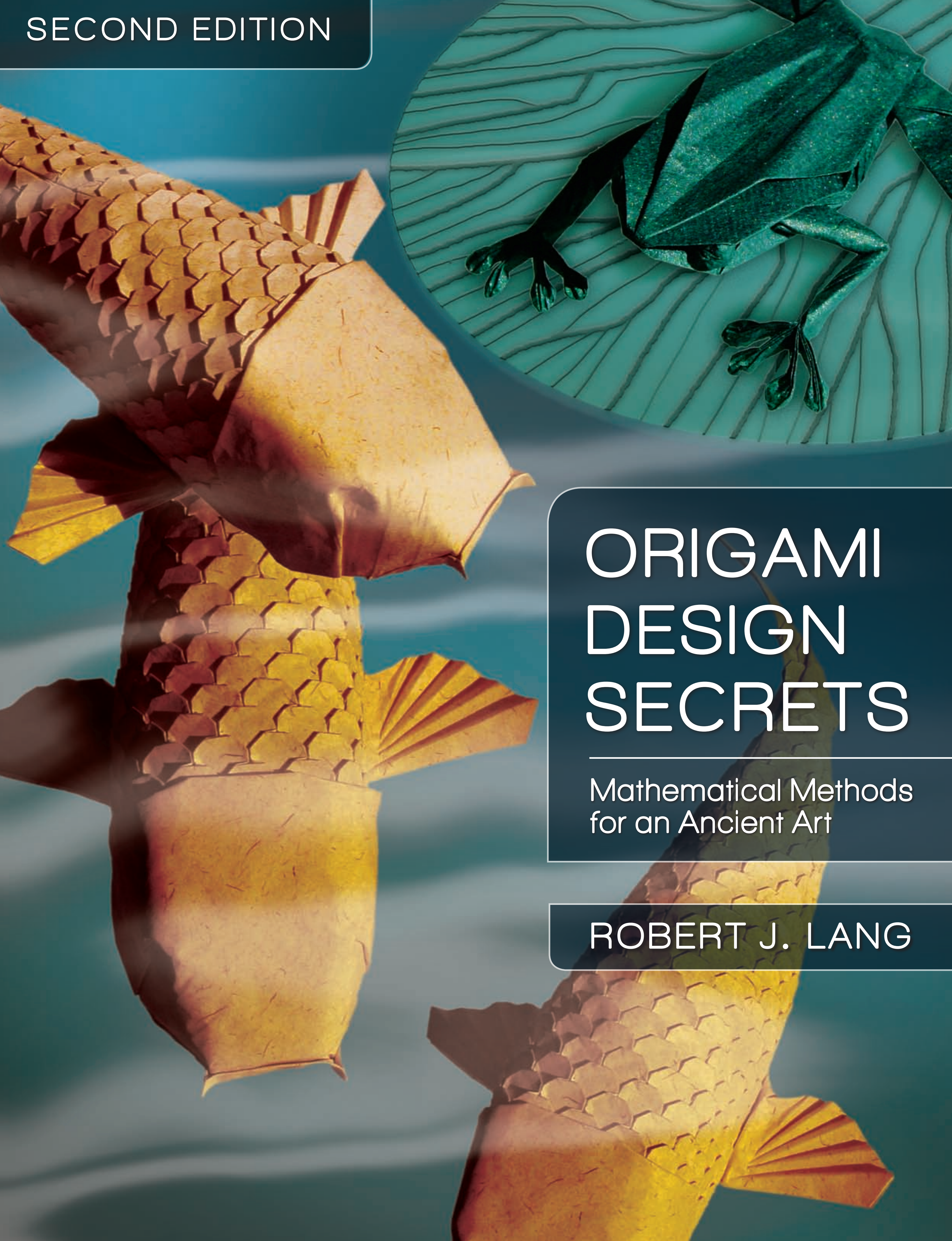
SECOND EDITION

ORIGAMI DESIGN SECRETS

Mathematical Methods for an Ancient Art

ROBERT J. LANG



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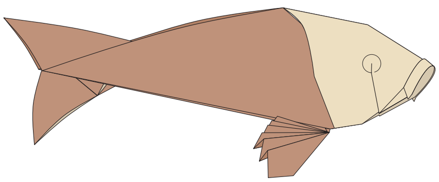

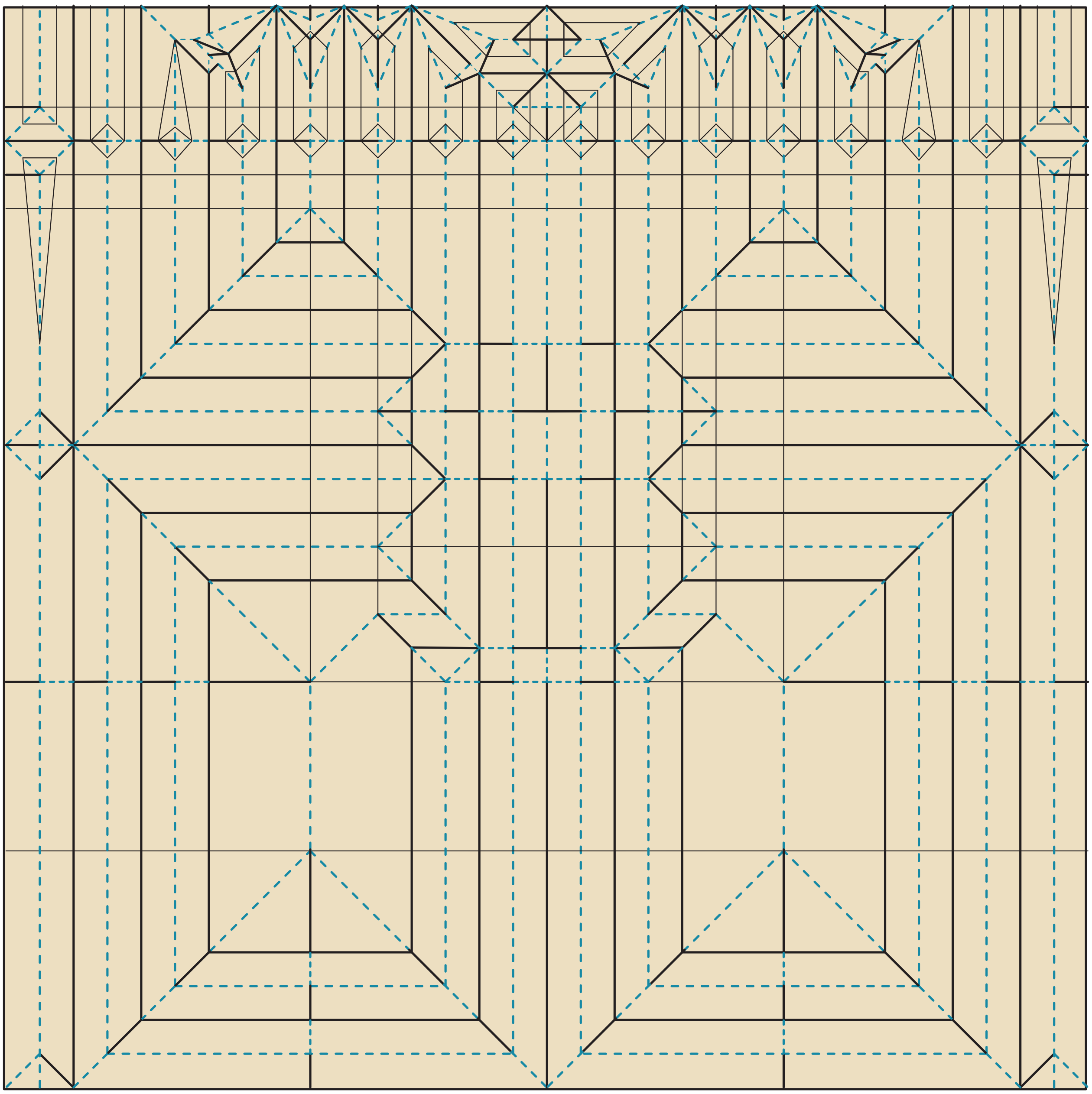

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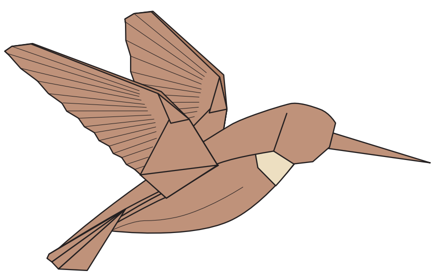

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Acknowledgements

This book was a labor of many years. It is both my earliest book and my latest book; I began writing up my ideas on how to design when I began what became my first published book in the early 1980s, but not until recently have I developed the framework for those ideas. Over the years, I have been influenced by many scientists and artists, both inside and outside of origami, all of whom contributed, one way or another, to the present tome. It is impossible for me to identify everyone who has contributed to my work, but some of the larger pieces come from the following, who I thank:

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Last, but most important, I must thank my wife, Diane, for her constant support and encouragement.

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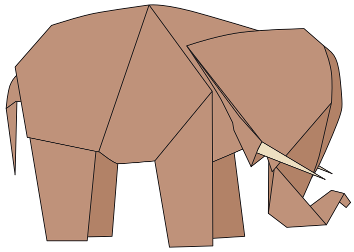

1 Introduction

In 1988, a French artist named Alain Georgeot prepared an exhibition of 88 elephants. They were made of folded paper, each different, and each one an example of *origami*, the Japanese art of paper folding. An art exhibition devoted entirely to origami is rare; one devoted to elephants is extremely unusual; and one devoted entirely to origami elephants was entirely unprecedented.

A display of 88 paper elephants illustrates both the remarkable attraction origami has for some people—after all, how many people would take the time to fold 88 versions of the same thing?—and the remarkable versatility of the art. Georgeot's collection of elephants represented only the tiniest fraction of the modern origami repertoire. Tens of thousands of paper designs exist for animals, plants, and objects, a regular abecedarium of subject matter. There are antelopes, birds, cars, dogs, elephants (of course), flowers and gorillas; horses, ibexes, jays, and kangaroos; lions, monkeys, nautiluses, octopi, parrots, quetzalcoatls, roses, sharks, trains, ukuleles, violinists, whelks, xylophones, yaks, and zebras, the last complete with stripes.

Innumerable innovations have been wrought upon the basic theme of folded paper. There are action figures: birds that flap their wings, violinists who bow their violins, inflatable boxes, clapping monkeys, snapping jaws. There are paper airplanes that fly—one won an international contest—and airplanes that don't fly, but are replicas of famous aircraft: the space shuttle, the SR-71 Blackbird, and the venerable Sopwith Camel. In some models, a single piece of paper is folded into several figures (a bull, bullfighter, and cape, for example) and in others, many identical pieces of

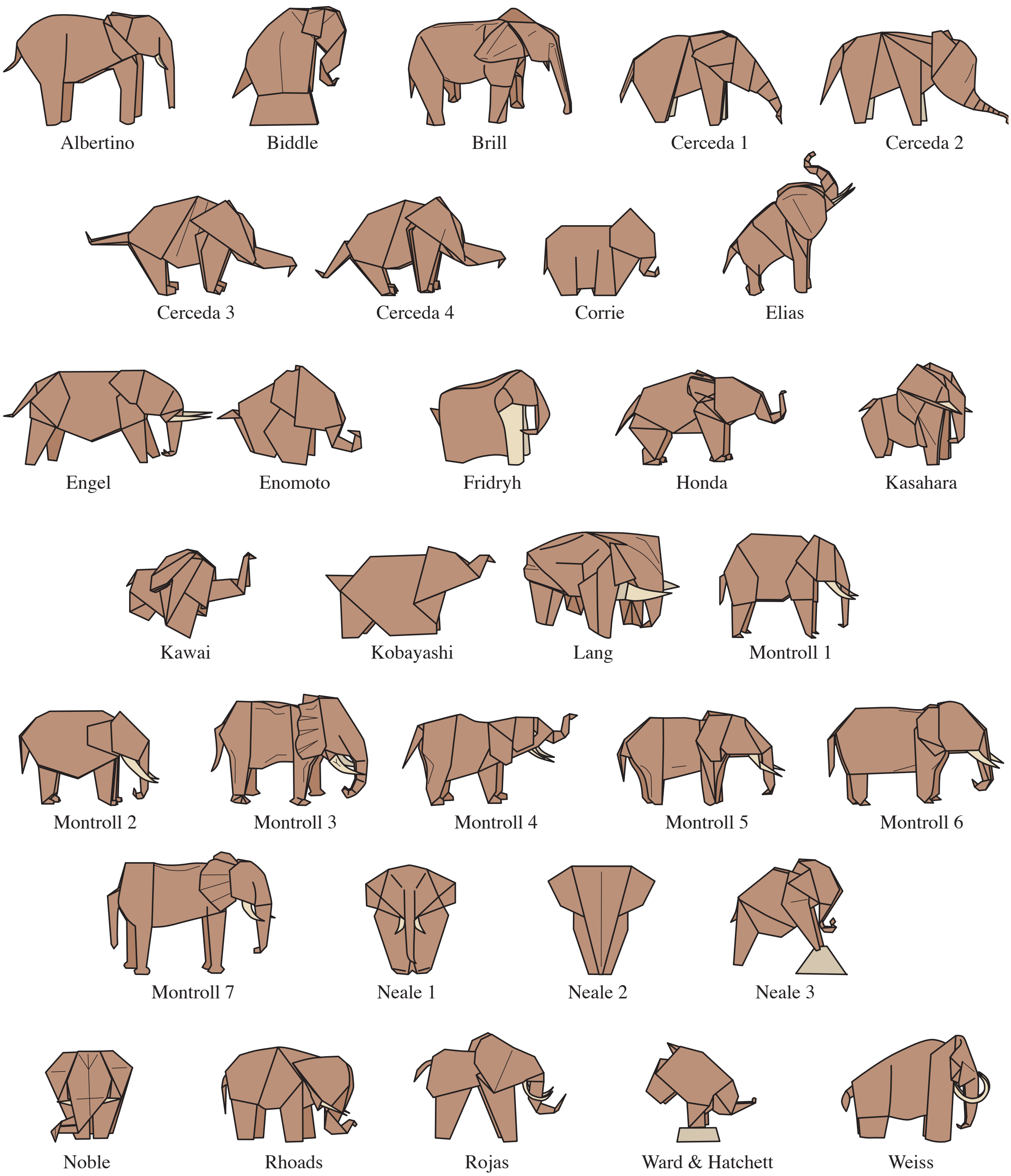

Figure 1.1.
A herd of origami elephants.

paper are assembled into enormous multifaceted polyhedra. If you can think of an object either natural or man-made, someone, somewhere, has probably folded an origami version.

The art of origami was originally Japanese, but the 88 elephants and the tens of thousands of other designs come from all over the world. Many figures originated in Japan, of course, but the U.S.A., England, France, Germany, Belgium, Argentina, Singapore, Australia, and Italy are major centers of origami activity. The designs range from simple figures consisting of only two or three folds to incredibly complex "test pieces" requiring hours to fold. Most of these thousands of designs have one thing in common, however: Nearly all were invented in the last 50 years.

Thus, origami is both an old art and a young art. Its youth is somewhat surprising. After all, folded paper has been an art form for some 15 centuries. It is ancient; one would not expect 98 percent of the innovation to come in the last 2 percent of the art's existence! Yet it has. Fifty years ago, all of the different origami designs in the world could have been catalogued on a single typed sheet of paper, had anyone had the inclination to do so. No model would have run over about 20 or 30 steps. Most could be folded in a few minutes, even by a novice. This is no longer the case. Today, in books, journals, and personal archives, the number of recorded origami designs runs well into the thousands; the most sophisticated designs have hundreds of steps and take several hours for an experienced folder to produce. The past 60 years in Japan, and 40 years worldwide, have seen a renaissance in the world of origami and an acceleration of its evolution.

And this has happened in the face of stringent barriers. The traditional rules of origami—one sheet of paper, no cuts—are daunting. It would appear that only the simplest abstract shapes are feasible with such rules. Yet over hundreds of years, by trial and error, two to three hundred designs were developed. These early designs were for the most part simple and stylized. Complexity and realism—insects with legs, wings, and antennae—were not possible until the development of specialized design methods in the latter part of the 20th century.

Although there are now many thousands of origami designs, there are not thousands of origami designers. In fact, there is only a handful of designers who have gone beyond basics, only a handful who can and do design sophisticated models. Although there is far more exchange of completed designs now than there used to be, there is not a similar exchange of design techniques.

This imbalance arises because it is much easier to describe how to replicate an origami figure than how to design one. Origami designs spread through publication of their folding sequence—a set of step-by-step instructions. The folding sequence, based on a simple code of dashed and dotted lines and arrows devised by the great Japanese master Akira Yoshizawa, transcends language boundaries and has led to the worldwide spread of origami.

While thousands of folding sequences have been published in books, magazines, and conference proceedings, a step-by-step folding sequence does not necessarily communicate how the model was designed. The folding sequence is usually optimized for ease of folding, not to show off design techniques or the structure of the model. In fact, some of the most enjoyable folding sequences are ones that obscure the underlying design of the model so that the appearance of the final structure comes as a surprise. "How to fold" is rarely "how to design." Folding sequences are widespread, but relatively few of the design techniques of origami have ever been set down on paper.

Over the last 40 years I have designed some 500+ original figures. The most common question I am asked is, "How do you come up with your designs?" Throughout the history of origami, most designers have designed by "feel," by an intuition of which steps to take to achieve a particular end. My own approach to design has followed what I suspect is a not uncommon pattern; it evolved over the years from simply playing around with the paper, through somewhat more directed playing, to systematic folding. Nowadays, when I set out to fold a new subject, I have a pretty good idea about how I'm going to go about folding it and can usually produce a fair approximation of my subject on the first try.

Hence the perennial question: How do you do that? The question is asked as if there were a recipe for origami design somewhere, a cookbook whose steps you could follow to reliably produce any shape you wanted from the square of paper. I don't think of origami design as a cookbook process so much as a bag of tricks from which I select one or more in the design of a new model. Here is a base (a fundamental folding pattern) with six legs: I'll use it to make a beetle. Here is a technique for adding a pair of points to an existing base: I'll combine these to make wings. Some designers have deeper bags of tricks than others; some, like John Montroll, have a seemingly bottomless bag of tricks. I can't really teach *the* way to design origami, for there is no single way to design, but what I can and will try to do in this book is to pass on some of the tricks from my bag. Origami design can indeed be pursued in a systematic fashion. There are now simple, codified mathematical and geometric techniques for developing a desired structure.

This book is a collection of those techniques. It is not a step-by-step recipe book for design. Origami is, first and foremost, an art form, an expression of creativity, and it is the nature of creativity that it cannot be taught directly. It can, however, be developed through example and practice. As in other art forms, you can learn techniques that serve as a springboard for creativity.

The techniques of origami design that are described in this book are analogous to a rainbow of colors on an artist's palette. You don't need a broad spectrum, but while one can paint beautiful pictures using only black and white, the introduction of other colors immeasurably broadens the scope of what is possible. And yet, the introduction of color itself does not make a painting more artistic; indeed, quite the opposite can happen. So it is with origami design. The use of sophisticated design techniques—sometimes called "technical folding," or *origami sekkei*—makes the resulting model neither artistic nor unartistic. But having a richer palette of techniques from which to choose can allow the origami artist to more fully express his or her artistic vision. That vision could include elements of the folding sequence: Does it flow naturally? Is the revelation of the finished form predictable or surprising? It could include elements of the finished form: Are the lines harmonious or jarring? Does the use of folded edges contribute to or detract from the appearance? Does the figure use paper efficiently or waste it? The aesthetic criteria to be addressed are chosen by the artist. Any given technique may contribute to some criteria (and perhaps degrade others). By learning a variety of design techniques, the origami artist can pick and choose to apply those techniques that best contribute to the desired effect.

These techniques are not always strict; they are sometimes more than suggestions, but less than commandments. In some cases, they are vague rules of thumb: "Beyond eight flaps, it is more efficient to use a middle flap." But they can also be as precise as a mathematical equation. In recent years, origami has attracted the attention of scientists and mathematicians, who have begun mapping the "laws of nature" that underlie origami, and converting words, concepts, and images into mathematical expression. The scientific fields of computer science, number theory, and computational geometry support and illuminate the art of origami; even more, they provide still more powerful techniques for origami design that have resulted in further advances of the art in recent years. Many design rules that on the surface apply to rather mundane aspects of folding, for example, the most efficient arrangements of points in a base, are actually linked to deep mathematical questions. Just a few of the subjects that bear on the process of origami design include the obvious ones of geometry and trigonometry, but also number theory, coding theory, the study of binary numbers, and linear algebra as well. Surprisingly, much of the theory is accessible and requires no more than high school mathematics to understand. I will, on occasion, bring out deeper connections to mathematics where they are relevant and interesting, and I will provide some mathematical derivations of important concepts, but in most cases I will refrain from formal mathematical proofs. My emphasis throughout this work will be upon usable rules rather than mathematical formality.

As with any art, ability comes with practice, whether the art is origami folding or origami design. The budding origami designer develops his or her ability by designing and seeing the result. Design can start simply by modifying an existing fold. Make a change; see the result. The repeated practice builds circuits in the brain linking cause and effect, independent of formal rules. Many of today's origami designers develop their folds by a process they often describe as intuitive. They can't describe how they design: "The idea just comes to me." But one can create pathways for intuition to take hold by starting with small steps of design. The great leap between following a path and making one's own path arises from the development of an understanding of why: Why did the designer do it that way? Why does the first step start with a diagonal fold rather than a square fold? Why do the first creases hit the corners? Why, in another model, do the first creases miss the corners only by a little bit? Why does a group of creases emanate from a spot in the interior of the paper? If you are a beginning designer, you should realize that *no design is sacred*. To learn to design, you must disregard reverence for another's model, and be willing to pull it apart, fold it differently, change it and see the effects of your changes.

Small ideas lead to big ideas; the concepts of design build upon one another. So do the chapters of this book. In each chapter, I introduce a few design principles and their associated terms. Subsequent chapters build on the ideas of earlier chapters. Along the way you will see some of my own designs, each chosen to illustrate the principles introduced in the chapter in which it appears.

Chapter 2 introduces the fundamental building blocks of origami: the basic folds. If you have folded origami before, you may already be familiar with the symbols, terms, and basic steps, but if not, it is essential that you read through this section. Chapter 2 also introduces a key concept: the relationship between the crease pattern and the folded form, a relationship that we will use and cultivate throughout the book.

Chapter 3 initiates our foray into design by examining a few designs. The first stage of origami design is modification of an existing design; in this chapter, you will have an opportunity to explore this approach by devising simple modifications to a few figures.

Chapter 4 introduces the concept of a base, a fundamental form from which many different designs may be folded. You will learn the traditional bases of origami, a number of variations on these bases, and several methods of modifying the traditional bases to alter their proportions.

Chapter 5 expands upon the idea of modifying a base by focusing upon modifications that turn a single point into two, three, or more simply by folding. This technique, called point-splitting, has obvious tactical value in designing, but it also serves as an introduction to the concept of modifying portions of a base while leaving others unchanged.

Chapter 6 introduces the concept of grafting: modifying a crease pattern as if you had spliced additional paper into it for the purpose of adding structural elements to an existing form. Grafting is the simplest incarnation of a broader idea, that the crease patterns for origami bases are composed of separable parts.

Chapter 7 then expands upon the idea of grafting and shows how multiple intersecting grafts can be used to create patterns and textures within a figure—scales, plates, and other textures. This set of techniques stands somewhat independently, as almost any figure can be "texturized."

Chapter 8 generalizes the concept of grafting to a set of techniques called tiling: figuratively cutting up and reassembling different pieces of crease patterns to make new bases. This chapter defines both tiles and matching rules that apply to the edges of tiles to insure that the assemblies of tiles can be folded into a flat shape. Chapter 8 also introduces the powerful concept of a uniaxial base—a family of structures that encompasses both the traditional origami bases and many of the most complex modern bases.

Chapter 9 shows how the tile decorations that enforce matching can be expanded into a design technique in their own right: the circle/river method, in which the solution of an origami base can be derived from packing circles into a square box. Circle/river packing is one of the most powerful design techniques around, capable of constructing figures with arbitrary configurations of flaps, and yet it can be employed using nothing more than a pencil and paper.

Chapter 10 explores more deeply the crease patterns within tiles; those that fit within circle/river designs are called molecules. The chapter presents the most common molecules, which are sufficient to construct full crease patterns for any uniaxial origami base.

Chapter 11 presents a different formulation of the circle/river packing solution for origami design, called tree theory, in which the design of the base is related to an underlying stick figure, and the packing problem is related to a set of conditions applying to paths along the stick figure. Although equivalent to circle/river packing, the approach shown here is readily amenable to computer solution. It is the most mathematical chapter, but is in many ways the culmination of the ideas presented in the earlier chapters for designing uniaxial bases.

Chapter 12 then introduces a particular style of origami called box pleating, which has been used for some of the most complex designs ever constructed. Box pleating in some ways goes beyond uniaxial bases; in particular, it can be used to construct fully three-dimensional figures by various combinations of box-like forms, pleats, flaps, and more.

Chapter 13 expands upon the flap concept of box pleating to introduce a new concept in design, called polygon packing, and a particular type of polygon packing, uniaxial box pleating, that ties together the concepts of box pleating and tree theory.

Chapter 14 continues the development of polygon packing and uniaxial box pleating, introducing the new design technique of hex pleating and methods of generalizing polygon packing further to arbitrary angles.

Chapter 15 continues to move beyond uniaxial bases, introducing the idea of hybrid bases, which combine elements from uniaxial bases with other non-uniaxial structures. The world of origami designs is enormously larger than the uniaxial bases that are the focus of this book, but as this chapter shows, elements from uniaxial bases can be combined with other structures, expanded, and extended, to yield ever-greater variety in origami figures.

The References section provides references and commentary organized by chapter with citations for material from both the mathematical and origami literature related to the concepts in each chapter.

Each chapter includes step-by-step folding instructions for one or more of my origami designs chosen to illustrate the design concepts presented in the chapter. I encourage you to fold them as you work your way through the book. Most have not been previously published. I have also, in several chapters, presented crease patterns and bases of models whose instructions have been published elsewhere; for many of them, you will find sources for their full folding sequences in the References section, though for some, the discovery of how to collapse the crease pattern into the base will be left as an exercise for the reader.

The concepts presented here are by and large my own discoveries, developed over some 40-plus years of folding. They were not developed in isolation, however. Throughout the book I have pointed out sources of influence and/or ideas I have adopted. In several cases others have come up with similar ideas independently (an event not without precedent in both origami and the sciences). Where I am aware of independent invention by others, I have attempted to identify it as such. However, the formal theory of origami design is very much in its infancy. Sources of design techniques are often unpublished and/or widely scattered in sometimes obscure sources. This work is not intended to be a comprehensive survey of origami design, and if it seems that I have left out something or someone, no slight was intended.

Technical folding, *origami sekkei*, is an edifice of concepts, with foundations, substructure, and structure. Because the organization of this book mirrors this structure, I encourage you to read the book sequentially. Each chapter provides the foundation to build concepts in the next. Let's start building.

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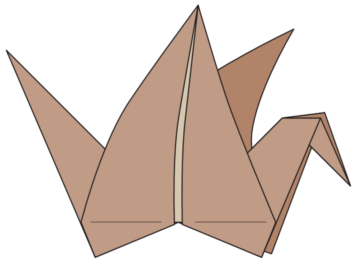


2 Building Blocks

Much of the charm of origami lies in its simplicity: There is the square, there are the folds. There are, it would appear, only two types of folds: mountain folds (which form a ridge) and valley folds (which form a trough). So, square + mountain folds + valley folds is the recipe for nearly all of origami. How simple can you get?

But is it true that there are two types of fold? Maybe there's only one; the mountain fold can be turned into a valley fold merely by turning the paper over.

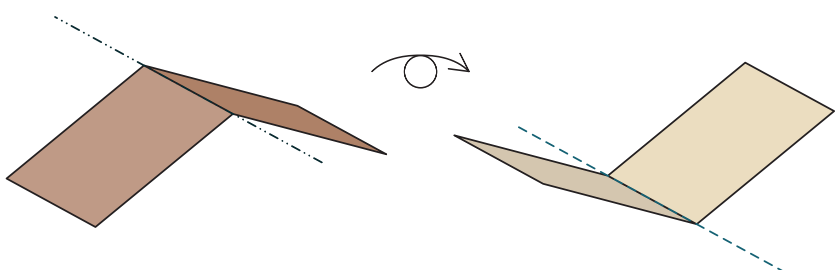


Figure 2.1.
A mountain fold is the same as a valley fold turned over.

On the other hand, perhaps there are *three* types of fold: valley folds, mountain folds, and unfolds. If we fold the paper in half and unfold it, we will be left with a line on the paper—a crease—which is also a type of fold. Creases are sometimes merely artifacts, leftover marks from the early stages of folding, but they can also be useful tools. Creases can provide reference points ("fold this point to that crease") and in the purest style of folding (no measuring devices, such as rulers, allowed) creases, folded edges, and their intersections are the only things that can serve as reference points. Creases are also commonly made in preparation for a complex maneuver.

Origami diagrammers attempt to break folding instructions into a sequence of simple steps, but some maneuvers are inherently complex and require bringing 5 or 6 (or 10 or 20) folds together at once. For such pleasant challenges, it's a big help to have all the creases already in place. Precreasing helps tame the dragon.

Valley, mountain, and crease are the three types of folds from which all origami springs. But even a valley fold is not necessarily the same as another valley fold if the layers of paper do not lie flat. When models move into three dimensions, both valley and mountain folds can vary in another way: the fold angle, which can take on many values. Imagine drawing a straight line across and perpendicular to the fold. The fold angle is the angular change in the direction of this line from one side of the fold to the other. This angle can vary continuously, from 180° (for a valley fold) to 0° (which is no fold at all) to –180° (for a mountain fold). By this measure, valley, mountain, and crease are all part of a continuum of fold angle.

There is yet more variation: A fold can be sharp or soft. The mathematical model of a "fold" is an infinitely sharp line, but with real paper, the sharpness of the fold is something the folding artist can choose. Sharp creases are not always desir-

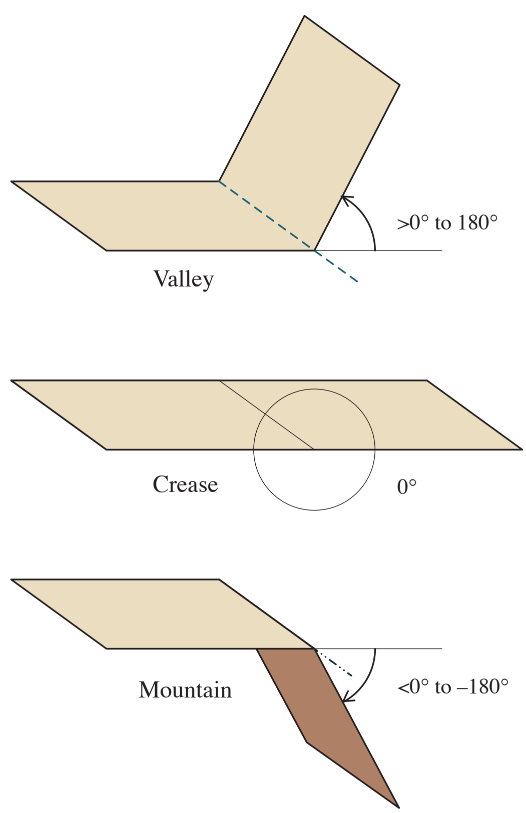


Figure 2.2.
Valley folds, creases, and mountain folds are all part of a continuum.

able. In a complex model with many folds, sharp creases can weaken the paper to the point that the paper rips. In a model of a natural subject, sharp lines can be harsh and unlifelike, whereas soft, rounded folds can convey an organic quality, a sense of life. On the other hand, when precision is called for, sharp folding may be required to avoid a crumpled mess down the road. Consequently, most models call for a mix of sharp and soft folding, and while the distinction can sometimes be given in the folding diagrams, in most cases, the artist must simply develop through experience a feel for how sharp a given crease must be.

2.1. Symbols and Terms

Origami instruction is conveyed through diagrams—a system of lines, arrows, and terms that become the *lingua franca* (or perhaps *lingua japonica*) of the worldwide arena. The modern system of origami diagrams was first devised by the great Japanese master Akira Yoshizawa in his books of the 1940s and 1950s, and was subsequently adopted (with minor variations) by the two early Western origami authors Samuel L. Randlett (United States) and Robert Harbin (U. K.). Despite occasional attempts by others at establishing a rival notation (e.g., Isao Honda, who used dashed lines everywhere, but distinguished mountain folds by a "P" next to the line), the Yoshizawa/Randlett/Harbin system caught on and has become the sole international system in the origami world.

No system is perfect, and over the years, various diagrammers have made their own additions to the system. Some, like open and closed arrows (to denote open and closed sink folds), died a quiet death; others, like Montroll's "unfold" arrow, have become firmly established in the origami diagrammatic lexicon (symbolicon?). Every author has his or her particular quirks of diagramming, but the core symbols and terms are nearly universal.

Odds are that you already have some familiarity with origami and have encountered the Yoshizawa diagramming system. It will, however, serve us to run through the basic symbols and terms, both to establish a fixed starting point and to start the wheels turning for origami design, which is as much a way of looking at origami as it is a set of codified tools.

The first thing to run through are origami terms, which include names, directions, and positions. Origami diagrams are ideally drawn so that the diagrams themselves are sufficient to enable the reader to fold the model (which allows people the world over to fold from them; a Japanese or Russian folder can fold from English diagrams and vice-versa). Nevertheless, many people find folding instructions more readily comprehensible with a verbal instruction attached, and so in the instructions in this book, you will find both words and pictures.

Origami verbal instructions are given as if the paper were flat on the page before you. Thus, words that say "fold the flap upward" mean that if you orient the working model the same way as the diagram on the page, you will fold the flap toward the top of the page. "Up," "down," and "to the side" all refer to directions with respect to the printed page. While directions are always given as if the paper were flat on the page, you may find it easier to pick the model up, fold in midair, or even turn it over to make the fold (mountain folds are commonly made by turning the paper over and forming a valley fold). If you do this, be sure that you always return it to the orientation shown in the next diagram.

As the folded model begins to accumulate multiple layers of paper, it becomes necessary to distinguish among the layers. By convention, the term "near" refers to the layers closest to you (i.e., those on top) and "far" layers are those on the bottom (thus, reserving the words "top" and "bottom" for directions with respect to the page).

Origami paper typically has a white side and a colored side. The two colors are featured in some models—there are origami skunks, pandas, and even zebras and chessboards whose coloration derives from skillful usage of the two sides of the paper. Even if only one side is visible at the end, it is helpful in keeping track of what's going on to show the two sides as distinct colors, and that is what I have done here.

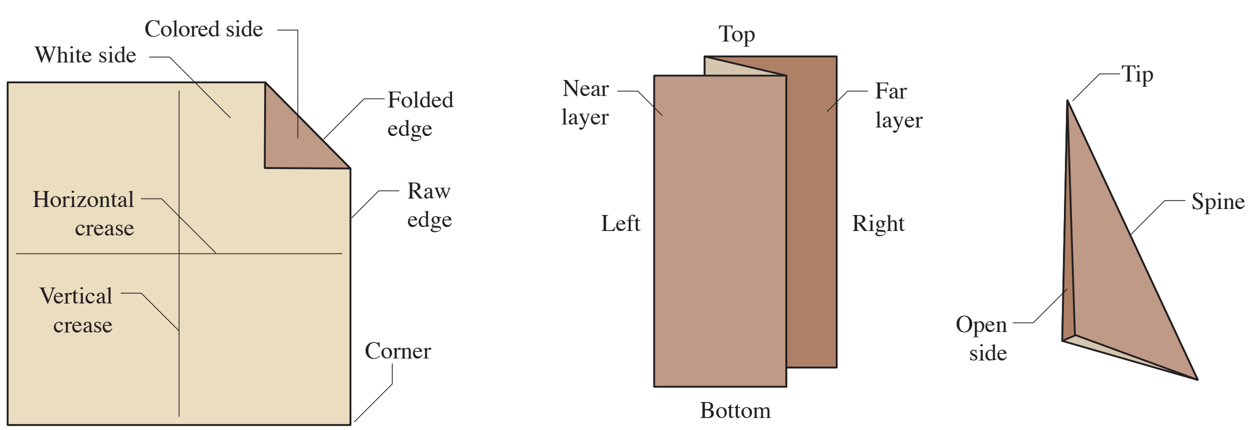


Figure 2.3.
Verbal terms that apply to origami diagrams.

Brightly colored origami paper often comes precut to squares. One of the small ironies of the art is that when precut square origami paper was introduced in Japan near the turn of the 20th century, it was made from inexpensive European machine-made paper, since handmade Japanese *washi* was far too expensive for most purposes. Thus, the origami paper that is considered the most authentically Japanese wasn't even originally from Japan!

For your own folding, there is no special requirement on paper other than it hold a crease and not easily rip. Traditional origami paper—available from most art and craft stores, via the Internet, and at many stores in the Japanese quarter of large cities—is relatively inexpensive and conveniently precut to squares. (It may not be precisely square, however. Like most machine-made papers, prepackaged origami paper has a definite grain and will change proportion slightly with humidity; a square in Florida will probably be a rectangle in Nevada.) Other papers that are useful are thin artist's foil (also available from art stores), foil wrapping papers, and various thin art papers you may run across with names like *unryu*, *kozo*, and *lokta*.

Origami diagrams are usually line drawings. Even in this day of three-dimensional computer rendering, line drawings convey the information of folding as well as anything. There are five types of lines that are used for different features of the folded shape. Paper edges, either raw (an original edge of the paper) or folded, are indicated by a solid line. Creases are indicated by a thinner line, and will often stop before they reach the edge of the paper. Valley folds are indicated by a dashed line; mountain folds by a chain (dot-dot-dash) line. The "X-ray line," a dotted line, is used to indicate anything hidden behind other layers, and could be used to represent a hidden edge (most often), fold, or arrow. It will usually be clear from context what the X-ray line is meant to represent.

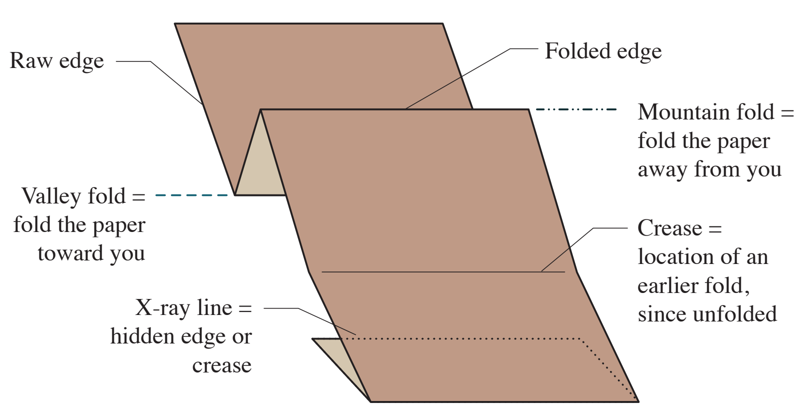
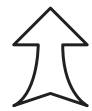


Figure 2.4.
The five types of lines used in sequential origami diagrams.

Actions are indicated by arrows that show the motion of the paper as a fold is made and sometimes show manipulations of the entire model. An open hollow arrow is used to show the application of pressure (usually in connection with a reverse or sink fold). See Figures 2.21–2.23 and 2.40–2.47 for examples.

Figure 2.5.
A hollow arrow indicates to "push here."

Push here.



An arrow that incorporates a loop indicates to turn the paper over—either from side to side (like turning the pages of a book) or from top to bottom (like flipping forward or backward on a wall calendar), with the direction specified by the orientation of the arrow.

Turn the paper over from side to side.

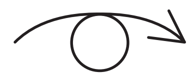
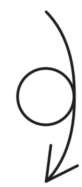


Figure 2.6.
A looped arrow indicates to turn the paper over.

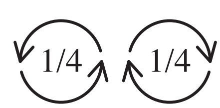
Turn the paper over from top to bottom.



If the model is to be rotated in the plane of the page, that is indicated by a fraction enclosed in two arrows showing the direction of rotation. The number inside the arrows is the fraction of a circle through which the rotation takes place. "1/2" is a half turn, i.e., the top becomes the bottom and vice-versa; "1/4" indicates a quarter-turn. Sometimes the amount of rotation is not a simple fraction; rather than putting something unwieldy like "21/34" in the arrows, I'll usually round it to the nearest quarter-turn and you can use the subsequent diagram to pin down the orientation precisely.

Figure 2.7.
A fraction inside a circle formed from two arrows indicates to rotate the paper.

Rotate the paper.



Most origami is folded flat at every step. When a model becomes three-dimensional, however, either because the final model is 3-D or one or more intermediate steps are 3-D, it frequently becomes necessary to show multiple views of the model to fully convey what is going on. In such cases, a small stylized eye indicates the vantage point from which a subsequent view is taken.

View from this vantage point. 

Figure 2.8.
An eye with a dotted line indicates the sightline used to specify a new point of view.

The next symbol indicates one of the most dreaded instructions in all of origami: repetition. You have worked through a long, tortuous sequence of folds, you think you're coming to the end, and there it is: "repeat steps 120–846 on the other 7 flaps." The bad news is usually conveyed in words, but for those who fold from the diagrams alone, repetition is conveyed by a symbol as well. Harbin, the great Western popularizer of origami, devised an arrow with hash marks to indicate repetition; however, this symbol is unnecessarily ambiguous, and I have preferred to use a boxed leader enclosing the range of steps to be repeated, as shown in Figure 2.9.

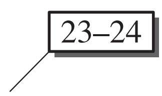
Repeat a range of steps. 

Figure 2.9.
A range of steps to be repeated is indicated by a boxed sequence of the numbered steps to be repeated.

Lastly, it frequently arises that a fold is to be made at 90° to another fold or to a folded edge. When this takes place and it is not obvious that the fold is at 90°, I will indicate it by a small right-angle symbol next to (and aligned with) the relevant intersection.

Right angle 

Figure 2.10.
A right angle is indicated by the geometer's symbol of a right angle located next to the relevant corner.

2.2. Basic Folding Steps

Now we turn to the basic folds of origami—single folds, or combinations of a few folds that occur over and over in origami figures. Most of these combinations date back hundreds of years in Spain and Japan as concepts, if not as recognized steps. These are, however, the building blocks from which nearly all origami models arise. The names are of much more recent vintage and vary from country to country, but in English-speaking countries, the names given here are widely accepted.

The first basic fold is the generic valley fold—a fold made with a single straight line, with the fold made concave toward the folder. The fold itself is indicated by a dashed line, which divides the paper into two parts, one stationary (usually), one moving. A symmetric double-headed arrow is used to indicate which part moves and the direction of motion. The moving part almost always must rotate up and out of the plane of the page; this motion is conveyed by curving the arrow.

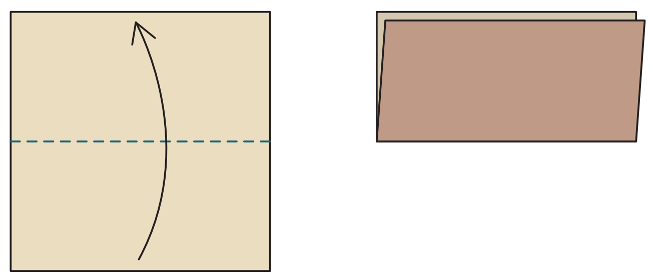


Figure 2.11.
A valley fold, as diagrammed, and the result.

The opposite of a valley fold is a mountain fold, which is called for when a portion of the paper is to be folded behind. The mountain fold is indicated by a chain line (dot-dot-dash), and the motion of the paper is indicated by a hollow single-sided arrowhead.

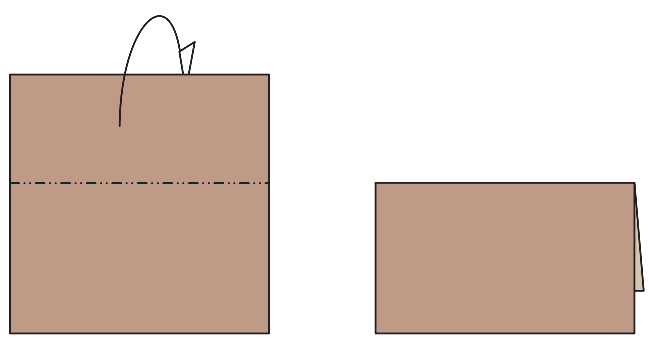


Figure 2.12.
A mountain fold, as diagrammed, and the result.

Quite often, a mountain fold is shown as a bit of shorthand for "turn the paper over, make a valley fold, and then turn it back to the original orientation," as in the example in Figure 2.12. However, mountain folds are frequently used to tuck paper into a pocket or between layers, situations where turning the paper over will not necessarily make a valley fold possible.

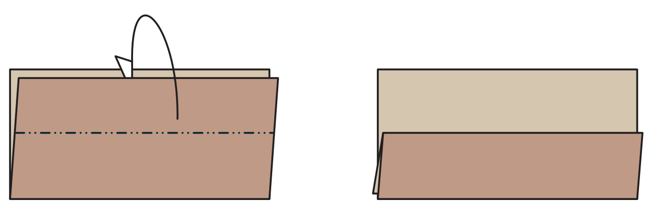


Figure 2.13.
A mountain fold is not always amenable to "turn the paper over and make a valley fold."

When a mountain fold (or, less often, a valley fold) is used to tuck one layer between two others, the layers will be separated as in Figure 2.13, and the arrow will be drawn between the two layers. If, when folding, you find that a flap can be folded into more than one location, examine the drawing closely, as the arrow will likely show where the layer should go.

Quite often, both a mountain fold and a valley fold will be called for on parallel layers, a maneuver that is commonly used for thinning legs and other appendages. This step is shown with two arrows and, if possible, both the mountain and valley fold. You may perform both a mountain and a valley fold if you wish, but many folders actually form both folds as mountain folds, making one from each side of the paper.

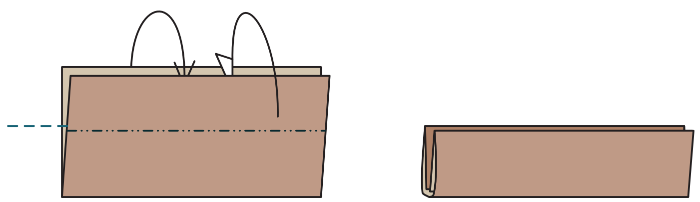


Figure 2.14.
Mountain and valley folds used to thin a flap.

Figure 2.14 illustrates several common subtleties of origami diagrams. The valley fold on the far layer is made clear by extending the fold line (the dashed line) beyond the edge of the paper. The valley fold is understood to run completely along the far layer of paper, even though it is not shown. (I could use an X-ray line to indicate the extension of the valley fold, but I don't in this figure because it would get mixed up with the overlaid mountain fold line). Both the mountain and valley fold layers get tucked into the middle of the model, which you can tell by observing that both arrowheads travel between the two layers. The resultant figure—the drawing to the right—shows the disposition of the layers along its edge, which makes this example unambiguous. It is often not possible to show such layers, however; you must rely upon the arrows between the layers, as in the figure on the left.

Folds, once made, do not always persist to the end of the model. It is a fairly frequent occurrence that folds are made to establish reference points or lines for future folds, or that a model is unfolded at some point to perform some manipulation upon hidden or interior lines. In either case, folds get unfolded. Unfolding is indicated by a symmetric hollow-headed arrow, as shown in Figure 2.15.

The same symbol is used to indicate when paper is to be pulled out from an interior pocket, as shown in Figure 2.16.

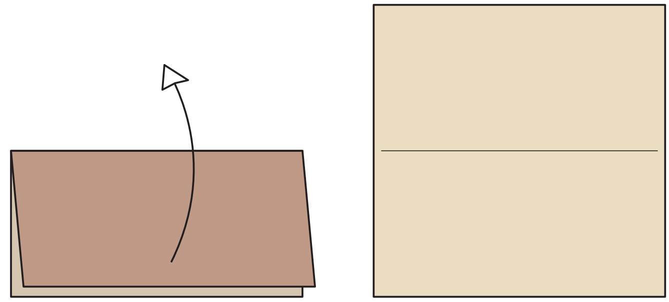


Figure 2.15.
The unfold arrow.

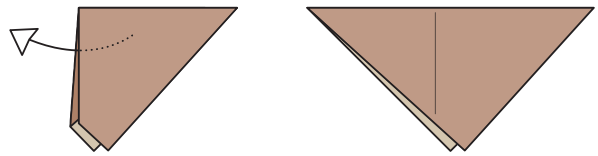


Figure 2.16.
The unfold arrow used to show pulling paper out from inside the model.

Particularly in the early stages of folding a model, one will make a fold and then immediately unfold it, for the purpose of establishing a crease that will be used in some future (usually more complicated) step. To keep the diagrams fairly compact, the fold-and-unfold action is commonly expressed in a single figure, and is indicated by a single double-headed arrow that combines the fold arrow (valley fold) and unfold arrow in a single arrow.

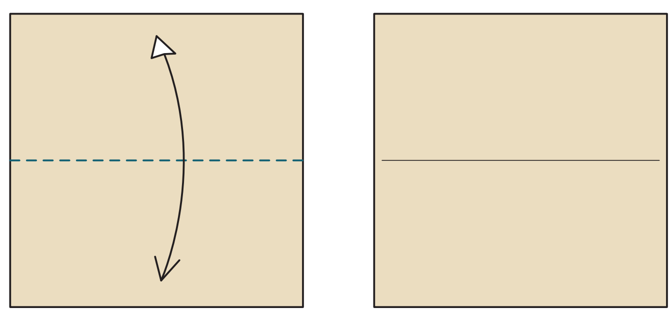


Figure 2.17.
Fold-and-unfold is indicated by a double-headed arrow that combines the "valley fold" and "unfold" arrowheads.

Most of the time, the fold in a fold-and-unfold step will be a valley fold, but on occasion, the desired crease is a mountain fold. Rather than diagramming this in three steps (turn the paper over, valley-fold-and-unfold, turn the paper back over), I will use the mountain fold arrow in combination with the unfold arrow, as shown in Figure 2.18. It should be understood that what is intended is to fold the moving flap behind, make the crease, and then unfold.

In the study of origami design, the crease pattern of the finished figure or a subset of same provides a great deal of information about the structure of the model—often more in-

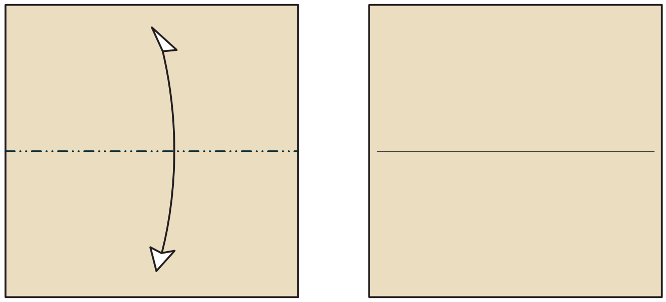


Figure 2.18.
Mountain-fold and unfold is indicated by a double-headed arrow that combines the "mountain fold" and "unfold" arrowheads.

formation than the sequence of folding instructions, because it shows the entire model (or folded form) at once. The simplest form of the crease pattern simply shows all creases as crease lines, as in Figure 2.19, which shows the crease pattern for the traditional Japanese flapping bird.

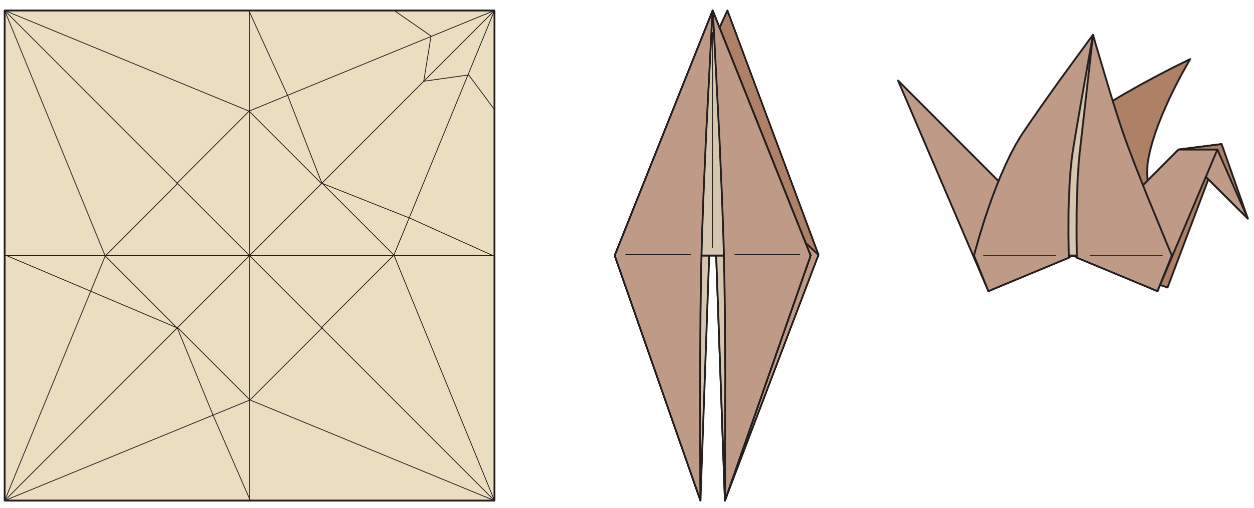


Figure 2.19.
Crease pattern, base, and folded model of the traditional Japanese flapping bird.

Knowing just the location of the creases, however, is not as useful as it could be; it is far more useful to know the directions of the creases, i.e., whether they are valley or mountain folds.

("More useful" is a bit of an understatement. In 1996, Marshall Bern and Barry Hayes proved that figuring out crease directions from a generic crease pattern is computationally part of a class of problems known as "NP-COMPLETE." As such problems grow in size, they quickly outstrip the abilities of any computer to solve.)

Thus, it is more helpful to give the direction—or crease assignment—of the creases: mountain, valley, or crease (that is, not folded at all). The traditional mountain and valley lines—chain and dashed—tend to lose their distinction in large crease patterns, dissolving into a morass of confusing clutter. Thus, in crease patterns, I will adopt a different convention that provides greater contrast. Creases that are valley fold lines will be indicated by dashed colored lines, while mountain folds will be solid black lines. Creases that lie flat will be indicated by thin gray lines. Flat creases that don't play an important role are not shown at all, but it is sometimes helpful to show creases that were important to the construction of the base. To see the difference between the two line styles, compare the two examples in Figure 2.20.

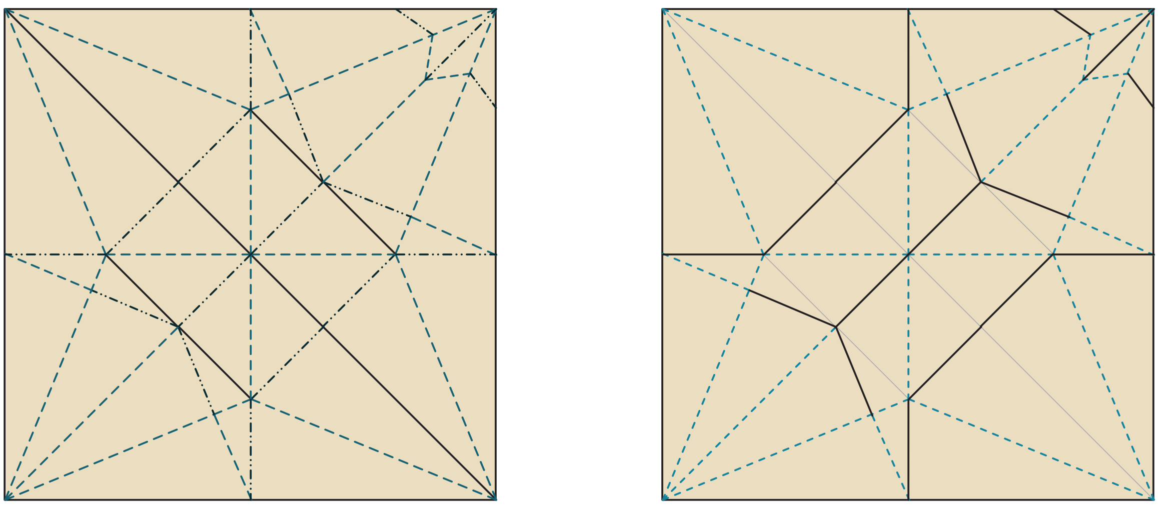


Figure 2.20.
Left: a crease pattern using the traditional patterned lines to indicate mountain and valley folds.
Right: the same crease pattern using contrasting lines specialized for crease patterns.

The use of dashed lines for valley folds and chain lines for mountain folds has been firmly established as a worldwide standard in origami for decades. The precise line styles used for crease patterns are less standardized. In general, because mountain folds are more visible in the unfolded paper, I choose solid, darker lines for them; valley folds are less visible, so they get lighter colors and their traditional dashing pattern, but a somewhat finer dashing so that the dashes themselves do not distract from the large-scale patterns of mountain and valley lines.

A crease pattern that has its mountain and valley folds distinctly labeled is said to be *assigned*, or *crease-assigned*. If we draw all of the fold lines with no distinction, then it is said to be *unassigned*. Not surprisingly, it can be far harder to fold an origami figure from an unassigned crease pattern than from an assigned one. The process of assigning creases can be thought of as labeling each of the fold lines with further information: namely, its fold direction. That's the first step in a much richer potential for labeling, and as we will eventually see, we can label creases with far more information, and far more significant information, than their mere mountain/valley status.

While all origami models are created entirely from mountain and valley folds, they often occur in distinct combinations, combinations that occur often enough that they have been given names of their own.

The first and simplest combination fold is the inside reverse fold, which is a fold used to change the direction of a flap. While either a mountain or valley fold could usually be used in the same place, a reverse fold combines both mountain and valley and is usually more permanent, since the tension of the paper tends to keep the reverse fold together. A reverse fold always takes place on a flap consisting of at least two layers of paper. In an inside reverse fold, the mountain fold line occurs on the near layer, a valley fold occurs on the far layer, and the "spine" above the fold lines is turned inside-out. It is indicated by a push arrow, since to form the reverse fold, the spine must be pushed and turned inside-out. If the far edges are visible, then the valley fold may be shown extending from the visible edge, as in Figure 2.21.

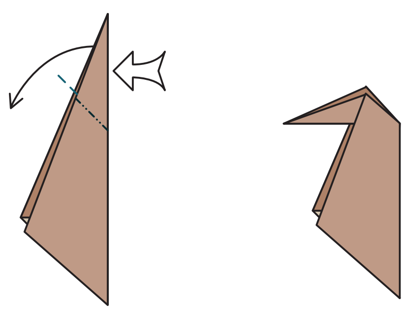


Figure 2.21.
The inside reverse fold.

In the inside reverse fold, the tip of the flap ends up pointing away from the spine; in Figure 2.21, the spine is the right side of the flap, so the tip must point to the left. If you wanted it to point to the right, then you would use the other type of reverse fold, the outside reverse fold, which is illustrated in Figure 2.22. Again, there is a mountain fold and a valley fold, but in the outside reverse fold, the valley fold occurs on the near layers and the mountain fold on the

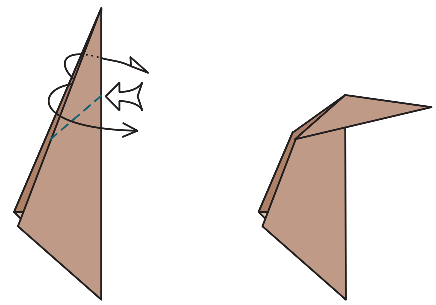


Figure 2.22.
The outside reverse fold.

far layers, opposite from what happens in the inside reverse fold. The outside reverse fold is also indicated by a push arrow, because it is typically made by pushing at the spine with one's thumb while wrapping the edges of the paper around to the right. Like the inside reverse fold, it is much more permanent than a simple mountain or valley fold would be.

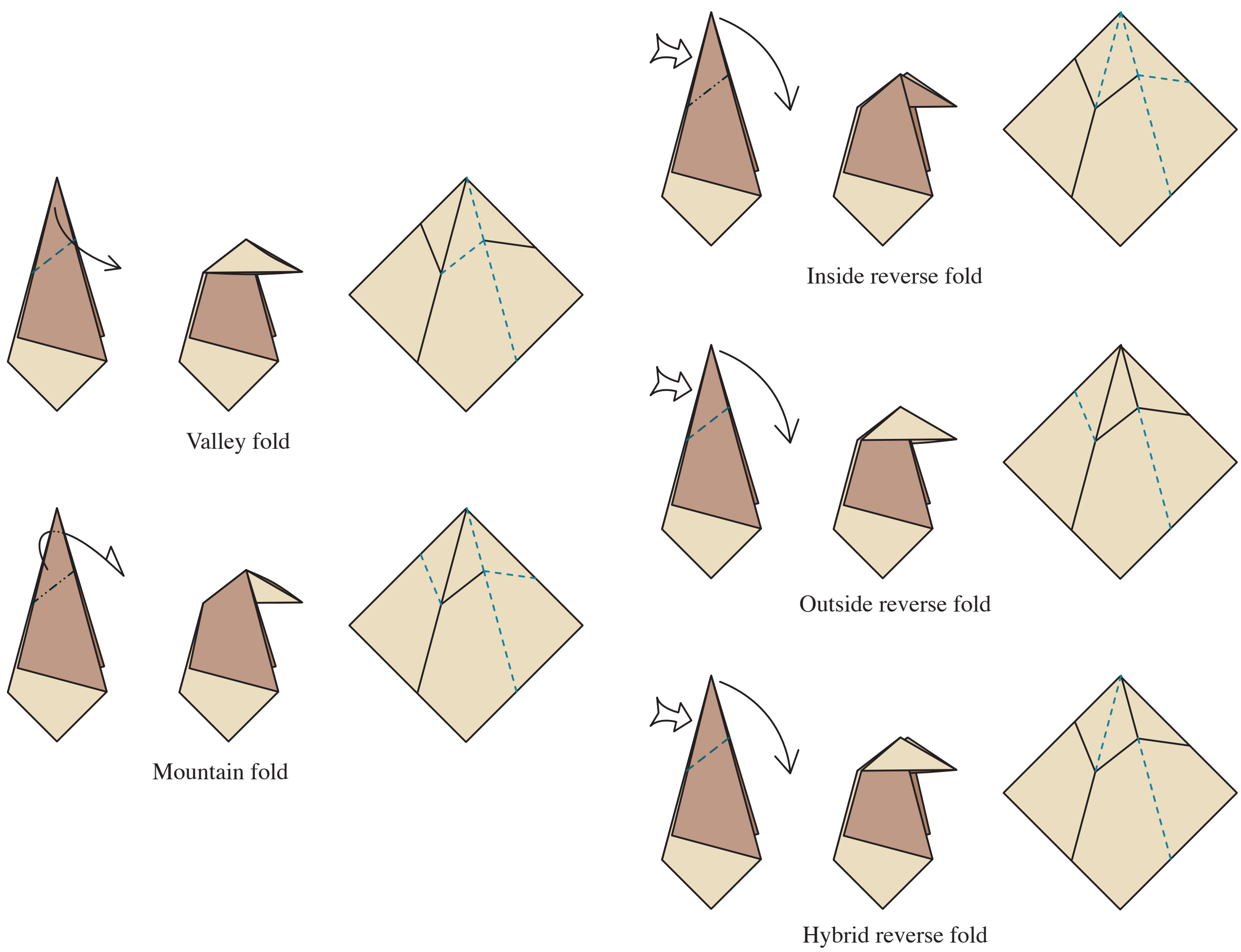


Figure 2.23.
The five possible ways to turn the tip of a three-layer flap.

In the verbal instructions, the term “reverse fold” (without an “inside” or “outside” qualifier) will generally mean “inside reverse fold.”

A simple flap with two layers has only two possible types of reverse fold: inside or outside. More complicated flaps with multiple layers can have multiple possibilities or even combinations of the two; for example, the triangular shape shown in Figure 2.23 (made by folding a square in thirds at one corner) can be either inside- or outside-reverse-folded to either the left or right; in addition, it is possible to make a sort of hybrid reverse fold that combines aspects of both. The silhouettes of all three shapes (and for that matter, the mountain- or valley-folded equivalents) are the same; they differ only in their crease patterns. In diagrams throughout the book, they will be distinguished by the presence or absence of push arrows (distinguishing reverse folds from mountain or valley folds) and/or the configuration of edges shown in subsequent diagrams.

Another combination fold that occurs with some regularity is the rabbit-ear fold (which acquired its name from some rabbit design long since lost in the mists of antiquity). The rabbit-ear fold is almost always performed on a triangular flap, and is characterized by three valley folds along the angle bisectors of the triangle, with a fourth fold, a mountain fold, extending from the point of intersection perpendicularly to one side.

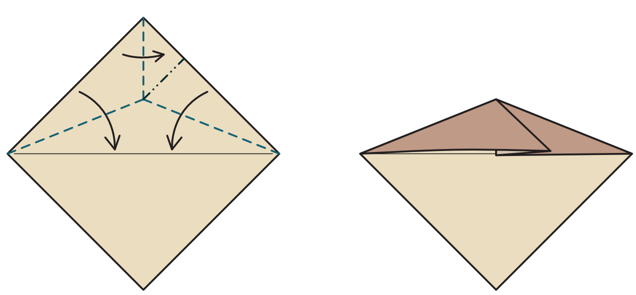


Figure 2.24.
The rabbit-ear fold.

When a rabbit-ear fold is formed, all of the edges lie on a common line. Remarkably, this procedure works for a triangle of any shape—or perhaps it is not so remarkable, since the rabbit ear is merely a demonstration of Euclid’s theorem that the angle bisectors of any triangle meet at a common point.

Rabbit-ear folds occur not only on isolated flaps. Bringing all the edges to lie on a common line is a special property; the rabbit ear is the simplest example of a molecule, which is the name for any crease pattern with this property. We will encounter rabbit-ear crease patterns and molecules in much

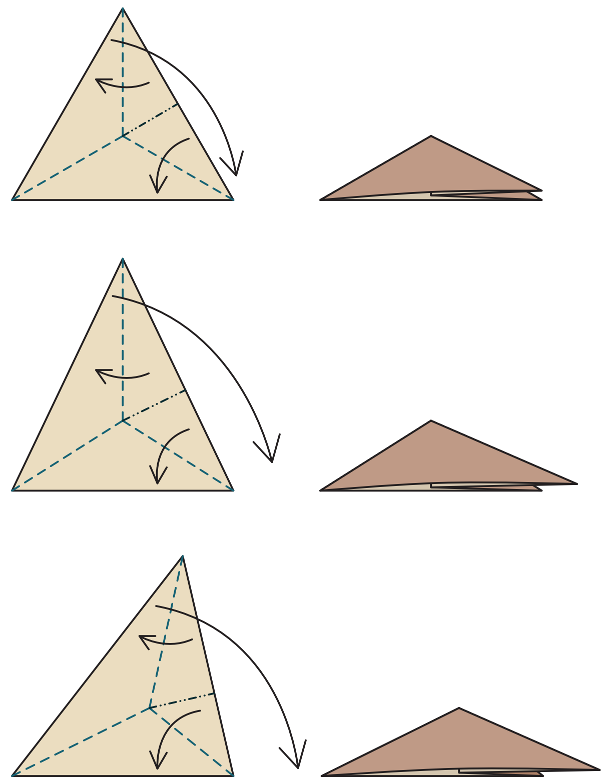


Figure 2.25.
The rabbit ear can be folded from any triangle.
Top: equilateral.
Middle: isosceles.
Bottom: scalene.

detail and many guises as we delve more deeply into systematic design.

In addition to the simple, straightforward rabbit ear, there are two variations that are regularly encountered. Figure 2.26 shows a variation in which the edges do not lie on a common line.

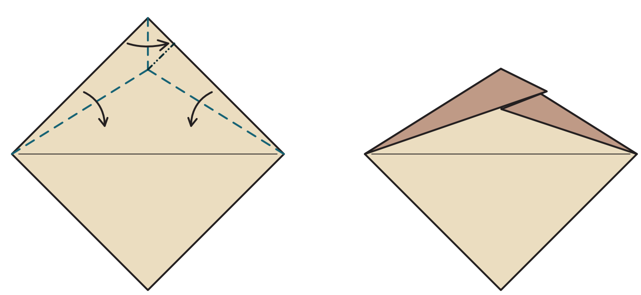


Figure 2.26.
A variation of a rabbit ear.

Figure 2.27 shows a combination of two rabbit ears made from the near and far layers of a two-layered flap. Known, appropriately, as a double rabbit ear, it is typically formed by pinching the near and far layers of the flap into rabbit ears and then swinging the tip over to the side.

Just as the reverse fold is a combination of a valley fold with its mirror image on another layer of a flap, the double rab-

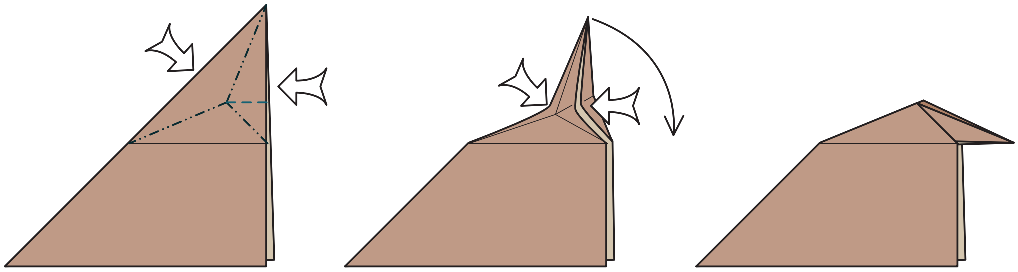


Figure 2.27.
A double rabbit ear.

bit ear is a combination of a rabbit ear with its mirror image also on another layer.

The next combination fold commonly encountered is the squash fold. In a squash fold, the layers of a flap are spread to the sides and the folded edge flattened.

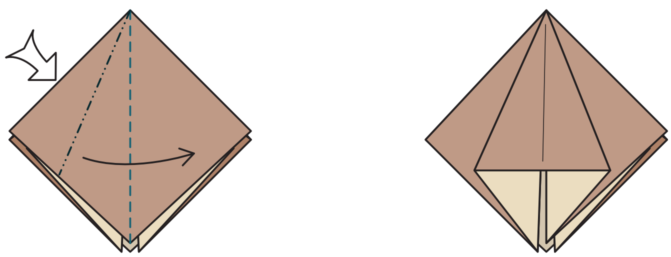


Figure 2.28.
The squash fold.

The squash fold is quite easy to perform (and sometimes very satisfying). It is nearly always formed symmetrically, that is, making equal angles on both the left and right. In the symmetric form, the crease that used to be the folded edge will be lined up with one or more raw edges underneath, as in Figure 2.28. It is also possible to squash-fold a point, as shown in Figure 2.29. Squash-folded points are harder to keep symmetric, because the point covers up the layers underneath, but you can make them symmetric by turning the paper over and checking the alignment on the other side before you make the creases sharp.

There are four creases involved in a squash fold: two valleys on each side of two mountains (usually, only one of each

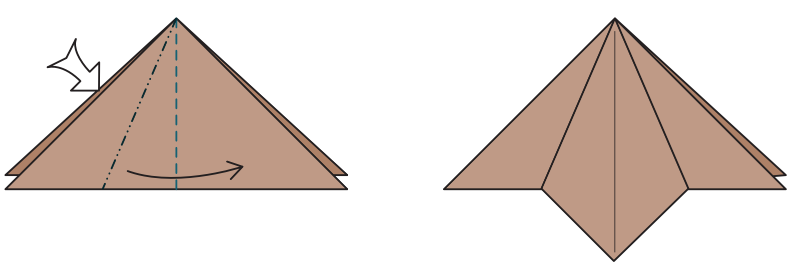


Figure 2.29.
Another version of a squash fold.

is visible on the near side of the flap). All four creases come together at a point. Most of the time, the two valley folds are side-by-side and the squash fold is symmetric about the valley fold. However, a squash fold can be made asymmetrically and it sometimes happens that the two valley folds are not side-by-side. When that happens, a portion of the visible flap can be seen to rotate (about the intersection of all the creases). This asymmetric version of a squash fold occurs often enough that it is given its own name: a swivel fold.

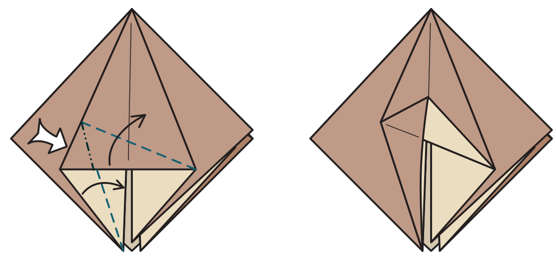


Figure 2.30.
A swivel fold.

We have seen that mountain, valley, and rabbit-ear folds have doubled forms where they are combined with their mirror images. Are there similarly doubled squash or swivel folds? The answer, surprisingly, is yes, and the combination is as difficult as the squash fold is easy. The combination of two swivel folds is called a petal fold (it is commonly used in origami flowers). However, instead of being formed on near and far layers (as in the reverse folds and double rabbit-ear fold), the two mirror-image swivel or squash folds are formed side by side. The petal fold is a very famous fold; it is the key step in the traditional Japanese flapping bird. It is diagrammed as two side-by-side squash folds that share a common valley fold.

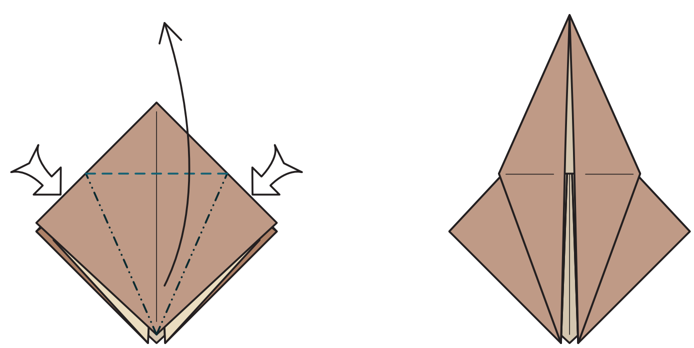
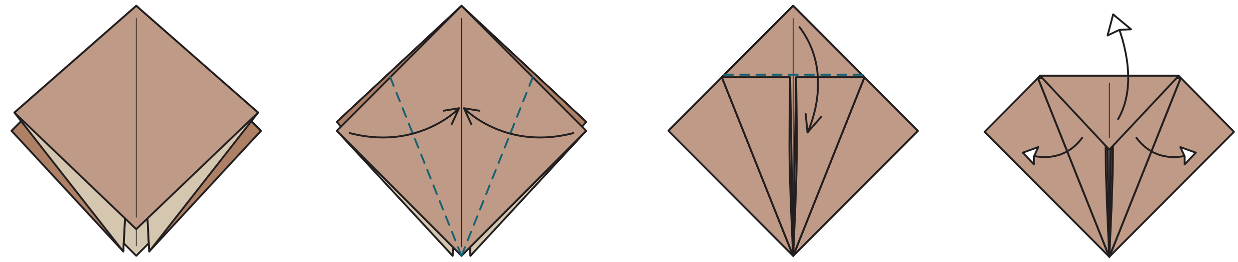


Figure 2.31.
The petal fold.

On the scale of origami difficulty (which runs simple, intermediate, complex, and now, super complex!), the petal fold is only considered an intermediate maneuver, but it is usually

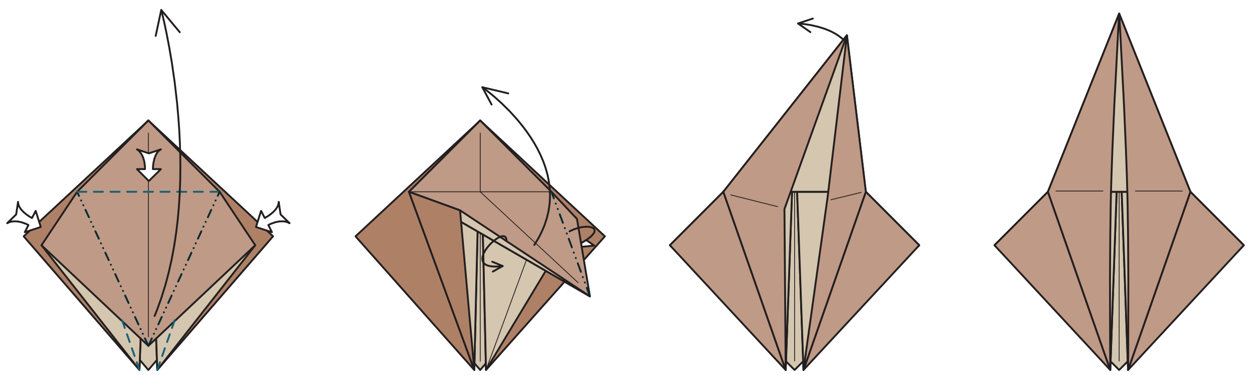


1. The most common petal fold starts with this shape, called the Preliminary Fold.

2. Fold the sides in so that the raw edges lie along the center line.

3. Fold the top point down over the other two flaps.

4. Unfold all three flaps.



5. To make the petal fold, lift up the first layer of the bottom corner while holding down the top of the model just above the horizontal crease. Allow the sides to swing in.

6. Continue lifting up the point; reverse the direction of the two creases running to its tip, changing valley folds to mountain folds.

7. Continue lifting the point all the way; then flatten.

8. Finished petal fold.

Figure 2.32.
The sequence to make a petal fold.

quite challenging for an origami novice to perform, and so is commonly broken down into several steps with some precreasing, as shown in Figure 2.32.

When you are a beginning folder, it is helpful to make the precreases as in steps 2 and 3 in Figure 2.32. However, as you become comfortable with folding, it's better to not precrease the sides as in step 2, because it is difficult to make the creases through both layers run precisely through the corners. It is neater to simply form the bisectors in each layer individually.

Petal-folding is usually performed on a flap to make it simultaneously narrow and longer. It is also possible to petal-fold an edge, creating a flap where there was none before, as shown in Figure 2.33.

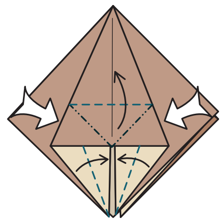
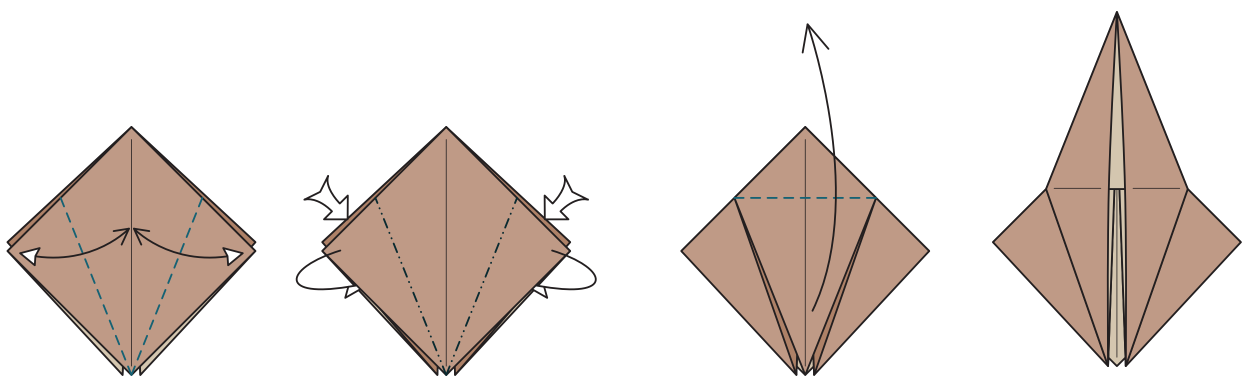


Figure 2.33.
Petal-folding an edge.

Petal folds, squash folds, reverse folds, and rabbit ears are all closely related to each other. It is often possible to reach the same end by more than one means. For example, the petal fold shown in Figure 2.33 can also be realized by making two reverse folds and a valley fold.



1. Fold the sides in to lie along the center line and unfold.

2. Reverse-fold the edges inside using the creases you just made.

3. Lift up the frontmost flap.

4. Finished petal fold.

Figure 2.34.
An alternative way to make a petal fold using reverse folds.

And if you were to cut apart the finished petal fold along the center line (cutting both slightly left and right of the center line to be sure to sever all layers that touch the center line), the petal-folded flap would turn out to be two rabbit ears!

Thus, the various combination folds are not distinct entities so much as convenient ways of getting two or four creases to come together at once. What is important in origami design is the underlying structure, not the specific sequence of steps one takes to get to the finished model (although it must be acknowledged that once the design is fixed, a sequence composed of simple combinations that flows from one to the next is far more aesthetically pleasing than a few precreases followed by, “Make these 150 creases come together at once”).

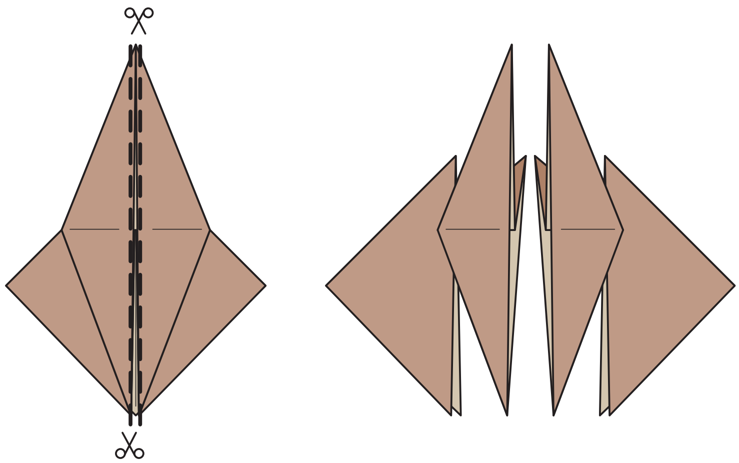


Figure 2.35.
A bisected petal fold reveals that it is composed of two rabbit-ear folds.

Reverse folds are commonly used to change the direction of a flap, for example, to do the final shaping. Another combination fold that is used to shape flaps is the pleat, which consists of side-by-side mountain and valley folds.

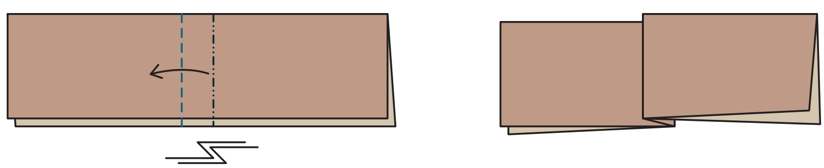


Figure 2.36.
Left: a pleat diagram.
Right: the finished pleat.

A pleat formed through a single layer of paper is unambiguous. However, when there are multiple layers present, there is a closely related fold, illustrated in Figure 2.37, which is called a crimp.

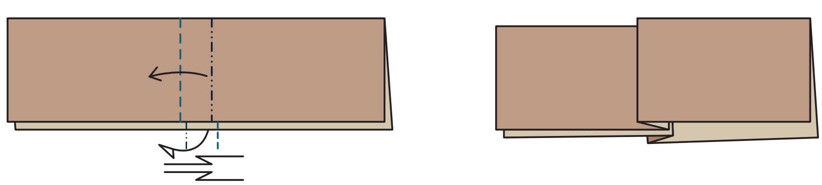


Figure 2.37.
Left: a crimp diagram.
Right: the finished crimp.

The crimp is a combination of a pleat with its mirror image on the far layer of paper. Thus, a crimp bears the same relationship to a pleat that an inside reverse fold bears to a mountain fold (or an outside reverse fold to a valley fold). Just as reverse folds do not come undone as easily as mountain or valley folds, crimps are more permanent than pleats. Both crimps and pleats are diagrammed by showing the fold lines on the near layers of paper; they can be distinguished by examining the edges of the flap. Sometimes it is not practical to show the edges, so a small set of zigzag lines is drawn next to the edge (as in Figures 2.36 and 2.37), which represents an edge-on view of the finished crimp or pleat.

The two folds of a pleat or crimp are often parallel, but they need not be. If they are not parallel, then the flap will change direction, with the net change of direction equal to twice the difference between the angles of the two creases.

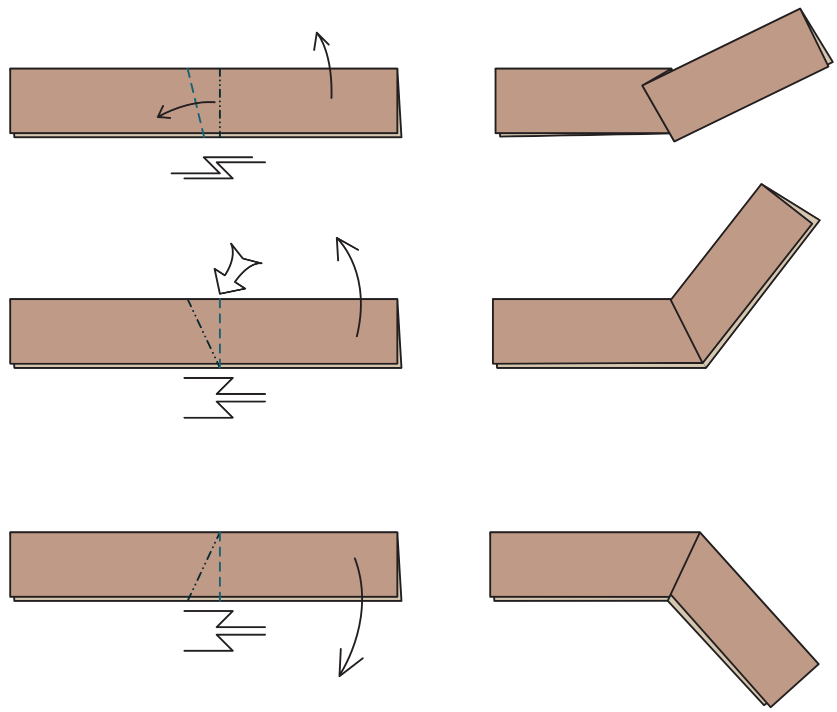
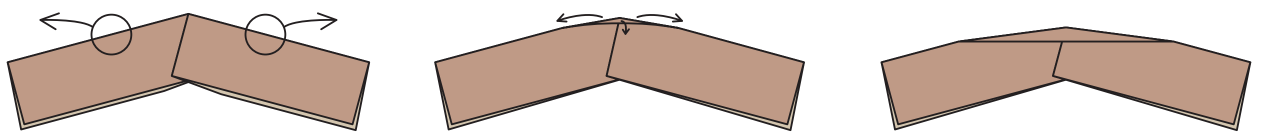


Figure 2.38.
Examples of angled pleats (top) and crimps (center, bottom).

The valley and mountain folds that make up a pleat or crimp can meet each other at one edge of the flap or the other, but cannot meet in the interior of the paper without adding additional creases. If you try to make them meet in the interior, which you can do by stretching the ends of an angled pleat or crimp away from each other, you will find that a small gusset must form that extends from the intersection point to the adjacent edges.

Stretching a pleat (or more commonly, a crimp) until it forms a gusset is a fairly common maneuver that is used to soften the change of angle to realize a more natural, rounded form. Stretching gussets is also the basis of some of the most powerful design techniques that we will see.

All of the combination folds we have encountered so far have involved edges, either the raw edge of the paper or folded edges on which the creases terminate. Their formation is somewhat eased by the ability to reach around behind each layer of

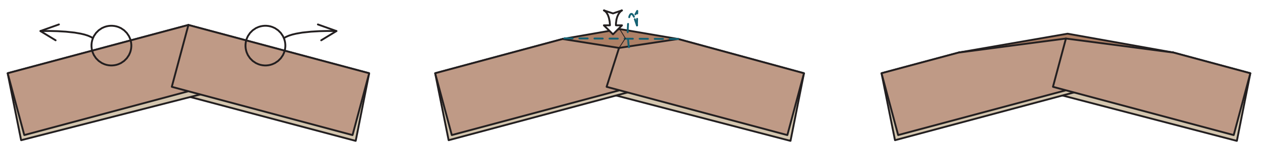


1. Example of a stretched pleat. Pull the two sides apart, keeping the angle fixed.

2. The top will form a little hood on either the front or back side.

3. Finished stretched pleat with gusset.

Figure 2.39.
Stretching an angled pleat forms a gusset on either the near or far layers.



1. Example of a stretched crimp. Pull the two sides apart, keeping the angle fixed.

2. The top forms a narrow diamond; dent the middle down between the layers.

3. Finished stretched crimp with gusset.

Figure 2.40.
Stretching an angled crimp forms a gusset between the layers of paper.

paper and work on the fold from either side. The next group of combination folds does not have this property—they are the family of sink folds. The inability to reach both sides of the paper makes them considerably harder to perform, since (usually) only one side of the paper is accessible, and usually puts any model including them well into the complex rating of difficulty. However, sink folds arise quite naturally from systematic methods of origami design, and so it is essential that they be learned and practiced.

The simplest of the various sink folds is the spread sink, which is only marginally more difficult than a squash fold. It works the same way; a flap is lifted up, its edges are spread symmetrically, and the result is flattened. What distinguishes a spread sink from a squash fold is that in the spread sink, at least two layers—an outer one and an inner one—are simultaneously squashed while remaining joined. Spread sinks are very satisfying to make; you start by flattening the very tip of the flap, then as the edges are stretched to the sides, the flattened region grows and reaches its maximum size when the paper is completely flat.

Spread sinks are most often formed from triangular corners, but there are analogous structures that form convex polygons of any size and shape.

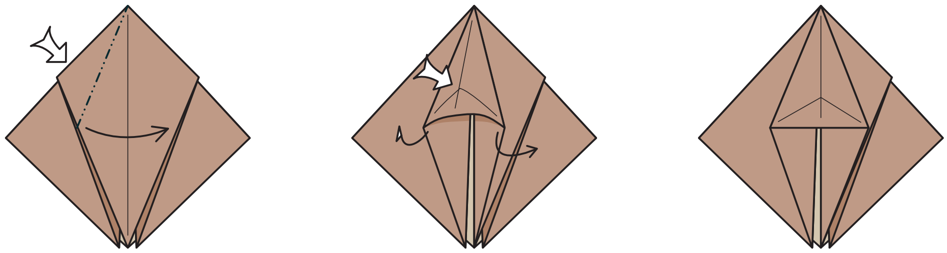


Figure 2.41.
A spread sink.

The next member of the sink family is the conventional, or open, sink. The open sink is a simple inversion of a corner formed from a region in the interior of the paper. Conceptually, it is quite simple: The line of the sink is a mountain fold, which runs all the way around the point being sunk like a road girdling a mountain peak. All of the creases above the sink line get converted to the opposite parity, mountain to valley, valley to mountain.

What makes an open sink "open" is that the part of the paper being sunk can (usually) be opened out entirely flat, which allows a relatively straightforward strategy for its formation: stretch the edges apart so that the tip of the point to be sunk flattens out, pinch a mountain fold all the way around, then push the middle down into the model and flatten the model. The creases in the sunk region will (again, usually) fall into the right place.

Figure 2.42 shows this process, including the intermediate stage, and the crease pattern of the result.

It is sometimes possible to make an open sink by performing a spread sink first, as Figure 2.43 shows.

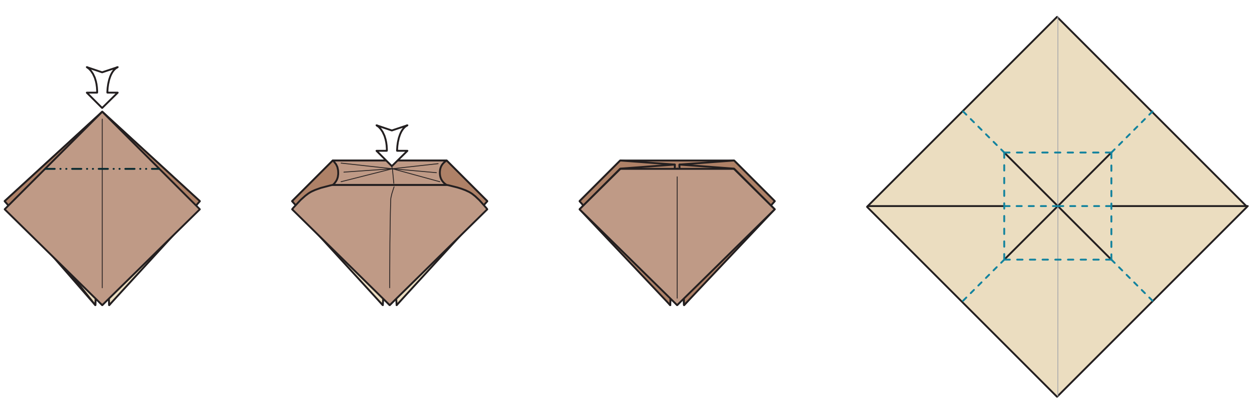
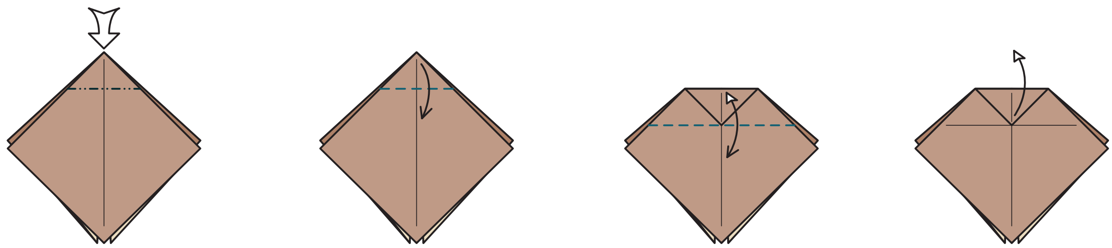


Figure 2.42.
The open sink, formation and crease pattern.

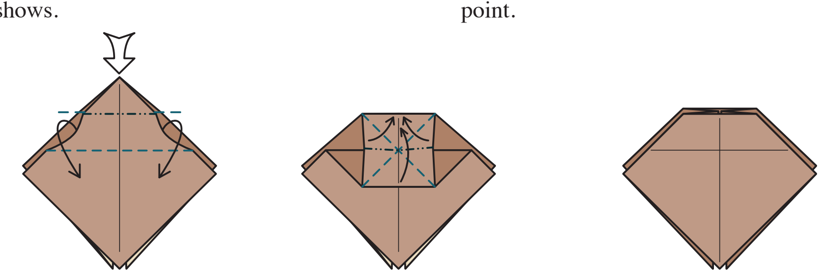


1. A sink fold can sometimes easily be made as a spread-sink, as this sequence shows.

2. Fold the point down along the sink line.

3. Fold and unfold along a crease that just touches the tip of the point.

4. Unfold the point.



5. Grasp the sides and fold them down while simultaneously stretching and pushing down on the top flap.

6. Bring the middle of the sides of the square region together at the top.

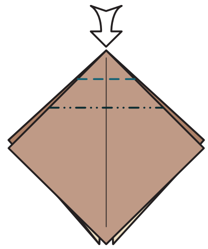
7. Completed sink fold.

Figure 2.43.
Folding sequence for making a sink folding using a spread sink.

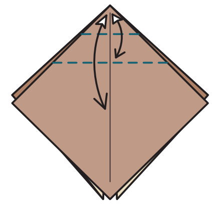
The example in Figure 2.43 is for a four-sided sink—one in which the point has four ridges coming down from it (and the polygon outlined by the mountain folds “going around the mountain” is a quadrilateral), but you can form three-, five-, and higher-sided sinks in a similar way.

As we have seen, a valley fold can combine with its mirror image to make a reverse fold, a squash fold can combine with its mirror image to make a petal fold, and a rabbit ear can combine with its mirror image to make a double rabbit ear. Can a sink fold be combined with its mirror image? Yes, in multiple ways, but the most common way happens when a point is sequentially sunk down and back up. The maneuver is called a double sink (or triple or quadruple sink, for more complicated generalizations).

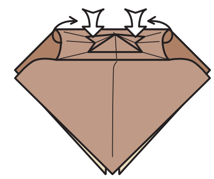
Although a multiple sink can be made sequentially—make the lowest sink, then reach inside and sink the point back upward—it’s usually easier to make them all together, first pinching the mountain folds around the point, then pinching the valley folds around before attempting to close up the model.



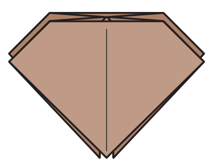
1. A double sink is indicated by parallel mountain and valley creases with a push arrow.



2. Always pre-crease all lines of a multiple sink before opening.



3. Form both the mountain and valley folds running all the way around the flattened polygon.



4. Finished double sink.

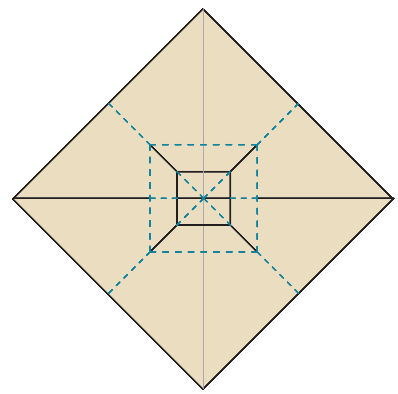
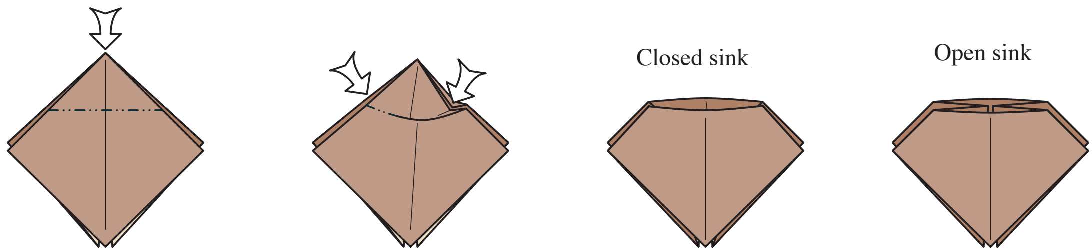


Figure 2.44.
A double sink, how to make it, and its crease pattern.

Sinks were recognized as distinct origami steps in the late 1950s and early 1960s. However, it took until the 1980s for a new variant to become common, the closed sink (whose recognition forced the division of sinks into “open” and “closed” varieties). A closed sink is also an inversion of a point, but in such a way that it is not possible to open the point flat while performing the maneuver. This makes closed sinks extremely hard to perform. In fact, from a strictly mathematical viewpoint, it is impossible to perform a closed sink using a finite number of folds (and what is impossible in mathematics is usually pretty hard in reality). That we can make closed sinks at all is due to the ability to “roll” a crease through one or more folded layers of paper.

Superficially, a closed sink is diagrammed the same way as an open sink: a push arrow and a mountain fold. However, in the closed sink, instead of forming the mountain fold all the way around every layer, some of the layers are held together, forming a cone, and the point is inverted through the cone without opening it out. Closed sinks are useful for locking layers together, as the edges of the pocket formed by a closed sink, unlike those of an open sink, cannot usually be opened up. The finished result can be distinguished by the presence of pleated layers inside the pocket of an open sink versus few or none in a closed sink.



1. A closed sink is also indicated by a push arrow, but is formed differently.

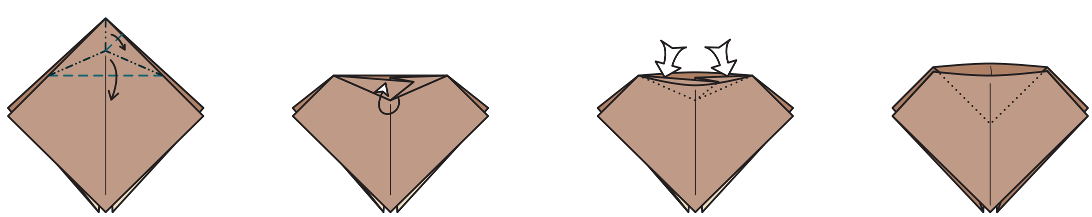
2. Open the point into a cone; starting at one side, start inverting the cone.

3. Flatten when fully inverted. In a closed sink, some edges are trapped at the top of the sink.

4. In an open sink, all edges are visible at the top of the sink.

Figure 2.45.
Formation of a closed sink. Right: the edges of an open sink for comparison.

In general, the more acute the point of a closed sink, the harder it is to carry out; anything narrower than a right angle is usually so difficult that it's more efficient to do it in two steps, as shown in Figure 2.46. First, fold the point into a rabbit ear, closed-sink the top of the rabbit ear, then once the sink is started, fully invert the rabbit ear back into the shape of the original point.



1. Another way to make a closed sink is to fold down the point and fold a rabbit ear from it.

2. Bring two layers of paper in front of the rabbit ear.

3. Push down inside the pocket, opening the point back up.

4. Finished closed sink.

Figure 2.46.
How to make a closed sink from a sharp point.

For any given corner, there is only one way of making an open sink, but there are multiple ways of forming closed sinks; in fact, a sink can be open at one end and closed at the other, an arrangement called a mixed sink. The different varieties are not always distinguishable from the outside, as different arrangements of interior (hidden) layers can have the same outward appearance. For a quadrilateral sink—one with four ridges running down from the top point—there are nine distinct configurations. They and their crease patterns are shown in Figure 2.47.

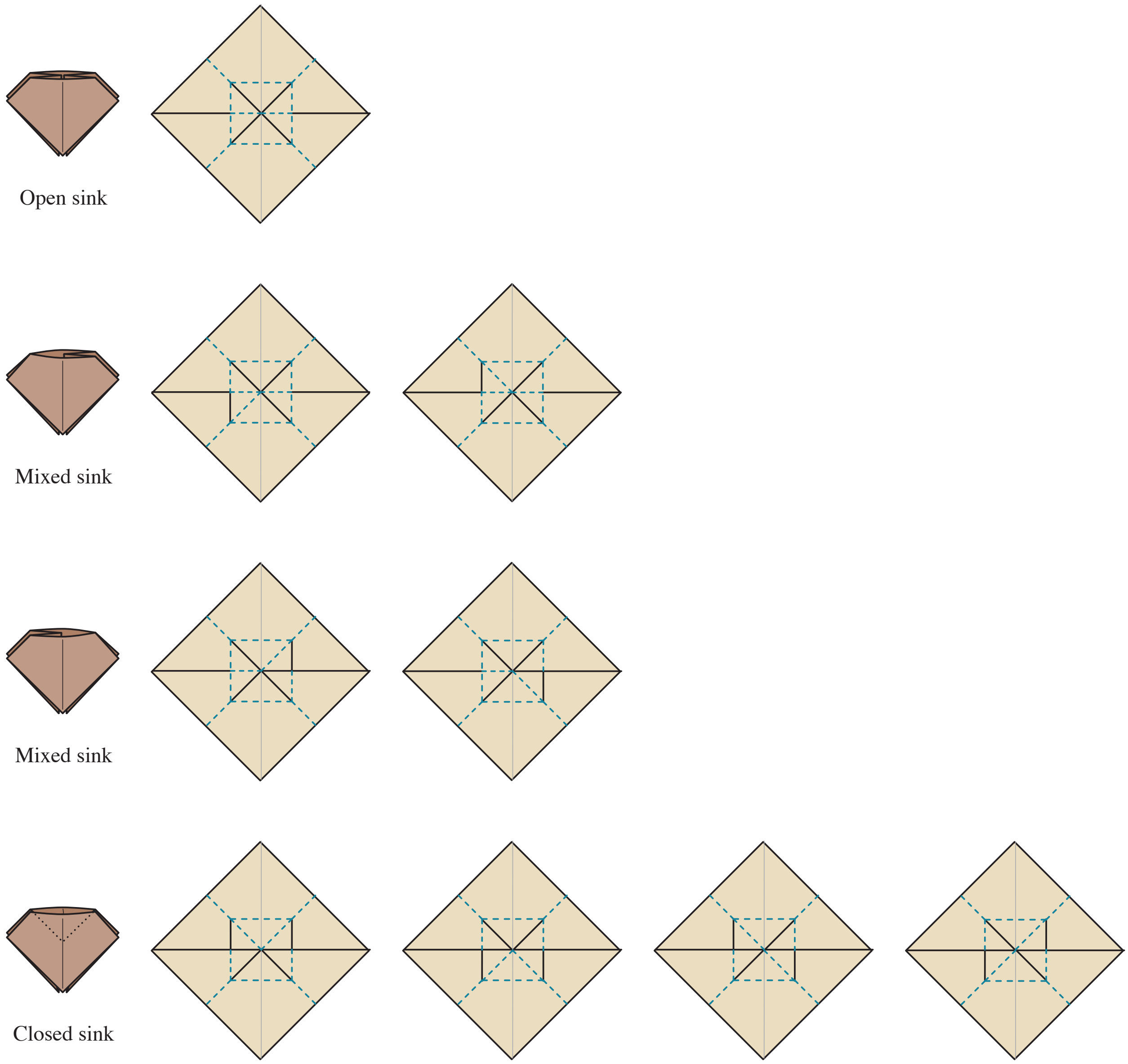


Figure 2.47.
The nine distinct types of sink for a four-ridged point.

In diagrams, which version of sink is desired is usually conveyed by the arrangement of edges in the subsequent views and/or by cut-away views of the interior layers.

The last—and by many accounts, the most challenging—of the sink folds goes by the name of unsink. As the name suggests, it is a reversal of a sink fold. That is, you are presented with an apparently sunken point and the object is to invert the point upward. The challenge here is that while you can always push a point downward to sink it, pulling a layer upward is problematic when there is nothing to grab onto.

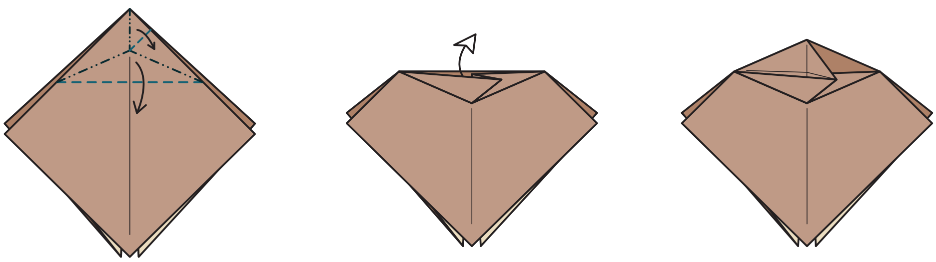


Figure 2.48.
An unsink fold.

Unsink folds come in open and closed varieties that are analogous to their similarly named sink brethren. The unsink is the youngest of the sink combination folds: It only began to be used in the late 1980s, and since then, only sporadically. It is not hard to imagine why. Most of the other combination folds arise naturally from the process of "playing with" the paper. If you want to change the direction of a point, the reverse fold naturally follows. Stretch a point to make it longer, and you are likely to (re)discover the petal fold. Shorten a flap—crimps and pleats fill the bill. And removal or rounding of a corner will lead you to reverse folds and sinks, both open and closed. But the unsink is something of an anomaly. It's unlikely to arise from simple doodling or shaping. But it does arise very directly from systematic origami design. In this chapter, we are—fortunately—still far away from being forced to learn to unsink, but we now, having enumerated the basic folds of origami, are ready to make our first forays into origami design.

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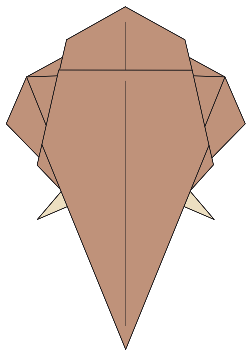
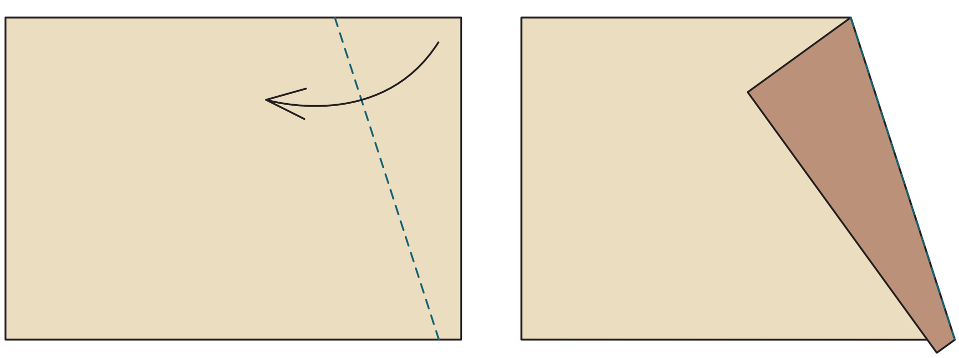


3 Elephant Design

In the beginning—at least, according to some mythologies—there was the Elephant. And so it is with the elephant that we begin our foray into origami design. The elephant—the subject of Georgeot's exhibition—is one of the most common subjects for origami. Presumably, this is because it is so readily suggested. Almost any large shape with a trunk is recognizable as an elephant. If the shape has four legs and large, floppy ears, so much the better. But all these features aren't needed; in fact, it is possible to fold an elephant using a single fold, as Figure 3.1 shows (designed by Dave Mitchell).



1. Begin with a sheet of writing paper. Fold the upper right corner down along an edge.

2. Finished One-Crease Elephant.

Figure 3.1.
Dave Mitchell's One-Crease Elephant.

Do you see it? The elephant is facing to the right.

Yes? Perhaps? This simple model—about as simple as you can get—illustrates one of the most important characteristics of origami models: They simplify the subject. Nearly all origami design is representational, but unlike, say, painting, the constraints of folding with no cuts make it nearly impossible to produce a truly accurate image of the subject. Origami is, as origami artist and architect Peter Engel has noted, an art of suggestion. Or put another way, it is an art of abstraction. The challenge to the origami designer is to select an abstraction of the subject that can be realized in folded paper.

You can also select a subject that lends itself to abstraction. Elephants are also popular subjects for origami design because they offer a range of challenges. What features do you include in the design? Is it a spare representation relying on a few lines to suggest a form, or is it necessary to capture all of the features of the subject? Getting the head and trunk may be sufficient for some folders, while others will be satisfied with nothing less than tusks, tail, and toenails. A somewhat more detailed elephant is shown in Figure 3.2.

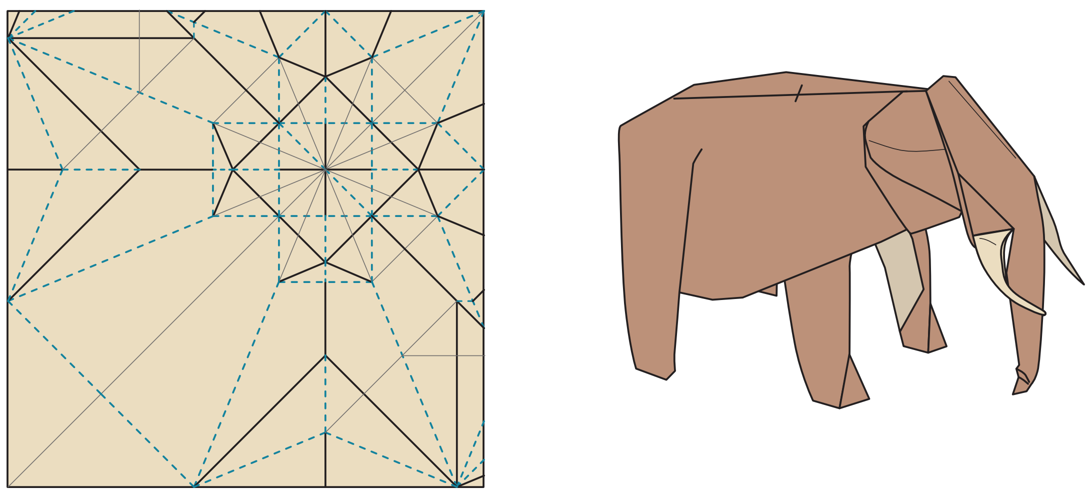


Figure 3.2.
Base crease pattern and finished folded model of my African Elephant.

These two designs illustrate the range of origami design: Every origami design falls somewhere along a continuum of complexity. Arguably, the one-crease elephant is the simplest possible origami elephant. But the complex elephant is almost assuredly *not* the most complex elephant possible. Complexity in origami is an open-ended scale; the title of “most complex” origami design (for any subject) is always transitory.

Furthermore, complexity carries with it a special burden. We do not denigrate the one-crease model for its abstraction; indeed, its abstract nature is part of its elegance and charm.

But a complex model creates a certain level of expectation in the viewer: an expectation that the model will convey a richer vision. The more folds we have in the model, the more we can reasonably expect from it. And thus, we must make every fold in the design count for something in the end result if elegance is to be attained.

Georgeot's exhibition consisted of 88 elephants ranging from simple to very complex indeed. But elephants, like rabbits, have a way of multiplying. Once he became known as "the origami elephant guy," origami elephants continued to come his way. He wrote that he had accumulated 155 different designs by the year 2000. Many folders have sent more than one, up to eight different designs from a single artist.

If you were to pick any two of Georgeot's elephants, you would find that they differ in many ways: One could be flat, the other three-dimensional; one in profile, one in front view. They might differ in the orientation of the paper relative to the model, in the number of appendages, or in what part of the paper those appendages come from. They may differ in the level of abstraction versus verisimilitude, in cartoonism versus realism, even in the use of curved versus straight lines (and which lines are chosen). All of these features are decisions that the designer makes along the way, whether consciously or unconsciously.

Of all the artistic criteria that may be applied to origami, one of the most important, yet elusive, is elegance. Elegance as it applies to origami is a concept not easily described. It implies a sense of fitness, of economy of effort. In origami, an elegant fold is one whose creases seem to go together, in which there is no wasted paper, whose lines are visually pleasing. Elegance cannot be easily quantified, but there is a property closely related to elegance that can: efficiency.

While elegance is a subjective measure of the quality of a design, efficiency is an objective measure. An efficient model is one in which all of the paper gets used for something; nothing is tucked out of the way. Inefficient models are those with unnecessary layers of paper. Such models are thick and bulky, often difficult to fold, and usually less aesthetically pleasing than a model without unnecessary layers of paper.

The most efficient models are the largest possible for a given sheet of paper. If you have folded two figures from ten-inch squares of paper and one figure is three inches across and the other is two inches across, then the smaller figure must by necessity have more layers of paper on average in any given flap. The smaller model will generally be thicker; it will hold together less well; and it will show more edges, which will break up the lines of the model. In short, the less efficient a model is, the poorer its visual appeal. Thus, efficiency is an aesthetic goal as well as a mathematical goal. For a base with a fixed number of flaps, the most efficient base is that base in which the flaps are as large as possible.

The tools of origami design cannot (yet) directly address elegance, but they can address its close relative, efficiency, by quantifying what is possible and impossible and providing direction for maximally efficient structures. To wield the tools of origami design, one must have some tools to start with. The way to build a set of tools is to examine some examples of design and deconstruct the model, identifying and isolating specific techniques. To get started and to illustrate some basic principles of origami design, let's add three more elephants to the roster.

3.1. Elephant Design 1

The first design shown in Figure 3.3 is for an Elephant's Head. It is very simple and takes only five creases.

This is very simple—it's perhaps one step up from the One-Crease Elephant, although, you might note, it took five steps. Can you devise an elephant using exactly two creases? Exactly three?

3.2. Elephant Design 2

On the scale of origami complexity, both the One-Crease Elephant and the Elephant's Head fall into the "simple" category. But as we add more features to a model, it almost invariably increases in complexity. As an illustration, let's take the same basic design as the Elephant's Head and add a pair of tusks to it.

The amount of folding increased substantially, just to create two tiny points for tusks. But I also added a few steps to give definition to the ears (step 9). Why? Why not just leave the face a flat surface as we did in the previous model? Two reasons. In the first Elephant's Head, the ears came almost for free—there were two flaps (the corners of the square) available to work with. But in this design, we needed to create side flaps (in steps 8 and 9) to define the ears, which required more folding.

There's a second reason, however, which is a bit more subtle. There is an aesthetic balance that needs to be maintained across an origami design. The tusks introduce some small, fine features into the model. The contrast between those fine features and the broad, flat, featureless expanse of

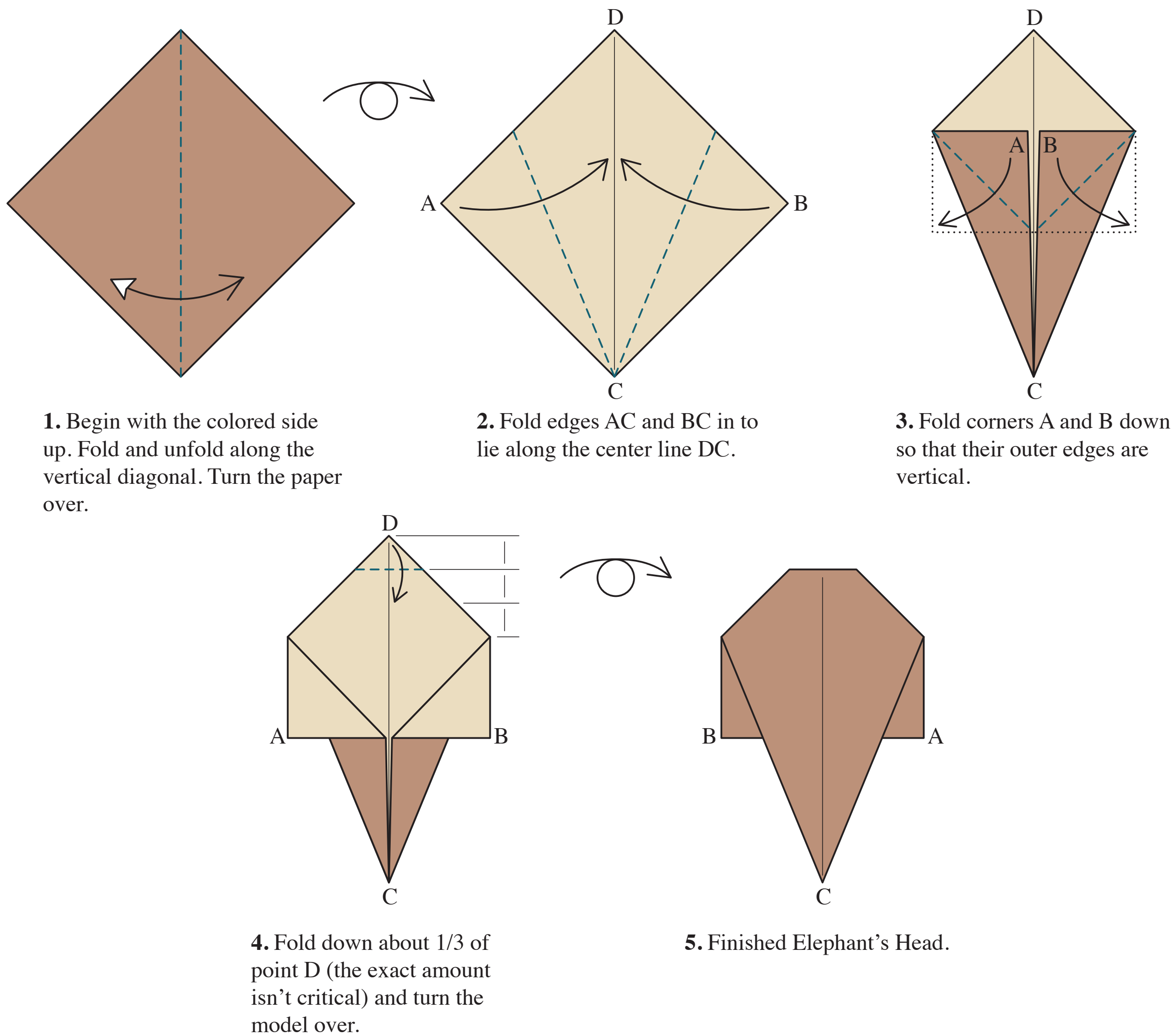


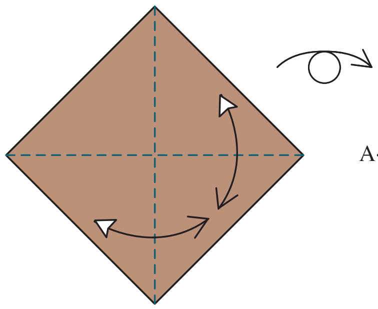
Figure 3.3.
Folding sequence for an Elephant's Head.

the face is jarring, so we introduced two folds to break up the surface of the face a bit and bring some balance to the lines of the model.

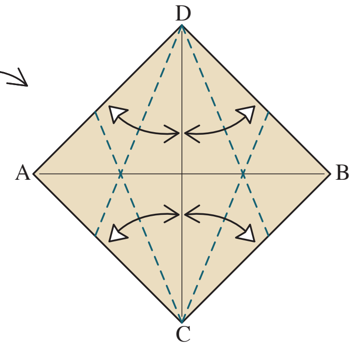
3.3. Elephant Design 3

We can take another step up the ladder of complexity. Now we'll make the tusks a bit longer.

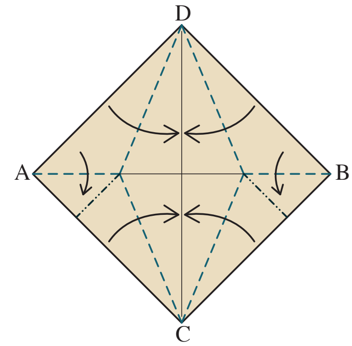
These three models depict the same subject, but with progressively greater anatomical accuracy (although they still leave a lot to be desired—like a body). They are simple,



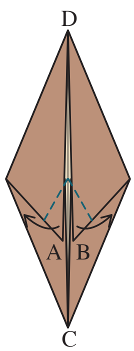
1. Begin with the colored side up. Fold and unfold along both diagonals. Turn the paper over.



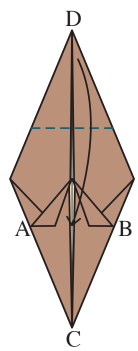
2. Fold edges AC and BC in to lie along the center line DC and unfold. Repeat with edges AD and BD.



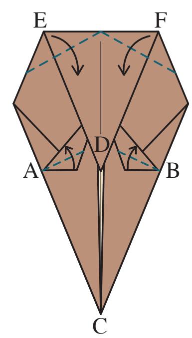
3. Fold rabbit ears from corners A and B.



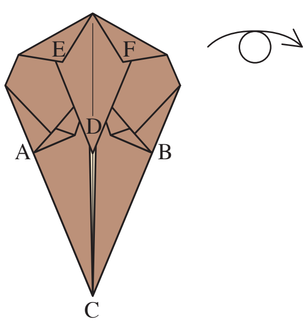
4. Fold corners A and B to the outside edges.



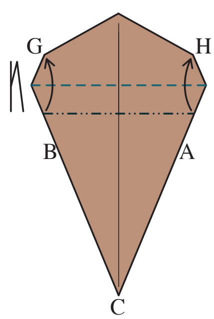
5. Fold corner D down so that it lies on an imaginary line running between points A and B.



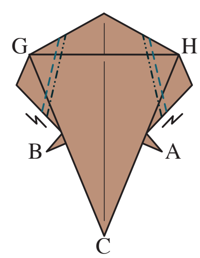
6. Fold corners A and B in half. Fold corners E and F down (the exact amount isn't critical).



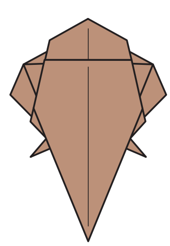
7. Turn the model over.



8. Pleat the front flap upward. The mountain fold will run from corner G to corner H.

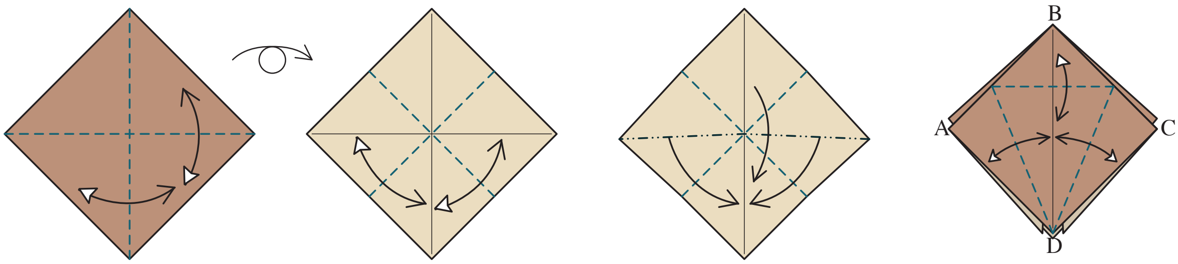


9. Pleat the sides of the head to form ears.



10. Finished Elephant's Head.

Figure 3.4.
Folding sequence for the more complex Elephant's Head.

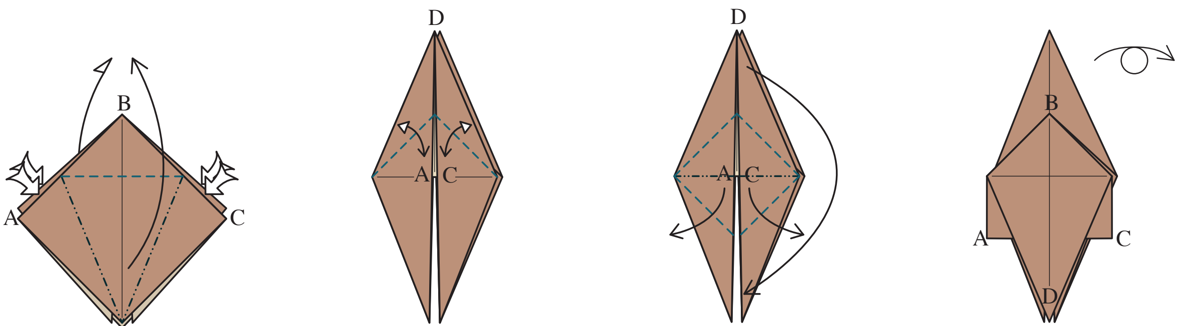


1. Begin with the colored side up. Fold and unfold along the diagonals. Turn the paper over.

2. Fold and unfold.

3. Bring the four corners together at the bottom to make a Preliminary Fold.

4. Fold edges AD and CD in to the center line and unfold. Then fold point B down and unfold.

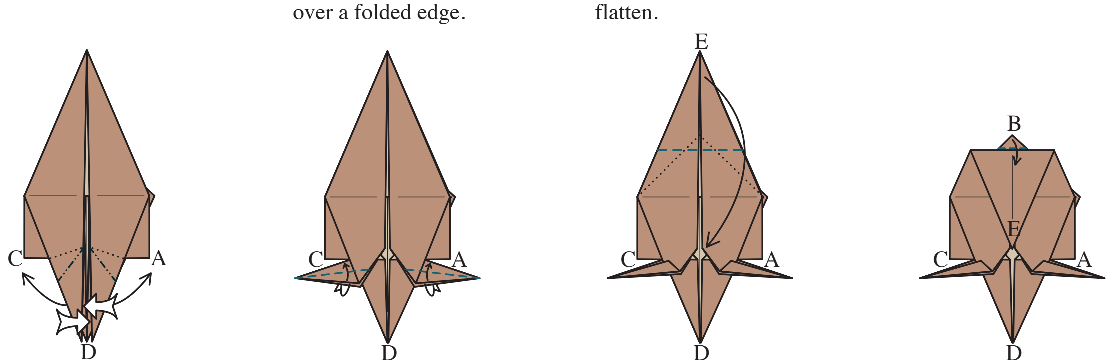


5. Petal-fold front and back to make a Bird Base.

6. Fold and unfold on the near flap. Each crease lies directly over a folded edge.

7. Fold corner D down while pulling points A and C out to the sides; flatten.

8. Turn the paper over.

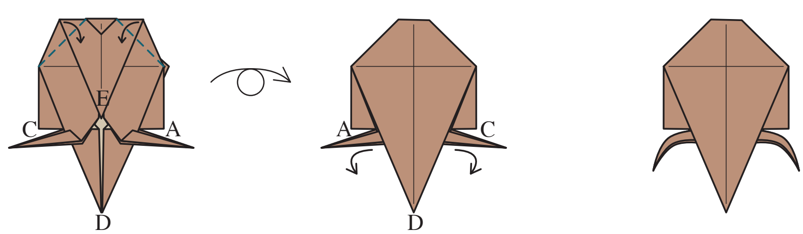


9. Reverse-fold the two bottom points out to the sides.

10. Narrow the two points with valley folds in front and behind.

11. Fold point E down.

12. Fold point B down.



13. Fold the corners down and turn the model over.

14. Curve the tusks.

15. Finished Elephant's Head.

Figure 3.5.
Folding sequence for yet another Elephant's Head.

but illustrate some basic principles of origami design that are worth identifying:

> *Generally, the more long points a model has, the more complex its folding sequence must be.*
>
> *Generally, the more long points a model has, the smaller the final model will be relative to the size of the square.*

These principles were widely known in the origami world of the 1960s and 1970s, but it was not until the 1980s and 1990s that they could be quantified. Those two decades saw the appearance of a new type of origami, the "technical fold." It is hard to define precisely what constitutes technical folding; technical folding tends to be fairly complex and detailed, encompassing insects, crustaceans, and other point-ridden animals. It is often geometric, as in box-pleated models and polyhedra. The early practitioners of what we call technical folding—Neal Elias, Max Hulme, Kosho Uchiyama, and a handful of others—were joined by a host of other folders—Montroll, Engel, and myself in the U.S., Fujimoto, Maekawa, Kawahata, Yoshino, Kamiya, Meguro, and many others in Japan—an expansion of the art that continues today. In fact, technical folding has its own name in Japan: *origami sekkei*. It is difficult to pin down a unique characteristic of a model that defines it as *origami sekkei*, but I have a candidate criterion: A fold is a technical fold when its underlying structure shows clear evidence of intentional design.

The first steps of design, however, do not require use of any specialized techniques or mathematical theorems. Anyone who can fold origami can design origami. In fact, if you folded one of the three elephant designs, you were calling upon your design skill. A sequence of folding diagrams—no matter how detailed—can still only provide a set of samples of what is a continuous process. In following a folding sequence, the reader must interpolate; he must connect the steps in his mind to form a continuous process. Depending on the amount of detail into which the steps are broken down, this process can be easy, as in Figure 3.6, or it can be difficult, as in Figure 3.7.

A good origami diagrammer, balancing the needs of brevity and clarity, strives to match the level of detail to the complexity of the fold and to the intended audience. In this book, I have aimed for a middle ground, along the lines of Figure 3.8.

When you begin following diagrams, you require each instruction to be broken down into the smallest possible steps.

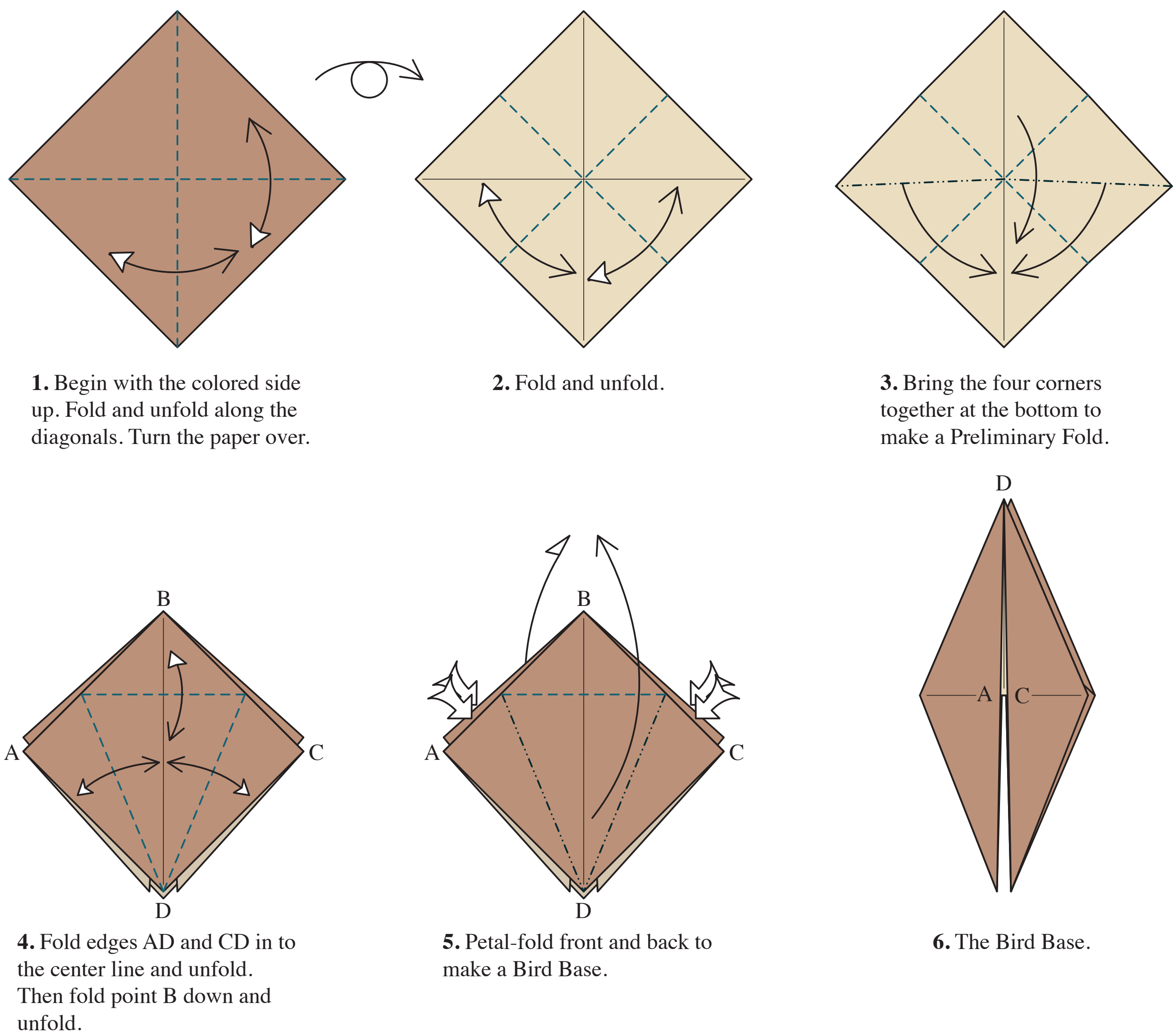
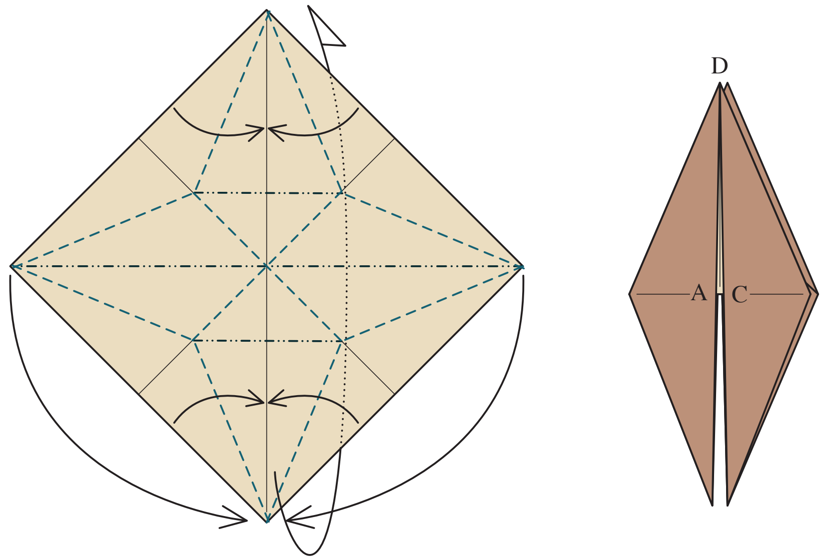


Figure 3.6.
Detailed sequence for folding a Bird Base.

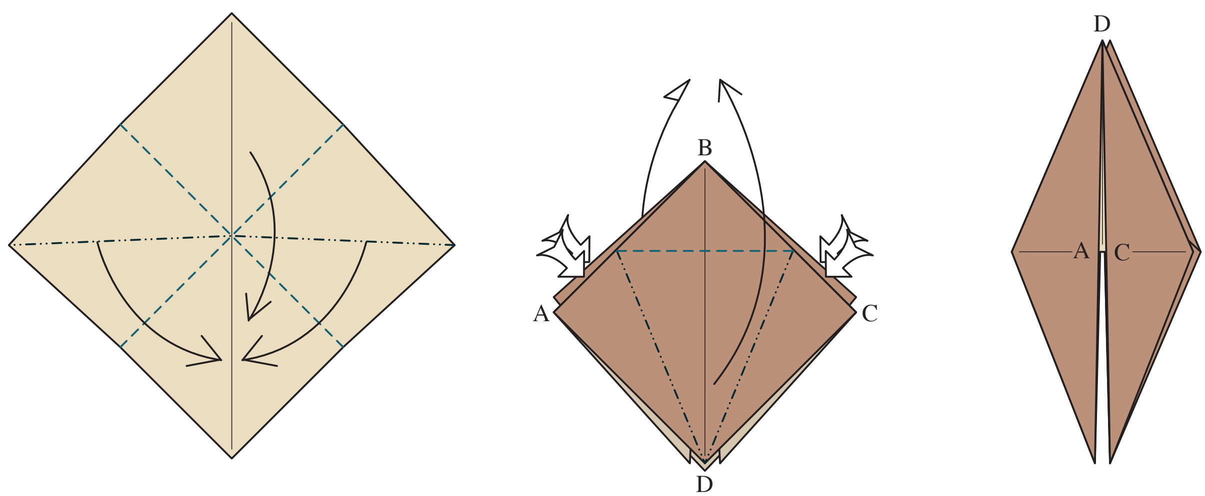
As you gain experience in following diagrams, the jumps between steps become larger. Instead of seeing every individual crease, the creases start to come in groups of two or three. As we have seen, the most common groups of creases have been given names: reverse folds, rabbit-ear folds, petal folds. More advanced folds may have groups of 10 or 20 creases that must be all brought together at once, or several different folds must occur simultaneously, or not all creases may be visible in the diagram. Following such a sequence is even more a process of design. Following a folding sequence is, in effect, resolving a series of small design problems going from one configuration of the paper to the next. Designing an entirely new model is the same task, merely scaled up.



1. Divide the square in half vertically and horizontally with creases. Crease all angle bisectors at the corners, then assemble using the creases shown.

2. The Bird Base.

Figure 3.7.
Compact sequence for folding a Bird Base.



1. Begin with a square with creased diagonals. Bring the four corners together at the bottom and flatten.

2. Petal-fold front and back.

3. The Bird Base.

Figure 3.8.
Intermediate sequence for folding a Bird Base.

Origami design runs along a continuous scale ranging from minor modification of an existing design to the "ground-up" creation of an entirely new model. Just as a beginning folder should begin to fold simple models from diagrams, the beginning designer should choose simple shapes to design.

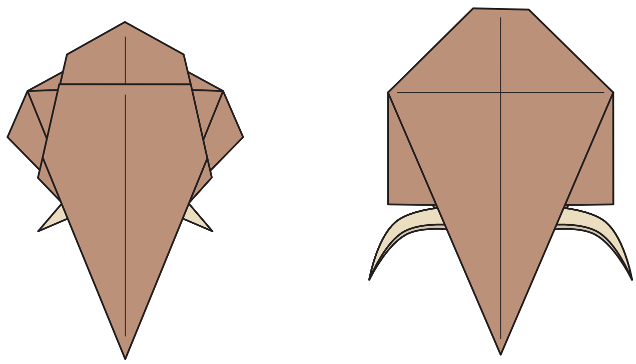
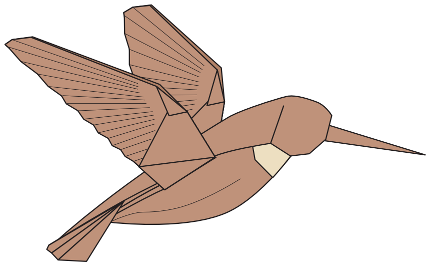


Figure 3.9.
Two variations on the Elephant's Heads.

And now is as good a time as any to start. The elephants in Figures 3.4 and 3.5 have colored tusks. Can you find a way to alter each model so that the tusks become white as shown in Figure 3.9? (Hint: Turn a flap inside-out.)

The first stage of origami design is to modify someone else's work, as you can with the elephants. Origami design is, in large part, built on the past. The origami designers of the present have created new techniques, but in doing so, they used techniques of those anonymous Japanese folders of history (as well as those of their contemporaries, of course). It behooves us to spend some time studying how prior generations of folders designed their models.



4 Traditional Bases

The design of an origami model may be broken down into two parts, folding the base, and folding the details. A *base* is a regular geometric shape that has a structure similar to that of the subject, although it may appear to bear very little resemblance to the subject. The *detail folds*, on the other hand, are those folds that transform the appearance of the base into the final model. The design of a base must take into account the entire sheet of paper. All the parts of a base are linked together and cannot be altered without affecting the rest of the paper. Detail folds, on the other hand, usually affect only a small part of the paper. These are the folds that turn a flap into a leg, a wing, or a head. Converting a base into an animal using detail folds requires tactical thinking. Developing the base to begin with requires strategy.

The traditional Japanese designs were, by and large, derived from a small number of bases that could be used to make different types of birds, flowers, and various other figures. For much of the 20th century, most new origami designs were also derived from these same basic shapes.

Bases have been both a blessing and a curse to inventive folding: a blessing because the different bases can each serve as a ready-made starting point for design, a curse because by luring the budding designer onto the safe, well-trodden path of using an existing base, he or she starts to feel that there's nothing new to do and never explores the wilds of base-free origami design.

We will, by the end of this book, do both. However, we will start with the traditional bases—first, to understand what our origami designer forebears had to work with, and second, because the traditional bases, despite being picked over by scores of origami designers for decades, still have some surprising life in them. While they may seem like unique constructions, the traditional origami bases are actually specific embodiments of quite broad and general design principles. By thoroughly understanding the traditional bases, we are prepared to understand the deeper principles of origami design.

4.1. The Classic Bases

So what, exactly, are the standard bases of the origami repertoire? Now, it must be admitted that any labeling scheme that dubs certain structures "the standard bases" is going to be somewhat arbitrary. But there are four shapes known for hundreds of years in Japan that are the basis of several traditional models. These shapes have a particularly elegant relationship with one another that takes on a special significance in origami design. They are often called the four *Classic Bases* of origami and are named for the most famous models that can be folded from them: the Kite, Fish, Bird, and Frog Base.

Perhaps not surprisingly, in many cases, more of the structure of an origami model is evident in the crease pattern than in the folded base. For one thing, in the crease pattern, all parts of the paper are visible, while in the folded model only the outermost layers are visible—perhaps 90% or more of them are hidden. Furthermore, certain structures appear over and over in a crease pattern, which you can recognize as features of the finished model. (Do a lot of creases come together at a single point? That point probably becomes the tip of a flap of the model.) With practice, you can learn to read the structure of a model in the crease pattern as if it were the entire folding sequence. The crease patterns, bases, and a representative model from each of the four Classic Bases are shown in Figure 4.1.

We have already encountered three of these in the Elephant's Head series—the Kite, Fish, and Bird Bases. (Challenge: Can you design an elephant that makes full use of the flaps of a Frog Base?) There is no precise definition of a base; perhaps a good working definition is "a geometric form with the same general shape and/or number of flaps as the desired subject."

In origami, a *flap* is a region of paper that can be manipulated relatively independently of other parts of the model. In origami design, bases supply flaps; major flaps on a base then get turned into major appendages of a final model. The Kite,

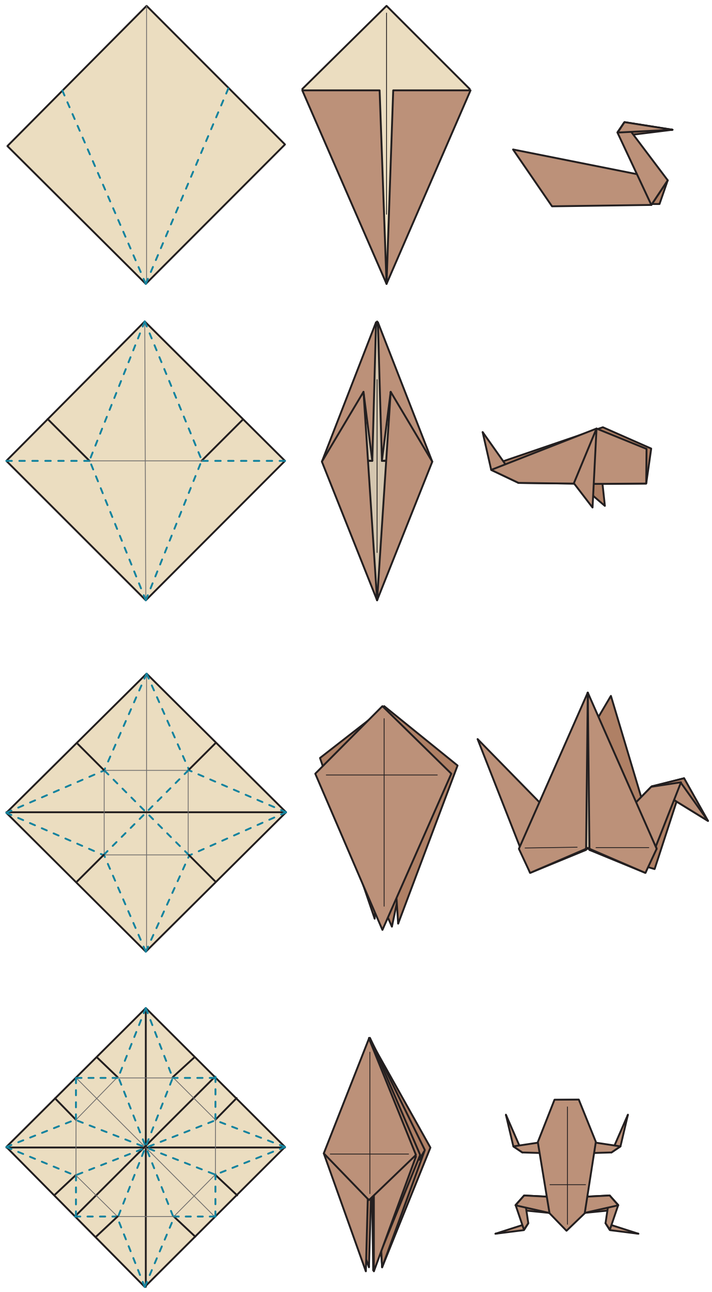


Figure 4.1.
Crease pattern, base, and a representative model for (top to bottom): Kite Base; Fish Base; Bird Base; Frog Base.

Fish, Bird, and Frog Bases have, respectively, one, two, four, and five large flaps and one, two, one, and four smaller flaps. To fold an animal, you usually need to start with a base that has the same number of flaps as the animal has appendages. A simple fish has two large flaps (head and tail) and two small ones (pectoral fins), which is why the Fish Base is so appropriate and so named. The average land-dwelling vertebrate has five major appendages (four legs and a head), which suggests the use of the Frog Base, but only if there is no long tail. The Frog Base does have five flaps, but the flap on the

Frog Base that is in a position to form a head is thick and difficult to work with. One of the four flaps of a Bird Base would be easier. But to use a Bird Base to fold a four-legged animal, you would have to represent two of the legs (usually the rear legs) with a single flap. In the 1950s and 1960s, there were a lot of three-legged origami animals hobbling around.

4.2. Other Standard Bases

The Classic Bases are not the only bases in regular use. There are a few other candidates for standard bases: the so-called Preliminary Fold (a precursor to the Bird and Frog Bases), the Waterbomb Base (obtainable from the Preliminary Fold

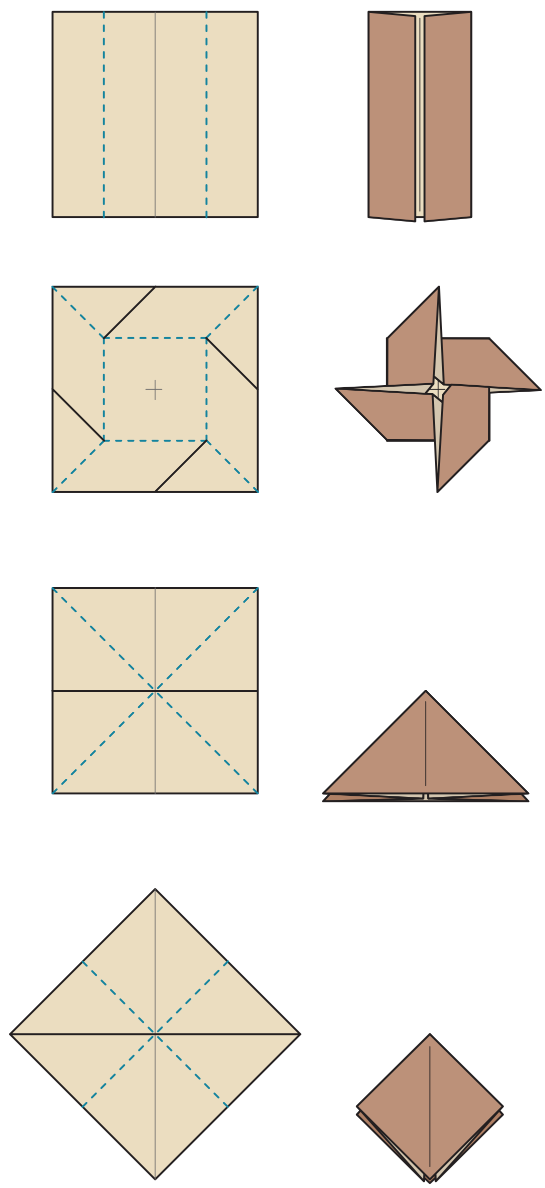


Figure 4.2.
Top to bottom: the Cupboard Base, Windmill Base, Waterbomb Base, and Preliminary Fold.

by turning it inside-out), the Cupboard Base (consisting of only two folds), and the Windmill Base (also known as the Double-Boat Base in Japan).

The Preliminary Fold was named Fold rather than Base by Harbin since it was a precursor to other bases, a somewhat artificial distinction that has stubbornly persisted in the English-speaking origami world.

Up through the 1970s, origami designers combined these bases with other procedures, known variously as blintzing, stretching, offsetting, and so forth—and we will learn some of these as well—resulting in a proliferation of named bases. It was not unheard-of to find, for example, a "double stretched Bird Base (type II)" as the starting form for a model. (Rhoads's Bat, *Secrets of Origami*). Of all the possible variants, two are sufficiently noteworthy as to deserve attention: the stretched Bird Base and the blintzed Bird Base are fairly versatile treatments of the classic Bird Base that have seen heavy use in modern times. Both are shown in Figure 4.3.

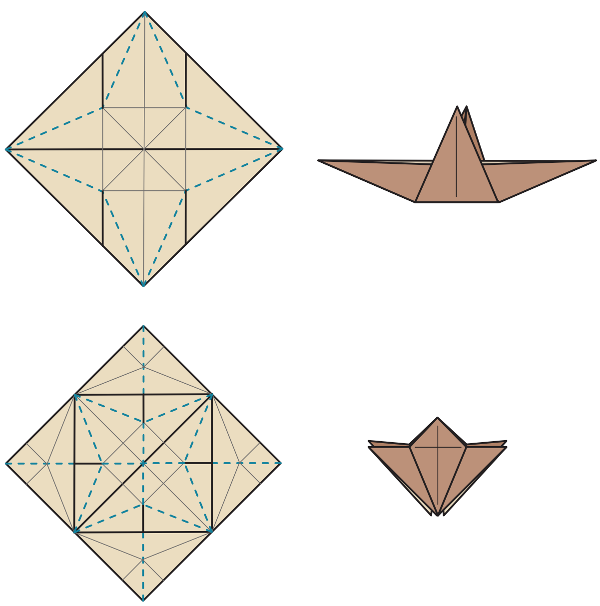


Figure 4.3.
Top: stretched Bird Base.
Bottom: blintzed Bird Base.

The stretched Bird Base is derived from the traditional Bird Base. It is obtained by pulling two opposite corners of the Bird Base as far apart as possible and flattening the result. Harbin recognized several variants of the stretched Bird Base, but the version shown in Figure 4.3 is the most common.

The blintzed Bird Base is also derived from the traditional Bird Base. It is obtained by folding the four corners to the center of a square, folding a Bird Base from the reduced square, and then unwrapping the extra paper to form new flaps. There are several ways of unwrapping the corners to make use of the extra flaps. The procedure of folding four corners to the center is called *blintzing*, named after the *blintz* pastry in which the four corners of a square piece of dough are folded to the center. For many years blintzing a base has been recognized as a straightforward way of increasing the number of flaps in a base.

Yet another named base system had been developed in Japan by Michio Uchiyama in the 1930s and thereafter. His system, carried on by his son Kosho, recognized two broad families of bases, one characterized by diagonal or radiating folds (type A) and the other by predominantly rectilinear folds (type B). Figures 4.4 and 4.5 show both families of bases. I have labeled them with Uchiyama's original numbering but rearranged them to better illustrate the relationships between bases.

Note that Uchiyama only gives the major creases for each base and does not specify the mountain/valley assignment; to make the shape flat, you will have to add additional creases on some of the patterns and work out the assignment for yourself.

Beginning with the development of subject-specific bases in the 1970s (Animal Base, Flying Bird Base, Human Figure Base), the variety of bases quickly proliferated to the point that naming every base began to seem a bit silly (the Great Crested Flycatcher Base). The net result was that most names were left by the wayside. Different authorities recognize different groups of bases as the standard set, but the four Classic Bases plus the Preliminary Fold, Waterbomb Base, Cupboard Base and Windmill Base are common to most.

4.3. Relationships between Bases

The standard bases are not wholly independent; some can be derived from others, as was suggested by Uchiyama's classification system and is illustrated more explicitly in Figure 4.6. Arrows indicate derivation. The square can be folded into a Cupboard Base, which can be further transformed into a Windmill Base. Similarly, the Kite Base is but a way station on the path to a Fish Base. The Preliminary Fold and Waterbomb Base are the same thing—one is just the inverse of the other—but while the Preliminary Fold alone can be turned directly into a Bird Base, either the Waterbomb Base or the Preliminary Fold can be used to make a Frog Base.

But the four Classic Bases—Kite, Fish, Bird, and Frog—share a deeper similarity that is only evident when one examines

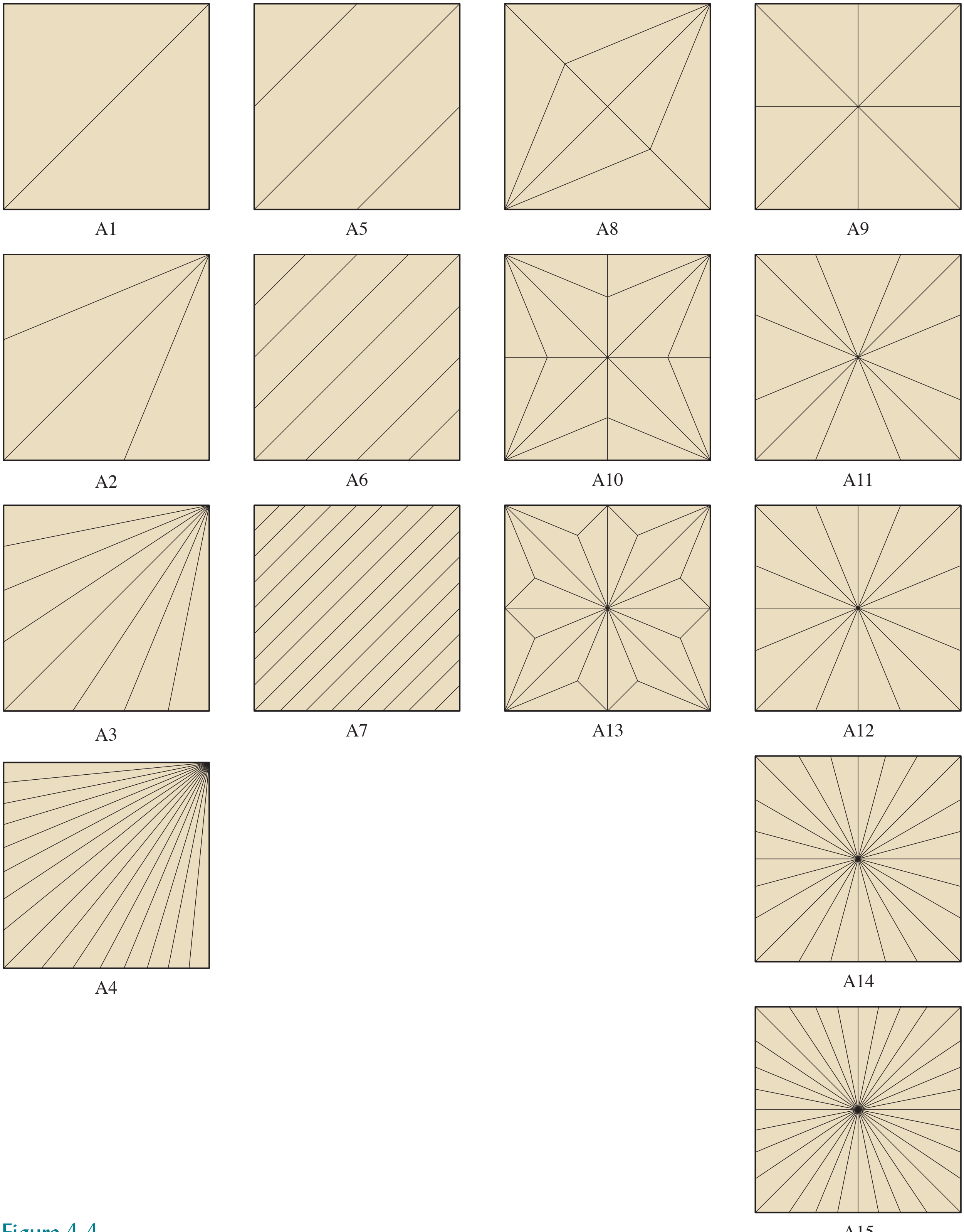


Figure 4.4.
The Uchiyama system of A bases, which are based primarily upon diagonal and/or radial folds. Note that the Kite Base, Fish Base, Bird Base, and Frog Base are among them.

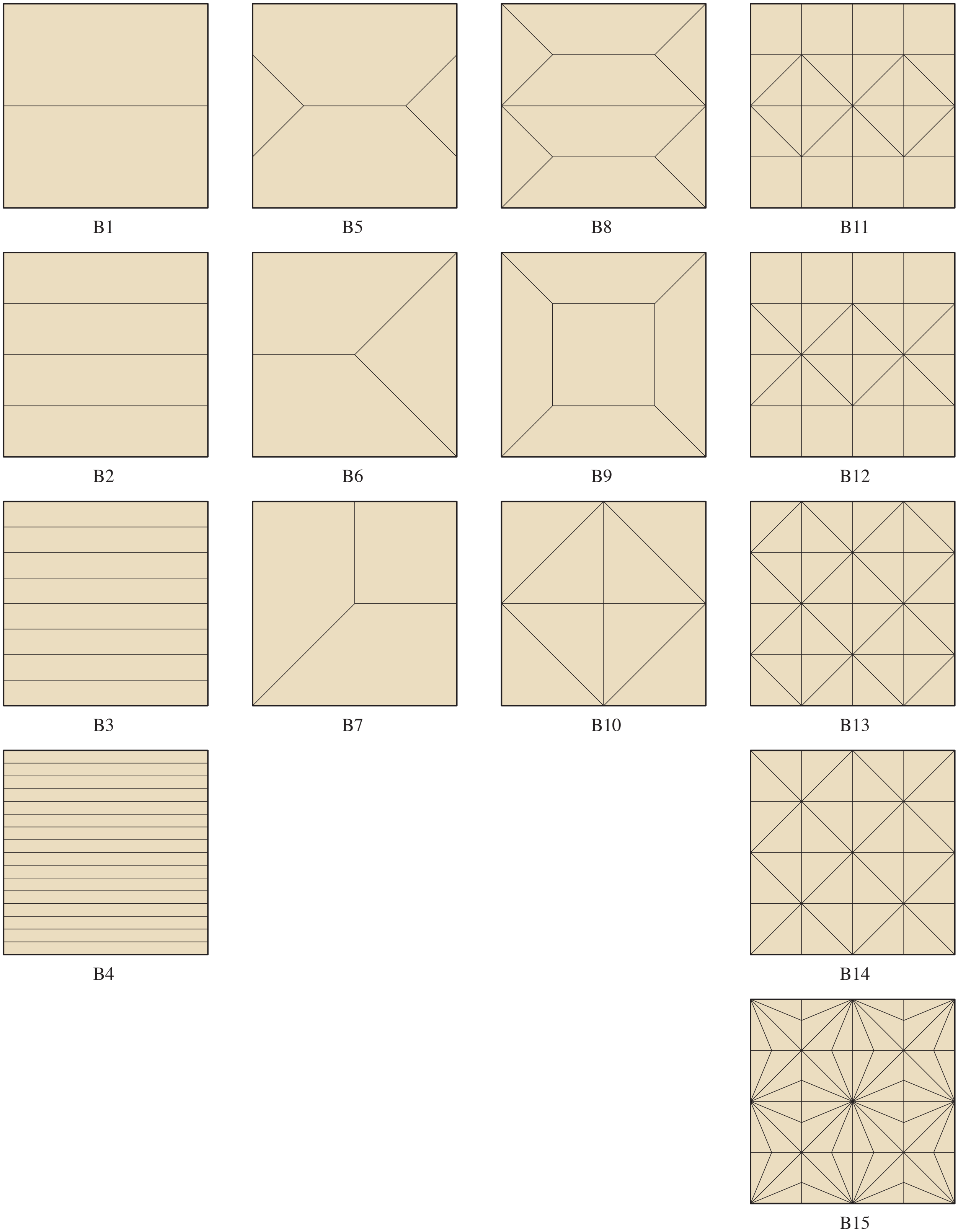


Figure 4.5.
Uchiyama's B bases.

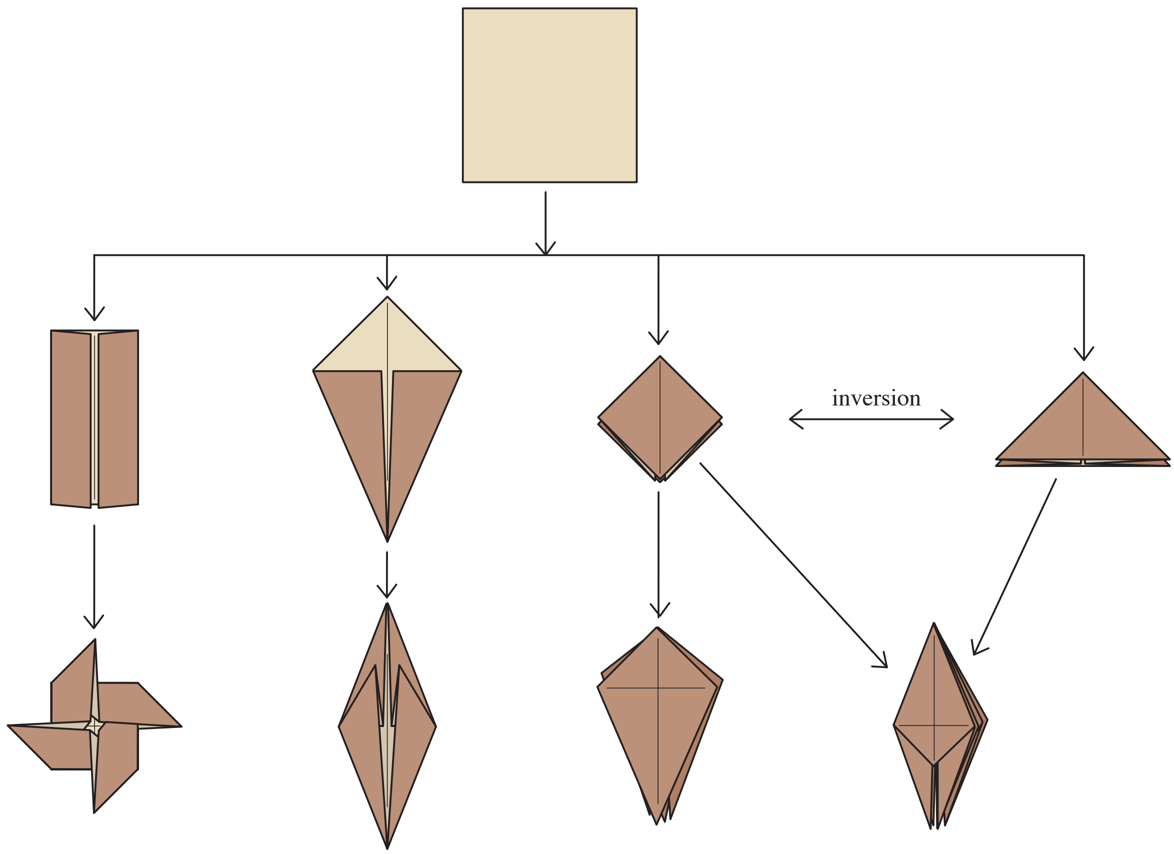


Figure 4.6.
Family tree of the standard bases.

their crease patterns. In these four bases, the same fundamental pattern appears in multiples of two, four, eight, and sixteen.

This reappearing shape is an isosceles right triangle with two creases in it; Figure 4.7 shows how it appears in each base in successively smaller sizes. Although the crease directions (mountain versus valley) may vary, the locations of the two creases within each triangle are the same. I have

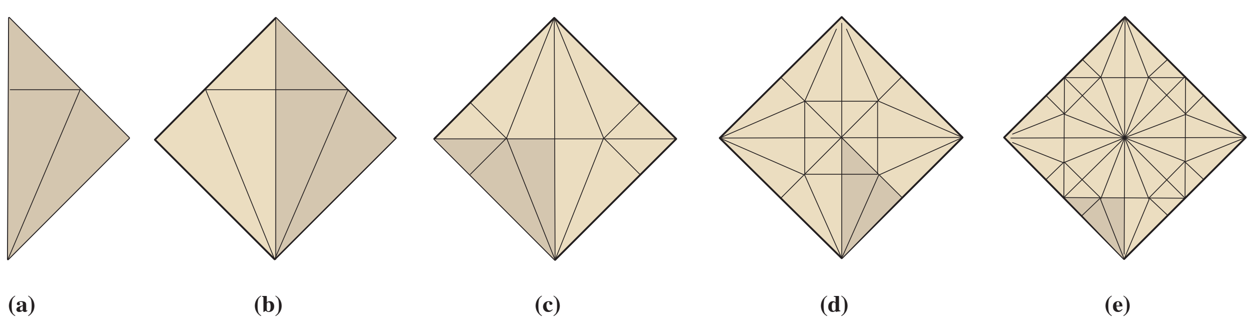


Figure 4.7.
(a) The basic triangle. (b) Kite Base. (c) Fish Base. (d) Bird Base. (e) Frog Base.

shown all creases as generic creases in the figure to emphasize this commonality.

Two of these isosceles triangles can be assembled into a square, yielding the Kite Base. Four give the Fish Base. Eight give the Bird Base. Sixteen give the Frog Base. The pattern is clear. We could easily go to 32, in which case we would end up with the blintzed Bird Base. There's no need to stop there, and origami designers haven't. In the mid-20th century Akira Yoshizawa devised a Crab based on the blintzed Frog Base, with 64 copies of the triangle; more recently, the crease pattern for my own Sea Urchin (Figure 4.8), which incorporates 128 copies of this triangle, creates a base with 25 equal-length flaps.

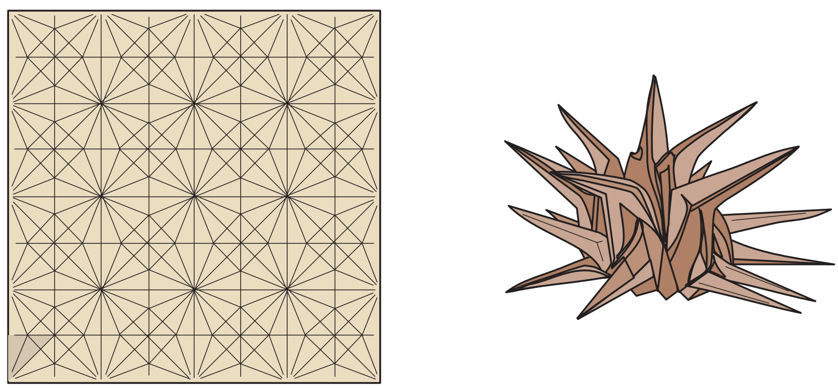


Figure 4.8.
Crease pattern and folded form of Sea Urchin.

The repeating pattern of triangles—first observed by Eric Kenneway in his column in *British Origami* magazine, "The ABCs of Origami"—is more than a geometrical curiosity. As we increase the number of triangles, we also increase the number of long flaps in the resulting base. The first three crease patterns suggest a simple relationship between the numbers of triangles and long flaps:

Base	Triangles	Flaps
Kite	2	1
Fish	4	2
Bird	8	4

These three crease patterns suggest that the number of long flaps is half the number of triangles in the base. But small numbers can be deceiving. A small number of examples can masquerade as many possible sequences—for the very next base breaks the pattern:

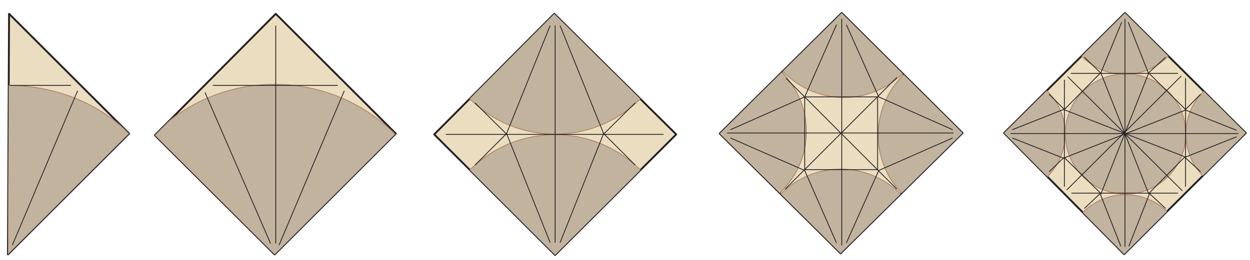
Base	Triangles	Flaps
Frog	16	5

So the Frog has five, rather than eight, flaps, as the simple pattern would suggest. And the Sea Urchin really messes things up:

Base	Triangles	Flaps
Urchin	128	25

You might find it an interesting experiment to fold the crease patterns that lie between the Frog and Sea Urchin into bases and count the number of long flaps in each (Hint: start with a blintzed Bird Base and blintzed Frog Base, but you will have to perform some additional manipulations to free the flaps).

So, there isn't a simple relationship between the number of triangles and the number of flaps. But there is a relationship nonetheless. Let us draw an arc of a circle in the triangular unit; then draw each arc in the unit as it appears in the crease pattern of the base.



(a) **(b)** **(c)** **(d)** **(e)**

Figure 4.9.
(a) The triangle unit, with inscribed circle. (b) Kite Base. (c) Fish Base. (d) Bird Base. (e) Frog Base.

The basic triangle unit contains 1/8 of a circle. When the units are combined, however, the circular arcs combine with each other to form quarter-, half-, and whole circles. If we count the number of distinct circular pieces, we get in the Kite Base, one quarter-circle; in the Fish Base, two quarter-circles; in the Bird Base, four quarter-circles; and in the Frog Base, four quarter-circles plus one whole circle, making five sections in all. One, two, four, and five circles—these are the same numbers as the number of long flaps in each of the Classic Bases. (If we do the same to the Urchin pattern, we will find

25 circles or circular segments—and of course there were 25 flaps as well.) Clearly, there is some relationship here between circles and flaps. But why circles? And what about the paper that is not part of any circle? Circles seem rather innocuous, but by drawing them onto a crease pattern, we have touched on a connection to the underlying structure of origami, which we will soon explore.

4.4. Designing with Bases

The Japanese designers of the past—and most of their modern successors—did not worry about units and circles, of course. For most of the history of folding, the Classic Bases were nothing more than starting points for origami design; you picked the base that had the right number of flaps. For a bird with folded wings, use the Bird Base. A human figure, with two arms, two legs, and a head, uses the Frog Base. But what about more complex figures? Insects and arthropods, with six, eight, ten, or more legs, wings, horns, pincers, and other appendages, became an enormous challenge. As early as the 1950s, far-sighted origami designers made forays into these more complex bases. Yoshizawa, using a multiply blintzed base, produced his remarkable Crab with 12 appendages, while the sculptor George Rhoads exploited the blintzed Bird Base for several distinctive animals, including his famous Elephant. But these were the exceptions.

And so, the early days of origami design saw the use of the same bases over and over, to the point that they began to seem worn out. There are only so many treatments that can be applied to this small number of basic shapes. A few designers—notably Neal Elias and Fred Rohm—developed innovative manipulations of the Classic Bases that opened up rich new veins of origami source material. For the most part, however, the Classic Bases are pretty well picked over.

Still, one occasionally finds a shiny nugget of originality among the tailings of the Classic Bases. Sometimes, a model's structure simply calls for the flaps and proportions of a Classic Base, as in the designs shown in Figures 4.10–4.15, which are folded from the Windmill, Kite, Bird, and Frog Bases. Take, for example, the Stealth Fighter shown in Figure 4.10 as crease pattern and folded model. It is folded from the Windmill Base, which can be seen in its crease pattern.

Or can it? The crease pattern, which typically shows the major creases of the model, contains more creases than just those of the base. But if you focus on the longer creases, you can probably pick out the creases of a Windmill Base with some