingly sought with computer programs.

We have based this book on the premise that it is now even more important to understand basic structural *modeling,* with strong emphasis on understanding behavior and interpreting results in terms of the limitations of the models being applied. In fact, we would argue that the generation of numerical analyses for particular cases is, in the "real world," increasingly a task performed by technicians or entry-level engineers, rather than by seasoned professional engineers. As numerical analysis becomes both more common and significantly easier, those structural analysts and designers who know *which* calculations to perform, *how to validate and interpret* those calculations, and *what the subsequent results mean* will be the most highly regarded engineers. The knowledge needed to do these tasks can often be encapsulated and illustrated with the ability to obtain and properly use analytical, closed-form estimates—or, in other words, the ability to obtain and properly use "back of the envelope" models and formulas.

We note that it is more than the outline of topics that sets apart this book from others. That outline, to be described immediately, is not what we would expect to find in a first course in structural analysis. In fact, much of what we have included in Chapters 3–7 derives from articles we have published in the various research journals (see the references and bibliographies at the end of each chapter). The common theme of these articles and of Chapters 3–7 is the development of effective analytical estimates of the responses of certain structural models. So, we hope to stretch the mold of traditional approaches to structural analysis—and especially how our colleagues teach structural analysis—to emphasize and more explicitly describe the modeling process, and thus present a more conscious view of estimating and assessing structural response.

We finally note that while this book is not intended as a text for a *first* course in structural analysis, we certainly think it is accessible to advanced undergraduates as well as to graduate students and practitioners. It does not require deep knowledge of advanced structural mechanics models or techniques:

- We use the principle of minimum total potential energy to derive governing equations and boundary conditions, but those equations can be derived in other ways or even simply accepted.
- We introduce extensions of the Castigliano theorems and Rayleigh quotients for discrete systems, laying a foundation for applying them to continuous systems.

The mathematical skills that will be exercised are more about applying techniques of dimensional analysis, reasoning about physical dimensions, and reasoning about the relative sizes of mathematical terms and using appropriate expansions to determine limits and limiting behavior.

Organization

This book is organized as follows. In Chapter 1 we outline some important principles and techniques of mathematical modeling, including dimensional analysis, scaling, linearity, and balance and conservation laws. In Chapter 2 we review basic structural models, including structural supports and materials, as well as some general considerations of load paths, redundancy, determinacy, and stability. We also review there the concept of *idealization,* and we complete the chapter by bringing *discretization* under the modeling umbrella as well.

In Chapter 3 we use subsets of two-dimensional elasticity theory to reconsider two classic structural mechanics problems so as to explore how we develop and express physical intuition. First, we rederive the traditional fourth-order Euler–Bernoulli beam equation and boundary conditions and then use these results to estimate ranges of validity for beam models. Intuition issues emerge as we interpret both boundary conditions, the beam's physical parameters, and the nature of the loading—in particular, the transition from sets of concentrated loads to a uniform load. We illustrate how planar truss configurations behave as beams and use two-dimensional elasticity to derive another classical problem, the static response of pressure-loaded cylinders, and show how our physical intuitions can lead us astray.

In Chapter 4 we demonstrate how the behavior of arches under lateral load can be tracked as it varies from beam behavior at small values of an arch parameter (i.e., arches with very small rises) to purely compressive arch behavior when the arch parameter is large (i.e., for large arch rises). It is also shown that the behavior "flips" when the load applied is axial, rather than lateral.

In Chapter 5 we introduce two methods of analyzing coupled discrete systems, in part to lay a foundation for their application to continuous systems in our two final chapters, and in part just to ensure a common background for readers who may not be familiar with either or both of the techniques described. First, we describe recently developed extensions of Castigliano's theorems, and then we introduce Rayleigh's quotient for estimating the fundamental frequencies of coupled spring-mass oscillators. Then, in Chapter 6 we apply the extension of Castigliano's second theorem to derive simple, yet quite accurate estimates of the transverse displacements of structures modeled in terms of coupled Timoshenko beams (e.g., tall buildings). Finally, in a similar vein, in Chapter 7 we use Rayleigh quotients to analyze the dimensional behavior of and calculate numerical values of fundamental frequencies of structures modeled in terms of Euler–Bernoulli, Timoshenko, and coupled-beam systems (e.g., again, potential models of tall buildings).

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Dr. Dym is a fellow of the Acoustical Society of America, the American Society of Mechanical Engineers, the American Society of Civil Engineers, and the American Society for Engineering Education, and is a member of the American Academy of Mechanics. Dr. Dym's awards include the Walter L. Huber Research Prize (ASCE, 1980), the Western Electric Fund Award (ASEE, 1983), the Boeing Outstanding Educator Award (first runner-up, 2001), the Fred Merryfield Design Award (ASEE, 2002), the Joel and Ruth Spira Outstanding Design Educator Award (ASME, 2004), the Archie Higdon Distinguished Educator Award (Mechanics Division, ASEE, 2006), and the Bernard M. Gordon Prize for Innovation in Engineering and Technology Education (NAE, 2012; co-winner).

Harry E. Williams is professor emeritus of engineering at Harvey Mudd College. After receiving his PhD from the California Institute of Technology, Dr. Williams joined the research staff of the Jet Propulsion Laboratory and then joined Harvey Mudd College as one of the founding faculty of its engineering program, serving there for 40 years. He has also been a Fulbright fellow at the University of Manchester, a liaison scientist for the Office of Naval Research, and a consultant to General Dynamics, Teledyne Microelectronics, the Naval Weapons Center, Aerojet-General, and the Boeing Company. He has published widely over the years, reflecting his interests in fluid mechanics, thermoelasticity, and the mechanics of solids and structures.

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Mathematical Modeling for Structural Analysis

Summary

The dictionary defines a *model* as "a miniature representation of something; a pattern of something to be made; an example for imitation or emulation; a description or analogy used to help visualize something (e.g., an atom) that cannot be directly observed; a system of postulates, data and inferences presented as a mathematical description of an entity or state of affairs." This definition suggests that *modeling* is an activity, a *cognitive activity* in which one thinks about and makes models to describe how devices or objects of interest behave. Thus, it is important to remember that when we describe or formulate a problem in words, draw a sketch (e.g., a free-body diagram), write down or derive a formula, and crank through to get some numbers, we are modeling something. In each of these activities we are formulating and representing a model of the problem in a *modeling language.* And as we go from words to pictures to formulas to numbers, we must be sure that we are translating our problem correctly and consistently. We have to maintain our assumptions, and at the right level of detail.

Since there are many ways in which devices and behaviors can be described—words, drawings or sketches, physical models, computer programs, or mathematical formulas—it is worth refining the foregoing dictionary definition to define a *mathematical model* as a "representation in mathematical terms" of the behavior of real devices and objects. Our primary modeling language is mathematics, so we must be able to translate fluently into and from mathematics.

Scientists use mathematical models to *describe* observed behavior or results, *explain why* that behavior and those results occurred as they did, and *predict* future behaviors or results that are as yet unseen or unmeasured. *Engineers* use mathematical models to describe and analyze objects and devices in order to predict their behavior because they are interested in *designing* devices and processes and systems. Design is a consequential activity for

engineers because every new airplane or building, for example, represents a model-based prediction that the plane will fly and the building stand without dire, unanticipated consequences. Further, as practicing engineers, we must always remember that we are dealing with models of a problem—*models of reality.* Thus, if our results do not match experimental data or intuitive expectations, we may well have a model that is simply wrong. So it is especially important in engineering to ask: How are such mathematical models or representations created? How are they validated? How are they used? Is their use limited and, if so, how?

To answer these and related questions, this chapter first sets out some basic principles of mathematical modeling and then goes on to describe briefly:

- abstraction and scaling
- dimensional consistency and dimensional analysis
- conservation and balance laws
- the assumption of linear behavior

Principles of Mathematical Modeling

Mathematical modeling is a principled activity that has principles behind it as well as methods that can be successfully applied. The principles are overarching or metaprinciples that are almost philosophical in nature, and they can be phrased as questions (and answers) about modeling tasks we need to perform and their purposes. That is, builders of mathematical (and other types of) models must **identify**

a. The need for the model: **Why** is this being done?
b. The data sought: **What** information is being sought?
c. The available relevant data: What is known (i.e., What is **given?**)
d. The circumstances that apply: What can be **assumed?**
e. The governing physical principles: **How** should this model be viewed?
f. The equations that will be used, the calculations that will be made, and the answers that will result: What will the model **predict?**
g. The tests to be made to validate the model and ensure its consistency with its principles and assumptions: Are the predictions **valid?**
h. The tests to be made to verify the model and ensure its usefulness in terms of the initial reason it was done: Can the predictions be **verified?**

i. Parameter values that are not adequately known, variables that should have been included, and/or assumptions that could be removed (i.e., can an iterative "model-validate-verify-improve-predict" loop be implemented? Can the model be **improved?**)
j. What will be done with the model: How will the model be **used?**

These identified tasks and questions can also be visually portrayed (see Figure 1.1).

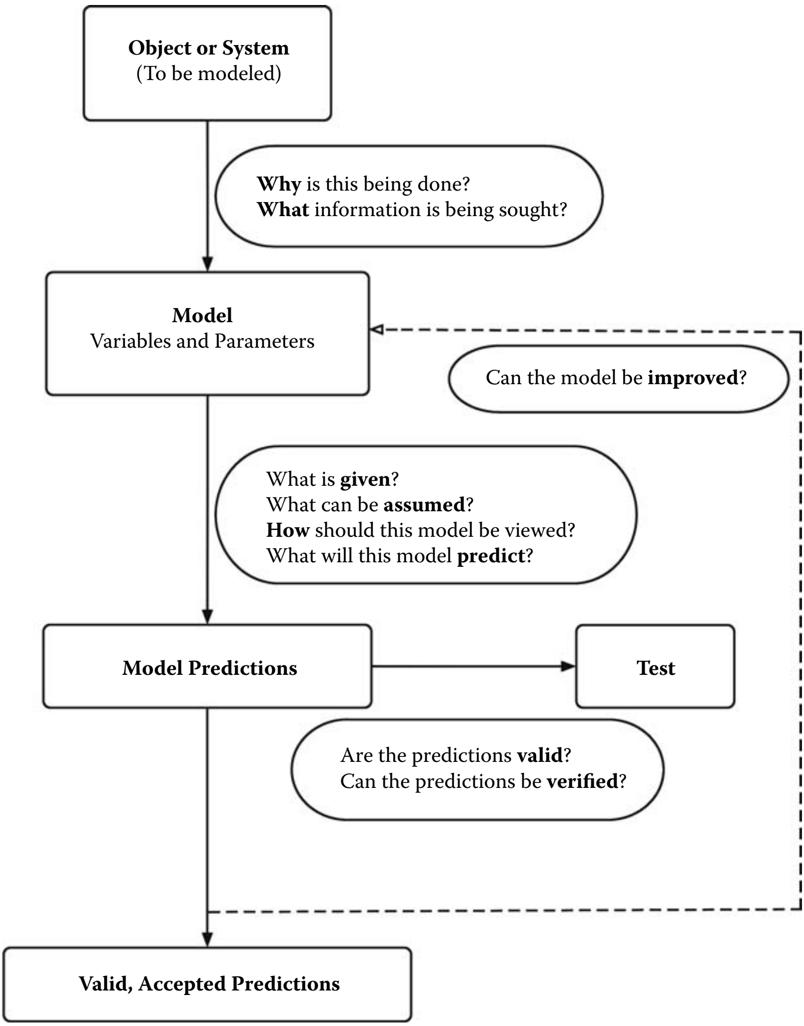

FIGURE 1.1
A graphical overview of *mathematical modeling* shows how the questions asked during a principled approach to model building relate to the development of that model. (Dym, C. L. 2004. *Principles of Mathematical Modeling*, 2nd ed. By permission of Elsevier Academic Press.)

It is worth noting that the last principle (**used?**) is often considered *early* in the modeling process, along with **why?** and **find?**, because the way a model is to be used is often intimately connected with the reason it is created. Note too that this list of questions and instructions is *not* an algorithm for building a good mathematical model. However, the underlying ideas are key to mathematical modeling, as they are key to problem formulation generally. Thus, the individual questions will recur often during the modeling process, so the list should be regarded as a general approach to *ways of thinking* about mathematical modeling.

It is most important to have a clear picture of why a model is wanted or needed. For example, a first estimate of the available power generated by a dam on a large river—say, the famed Three Gorges Dam on the Yangtze River in the People's Republic of China—would not require a model of the dam's thickness or the strength of its foundation. On the other hand, its height would be essential, as would some model and estimates of river flow quantities. By way of contrast, a design of the actual dam would need a model that incorporates all of the dam's physical characteristics (e.g., dimensions, materials, foundations) and relates them to the dam site and the river flow conditions. Thus, defining the task is the first essential step in model formulation.

The next step would be to list what is known—for example, river flow quantities and desired power levels—as a basis for listing variables or parameters that are not yet known. One should also list any relevant assumptions. For example, levels of desired power may be linked to demographic or economic data, so any assumptions made about population and economic growth should be spelled out. Assumptions about the consistency of river flows and the statistics of flooding should also be spelled out.

Which physical principles apply to this model? The mass of the river's water must be conserved, as must its momentum, as the river flows, and energy is both dissipated and redirected as water is allowed to flow through turbines in the dam (or spill over the top!). Mass must be conserved, within some undefined system boundary, because dams do accumulate water mass from flowing rivers. There are well-known equations that correspond to these physical principles. They could be used to develop an estimate of dam height as a function of power desired. The model can be validated by ensuring that all equations and calculated results have the proper dimensions, and it can be exercised against data from existing hydroelectric dams to get empirical data and validation.

If the model is inadequate or fails in some way, an *iterative loop* is then entered in which one cycles back to an earlier stage of the model building to reexamine any assumptions, known parameter values, the principles chosen, the equations used, the means of calculation, and so on. This iterative process is essential because it is the only way that models can be improved, corrected, and validated.

Abstraction and Scale (I)

Consider now issues of *scale,* of *relative size.* Size, whether absolute or relative, is very important because it affects both the form and the function of those objects or systems being modeled. Scaling influences—indeed, often controls—the way objects interact with their environments, for objects in nature, the design of experiments, or the representation of data by smooth, nice-looking curves. This section briefly discusses the ideas behind abstraction and scale, with further details to follow later.

Abstraction, Scaling, and Lumped Elements

An important decision in modeling is choosing an appropriate level of detail for the problem at hand and thus knowing what level of detail is prescribed for the attendant model. This process is called *abstraction* and it typically requires a thoughtful and organized approach to identifying those phenomena that will be emphasized—that is, to answering the fundamental question about why a model is being sought or developed. Further, thinking about finding the right level of abstraction or the right level of detail often requires finding the right *scale* for the model being developed. Stated differently, thinking about *scaling* means thinking in terms of the magnitude or size of quantities measured with respect to a standard that has the same physical dimensions.

For example, the linear elastic spring is used to model more than just the force–extension relation of simple springs such as old-fashioned butcher's scales or automobile springs. For example, $F = kx$ can be used to describe the static load-deflection behavior of a diving board, where the spring constant k will reflect the stiffness of the diving board taken as a whole, which in turn reflects more detailed properties of the board, including the material of which it is made and its own dimensions. The validity of using a linear spring to model the board can be confirmed by measuring and plotting the deflection of the board's tip as it changes with standing divers of different weights.

The classic spring equation is also used to model the static and dynamic behavior of tall buildings as they respond to wind loading and to earthquakes. These examples suggest that a simple, highly abstracted model of a building can be developed by aggregating various details within the parameters of that model. That is, the stiffness k for a building, as with the diving board, would be a *lumped element* that aggregates a great deal of information about how the building is framed, its geometry, its materials, and so on. For both the diving board and the tall building, detailed expressions of how their respective stiffnesses depended on their respective properties would be needed. It is not possible to do a detailed design