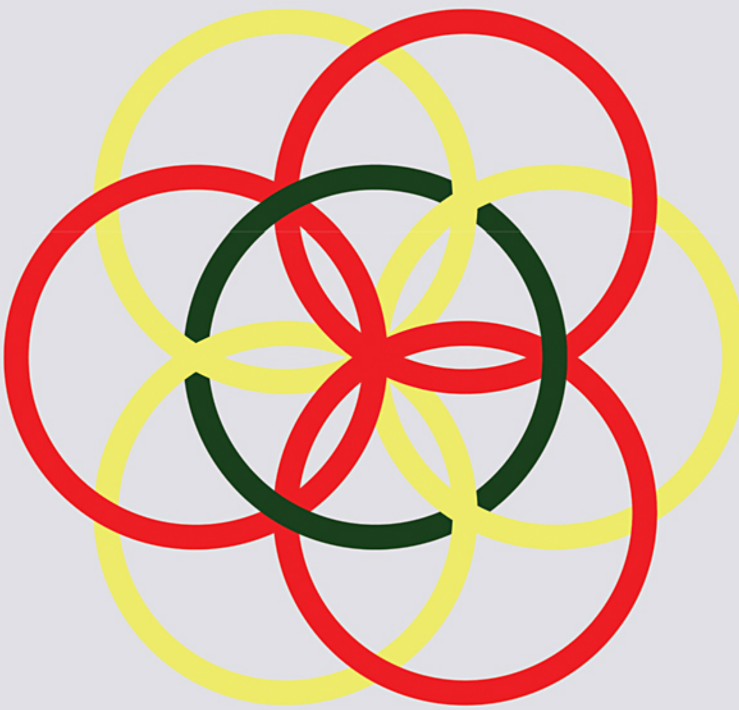
DISCRETE MATHEMATICS AND ITS APPLICATIONS

Series Editor KENNETH H. ROSEN

A STUDENT'S GUIDE TO THE STUDY, PRACTICE, AND TOOLS OF MODERN MATHEMATICS



Donald Bindner

Martin Erickson

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Preface

When you study mathematics at the university level, there is much to learn. You must learn a new language (the language of mathematics) and several new skills, including the use of mathematical software and other resources. You should also learn about "math culture." Wouldn't it be nice if there was a guidebook that you could turn to for help?

This is that book. Here you can find answers to such questions as: How do I study mathematics? How do I write a mathematical proof? How do I write a mathematical paper? How do I do mathematical research? How do I give a mathematical presentation? These issues, and many others, are discussed in short chapters on each topic. You can read a chapter in its entirety, or just get the facts you need and move on. Either way, you learn something new and clear a hurdle toward doing what you want to do mathematically.

You can get more out of your mathematical studies by knowing how to use mathematical tools. How do I compose a LaTeX file? How do I give a math talk using Beamer? How do I use a computer algebra system? How do I create mathematical diagrams? How do I display my mathematical ideas on a Web page? These are tasks that undergraduate and graduate students, as well as professional mathematicians, must deal with. This book explains the use of popular mathematical tools.

This book can help you succeed in your mathematical endeavors. It is a reference that you can consult again and again as you progress in your studies. When you are a beginning math student, you may be interested in learning how to study for math tests and not so interested in learning how to do mathematical research. As time goes on, you will want to engage in advanced activities, such as computer programming, mathematical typesetting, and mathematical research. This book can accompany you as you grow mathematically.

We (the authors) have written the kind of guide that we wish had existed when we were students. We usually had to seek out a "guru" on Linux or Maple or MATLAB, or read a different book on each subject. We would have been happy to find a lot of relevant information in an easy-to-use format. Accordingly, we cover topics that we believe will be of most benefit to most mathematics students. We hope that you will be saved many hours of unnecessary toil by our advice and tutorials. We have attempted to make learning fun by including exercises and challenges that we hope will stimulate your creativity and problem-solving ability. Mathematics is not going to be easy. As Euclid said, "There is no royal road to Geometry." But at least the road can have guideposts and tourist information centers along the way.

We have written this book to help students get started in mathematics. It is our way of saying "thank you" to the mentors who helped us learn what we are now passing along to you. Besides this general acknowledgment, it is a pleasure to thank Kenneth H. Rosen for encouragement, guidance, and contribution of exercises; Robert B. Stern for support and advice; Jerrold W. Grossman and Serge G. Kruk for reviews and suggestions; Daniel Jordan and Anthony Vazzana for input about what to cover; Linda Bindner and David Garth for proofreading; and the students in our Topics in Mathematics Education: Technology class for field testing. Finally, we wish you, the reader, the very best in your mathematical development.

Part I

The Study and Practice of Modern Mathematics

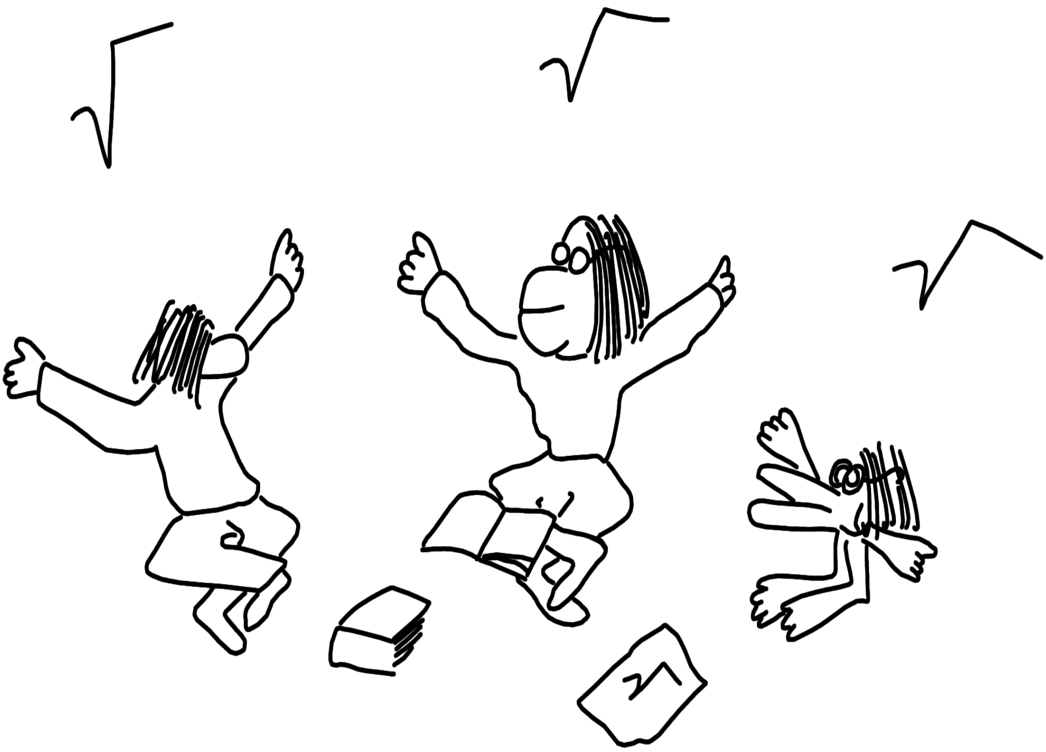
Introduction

What is it like being a mathematics student? How do you study mathematics? How do you write mathematics? How do you do mathematical research and present your findings? The following eight chapters give advice on how to be an outstanding mathematics major. The purpose of this discussion is to help you succeed in your coursework and be prepared to go on to graduate school or employment as a professional mathematician.

Learning mathematics is partly about learning to think like a mathematician. Mathematicians are interested in mathematical topics, such as geometry and algebra. Mathematicians also have a way of thinking logically about problems. As you study math, you will develop and enhance your mathematical thought processes. You may notice that concepts and problems that were once difficult for you become easier and you can explain them to others. Thus, you climb the ladder of mathematical attainment. We (the authors) hope that this book helps you achieve success in your mathematical study and practice.

Chapter 1

How to Learn Mathematics



The purpose of this chapter is to help you learn mathematics more efficiently, more thoroughly, and ultimately more enjoyably.

1.1 Why learn mathematics?

Most of us study mathematics because we are taking courses in it. But why are we enrolled in these courses? Ideally, it is because we are interested in mathematics, we are intrigued by mathematical problems, we like the lore of mathematics history (what we have heard of it so far), we appreciate the beauty of the subject, we are inspired by the possibility of applying mathematics to real-world situations, and we enjoy talking about mathematics with others, perhaps even teaching others. These are some of the reasons why people are motivated to study mathematics. Another valid reason is that there are employment opportunities in mathematics.

Given these reasons for studying mathematics, it is natural to ask: How can we study and learn mathematics most effectively? In this chapter, you will learn some basic techniques to help you increase your acquisition of knowledge, improve your course grades, and enhance your overall mastery of mathematics.

In our experience, students can dramatically improve their success in mathematics studies by following the pointers in this chapter. It takes practice and discipline, but you can do it!

Please see [50] for a good discussion of effective mathematics study. A good resource about beginning college-level mathematics studies is [24]. Some good textbooks on mathematical "foundations" are [51] and [46].

1.2 Studying mathematics

The key to studying any subject is to continually apply yourself, day by day. In mathematics, as in many other academic subjects, instructors build on the previous lessons in order to go on to the next material. **It is imperative that you keep current with reading and homework assignments**.

What are some other principles of good study practice in mathematics? Mathematics is an active endeavor. You can't learn it by watching it, listening to it, or reading it. You have to *do* it. Form the habit of reading with pen and paper, trying to find examples, counterexamples, even errors.

A good way to begin is by copying statements of definitions and theorems from your text, in order to scrutinize them. For example, write out the following definitions.

> Definition. A sequence $\{a_n\}$ of real numbers is *increasing* if $a_n < a_{n+1}$ for all $n \geq 1$.

> Definition. A sequence $\{a_n\}$ of real numbers is *bounded above* if there exists a real number M such that $a_n \leq M$ for all $n \geq 1$.

When you copy mathematical statements, you begin to memorize them and make them part of your mathematical thinking. **Your instructors expect you to know the definitions and theorems.**

When you encounter a new mathematical definition, think of an example that illustrates it. For instance, the sequence $\{n\}$ is increasing and the sequence $\{1/n\}$ is bounded above (by $M = 1$).

Notice that in both of the above definitions, the quantifier "for all" means that the statements must be true for all $n \geq 1$. If they fail for even one value of n, then a sequence doesn't have the stated property.

Definitions are key ingredients in theorems. Consider this theorem.

> Theorem. A sequence of real numbers that is increasing and bounded above converges.

When you encounter a new theorem, test it. Think of an instance when the theorem holds. The sequence $\{2 - (1/n)\}$ is increasing and bounded above (by $M = 2$), and it converges to 2. Also, think of an instance where the hypothesis of the theorem isn't satisfied and the conclusion of the theorem isn't true. What if the sequence isn't increasing? Does the conclusion still hold? What if the sequence isn't bounded above? Try to find a counterexample in these situations. Doing so will help you appreciate the importance of the two ingredients in the hypothesis of the theorem.

How does one remember all the definitions and theorems? The way to remember something is to apply it, and one way to apply your knowledge is by solving problems. Perhaps the most important activity in learning mathematics is problem solving. When you solve a problem, you put your understanding to a test.

Another way to learn definitions and theorems is by studying in groups. You can quiz each other on the statements of definitions and theorems. You can also check each others' homework solutions and learn from each other when you construct and write proofs.

1.3 Homework assignments and problem solving

As you progress to higher-level mathematics courses, you will find that your instructors expect you to write your homework assignments in complete sentences, with more complete explanations than you were used to giving in lower-level courses. Thus, you face two distinct but related tasks: solving the problems and writing the solutions well.

We'll talk more about mathematical writing in the next chapter, but we would like to emphasize the most important point here: **revise your work**. When you get a homework assignment, start trying to solve the problems right away. Write down your initial ideas. Then take a second look at what you've written and revise your explanations to make them easier to follow and more elegant. Plan to work in stages, solving more of the problems and revising your written work as you go along.

A good way to improve your performance on homework assignments is to pay close attention when you are attending a mathematics lecture or reading a mathematical explanation or proof. In particular, keep in mind that each step should have a HOW and a WHY. The HOW is the justification for a step. The WHY is why you are performing that particular step (the reason it leads to the overall solution or proof). You should incorporate this "how and why" mentality into your mathematical thinking and writing. Make sure that your audience (instructor or peers) knows how and why you are performing the steps that you are performing.

Problem solving is a skill (or art) best learned by practicing on examples. Let's consider a sample homework problem related to the definitions and theorem of the previous section.

Example 1.1. Problem. Let the sequence $\{a_n\}$ be defined by the recurrence formula $a_1 = 0$, and $a_n = \sqrt{a_{n-1} + 2}$, for $n \geq 2$. Prove that $\{a_n\}$ converges.

We use a calculator or computer to find approximate values for the first few terms of the sequence:

$$0,\ 1.4142,\ 1.8477,\ 1.9615,\ 1.9903,\ 1.9975,\ 1.999397637.$$

The idea of generating **data** and looking for patterns is very important in mathematical problem solving. Based on our data, it is natural to conjecture that the sequence is increasing and bounded above by 2. Moreover, the sequence apparently converges to 2.

If we are to use the theorem of the preceding section, then we must prove that the sequence $\{a_n\}$ is increasing and bounded above. How can we prove that the sequence is increasing? Remember the definition of an increasing sequence. A sequence $\{a_n\}$ is increasing if $a_n < a_{n+1}$ for all $n \geq 1$. By the definition of $\{a_n\}$, this inequality is equivalent to the inequality

$$\sqrt{a_{n-1} + 2} < \sqrt{a_n + 2}.$$

Simplifying terms, this becomes $a_{n-1} < a_n$, which is the same as the statement to be proved, but with smaller values of the index. This gives us the idea for a proof by mathematical induction.

Can you complete a proof by mathematical induction that the sequence $\{a_n\}$ is increasing? Can you also give a proof by mathematical induction that the sequence is bounded above by 2?

If you prove these two assertions, then it will follow by the theorem that the sequence $\{a_n\}$ converges. Furthermore, we can show that the sequence converges to 2 by taking the limit of both sides of the recurrence relation. Suppose that $\lim_{n\to\infty} a_n = L$. Then

$$\begin{aligned}
\lim_{n\to\infty} a_n &= \lim_{n\to\infty} \sqrt{a_{n-1} + 2} \\
L &= \sqrt{\lim_{n\to\infty} (a_{n-1} + 2)} \\
L &= \sqrt{\lim_{n\to\infty} a_{n-1} + 2} \\
L &= \sqrt{L + 2}.
\end{aligned}$$

Solving this equation, we find that $L = 2$.

Let's consider two more definitions and another sample problem.

> Definition. A *limit point* of a set of real numbers is a real number to which a sequence of other elements of the set converges.

> Definition. A set of real numbers is *closed* if it contains its limit points.

Notice the implicit quantifier *all* in the second definition. To be closed, a set must contain *all* its limit points. If there is even one limit point that it doesn't contain, then the set isn't closed. Furthermore, the definition doesn't require the set to actually have any limit points in order to be closed, only to contain them if there are any. For more on the logic of mathematical statements, a good resource is [47].

Example 1.2. Here is a sample homework problem based on the concept of a closed set.

Problem. Given sets of real numbers A and B, define $A + B = \{a + b \mid a \in A, b \in B\}$. If A and B are closed, is $A + B$ necessarily closed?

This is a "prove-or-disprove" type of problem. The time-tested solution method is to try to find a counterexample, for if you find one then you can report that the result doesn't always hold. On the other hand, as you search for a counterexample, you might start to believe that you can't find one because the result is true. So you try to prove that the result is true. If you get stuck in the proof, you can sometimes pinpoint the reason why you are stuck and this will help you find a counterexample.

Of course, to get anywhere on this problem, you have to know the definition of a closed set.

In trying to answer the problem in the affirmative (that is, prove the result), you may get stuck if the sets A and B are unbounded. Indeed, there is a simple counterexample of this type:

$$A = \{1, 2, 3, 4, \ldots\} \quad \text{and} \quad B = \left\{-1\frac{1}{2}, -2\frac{1}{3}, -3\frac{1}{4}, -4\frac{1}{5}, \ldots\right\}.$$

The sets A and B are closed (they have no limit points), but 0 is a limit point of the set $A+B$, yet 0 is not in this set. Hence, the set $A+B$ isn't closed.

There are several different types of mathematical proofs, e.g., proof by mathematical induction and proof by contradiction. When studying mathematics, you should cultivate the habit of paying attention to the kinds of proofs you learn. Try to understand their structures and you may be able to use the same structures in your own work, whether it's a homework assignment, a test, or mathematical research. More on the art and strategy of proofs can be found in [57].

Let's take a look at another problem.

Example 1.3. Problem. Find a formula for

$$1^3 - 3^3 + 5^3 - 7^3 + \cdots + (-1)^{n+1}(2n-1)^3, \quad n \geq 1.$$

We need data to solve this problem. Let $f(n)$ be the expression we are trying to find a formula for. In this case, "formula" means an expression without a summation. Here is a table of values of $f(n)$ for small n.

n	1	2	3	4	5	6	7	8
$f(n)$	1	−26	99	−244	485	−846	1351	−2024

We see that the values of $f(n)$ alternate in sign, and we also notice that n divides $f(n)$, for each value of n. Hence, it makes sense to consider the "reduced" values $f(n)/n$, as shown below.

n	1	2	3	4	5	6	7	8
$f(n)/n$	1	−13	33	−61	97	−141	193	−253

We see that the absolute values of these numbers are all odd numbers. So let's make a table of the absolute values minus 1.

n	1	2	3	4	5	6	7	8
$\lvert f(n)/n\rvert - 1$	0	12	32	60	96	140	192	252

It's unmistakable that these numbers are divisible by 4. So let's divide them by 4.

n	1	2	3	4	5	6	7	8
$(\lvert f(n)/n\rvert - 1)/4$	0	3	8	15	24	35	48	63

A little concentration reveals the pattern of these numbers: the nth number is $n^2 - 1$. Putting all our observations together, we make a conjecture:

Conjecture: $f(n) = (-1)^{n+1}n[4(n^2-1)+1], \quad n \geq 1.$

Knowing what the formula (probably) is, you can prove it by mathematical induction (see Chapter 2).

1.4 Tests

It can be scary facing a mathematics test, but preparation can make the task agreeable and even fun. The best advice we can give is to try to predict the test questions and make sure that you can answer them. Imagine the good feeling you will have if those problems are presented on the test and you have practiced the answers. Even if the same problems are not posed, you will have improved your math knowledge by studying similar problems or at least ones in the same ballpark.

You should make a special point of writing down the relevant definitions, theorems, and examples that you have covered in class. Your instructor expects that you know these, so don't disappoint her or him. Knowing the basics will help you take the next step and solve the problems based on these basics.

Finally, use all of the available time on a test. You would be surprised (like your instructors) at how many students don't do this.

1.5 Inspiration

Mathematics can be a rewarding, challenging, difficult, fascinating endeavor. We are privileged to be able to study and learn mathematics, for mathematics is one of the great creations of human minds. Mathematics is a growing body of knowledge that you can be a part of, too. Every time you learn mathematics, teach someone else, or discover something new, you are adding to the total sum of mathematical thought.

Let's conclude with some perspectives on mathematics:

- "The highest form of pure thought is in mathematics." (Plato)
- "We could use up two Eternities in learning all that is to be learned about our own world and the thousands of nations that have arisen and flourished and vanished from it. Mathematics alone would occupy me eight million years." (Mark Twain)
- "Pure mathematics is, in its way, the poetry of logical ideas." (Albert Einstein)
- "[At family celebrations] when it came time for me to blow out the candles on my birthday cake, I always wished, year after year, that [Hilbert's] Tenth Problem would be solved—not that I would solve it, but just that it would be solved. I felt that I couldn't bear to die without knowing the answer." (Julia Robinson)
- "Mathematics is like looking at a house from different angles." (Thomas Storer)
- "Pure mathematics is the world's best game. It is more absorbing than chess, more of a gamble than poker, and lasts longer than Monopoly. It's free. It can be played anywhere—Archimedes did it in a bathtub." (Richard J. Trudeau)

The authors wish to add a little advice: Work on your mathematics every day. When you're at rest, the ideas will continue to click and the next day you will go further and learn more.

Exercises

1. Copy the statements of five definitions and five theorems from one of your math textbooks. Identify the use of the defined words in the statements of the theorems. Give examples that illustrate the theorems. Show how the conclusions of the theorems don't necessarily hold if the hypotheses are not satisfied.

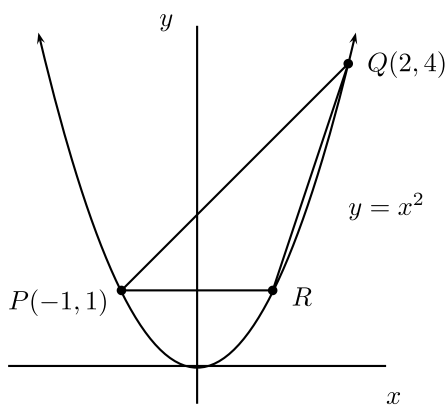
2. Examine the proofs of three or four mathematical theorems. What is the structure of these proofs? Identify where the hypotheses of the theorems are used in the proofs.

3. Look up quotes about mathematics or mathematicians. What are your favorites?

4. What are your "secrets" for learning mathematics? What works best for you?

5. Complete the proof outlined in Example 1.1.

6. Given points $P(-1, 1)$ and $Q(2, 4)$ on the parabola $y = x^2$, where should the point R be on the parabola (between P and Q) so that the triangle PQR has the maximum possible area?



7. A license consists of two digits (0 through 9), followed by a letter (A through Z), followed by another two digits. How many different licenses are possible?

8. Find and prove a formula for

$$1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{n+1}n^2, \qquad n \geq 1.$$

9. Prove the inequalities

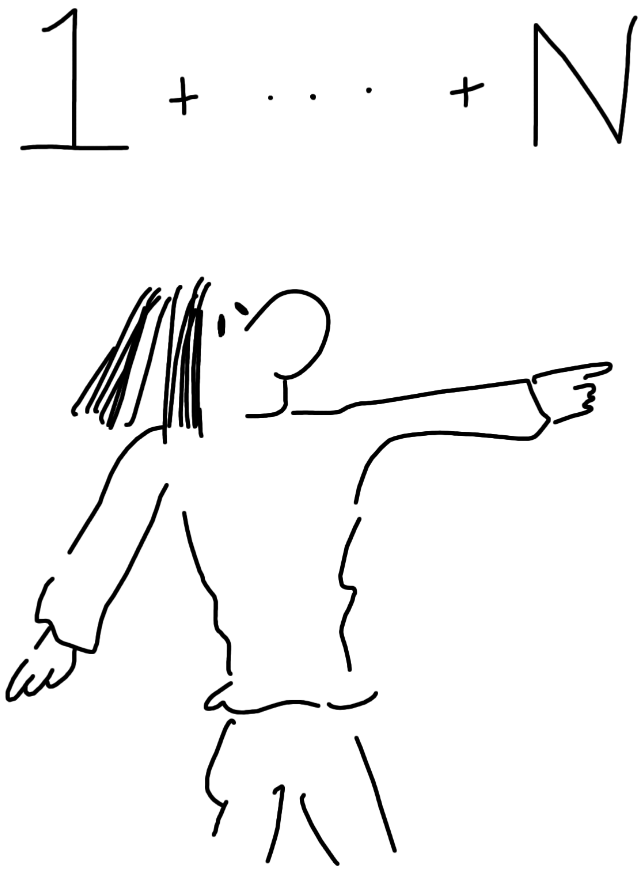
$$2\sqrt{n+1} - 2 < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} < 2\sqrt{n}, \qquad n \geq 1.$$

   Hint: Use integrals.

10. Which prime numbers are sums of two squares of integers? Hint: Remember to generate data.

Chapter 2

How to Write Mathematics



The purpose of this chapter is to help you write mathematical explanations and proofs. Good mathematical writing takes practice. It's also necessary to know some basic rules. Perhaps the most important feature of good mathematical writing is the revision process: writing and rewriting. This chapter discusses, with examples, the principles of mathematical writing.

2.1 What is the goal of mathematical writing?

As a mathematics student, you need to know many things in order to get started in the world of mathematics. These things include mathematical concepts, definitions, theorems, and proofs. Equally important is the knowledge of how to *write* your ideas, solutions, and proofs. Some students are surprised by this. They ask, "Why do I need to learn to write to do mathematics?" The answer is that writing is important in nearly all fields, and certainly in mathematics. You need to learn to write well so that others can follow your work.

The goal of mathematical writing is clear communication of mathematical ideas. Mathematical writing is accurate, precise, and concise. In general, writing is a skill that should be worked on and can be improved with practice. Furthermore, writing about a subject goes hand-in-hand with learning about the subject.

2.2 General principles of mathematical writing

Here we give an overview of the principles of mathematical writing. We will cover these principles in more detail in the later sections. Good resources on mathematical writing are [25] and [31].

Remember to practice the three most important principles that apply to all types of writing:

- Say something worthwhile.
- Proofread.
- Revise.

Mathematical writing has some further requirements:

- Write complete sentences. Writing complete sentences helps you to organize your thoughts and convey what you want to convey in the clearest way.
- Write accurately, precisely, and concisely. Don't write opinions, meaningless examples, or extraneous expressions. Avoid using the words "you" and "I" in math proofs.
- Avoid overly wordy and overly symbolic writing. Use a balance of words and symbols.
- Use mathematical terms and expressions properly.
- In long solutions or proofs, tell the reader in advance what you are trying to accomplish.
- Pay attention to all aspects of your writing: punctuation, spelling, mathematical content, readability, etc.

Keep your audience in mind, and write so that it is a pleasure for your audience to read your work.

2.3 Writing mathematical sentences

The basic unit of mathematical writing is the sentence. A good mathematical argument consists of sentences arranged in paragraphs that prove a theorem or give an example or