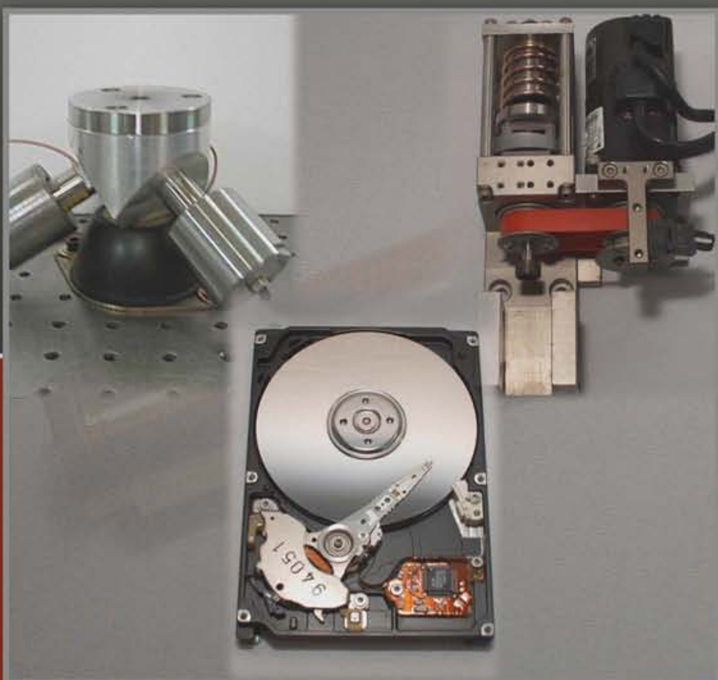
Piezoelectric Actuators

Control Applications of Smart Materials



Seung-Bok Choi
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Preface

The new generation of smart materials technology, featuring a network of sensors and actuators, control capability, and computational capability, will have a tremendous impact on the design and manufacture of the next generation of products in diverse industries such as aerospace, manufacturing, automotive, sporting goods, medicine, and civil engineering. Some classes of smart materials will be able to execute specific functions autonomously in response to changing environmental stimuli. Self-repair, self-diagnosis, self-multiplication, and self-degradation are some of the anticipated principal characteristics of the supreme classes of smart materials. These inherent properties of smart materials will only eventually be realized in practice by incorporating appropriate control techniques. Currently, there are several smart materials that exhibit one or more functional capabilities. Among them, electrorheological fluid, magnetorheological fluid, piezoelectric materials, and shape memory alloys are effectively employed in various engineering applications.

This book is a collection of our recent research and development on the control strategies of smart material systems using piezoelectric actuators and sensors. More specifically, this book is a reflection of prestigious refereed international journal papers that we have recently written. This book consists of eight chapters. Chapter 1 briefly describes the piezoelectric effect from a microscopic point of view and discusses some general requirements to achieve successful piezoelectric devices. Chapter 2 describes some control methodologies that are very effective in controlling systems that feature piezoelectric actuators. Chapter 3 focuses on the active vibration control of flexible structures utilizing piezoelectric actuators. A vibration control of a flexible beam is presented by implementing piezoceramic actuators associated with the quantitative feedback theory (QFT) control technique, in which system uncertainties such as nonlinear hysteresis behavior of the piezoactuators are treated. An active vibration control of hull structure, which is commonly used in aerospace and underwater vehicles, using self-sensing piezoelectric actuators is also presented by adopting the linear quadratic Gaussian (LQG) control technique. In addition, a hybrid mount featuring the passive rubber element and active piezoactuator is adopted to a vibration control of a flexible beam structure. Its control performance by adopting robust sliding mode controller (SMC) is presented at both resonant and nonresonant regions. Chapter 4 presents two vibration control cases utilizing different active mounts associated with piezoelectric actuators. In the first case, a one-axis active mount is used. An active mount associated with passive rubber element and piezostack actuators is introduced and implemented to suppress vertical vibration via sliding mode controller. In the second case, a three-axis active mount is used. Three inertial-type piezoelectric actuators are integrated with a rubber mount. Under consideration of practical dynamic systems, its vibration control performance is presented via the linear quadratic regulation (LQR) control algorithm. Chapter 5 deals with the effective control of various flexible robotic manipulators featuring piezoelectric actuators in their operating conditions. A flexible two-link manipulator, which has flexible links associated with piezoactuators, is adopted, and vibration-regulating and position-tracking controls are achieved by employing a sliding mode controller. A hybrid control scheme for a two-link flexible manipulator is realized by implementing servomotors for commanded motion and piezoactuators for vibration control during executed dynamic motions. Furthermore, a gantry-type robot featuring an X-Y table system and a flexible robot arm is introduced and its control is presented by adopting a loop-shaping H_∞ control technique. A piezoceramic is utilized as an actuator for the vibration suppression of the flexible arm while a bidirectional-type electrorheological (ER) clutch actuator is adopted to drive the X-Y table system. Competent position tracking control and vibration control are demonstrated for the required planar motion of an X-Y table system and a flexible robot arm, respectively. Chapter 6 presents two application cases of fine motion control system utilizing piezoelectric actuators. The first case uses a piezoactuator-driven optical pick-up for a CD-ROM (compact disc read-only memory) drive. A bimorph type of the piezoceramic actuator is employed to achieve accurate position tracking control of an optical pick-up device in a CD-ROM. This is accomplished by adopting a robust sliding mode controller. The second case uses a dual servo stage control system. Its fine motion is accomplished by a piezostack actuator associated with displacement amplifier, while its coarse motion is accomplished by a bidirectional ER clutch. A Preisach model–based feed-forward compensator with a proportional-integral-derivative (PID) feedback controller is employed to compensate for the hysteresis nonlinearity of the fine positioning system. On the other hand, a sliding mode controller with a friction compensator is adopted to achieve robust control performance in the coarse positioning stage. Chapter 7 presents two application cases of hydraulic control systems utilizing piezoelectric actuators. The first case depicts a hydraulic position control system. A hydraulic pump operated by the motion of a piezoactuator-driven diaphragm is introduced and integrated with a cylinder system. Its position control is accomplished by a sliding mode controller. The second case depicts a dispensing control system of micro-volume of liquid adhesives at high flow rate in chip-packaging processes. A jetting dispenser driven by piezostack with displacement amplification device is introduced and its control performance via a simple PID control algorithm is presented by evaluating the dispensing amount. Chapter 8 introduces piezoelectric shunt technology and its application to the vibration control of information storage device such as CD-ROM device and hard disk drive (HDD). The piezoelectric transducer converts the mechanical energy of the vibrating structure to electrical energy, which is then dissipated by Joule heating in the external shunt circuit networked to the piezoelectric material. In the CD-ROM drive, base structure is integrated with piezoelectric shunt circuit and shunt damping performance, such as vibration suppression, and is evaluated in both frequency and time domains. In the HDD, a piezoelectric bimorph, in which two piezoelectric annular plates are mounted on opposite sides of the very thin aluminum plate, is designed for drive shunt damping based on the dynamic analysis of HDD disk–spindle system. The shunt-damping performance of the rotating disk–spindle system is experimentally evaluated in frequency domain.

This book can be used as a textbook for graduate students who may be interested in the control methodology of smart structures and smart systems associated with piezoelectric actuators and sensors. The students, of course, should have some technical and mathematical background in vibration, dynamics, and control to be able to comprehend the content of this book. This book can also be used as a professional reference for scientists and practical engineers who would like to create new machines or devices featuring smart material actuators and sensors integrated with piezoelectric materials.

We would like to express our gratitude to the following individuals: Professors B. S. Thompson and M. V. Gandhi at Michigan State University, East Lansing, who were a source of knowledge in the field of smart material technology; Professor N. M. Wereley at the University of Maryland, College Park, who collaborated with us in the field of smart materials in recent years; and many talented graduates with MSs and/or PhDs from Smart Structures and Systems Laboratory in the Department of Mechanical Engineering at Inha University, Incheon, South Korea.

Many of the experimental results presented in this book are due to our research endeavors, which were funded by several agencies. We would like to acknowledge the financial support provided by the Korea Agency for Defense Development (Program Monitor, Dr. M. S. Suh), Center for Transportation System of Yellow Sea at Inha University (Director, Professor J. W. Lee), Center for Information Storage Device at Yonsei University, Seoul, South Korea (Director, Professor Y. P. Park), National Research Laboratory Program directed by Korea Science and Engineering Foundation, Korea Research Foundation, Protec Company, and Research Fund from Inha University.

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Young-Min Han received his PhD in mechanical engineering from Inha University, Incheon, South Korea, in 2005. Since 2006, he has been a research professor at Inha University. His current research interest includes the design and control of functional mechanisms utilizing smart materials such as active mounts, dispensing systems, shock absorbers, robotic manipulators, and human–machine interfaces. Dr. Han is the author of over 30 international journal papers and 20 international conference proceedings.

1 Introduction

1.1 PIEZOELECTRIC EFFECT

Since the discovery of the piezoelectric effect of crystals by Pierre and Jacques in 1880, a significant progress has been made both in terms of the material itself and its applications. Piezoelectricity is an electromechanical phenomenon that involves interaction between the mechanical (elastic) and the electrical behavior of a material. A typical piezoelectric material produces an electric charge or voltage in response to a mechanical stress, and vice versa. The former is known as the direct piezoelectric phenomenon, while the latter is known as the converse piezoelectric phenomenon. In the application of piezoelectric materials, the direct effect is normally used for sensing technology, while the converse effect is used for actuating technology. The direct and converse effects of commercial piezoelectric materials are achieved by a so-called poling process, which involves exposing the material to high temperatures while imposing high electric field intensity in a desired direction. Before the poling process, the piezoelectric material exhibits no piezoelectric properties, and it is isotropic because of the random orientation of the dipoles, as shown in Figure 1.1a. However, upon developing a poling voltage in the direction of the poling axis, the dipoles reorientate to form a certain class of anisotropic structures as shown in Figure 1.1b. Then, a driving voltage with a certain direction of polarity causes that the cylinder deforms. For example, a driving voltage with an opposite polarity to the poling axis causes that the cylinder elongates.

Macroscopically, piezoelectric materials exhibit field–strain relation, as shown in Figure 1.2. The relation is nearly linear for low electric field, which may provide many advantages when employing piezoelectric materials in system modeling and control realization. However, the polarization saturates at high electric field, and domains expand and switch. This causes significant nonlinear hysteresis behavior that can be detrimental when employing piezoelectric materials in control implementation associated with high electric field. In control implementation, the nonlinear hysteresis behavior is normally treated by means of two methodologies: feedforward compensator and closed-loop robust control scheme. The former is achieved by establishing an accurate nonlinear hysteresis model, while the latter is achieved by considering the hysteresis as actuator uncertainty. However, in many applications of piezoelectric materials to the continuous structures, the linear constitutive model is adopted despite the nonlinear hysteresis behavior at high electric field.

The direct and converse piezoelectric phenomena, involving an interaction between the mechanical behavior of a material, can be usefully modeled by linear constitutive equations involving two mechanical variables and two electrical variables. Thus, in

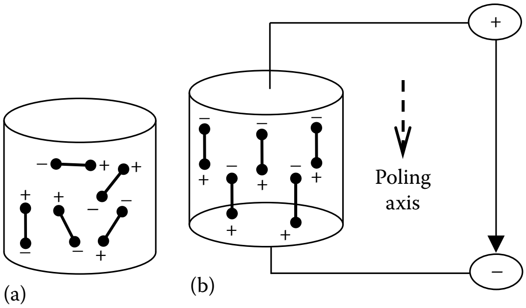

FIGURE 1.1 The micromechanism of the piezoelectric effect. (a) No voltage and (b) poling voltage.

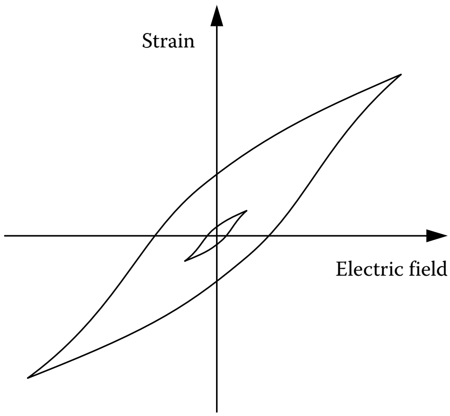

FIGURE 1.2 Field–strain relation of a typical piezoelectric material.

matrix form, the equations governing the direct piezoelectric effect and the converse piezoelectric effect are written, respectively, as

$$\{D\} = [e]^{\mathrm{T}}\{S\} + [\alpha^{\mathrm{S}}]\{E\} \tag{1.1}$$

$$\{T\} = [c^{\mathrm{E}}]\{S\} - [e]\{E\} \tag{1.2}$$

where

$\{D\}$ is the electric displacement vector
$[e]^{\mathrm{T}}$ is the transpose of the dielectric permittivity matrix $[e]$
$\{S\}$ is the strain vector
$[a^{\mathrm{S}}]$ is the dielectric matrix at constant mechanical strain
$\{E\}$ is the electric field vector
$\{T\}$ is the stress vector
$[c^{\mathrm{E}}]$ is the matrix of elastic coefficients at constant electric field strength

Equation 1.1 is the electrical expression governing an unstressed material subjected to an electrical field. Since the strain vector contains zeros, Equation 1.1 reduces to a relationship relating the field strength to electrical displacement. Equation 1.2 is the mechanical expression governing the material at zero field strength. Thus, since the electric field vector is only populated by zero elements, Equation 1.2 reduces to a relationship relating the stress and strain components of deformation. Since piezoelectric materials possess anisotropic properties, their mechanical and electrical behavior is dependent upon the direction of the external electric field relative to a set of axes fixed in the material. Thus, design methodologies involving piezoelectric materials must carefully accumulate these anisotropic features. It is noted that the direct relationship given by Equation 1.1 is normally used when modeling the sensing capability of the piezoelectric material, whereas the actuator capability is modeled using the converse relationship given by Equation 1.2.

1.2 GENERAL REQUIREMENTS FOR CONTROL DEVICES

So far, many natural and synthetic materials exhibiting piezoelectric properties have been proposed and developed. Natural materials include quartz, ammonium phosphate, paraffin, and bone, while synthetic materials include lead zirconate titanate (PZT), barium titanate, lead niobrate, lithium sulfate, and polyvinylidene fluoride (PVDF). Among these materials, PZT and PVDF are most popular and commercially available. Both classes of materials are available in a broad range of properties to suit diverse applications. PZT is normally used as actuators, while PVDF as sensors. One of the salient properties of PZT or PVDF is that it has very fast response characteristic to the voltage, and hence wide control bandwidth. In addition, we can fabricate simple, compact, low power–consuming devices featuring a set of piezoelectric actuators and/or sensors. Application devices utilizing piezoelectric materials include the vibration control of flexible structures such as the beam, the plate, and the shell; the noise control of cabin; and the position control of structural systems such as the flexible manipulator, the engine mount, the ski, the snowboard, the robot gripper, ultrasonic motors, and various types of sensors including the accelerometer, the strain gage, and sound pressure gages.

The successful development of a technology incorporating piezoelectric materials requires several issues. When we fabricate smart structures utilizing piezoelectric actuators and sensors, the following points have to be considered: the fabrication method (surface bonding or embedding), curing temperature in case of embedding, insulating between piezoelectric layers, and harness of electric wires. The important issues to be considered in the modeling of piezoelectric-based smart structures include structure dynamics, actuator dynamics, sensor dynamics, bonding effect, hysteresis phenomenon, the optimal location of actuators and sensors, and the number of actuators and sensors. Figure 1.3 presents a general block diagram of the control system featuring piezoelectric actuators and sensors. The control action is very similar to the conventional control system except for the high-voltage amplifier. The response time of the high-voltage amplifier, which normally has an amplification factor of 200, should be fast enough in order not to deteriorate the dynamic bandwidth of piezoelectric actuators. The microprocessor with analog-to-digital (A/D) and digital-to-analog (D/A) signal converters needs to have at least 12 bit, and also takes account of a high sampling frequency up to 10 kHz. Most of the currently available control algorithms for the piezoelectric actuators are realized in an active

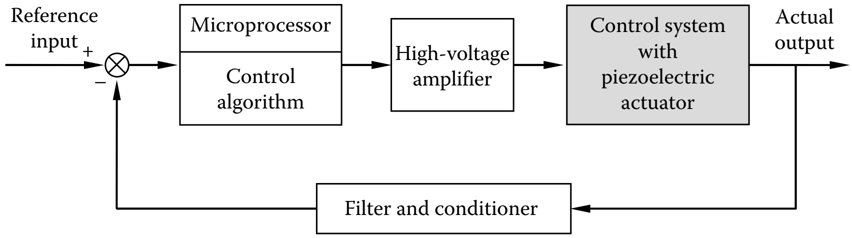

FIGURE 1.3 A typical control system featuring piezoelectric actuators.

manner. Potential candidates for active controller include negative velocity feedback controller, proportional–integral–derivative (PID) controller, optimal controller (linear quadratic regulator, LQR), sliding mode controller, H_∞ controller, and quantitative feedback theory (QFT) controller.

On the other hand, it is well known that each smart material actuator and sensor has diverse characteristics with distinct advantages and disadvantages. For instance, the piezoelectric actuator provides a very high broadband frequency response, but has relatively low control force compared with the shape memory alloy (SMA) actuator. Therefore, in order to achieve an optimal control performance under any constraints, such as weight, geometry, rigidity, dynamic bandwidth, sensitivity, and power consumption, a hybrid design philosophy of smart material actuators and sensors is required. By performing a judicious selection, control design engineers can synthesize numerous classes of hybrid actuating and/or sensing systems to satisfy a

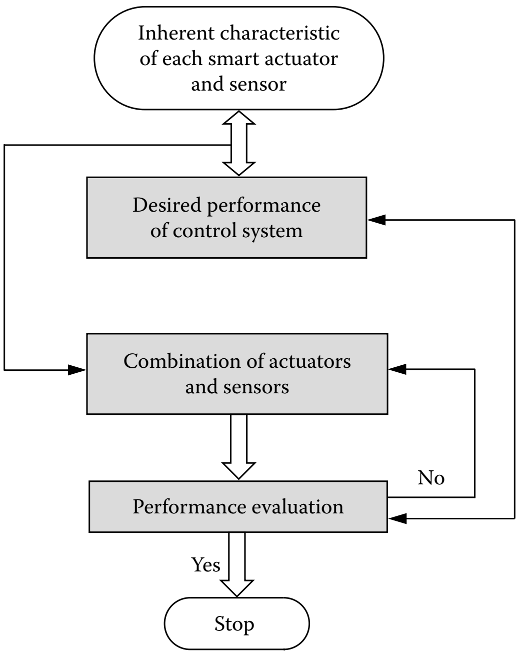

FIGURE 1.4 A design concept of hybrid actuating and sensing system.

broad range of specifications inaccessible by a single class of actuator and/or sensor system alone. Figure 1.4 presents a flowchart showing the design concept of a hybrid smart system consisting of more than two actuators and/or sensors. After thoroughly evaluating the inherent characteristics of each actuator and sensor, the desired control specifications are imposed under the operating condition. An appropriate combination of smart material actuator or sensor is then determined on the basis of the imposed specifications. This can be achieved by analyzing control performance, and hence comparing with the performance of a single actuating or sensing system. This indicates that the establishment of a set of variable analytical tools for predicting the control performance is a prerequisite for hybrid optimal control strategies.

In order to successfully develop a hybrid smart system, many factors need to be considered in the modeling and control process. These include actuating force, response time, cost, networkability, embeddability, linearity, sensitivity, and operating temperature. Moreover, an interaction phenomenon between more than two actuators and/or sensors needs to be carefully treated. Some difficulties that frequently occur in multi-output control systems should also be appropriately resolved. The hybridization of smart material actuators and sensors truly indicates an intelligent system that learns and adapts its behavior in response to the external stimuli provided by the environment in which it operates.

2 Control Strategies

2.1 INTRODUCTION

In general, an automatic controller compares the actual output of the plant with the reference input (desired value), and hence produces a control signal that will reduce the deviation between the actual and desired values to zero or to a small value. Figure 2.1 presents a block diagram of a typical control system that is integrated with piezoelectric actuators. The controller detects the actuating error signal that is usually at a very low power level, and amplifies it to a sufficiently high level via the voltage (or current) amplifier for the actuators. The actuator is a power device that produces the input to the application device according to the control signal. Piezoelectric actuators are used in the active control system. Control energy cannot only be taken away, but also be inserted into the plant with the active type of actuator. Most of the currently available sensors such as accelerometer can be adapted to measure dynamic responses of the control system associated with piezoelectric actuators. In this chapter, some control methodologies that are very effective in controlling the system featuring piezoelectric actuators are introduced.

2.2 PID CONTROL

One attractive controller to achieve a desired position of force using the piezoelectric control system is the proportional–integral–derivative (PID) controller. As well known, the PID controller is easy to implement in practice, but very effective with robustness to system uncertainties. The control action of each P, I, and D is shown in Figure 2.2. From the block diagram, the input is expressed by [1]

$$
\begin{aligned}
u(s) &= k_{\mathrm{p}}E(s), && \text{for P action} \\
u(s) &= \frac{k_{\mathrm{i}}}{s}E(s), && \text{for I action} \\
u(s) &= k_{\mathrm{d}}sE(s), && \text{for D action}
\end{aligned}
\tag{2.1}
$$

where
- s is the Laplace variable
- k_{p}, k_{i}, and k_{d} are control gains for the P, I, and D components, respectively
- $E(s)$ is the feedback error signal between the desired value and the actual output value

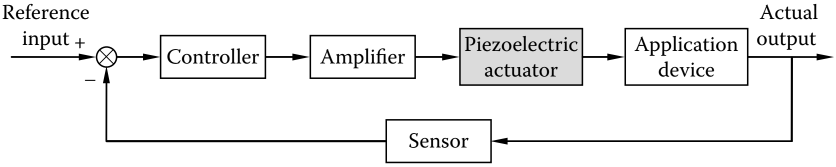

FIGURE 2.1 Block diagram of a typical control system featuring piezoelectric actuators.

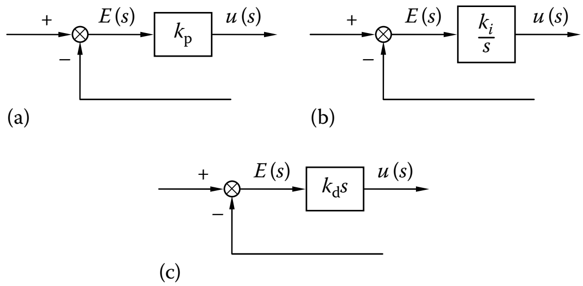

FIGURE 2.2 Control action of P, I, and D components. (a) P action, (b) I action, and (c) D action.

Consequently, the form of PID controller is given by

$$u(s) = k_p E(s) + \frac{k_i}{s} E(s) + k_d s E(s) \tag{2.2}$$

The P controller is essentially an amplifier with an adjustable gain of k_p. If the k_p is increased, the response time of the control system becomes faster. But instability of the control system may occur using very high feedback gains of k_p. The value of the control $u(t)$ is changed at a rate proportional to the actuating error signal $e(t)$ by employing the I controller. For zero actuating error, the value of $u(t)$ remains stationary. By employing the I controller action, the steady-state error of the control system can be effectively alleviated or eliminated. This is a very significant factor to be considered in the tracking control problem. In general, we can increase system stability by employing the D controller. However, the D control action may amplify noise signals and cause a saturation effect in the piezoelectric actuator. It is also noted that the D control action can never be implemented alone because the control action is effective only during transient periods. An appropriate determination of control gains k_p, k_i, and k_d to achieve superior control performance can be realized by several methods: the Ziegler–Nichols method, the adaptive method, and the optimal method.

2.3 LQ CONTROL

The linear quadratic (LQ) control is one of most popular control techniques that can be applied to many control systems including piezoelectric actuator-based control system. In this control method, the plant is assumed to be a linear system in the state space form and the performance index is a quadratic function of the plant states and control inputs. One of salient advantages of LQ control method is that it leads to linear control laws that are easy to implement and analyze. For the linear quadratic regulator (LQR) type optimal control, the following state equation is considered [2]:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \tag{2.3}$$

where
- $\mathbf{x}$ is the state vector
- $\mathbf{u}$ is the input vector
- $\mathbf{A}$ is the system matrix
- $\mathbf{B}$ is the input matrix

The impending problem is to determine the optimal control vector

$$\mathbf{u(t)} = -\mathbf{Kx(t)} \tag{2.4}$$

so as to minimize the performance index

$$J = \int_0^\infty (\mathbf{x^TQx} + \mathbf{u^TRu})\ \mathrm{d}t \tag{2.5}$$

where
- $\mathbf{Q}$ is the state weighting matrix (positive-semidefinite)
- $\mathbf{R}$ is the input weighting matrix (positive-definite)

The matrices $\mathbf{Q}$ and $\mathbf{R}$ determine the relative importance of the error and the expenditure of the control energy. If ($\mathbf{A}$, $\mathbf{B}$) is controllable, the feedback control gain is obtained by

$$\mathbf{K} = \mathbf{R^{-1}B^TP} \tag{2.6}$$

where $\mathbf{P}$ is the solution of the following algebraic Riccati equation:

$$\mathbf{A^TP} + \mathbf{PA} - \mathbf{PBR^{-1}B^TP} + \mathbf{Q} = \mathbf{0} \tag{2.7}$$

If the performance index is given in terms of the output vector rather than the state vector, that is,

$$J = \int_0^\infty (\mathbf{y^TQy} + \mathbf{u^TRu})\ \mathrm{d}t \tag{2.8}$$

then the index can be modified by using the output equation

$$\mathbf{y} = \mathbf{Cx} \tag{2.9}$$

to

$$J = \int_0^\infty (\mathbf{x^TC^TQCx} + \mathbf{u^TRu})\ \mathrm{d}t \tag{2.10}$$

The design step to obtain the feedback gain **K** that minimizes the index in Equation 2.10 is same as the step for the feedback gain **K** that minimizes the index in Equation 2.5. The LQ optimal control can be easily extended to the linear quadratic Gaussian (LQG) problem if the control system and the performance index are associated with white Gaussian noise as follows [2]:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu} + \Gamma w \\ \mathbf{y} &= \mathbf{Cx} + \nu\end{aligned} \tag{2.11}$$

$$\lim_{T\to\infty} \frac{1}{2T}\mathbf{E}\left\{\int_{-T}^{T} \left(\mathbf{x}^{\mathbf{T}}\mathbf{Qx} + \mathbf{u}^{\mathbf{T}}\mathbf{Ru}\right)\mathrm{d}t\right\} \tag{2.12}$$

where
 w stands for random noise disturbance
 v represents random measurement (sensor) noise

Both w and v are white Gaussian zero-mean stationary processes. It is noted that because the states and control are both random, the performance index will be random. Thus, the problem is to find the optimal control that will minimize the average cost. Using the same procedure as for the LQR problem, the solution is achieved as follows:

1. Controller

$$\begin{aligned}u &= -\mathbf{K}\tilde{\mathbf{x}} \\ \mathbf{K} &= \mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{P} \\ \mathbf{A}^{\mathbf{T}}\mathbf{P} + \mathbf{PA} &- \mathbf{PBR}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{P} + \mathbf{Q} = \mathbf{0}\end{aligned} \tag{2.13}$$

2. Estimator

$$\begin{aligned}\dot{\tilde{\mathbf{x}}} &= \mathbf{A}\tilde{\mathbf{x}} + \mathbf{Bu} + \mathbf{K_e}(\mathbf{y} - \mathbf{C}\tilde{\mathbf{x}}) \\ \mathbf{K_e} &= \mathbf{P_1}\mathbf{C}^{\mathbf{T}}\mathbf{R_1^{-1}} \\ \mathbf{AP_1} + \mathbf{P_1}\mathbf{A}^{\mathbf{T}} &- \mathbf{P_1}\mathbf{C}^{\mathbf{T}}\mathbf{R_1^{-1}}\mathbf{CP_1} + \Gamma\mathbf{W}\Gamma^{\mathbf{T}} = \mathbf{0}\end{aligned} \tag{2.14}$$

 where
 $\tilde{\mathbf{x}}$ denotes the estimated state
 K and $\mathbf{K_e}$ are the controller gain matrix and the Kalman filter gain matrix, respectively
 P and $\mathbf{P_1}$ are the positive-definite solutions of the Riccati equations
 W and $\mathbf{R_1}$ are covariance matrices of disturbance and noise, respectively

It is noted that the problem can be solved in two separate stages; controller gain **K** and estimator gain $\mathbf{K_e}$. Figure 2.3 shows the corresponding control block diagram.

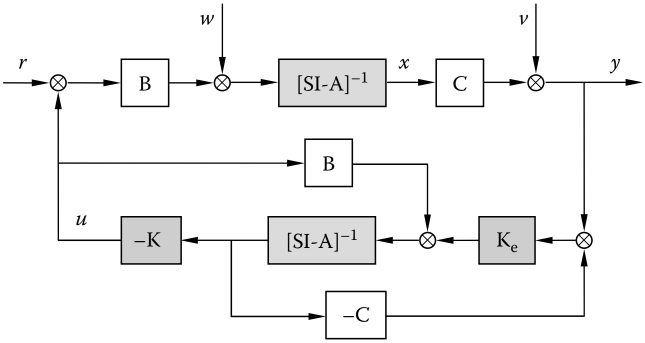

FIGURE 2.3 Control block diagram of the LQG.

2.4 SLIDING MODE CONTROL

Despite many advantages of the feedback control systems, there exist some system perturbations (uncertainties) associated with piezoelectric actuators. For instance, there exists nonlinear hysteresis in the behavior of piezoelectric actuators. Therefore, in order to guarantee control robustness of the control system featuring piezoelectric actuators, a robust controller needs to be implemented to take account for system uncertainties.

A sliding mode controller (SMC), also called variable structure controller, is well known as one of the most attractive candidates that assumes control robustness against system uncertainties and external disturbances. The SMC has its roots in the literature of the former Soviet Union. Today, throughout the world, the research and development on the SMC continue to apply it to a wide variety of engineering systems [3,4]. Sliding modes that can be obtained by appropriate discontinuous control laws are the principal operation modes in the variable structure systems. The systems have invariance properties to the parameter variations and external disturbances under the sliding mode motion. In order to demonstrate the invariance property under the sliding mode motion, consider the following second-order system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= cx_2 - kx_1, \quad c > 0, \quad k > 0 \quad \text{or} \quad k < 0 \end{aligned} \tag{2.15}$$

When $k<0$, the eigenvalues of the system become $\lambda_{1,2} = (c/2) \pm \sqrt{(c^2/4) - k}$. Therefore, the phase portrait of the system is a saddle showing unstable motion except stable eigenvalue linear (refer to Figure 2.4a). When $k > 0$, the eigenvalues become $\lambda_{1,2} = (c/2) \pm \sqrt{(c^2/4) - k}$. Thus, the phase portrait of the system is a spiral source showing unstable motion (refer to Figure 2.4b). If the switching occurs on the line $s_g = cx_1 + x_2 = 0$ and on $x = 0$ with the following switching logic,

$$k = \begin{cases} \text{positive}, & x_1 s_g < 0; \text{(I)} \\ \text{negative}, & x_1 s_g > 0; \text{(II)} \end{cases} \tag{2.16}$$

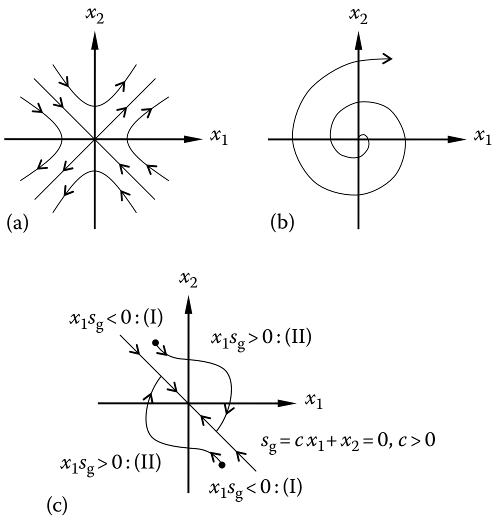

FIGURE 2.4 Invariance property of the SMC. (a) Saddle, (b) spiral source, and (c) with switching logic.

then the system becomes asymptotically stable for any arbitrary initial conditions as shown in Figure 2.4c. Two subsystems converge to a line s_g (called switching line [surface] or sliding line [surface]). Once hitting the sliding line, the system can be described by

$$s_g = cx_1 + x_2 = 0; \quad \text{sliding mode equation} \tag{2.17}$$

This implies that the original system response is independent of system parameters on the sliding line (sliding mode motion). This guarantees the robustness of the system to system uncertainties and external disturbances. In general, the sliding mode motion can be achieved by satisfying the following so-called sliding mode condition [3]:

$$s_g \dot{s}_g < 0 \tag{2.18}$$

The above condition can be interpreted as the condition for Lyapunov stability. In order to provide design steps for the SMC, consider the following control system subjected to the external disturbance:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= ax_1 + x_2 + u + d \end{aligned} \tag{2.19}$$

where
- d is external disturbance
- a is parameter variation

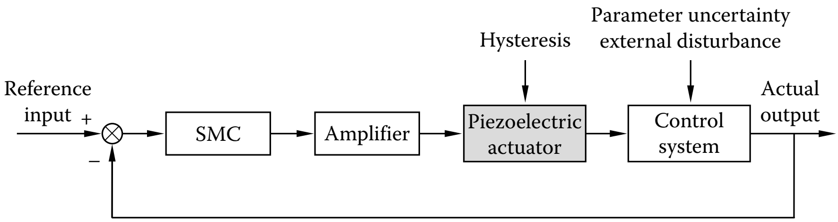

FIGURE 2.5 Control block diagram of the SMC.

These are bounded by

$$|d| \le \varepsilon, \quad a_1 \le a \le a_2 \tag{2.20}$$

As a first step, we choose a stable sliding line as follows:

$$s_g = cx_1 + x_2 = 0, \quad c > 0 \tag{2.21}$$

Then the sliding mode dynamics become

$$\dot{s}_g = cx_2 + ax_1 + x_2 + u + d \tag{2.22}$$

Thus, if we design the SMC, u, by

$$\begin{aligned} &u = -cx_2 - x_2 - a_0 x_1 s_g - \left(k + |a_m||x_1|\right)\mathrm{sgn}(s_g) \\ &k > \varepsilon, \quad a_0 = \frac{(a_1 + a_2)}{2}, \quad a_m = a_2 - a_0 \end{aligned} \tag{2.23}$$

The sliding mode condition in Equation 2.18 can be satisfied as follows:

$$s_g \dot{s}_g = (a - a_0)x_1 s_g - \left(k + |a_m||x_1|\right)|s_g| < 0 \tag{2.24}$$

In the controller given by Equation 2.22, k is the discontinuous control gain, and sgn(·) is a signum function. This design step can be easily extended to higher-order control systems [4]. Figure 2.5 represents the block diagram of SMC for a control system utilizing piezoelectric actuators.

2.5 H_∞ CONTROL

H_∞ controller is a robust control technique that can be applicable to piezoelectric actuator-based control systems. In this control method, all of the information about the system is expressed in linear fractional transformation (LFT) framework as follows [5]:

$$\begin{bmatrix} z \\ y \end{bmatrix} = P \begin{bmatrix} w \\ u \end{bmatrix}, \quad u = Ky \tag{2.25}$$