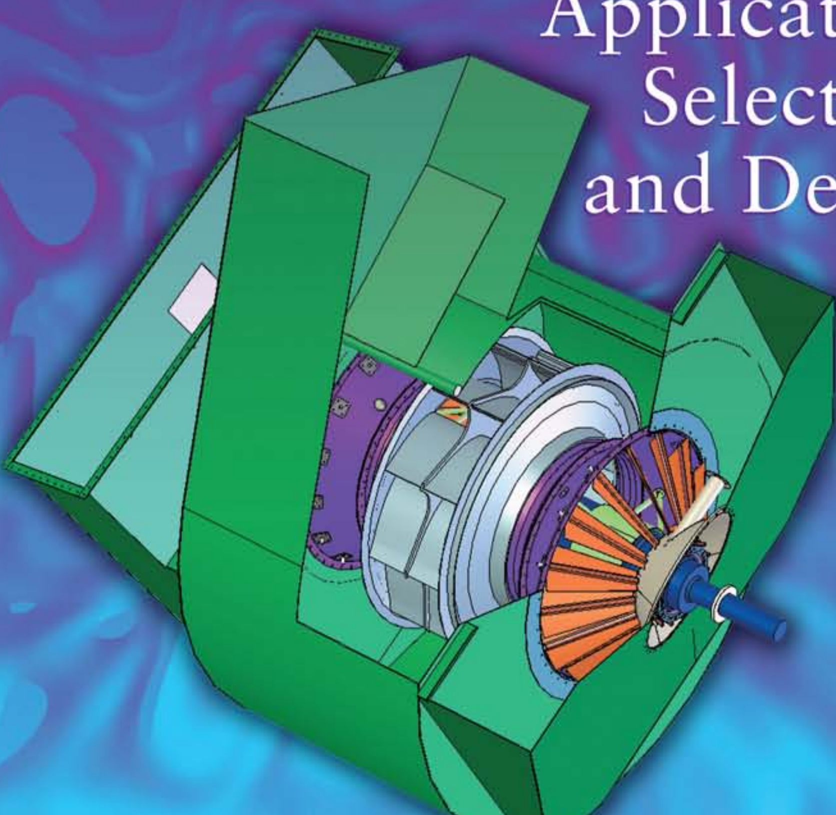
SECOND EDITION

FLUID MACHINERY

Application, Selection, and Design



Terry Wright
Philip M. Gerhart

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Preface

This book is in fact the second edition of *Fluid Machinery: Performance, Analysis and Design* by Terry Wright. The subtitle change is thought to more adequately reflect the emphases of both the current work and the first edition. Unlike most books on the subject, which seem to emphasize only design of turbomachinery, at least half of the current work is dedicated to the more widespread engineering tasks of application of turbomachines and selection of the proper machine for a particular application.

The preface to the first edition lays out the philosophy for the work in some detail and there is no need to repeat it here as there have been few changes. What has changed is the micro-organization of the material. There has been substantial reorganization within the chapters, hopefully allowing a more logical flow for the learner. It would be fair to say that the reader, be it a university student or a practicing engineer, has been at the front of our minds during this revision.

A particularly vigorous effort has been made with the mathematical symbols, with the aim of keeping notation consistent within and across chapters. At times, this has required us to abandon customary usage in the field. Probably, the best example is the use of the vector-diagram angle β. Traditional use has β as the angle between the relative vector and the blade speed for radial flow machines and as the angle between the relative vector and the axial (throughflow) direction in axial machines. We decided to keep β consistently as the relative velocity–speed angle in order to avoid confusion.

As might be expected, we have updated the material where appropriate. This presents a particular challenge in the area of computational fluid dynamics (CFD) in turbomachinery. There has certainly been astounding growth in this field over the years since the first edition was published in 1999; however, even the leaders in this field state that a good design must begin with an essentially one-dimensional layout of the type that we emphasize. As a result, we only "point the way" toward using CFD for turbomachinery design and analysis.

A particularly strong feature of the book is the inclusion of a significant number of exercise problems at the end of the chapters. There are nearly 350 problems, with about a third of them new to this edition. A solutions manual is also available to instructors.

Preface to the First Edition

The purpose of this book is to provide a fairly broad treatment of the fluid mechanics of turbomachinery. Emphasis is placed on the more utilitarian equipment, such as compressors, blowers, fans, and pumps, that will be encountered by most mechanical engineers as they pursue careers in industry. This emphasis is intended to allow the text to serve as a useful reference or review book for the practicing engineer. Both gas and hydraulic turbines are considered for completeness, and the text inevitably includes material from the large literature on gas turbine engines. These machines traditionally have been treated as aerospace equipment and are considered at length in the literature (Oates, 1984; Wilson, 1984; Oates, 1985; Bathie, 1996; Lakshiminarayana, 1996; Mattingly, 1996). Although recent developments in power generation for either load-peaking, distributed generation or process cogeneration have significantly increased the chances that an engineering graduate will encounter gas turbine engines, this text will focus primarily on the more commonly encountered industrial equipment.

The performance parameters of fluid machinery are carefully developed and illustrated through extensive examples. The relationship of the inherent performance of a machine, in terms of the flow rate, head change, and sound power or noise generation through the rotating impeller, is discussed and treated as it relates to the fluid system with which the machine interacts. The dependence of machine performance on the resistance characteristics of the fluid system is emphasized throughout by examining the machine and the system simultaneously through the text. The characteristic sound pressure and sound power levels associated with a fluid machine are treated in this text as a basic performance variable—along with flow and pressure change.

The fundamental relationship between the shape and internal geometry of a turbomachine impeller and its inherent performance is treated from the beginning of the text. In the early chapters, the shape and size of a machine are related through the concepts of similarity parameters to show how the head and the flow combine with shape and size to yield unique relationships between the geometry and performance. The development of these "specific" speed, noise, and size relations is set out in an empirical, traditional manner as correlations of experimental data. The concepts are used to achieve a basic unification of the very broad range and variety of machine types, shapes, and sizes encountered in engineering practice.

In the later chapters, the theme of geometry and performance is continued through the approximate treatment of the flow patterns in the flow passages of the machine. The fundamental consideration of the equations for mass and angular momentum leads to the governing relations for turbomachinery flow and performance. Again, the process is related to the machine geometry, size, speed, and flow path shape. This higher level of detail is related as closely as possible to the overall consideration of size and speed developed earlier.

Following extensive examples and design exercises for a broad range of equipment, applications, and constraints, the later chapters begin tightening the rigor of the calculational process. The simplifying assumptions used to develop the earlier illustrations of fundamental performance concepts are replaced with a more complex and more rigorous analysis of the flow fields. The more thorough treatment of the flow analyses provides a more realistic view of the complexity and difficulties inherent in understanding, analyzing, and designing turbomachinery components.

Following the development of greater rigor and greater calculational complexity, near the end of the text, some of the more vexing problems associated with turbomachinery analysis and design are introduced as advanced design topics. The influence of very low Reynolds numbers and high levels of turbulence intensity is considered as they influence the design and geometric requirements to generate specified levels of performance. Limitations on performance range and acceptable operation are introduced in these later chapters through consideration of compressibility, instability, and stalling phenomena and the inherent degradation of machine and system interaction.

Some attention is given throughout the text to the need to apply advanced analytical techniques to turbomachinery flow fields in a final design phase. However, the more approximate techniques are emphasized throughout most of the book. Here, the sense of a preliminary design approach is employed to promote a basic understanding of the behavior of the machinery and the relation between performance and geometry. The final chapter provides an overview of the calculational techniques that are being used to provide a rigorous, detailed analysis of turbomachinery flows. Beginning with a reasonable geometry, these techniques are used to examine the influence of detailed geometric refinements and allow the designer to achieve something of an optimized layout and performance for a machine. The chapter is used to emphasize the importance of current and future computational capabilities and to point the reader toward a more rigorous treatment of fluid mechanics in machine design.

Throughout most of the text, the examples and problem exercises are either partially or totally concerned with the design or selection process. They deal with system performance requirements or specifications, along with size, speed, cost, noise, and efficiency constraints on the problem solution. The purpose of this pragmatic design approach to turbomachinery applications is to expose the reader—either a student or a practicing engineer—to the most realistic array of difficulties and conflicting requirements possible within the confines of a textbook presentation. By using examples from a fairly large range of industrial applications, it is hoped that the reader will see the generality of the basic design approach and the common ground of the seemingly diverse areas of application.

Authors

Philip M. Gerhart holds a BSME degree from Rose-Hulman Institute of Technology and MS and PhD degrees from the University of Illinois at Urbana-Champaign. He is a registered professional engineer in Indiana and Ohio. He was a professor of mechanical engineering at the University of Akron from 1971 to 1984, chair of the Department of Mechanical and Civil Engineering at the University of Evansville from 1985 to 1995, and has been dean of the College of Engineering and Computer Science since 1995.

Dr. Gerhart has written two books and more than 35 scholarly papers and reports. He has been principal investigator on grants from the United States Army, NASA, the National Science Foundation, and the Electric Power Research Institute. He has served as a consultant to several firms in the power and process industries. He serves as an associate director of the Indiana Space Grant Consortium.

Dr. Gerhart is a member of the American Society for Engineering Education and a fellow member of the American Society of Mechanical Engineers. He served as ASME's vice-president for Performance Test Codes from 1998 to 2001. He has served many years on the Performance Test Codes Standards Committee and the technical committees on fans and fired steam generators. He was awarded the ASME's Performance Test Codes Gold Medal in 1993 and the Silver Beaver award from the Boy Scouts of America in 2001.

Terry Wright holds BS, MS, and PhD degrees in aerospace engineering from the Georgia Institute of Technology and is a registered professional engineer (retired) from Alabama. He initially joined the Westinghouse Research Laboratories and served there for many years as a research scientist and fellow engineer. Much of his effort in this period was in working with the Sturtevant Division of the Westinghouse Corporation, involved with their design and manufacture of turbomachinery.

Dr. Wright became a professor of mechanical engineering at the University of Alabama at Birmingham in the mid-1980s and was active in teaching and mentoring in fluid mechanics and applications in turbomachinery and minimization of turbomachinery-generated noise. While at the university, he consulted with industrial manufacturers and end users of turbomachinery equipment. In addition to his academic and research activities, he also served as chairman of the department of mechanical engineering through most of the 1990s.

He has acted as a technical advisor to government and industry, has published over 90 research and industrial reports (of limited distribution), and has also published over 40 papers in engineering journals, pamphlets, and proceedings of the open literature. He has been active on technical committees on turbomachinery and turbomachinery noise in the American Society of Mechanical Engineers.

Dr. Wright has served as an emeritus professor of the University of Alabama at Birmingham and is active in writing and society activities. He continues to interact with the manufacturers and industrial users of turbomachinery equipment and is a current advisor/consultant to the ASME PTC 11 committee on fan inlet flow distortion.

1

Introduction

1.1 Preliminary Remarks

For convenience of review and quick reference, this introduction includes the basic fundamentals of thermodynamics and fluid mechanics needed to develop and manipulate the analytical and empirical relationships and concepts used in turbomachinery. The standard nomenclature for turbomachinery will be used where possible, and the equations of thermodynamics and fluid mechanics will be particularized to reflect practice in the industry.

1.2 Thermodynamics and Fluid Mechanics

The physics and properties of the common, simple fluids considered in this book include those of many gases, such as air, combustion products, dry steam, and others, and liquids, such as water, oils, petroleum products, and other Newtonian fluids transported in manufacturing and energy conversion processes. These fluids and the rules governing their behavior are presented here in terse fashion for only the simple working fluids, as needed for examples and problem solving in an introductory context.

More complete coverage of complex fluids and special applications is readily available in textbooks on thermodynamics and fluid mechanics, as well as in more specialized engineering texts and journals. See, for example, White (2008); Fox et al. (2009); Munson et al. (2009); Gerhart et al. (1992); Van Wylen and Sonntag (1986); Moran and Shapiro (2008); Baumeister et al. (1978); the journals of the American Society of Mechanical Engineers and the American Institute of Aeronautics and Astronautics, and the *Handbook of Fluid Dynamics and Turbomachinery*, Schetz and Fuhs, 1996. Here, there will be no extensive treatment of multiphase flows, flows of mixtures such as liquid slurries or gas-entrained solids, fluids subject to electromagnetic effects, or ionized or chemically reacting gases or liquids.

1.3 Units and Nomenclature

Units will generally be confined to the International System of units (SI) and British Gravitational system (BG) fundamental units as shown in Table 1.1. Unfortunately, turbomachinery performance variables are very frequently expressed in industry-specific units that will require conversion to the fundamental unit systems. Units for some of these performance parameters, based on pressure change, throughflow, and input or extracted power, are given in Table 1.2. Conversion factors between the more common units are available in Appendix B.

As seen in Table 1.2, often the units are based on instrument readings, such as manometer deflections, or electrical readings rather than the fundamental parameter of interest. As an engineer, one must deal with the nomenclature common to the particular product or industry at least some of the time. Hence, this book will include the use of gpm (gallons per minute) for liquid pumps, hp [horsepower = (ft × lb/s)/550] for shaft power, and some others as well. However, this book will typically revert to fundamental units for analysis and design and convert to the traditional units if necessary or desirable.

TABLE 1.1

Fundamental Units in SI and BG

Length	Meter (m)	Foot (ft)
Mass	Kilogram (kg)	Slug (slug)
Time	Second (s)	Second (s or sec)
Temperature	Kelvin (K)	°Rankine (°R) or °F
Force	Newton (N); $N = kg \times m/s^2$	Pound (lb); $lb = slug \times ft/s^2$
Pressure	Pascal (Pa); $Pa = N/m^2$	lb/ft^2
Work	Joule (J); $J = N \times m$	ft × lb
Power	Watt (W); $W = J/s$	ft × lb/s

TABLE 1.2

Traditional (Industrial) Performance Units in BG and SI

Pressure	lb/ft^2 (psf); in. wg; in. Hg; lb/in^2 (psi)	N/m^2 (Pa); mm H_2O; mm Hg
Head	foot (ft)	m; mm
Volume flow rate	ft^3/s (cfs); ft^3/min (cfm); gal/min (gpm)	m^3/s; l/s (liter $l = 10^{-3}$ m^3); cc/s
Mass flow rate	slug/s; lbm/s	kg/s
Weight flow rate	lb/s; lb/hr	N/s; N/min
Power	Watts; kW; hp; ft × lb/s	N × m/s; J/s; kJ/s; Watts; kW

1.4 Thermodynamic Variables and Properties

The variables and properties frequently used include the state variables: pressure, temperature, and density (p, T, and ρ). They are defined, respectively, as: p is the average normal stress in the fluid; T is a measure of the internal energy in the fluid (actually, a measure of the kinetic energy of molecular motion); and ρ is the mass per unit volume of the fluid (in thermodynamics, the specific volume, $v = 1/\rho$, is more often used). These are the fundamental variables that define the state of the fluid. When considering (as one must) work and energy inputs to the fluid, then the specific energy (e), internal energy (u), enthalpy ($h \equiv u + p/\rho$), entropy (s), and specific heats of the fluid (c_{p} and c_{v}) must be included. Consideration of fluid friction and heat transfer will involve two transport properties: the viscosity (μ) and the thermal conductivity (κ). In general, all of these properties are interrelated in the state variable functional form, for example, $\rho = \rho(p, T)$, $h = h(T, p)$, and $\mu = \mu(T, p)$; that is, they are functions of the state properties of the fluid.

The internal energy u is a measure of the thermal energy of the fluid; the specific energy, e, includes thermal energy as well as potential and kinetic energies, such that $e = u + V^2/2 + gz$. Here, g is the gravitational force per unit mass, commonly referred to as the "acceleration due to gravity," V is the local velocity, and z is a coordinate above a specified datum (positive in the upward direction).

For gases, this book restricts attention to those gases whose behavior can be described as thermally and calorically perfect. That is, the properties are related according to $p = \rho RT$, $u = c_{\mathrm{v}}T$, and $h = c_{\mathrm{p}}T$ with R, c_{v}, and c_{p} constant properties of the particular fluid. R is defined in terms of the molecular weight, M, of the gas; $R = R_{\mathrm{u}}/M$. R_{u} is the universal gas constant, $8310\,\mathrm{m}^2/(\mathrm{s}^2\,\mathrm{K})$ in SI units and $49{,}700\,\mathrm{ft}^2/(\mathrm{s}^2\,{}^\circ\mathrm{R})$ in BG units. Appendix A provides limited information on these and other fluid properties for handy reference in examples and problem solving. R, c_{p}, and c_{v} are related by $R = c_{\mathrm{p}} - c_{\mathrm{v}}$ and by the specific heat ratio, $\gamma = c_{\mathrm{p}}/c_{\mathrm{v}}$. Other relations between the gas constants are $c_{\mathrm{v}} = R/(\gamma - 1)$ and $c_{\mathrm{p}} = \gamma R/(\gamma - 1)$. For most turbomachinery air-moving applications, one can accurately assume that c_{v}, c_{p}, and γ are constants, although for large temperature excursions, such as that might occur in a high-pressure compressor or gas turbine, c_{v}, c_{p}, and γ increase with temperature.

In addition to these thermodynamic state variables, it is necessary to define the "real-fluid" transport properties: dynamic and kinematic viscosity. Dynamic viscosity, μ, is defined in fluid motion as the constant of proportionality between shear stress and strain rate in the fluid. For simple shear flows, the relationship is $\tau = \mu(\partial V/\partial n)$, where n is the direction normal to the velocity V. Dynamic viscosity has the units of stress over velocity gradient (i.e., $\mathrm{Pa} \times \mathrm{s}$ or $\mathrm{lb} \times \mathrm{s}/\mathrm{ft}^2$). Dynamic viscosity is virtually independent of pressure for most fluids, yet it can be a fairly strong function of the fluid temperature. Typical variation of dynamic viscosity in gases can be approximated in a power law form such as $\mu/\mu_0 \approx (T/T_0)^n$. For air, the values $n = 0.7$ and $T_0 = 273\,\text{K}$ can be used with $\mu_0 = 1.71 \times 10^{-5}\,\text{kg/m s}$. Other approximations are available in the literature, and data are presented in Appendix A. The kinematic viscosity, $\nu \equiv \mu/\rho$, is useful in incompressible flow analysis and is convenient in forming the Reynolds number, $Re = \rho V d/\mu = V d/\nu$.

Liquid viscosities decrease with increasing temperature, and White (2008) suggests the following as a reasonable estimate for pure water:

$$\ln\left(\frac{\mu}{\mu_0}\right) = -1.94 - 4.80\left(\frac{273.16}{T}\right) + 6.74\left(\frac{273.16}{T}\right)^2, \tag{1.1}$$

with $\mu_0 = 0.001792\,\text{kg/ms}$ and T in Kelvin (within perhaps 1% error).

When dealing with the design and selection of liquid-handling machines, the prospect of "vaporous cavitation" within the flow passages of a pump or turbine is an important consideration. The critical fluid property governing cavitation is the vapor pressure of the fluid. The familiar "boiling point" of water (100°C $\sim$ 212°F) is the temperature required to vaporize water at *standard atmospheric pressure*, 101.3 kPa. The vapor pressure of water (or any other liquid) is lower at lower temperatures, so boiling or cavitation can occur with a reduction in the pressure of a liquid, even at temperatures near atmospheric. As pressure is prone to be significantly reduced in the entry (or suction) regions of a pump, if the fluid pressure becomes more or less equal to the vapor pressure of the fluid, boiling or cavitation may commence there. The vapor pressure is a strongly varying function of temperature. For water, p_v ranges from nearly zero (0.611 kPa) at 0°C to 101.3 kPa at 100°C. Figures A.3 and A.4 showing variation of p_v with T are given in Appendix A for water and some fuels. A rough estimate of this functional dependence (for water) is given as $p_v \approx 0.61 + 10^{-4}T^3$ (p_v in kPa and T in °C). This approximation (accurate to only about 6%) illustrates the strong nonlinearity that is typical of liquids. Clearly, fluids at high temperature are easy to boil with pressure reduction and cavitate readily.

The absolute fluid pressure associated with the onset of cavitation depends, in most cases, on the local barometric pressure. Recall that this pressure varies strongly with altitude in the atmosphere and can be modeled, using elementary hydrostatics, as $p_b = p_{SL}\exp\left[(-g/R)\int \mathrm{d}T/T(z)\right]$, integrated from sea level to the altitude z. The function $T(z)$ is accurately approximated by the linear lapse rate model, $T = T_{SL} - Bz$, which yields $p_b = p_{SL}(1 - Bz/T_{SL})(g/RB)$. Here, $B = 0.0065\,\text{K/m}$, $p_{SL} = 101.3\,\text{kPa}$, $g/RB = 5.26$, and $T_{SL} = 288\,\text{K}$. This relation allows approximation of the absolute inlet-side pressure for pump cavitation problems with vented tanks or open supply reservoirs at a known altitude.

1.5 Reversible Processes, Irreversible Processes, and Efficiency with Perfect Gases

In turbomachinery flows, not only must the state of the fluid be known, but also the process or path between the end states is of interest in typical expansion and compression processes. Because turbomachinery flows are essentially adiabatic, the ideal process relating the end-state variables is the isentropic process. Recalling the combined first and second law of thermodynamics,

$$T\,\mathrm{d}s = \mathrm{d}h - \frac{\mathrm{d}p}{\rho}, \tag{1.2}$$

and using the perfect gas relations (c_p and c_v constant, $\mathrm{d}h = c_\mathrm{p}\,\mathrm{d}T$, $\mathrm{d}u = c_\mathrm{v}\,\mathrm{d}T$, $R = c_\mathrm{p} - c_\mathrm{v}$, and $p = \rho RT$), this becomes

$$\mathrm{d}s = \frac{c_\mathrm{p}\,\mathrm{d}T}{T} - \frac{R\,\mathrm{d}p}{p}. \tag{1.3}$$

On integration between end states 1 and 2, one can write the change in entropy as

$$s_2 - s_1 = c_\mathrm{p}\,\ln\left(\frac{T_2}{T_1}\right) - R\,\ln\left(\frac{p_2}{p_1}\right). \tag{1.4}$$

If the fluid flow process is adiabatic and reversible (without heat addition or friction), the process is isentropic, $s_2 - s_1 = 0$ and

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}. \tag{1.5}$$

This relationship between the fluid properties has the form of the well-known *polytropic process*, for which

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{T_2}{T_1}\right)^{n/(n-1)} = \left(\frac{\rho_2}{\rho_1}\right)^{n}. \tag{1.6}$$

Many different processes can be described by varying the exponent n; for example, $n = 1$ represents an isothermal process, $n = 0$ represents an isobaric (constant pressure) process, $n = \gamma$ represents an isentropic process, and $n = \infty$ represents an isochoric (constant volume) process. Any *reversible* process with $n \neq \gamma$ would involve heat transfer.

All real processes are irreversible because of fluid friction, turbulence, mixing, and so on. A real process is characterized by an *efficiency*, η:

For a compression (pumping) process, $\eta \equiv$ (work input in ideal [reversible] process/work input in real process).

For an expansion (turbine) process, $\eta \equiv$ (work output in real process/work output in ideal [reversible] process).

Limiting consideration for the time being to compression processes, the *isentropic efficiency* is defined by

$$\eta_{\mathrm{s,compression}} \equiv \frac{w_{\mathrm{s}}}{w} = \frac{c_{\mathrm{p}} T_1 [(p_2/p_1)^{(\gamma-1)/\gamma} - 1]}{c_{\mathrm{p}}(T_2 - T_1)} = \frac{(p_2/p_1)^{(\gamma-1)/\gamma} - 1}{(T_2/T_1) - 1}. \quad (1.7)$$

The isentropic efficiency compares the work input along the actual (irreversible) path of compression with the work that would be input in an ideal compression that follows a different thermodynamic path between the initial and final pressures. Alternatively, the so-called *polytropic efficiency* compares the real and ideal work inputs along the actual (adiabatic but irreversible) path of compression. This efficiency is defined by considering a specific point on the compression path as

$$\eta_{\mathrm{p,compression}} \equiv \frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}h} = \frac{\gamma - 1}{\gamma}\frac{T}{p}\frac{\mathrm{d}p}{\mathrm{d}T}.$$

To express η_{p} in terms of the endpoints of the compression process, η_{p} is assumed constant and the equation is integrated to give

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\eta_{\mathrm{p}}\gamma/(\gamma-1)}.$$

This equation is equivalent to Equation 1.5, if we put

$$\frac{n}{n-1} = \frac{\eta_{\mathrm{p}}\gamma}{\gamma - 1}, \quad (1.8)$$

in other words

$$\eta_{\mathrm{p}} = \frac{n}{n-1}\frac{\gamma - 1}{\gamma}. \quad (1.9)$$

Isentropic and polytropic efficiencies can be related by the following pair of equations:

$$\eta_{\mathrm{s}} = \frac{(p_2/p_1)^{(\gamma-1)/\gamma} - 1}{(p_2/p_1)^{(\gamma-1)/\eta_{\mathrm{p}}\gamma} - 1}, \quad (1.10a)$$

$$\eta_{\mathrm{p}} = \frac{\gamma - 1}{\gamma}\frac{\ln(p_2/p_1)}{\ln\left(1 + [(p_2/p_1)^{(\gamma-1)/\gamma} - 1]/\eta_{\mathrm{s}}\right)}. \quad (1.10b)$$

For expansion processes (turbines), the equations are

$$\eta_s = \frac{(T_1/T_2) - 1}{1 - (p_2/p_1)^{(\gamma-1)/\gamma}}, \tag{1.11a}$$

$$\eta_p = \frac{n-1}{n}\frac{\gamma}{\gamma - 1}, \tag{1.11b}$$

$$\eta_s = \frac{1 - (p_2/p_1)^{\eta_p(\gamma-1)/\gamma}}{1 - (p_2/p_1)^{(\gamma-1)/\gamma}}, \tag{1.11c}$$

$$\eta_p = \frac{\gamma}{\gamma - 1}\frac{\ln\left[1 - \eta_s\left(1 - [p_2/p_1]^{(\gamma-1)/\gamma}\right)\right]}{\ln(p_2/p)}. \tag{1.11d}$$

In many cases (e.g., liquid pumps and low pressure fans), fluid density changes are either absent or negligibly small. In such cases, the distinction between isentropic and polytropic efficiencies vanishes and one uses the hydraulic or aerodynamic efficiency, defined for pumping machinery as the ratio of pressure change over density to the actual work done on the fluid (η_a (or η_H) $= (\Delta p/\rho)/w$) and for work-producing machinery as the ratio of actual work to the change in pressure divided by density. In other cases, in which density change is small but significant, the polytropic process model is used but the polytropic efficiency is approximated by the aerodynamic efficiency. This will be discussed in a later chapter.

1.6 Equations of Fluid Mechanics and Thermodynamics

Fluid mechanics analyses use the natural laws that govern Newtonian physics. That is, the flow must satisfy: conservation of mass, $\mathrm{d}m/\mathrm{d}t = 0$; Newton's second law of motion, $\boldsymbol{F} = \mathrm{d}(m\boldsymbol{V})/\mathrm{d}t$ (here, the bold letters indicate the vector character of the terms); which in terms of angular momentum is $\boldsymbol{M} = \mathrm{d}\boldsymbol{H}/\mathrm{dt} = \mathrm{d}(\Sigma(\delta m)\boldsymbol{r} \times \boldsymbol{V})/\mathrm{d}t$, where δm is the mass of each term being included in the sum; and conservation of energy, $\mathrm{d}Q'/\mathrm{d}t - \mathrm{d}W/\mathrm{d}t - \mathrm{d}E/\mathrm{d}t = 0$. In the energy equation, the first law of thermodynamics, Q' is the heat transferred to the fluid, W is the work done by the fluid, and E is the energy of the fluid. These equations, along with the second law of thermodynamics and state equations mentioned above, complete the analytical framework for a fluid flow.

In the study of fluid mechanics, these basic forms are converted to a control volume formulation using the Reynolds transport theorem. Conservation of mass for *steady flow* becomes

$$\iint_{\mathrm{cs}} \rho(\boldsymbol{V} \cdot \boldsymbol{n})\,\mathrm{d}A = 0, \tag{1.12}$$

where "cs" indicates integration over the complete surface of the control volume and $(\boldsymbol{V} \cdot \boldsymbol{n})$ is the scalar product of the velocity with the surface unit normal vector, $\boldsymbol{n}$ (i.e., it is the "flux term"). For any single surface, the mass flow rate is

$$\dot{m} = \int \rho(\boldsymbol{V} \cdot \boldsymbol{n})\mathrm{d}A. \tag{1.13}$$

Equation 1.12, the so-called continuity equation, says simply that in order to conserve mass, what comes into the control volume must leave it. For simple inlets and outlets, with uniform properties across each inlet or outlet, the continuity equation becomes

$$\sum (\rho VA)_{\text{out}} - \sum (\rho VA)_{\text{in}} = 0. \tag{1.14}$$

For incompressible flow (ρ = constant), the continuity equation reduces to

$$\sum (VA)_{\text{out}} = \sum Q_{\text{out}} = \sum (VA)_{\text{in}} = \sum Q_{\text{in}}, \tag{1.15}$$

where Q is the volume flow rate VA and $\dot{m} = \rho VA = \rho Q$.

For steady flow, Newton's second law becomes

$$\sum \boldsymbol{F} = \iint_{\text{cs}} \boldsymbol{V}\rho(\boldsymbol{V} \cdot \boldsymbol{n})\,\mathrm{d}A, \tag{1.16}$$

retaining the vector form shown earlier. Again, for simple inlets and outlets

$$\sum \boldsymbol{F} = \sum \dot{m}\boldsymbol{V}_{\text{out}} - \sum \dot{m}\boldsymbol{V}_{\text{in}}. \tag{1.17}$$

The incompressible form can be written with $\dot{m} = \rho Q$ as

$$\sum \boldsymbol{F} = \rho \sum \Big(Q\boldsymbol{V}_{\text{out}} - \sum Q\boldsymbol{V}_{\text{in}}\Big). \tag{1.18}$$

For a steady, compressible flow, through a control volume, the conservation of energy equation is

$$\dot{Q} - \dot{W}_{\text{sh}} = \iint_{\text{cs}} \rho\left(h + \frac{V^2}{2} + gz\right)(\boldsymbol{V} \cdot \boldsymbol{n})\,\mathrm{d}A, \tag{1.19}$$

where $\dot{Q}$ is the rate of heat transfer and $\dot{W}_{\text{sh}}$ is the rate of shaft work.

Using Equation 1.2 and the second law of thermodynamics, a *mechanical energy equation* can be developed; that is

$$-\dot{W}_{\text{sh}} - \dot{\Phi} = \dot{m}\left(\int_1^2 \frac{\mathrm{d}p}{\rho} + \frac{V_2^2 - V_1^2}{2} + gz_2 - gz_1\right), \tag{1.20}$$

where $\dot{\Phi}$ is the dissipation of useful energy by viscosity and the integral is taken along the thermodynamic process path between the inlet and the outlet.

The well-known Bernoulli equation for steady, incompressible, frictionless flow without shaft work is frequently a very useful approximation to more realistic flows and can be developed from the mechanical energy equation by dropping work and loss terms and assuming constant density; that is

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \text{constant} = \frac{p_T}{\rho}, \tag{1.21}$$

where p_T (sometimes written as p_0) is the total pressure of the flowing fluid. In turbomachinery flows, where work always takes place in the flow process, Bernoulli's equation is not valid when the end states are located across the region of work addition or extraction and the total pressure rises or falls.

If shaft work or frictional losses are to be included, then the Bernoulli equation must be replaced by the incompressible mechanical energy equation,

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 + \frac{w_{\text{sh}}}{\rho} + \frac{\phi_{\text{v}}}{\rho}. \tag{1.22}$$

Here, w_{sh} is the shaft work per unit mass (positive for work output) and ϕ_{v} is the viscous dissipation per unit mass. This equation is frequently rewritten in "head" form as

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{\text{sh}} + h_{\text{f}}, \tag{1.23}$$

where each term in the equation has units of length. h_{sh} and h_{f} are the shaft head addition or extraction and the frictional loss, respectively.

Equations 1.12 through 1.23 are the basic physical relationships for analyzing the flow in turbomachines and their attached fluid systems are considered in this book. If further review or practice with these fundamentals is needed, see White (2008); Fox et al. (2004); Gerhart et al. (1992); Schetz and Fuhs (1996); and Baumeister et al. (1978).

1.7 Turbomachines

This book will be restricted to the study of fluid mechanics and thermodynamics of turbomachines. This requires that a clear definition of turbomachinery be established at the outset. Paraphrasing from other authors (Balje, 1981; White, 2008), turbomachines can be defined as follows:

> A *turbomachine* is a device in which energy is transferred to or from a continuously moving fluid by the action of a moving blade row. The blade

> row rotates and changes the stagnation pressure of the fluid by either doing work on the fluid (as a pump) or having work done on the blade row by the fluid (as a turbine).

This definition excludes a large class of devices called positive displacement machines. They have moving boundaries that either force the fluid to move or are forced to move by the fluid. Examples include piston pumps and compressors, piston steam engines, gear and screw devices, sliding vane machines, rotary lobe pumps, and flexible tube devices. These are not turbomachines, according to the definition, since flow does not move continuously through them and they will not be further considered here. See Balje (1981) for more detailed material on these types of machines. Also no treatment will be provided here for the very broad areas of mechanical design: dynamics of rotors, stress analysis, vibration, bearings and lubrication, or other vital mechanical topics concerning turbomachinery. Others works may be consulted for further study of these important topics (Rao, 1990; Beranek and Ver, 1992; Shigley and Mischke, 1989). Because of its overriding importance in selection and siting of turbomachines, the subject of noise control and acoustics of turbomachinery will be included in our treatment of the performance and fluid mechanics of turbomachines.

A variety of names are used for the component parts of a turbomachine. The rotating element is variously called the *rotor*, the *impeller*, the *wheel*, and the *runner*. Whatever it is called, the rotating element carries a number of *blades*. Sometimes, turbine blades are called *buckets*. Nonrotating blade rows that direct or redirect the flow are called *stators* or *vanes*. If they accelerate the flow, such as in a turbine, they may be called *nozzles*, and fixed blades that decelerate the flow might be called *diffusers*. Finally, the rotating elements are mounted on a *shaft* and the working parts of the machine are typically enclosed in a *casing*.

1.8 Classifications

Much has been written on classifying turbomachinery, and a major subdivision is implied in the definition stated above. This is the power classification, identifying whether power is added to or extracted from the fluid. Pumps, which are surely the most common turbomachines in the world, are power addition machines and include liquid pumps, fans, blowers, and compressors. They operate on fluids such as water, fuels, waste slurry, air, steam, refrigerant gases, and a very long list of others. Turbines, which are probably the oldest type of turbomachines, are power extraction devices and include windmills, waterwheels, modern hydroelectric turbines, the exhaust side of automotive engine turbochargers, and the power extraction end of an aviation gas turbine engine. Again, they operate on a seemingly endless list of

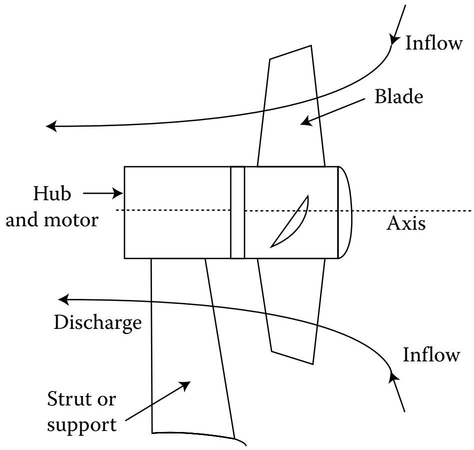

FIGURE 1.1 Example of an open flow turbomachine. No shroud or casing defines the limits of the flow field.

fluids, including gases, liquids, and mixtures of the two, as well as slurries and other particulate-laden fluids.

The manner in which the fluid moves through and around a machine provides another broad means of classification. For example, some simple machines are classed as open or open flow, as illustrated in Figure 1.1. Here, there is no casing or enclosure for the rotating impellers and they interact rather freely with the flowing stream—most often, the atmosphere. Consistent with the power classification, the propeller is an open flow pumping device, and the windmill is an open flow turbine. Figure 1.2 shows examples of enclosed or encased turbomachines where the interaction between

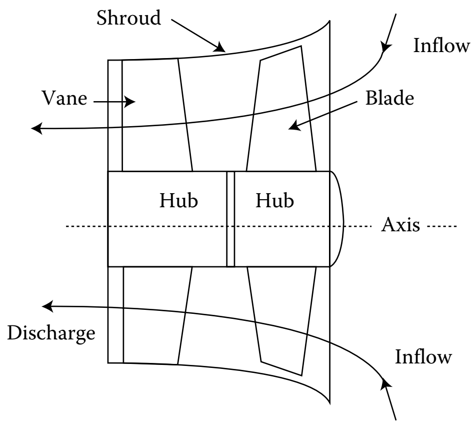

FIGURE 1.2 Example of enclosed or encased turbomachine with the shroud controlling the outer streamlines.

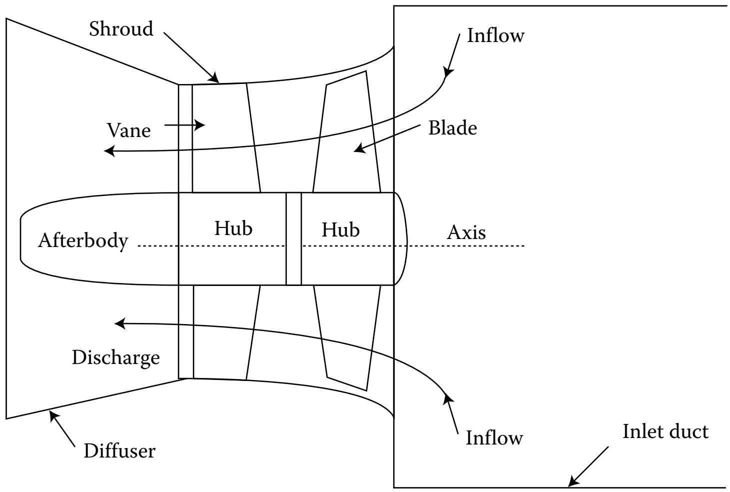

FIGURE 1.3 Layout of an axial fan with the major components.

the fluid and the device is carefully controlled and constrained by the casing walls. Again, these examples are power classified—in this case, as pumps, fans, or compressors.

Since all turbomachines have an axis of rotation, the predominant organization of the mass flow relative to the rotating axis can be used to further refine the classification of machines. This subdivision is referred to as flow path or throughflow classification and deals directly with the orientation of the streamlines that carry the mass flow. In *axial flow machines*—pumps, fans, blowers, or turbines—the fluid moves through the machine on streamlines or surfaces approximately parallel to the axis of rotation of the impeller. Figure 1.3 shows an axial (or axial flow) fan characterized by flow parallel to the fan axis of rotation.

Radial flow machines, with flow that is predominantly radial in the working region of the moving blade row, are illustrated in Figure 1.4, which shows a cutaway or sectioned view. The device is a radial flow (or "centrifugal") fan or pump and illustrates typical geometrical features of such machines. If the flow direction was reversed, the geometry would be typical of radial inflow turbines.

Since nothing is ever as simple or straightforward as one would like, there must be a remaining category for machines that fail to fit the categories of predominantly axial or predominantly radial flow. *Mixed flow machines* such as pumps, fans, turbines, and compressors may all fall into this class and are illustrated by the compressor shown in Figure 1.5. Flow direction for a pump is generally from the axial path to a conical path moving upward at an angle roughly between 20° and 65°. Again, reversing the direction of flow yields a path that is typical of a mixed flow turbine.

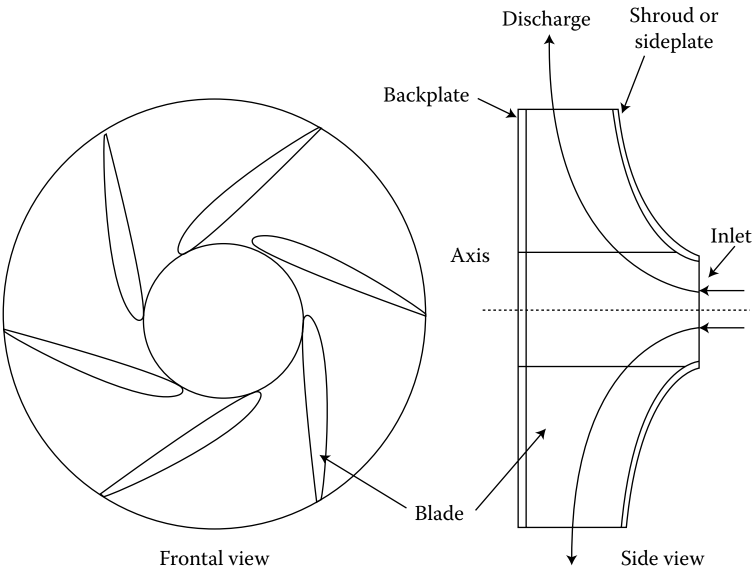

FIGURE 1.4 A radial throughflow turbomachine. The flow moves through the blade row in a primarily radial direction.

The simple flow paths shown here may be modified to include more than one impeller. For radial flow machines, two impellers can be joined back-to-back so that flow enters axially from both sides and discharges radially as sketched in Figure 1.6. These machines are called *double suction* (for liquid pumps) or *double inlet* (for gas movers such as fans or blowers). The flow

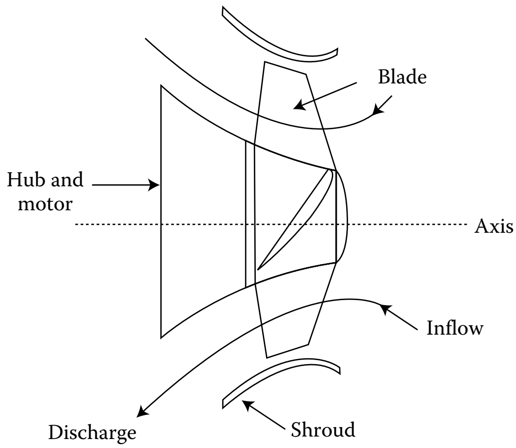

FIGURE 1.5 A mixed throughflow machine. The flow can enter axially and exit radially or at an angle from the axis, as shown.

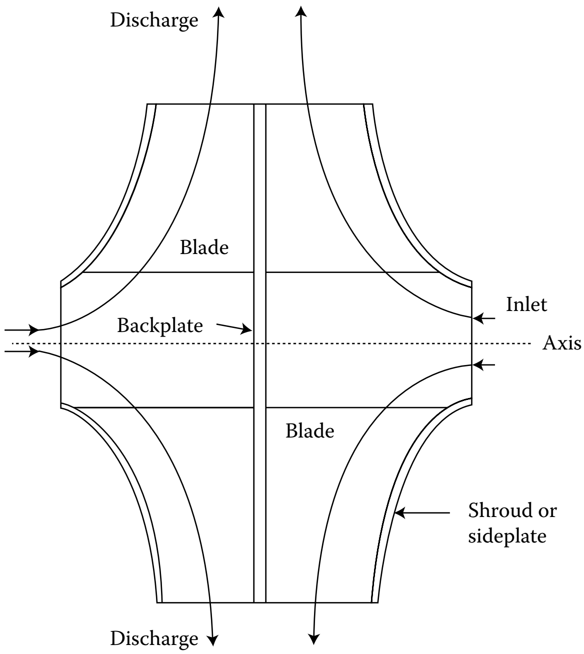

FIGURE 1.6 A double-inlet, double-width centrifugal impeller.

paths are parallel to each other, and flow is usually equal in the two sides, with equal energy addition occurring as well.

A *double-flow* design is often used in large (axial flow) steam turbines; in these devices, the fluid enters at the center and splits to flow toward both ends in the axial direction. Other machines might consist of two or more impellers in axial flow, radial flow, or even in mixed flow configurations. In these machines, the flow proceeds serially from one impeller to the next, with energy addition occurring at each stage. These multistage machines are illustrated in Figure 1.7.

Further breakdown of these classifications can include the compressibility of the fluid in the flow process. If the density is virtually constant in the entire flow process, as in liquid pumps and turbines, the *incompressible flow* label can be added. For gas flows, if there are large absolute pressure changes or high speeds or large Mach numbers involved that lead to significant changes in density, the machines can be labeled as *compressible flow* or simply as compressors. This book will try to keep the range of names for turbomachines as nearly unified as possible and make distinctions concerning gas–liquid and compressible–incompressible when it is convenient or useful to do so.

A set of photographs of turbomachines is included at the end of this chapter (Figures 1.19 through 1.26). They should help to relate the various flow paths to actual machines.

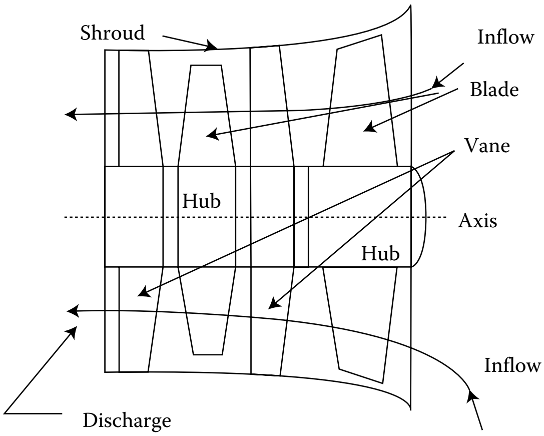

FIGURE 1.7 A two-stage axial fan configuration.

1.9 Turbomachine Performance and Rating

The *performance parameters* for a turbomachine are typically taken to be (1) the fluid flow rate through the machine, (2) a measure of the specific energy change of the fluid (pressure rise or drop, head, or pressure ratio), (3) shaft power, and/or (4) efficiency. In this book, the sound (or noise) generated by the machine is also treated as a performance variable. These performance parameters are related to each other and to the machine *operating parameters*, which are typically (1) rotational speed, (2) fluid density, and (sometimes) (3) fluid viscosity. The relationships between these parameters are the most important information about any machine; these relationships are loosely termed the "performance" of the machine.

Engineering information on the performance of a turbomachine is almost always determined experimentally, in a process called *performance test*. Performance testing is sometimes called "performance rating" or simply "rating," especially if done by the machine manufacturer in their shop or laboratory. Consideration of the process of rating a machine gives valuable insight into machine performance. A schematic layout of a rating facility is shown in Figure 1.8. This figure illustrates one of several possible layouts that may be used to rate a fan. Performance measurement is typically done with equipment and procedures specified by recognized standards to ensure accuracy, acceptance, and reproducibility of test results. For fans, the appropriate standard is "Laboratory Methods of Testing Fans for Rating" (AMCA, 1999). If a field test were desired, the standard might be ASME-PTC 11 Fans (ASME, 2008).

Referring again to Figure 1.8, shown on the left is a centrifugal fan; this is the machine being rated. This fan draws air from the room into the unrestricted

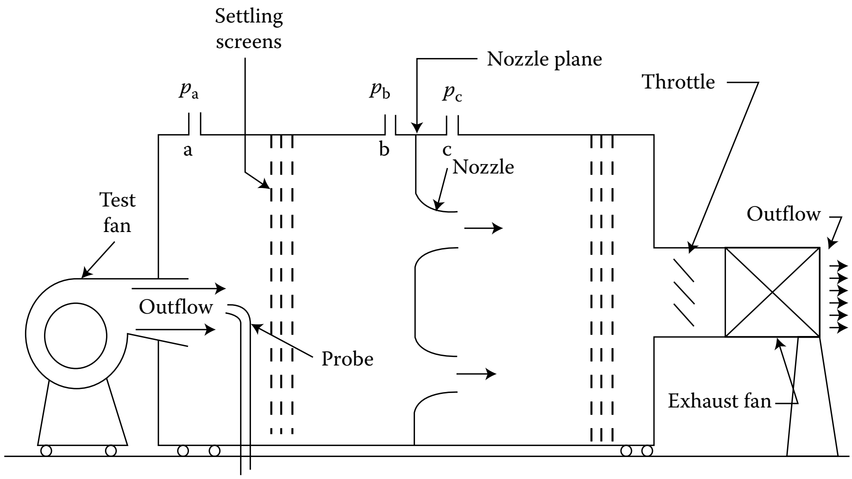

FIGURE 1.8 Schematic of a fan performance test facility, based on AMCA 210. (From AMCA, 1999. *Laboratory Methods of Testing Fans for Aerodynamic Performance Rating*, ANSI/AMCA 210-99, ANSI/ASHRAE 51-99, Air Moving and Conditioning Association. With permission.)

intake and discharges the air into the sealed flow box, or (more formally) plenum chamber. There are numerous pressure taps located along the flow path through the plenum (shown as "a," "b," and "c"). If compressibility is significant, the alternate instrumentation for determining total pressure and temperature at the fan discharge would be the Pitot-static probe shown, with a thermocouple for local temperature measurement. At "a," (or on the Pitot-static probe), the discharge pressure of the fan is monitored. Because the fan is doing work on the air passing through it, the pressure, p_a (total or static), will be greater than the room ambient pressure (as measured outside the plenum or near the intake). The fan discharge pressure is used to identify the pressure increase imparted by the fan to the air by calculation of the total pressure change ($p_{02} - p_{01}$). These pressures include the kinetic energy term associated with the fan discharge velocity. If the pressure change across the machine is sufficiently small (less than about 1% of the barometric pressure), the static pressure change ($\Delta p_s = p_a - p_{ambient}$) is used. This value is called the *static pressure rise* of the fan. The fan *total pressure rise* would include the velocity pressure of the discharge jet (only) to yield $\Delta p_T = \Delta p_s + \rho V_j^2/2$, where ρ is the ambient density. (This use of a total-to-static pressure rise as a performance variable is unique to fans.)

The jet of air being discharged by the fan is spread out (or "settled") as it moves through the first chamber of the box and the row of resistive screens through which the air must pass. The amount of resistance caused by these screens is specified by the test standard to ensure that the flow is smoothly distributed across the cross-section of the plenum chamber on the downstream side of the screens. In other test arrangements such as constant area pipes or ducts, the flow settling may rely on flow straighteners such as nested tubes, honeycomb, successive perforated plates, or fine-mesh screens. Any particular arrangement must yield a nearly uniform approach velocity as the flow nears the inlet side of the flow metering apparatus. The meter might be a set of precision-built or calibrated flow nozzles (e.g., ASME long radius nozzles), as sketched, mounted in the center plane of the plenum chamber (see ASME, 2004; Holman and Gadja, 1989; Granger, 1988; or Beckwith et al., 1993 for details of the construction of these nozzles). The meter may also be a nozzle or a sharp-edged orifice plate in a pipe or duct or sometimes a precision Venturi meter.

The pressure tap at point "b" supplies the value upstream of the flow nozzles and the tap at "c" gives the downstream pressure. If compressibility is significant, the total pressure and temperature at the flow meter must be established to provide accurate information for the calculation of the mass flow rate. The difference in pressure across the meter, perhaps read across the two legs of a simple U-tube manometer or pressure transducer, supplies the differential pressure or pressure drop through the nozzle, $\Delta p_{\mathrm{b-c}} = p_{\mathrm{b}} - p_{\mathrm{c}}$. This differential pressure is proportional to the square of the velocity of the air being discharged by the nozzles. The product of the velocity and the nozzle area provides the volume flow rate handled by the fan.

Another item of performance data required is a measure of the power being supplied to the test fan. This can be determined by a direct measure of the electrical power, in watts, being supplied to the fan motor, or some means may be provided to measure torque to the fan impeller along with the rotating speed of the shaft. The product of the torque and speed is, of course, the actual power supplied to the fan shaft by the driving motor. To complete the acquisition of test data in this experiment, it remains to make an accurate determination of the air density. In general, the density at the inlet of the blower and at the flow meter must be known.

Fluid velocity at the discharge of the nozzle is calculated from

$$V_{\mathrm{n}} = c_{\mathrm{d}}\left(\frac{2\Delta p_{\mathrm{b-c}}}{\rho(1-\beta^4)}\right)^{1/2}, \tag{1.24}$$

where c_{d} is the nozzle discharge coefficient used to account for viscous effects in the nozzle flow and β is the ratio of nozzle diameter to the diameter of the duct upstream of the nozzle. (In the system illustrated in Figure 1.8, $\beta \approx 0$.) c_{d} is a function of the diameter-based Reynolds number for the nozzles. One correlation (Beckwith et al., 1993) for c_{d} is

$$c_{\mathrm{d}} = 0.9965 - 0.00653\left(\frac{10^6}{Re_{\mathrm{d}}}\right)^{1/2}. \tag{1.25}$$

The Reynolds number is $Re_{\mathrm{d}} = V_{\mathrm{n}} d/\nu$ (ν is the fluid kinematic viscosity).

The inlet density must be accurately determined from the barometric pressure, p_{amb}, ambient dry-bulb temperature, T_{amb}, and the ambient wet-bulb temperature, T_{wb}. The correction to the air density, ρ, can be carried out using a psychometric chart (see, e.g., Moran and Shapiro, 2008; AMCA, 1999). The psychometric chart from the AMCA standard is reproduced in Appendix A. The density at the flow meter can be determined from the inlet value and the total pressure ratio according to

$$\rho_{\text{meter}} = \rho_{\text{inlet}}\left(\frac{p_{02}}{p_{01}}\right)\left(\frac{T_{01}}{T_{02}}\right), \tag{1.26}$$

where the pressure and temperature are in absolute units. For "incompressible" test conditions, the two densities will be essentially the same.

To complete the data gathering from the fan, the noise generated by the test fan should be measured using a microphone and suitable instrumentation to provide values of L_{W}, the sound power level in decibels. Turbomachinery noise will be considered in detail in Chapter 4; it is mentioned here because of the frequently overriding importance of this performance parameter.

The efficiency of the fan is calculated from the main performance variables as the output fluid power divided by the shaft input power, P_{sh}. This is the traditional definition for pumps, fans, and blowers, even into the compressible regime. It should be noted that this yields the so-called *overall efficiency* wherein the difference between fluid power and shaft power (the *losses*) includes mechanical losses in both bearings and seals, and aerodynamic/hydraulic losses in the flow through the impeller. Mechanical losses can be isolated by defining a mechanical efficiency, $\eta_{\text{M}} = P_{\text{fluid}}/P_{\text{sh}}$. The (overall) efficiency is

$$\eta_{\text{To}} = \dot{m}\frac{([p_{02} - p_{01}]/\rho)}{P_{\text{sh}}}. \tag{1.27}$$

This is equivalent, for incompressible flows, to

$$\eta_{\text{T}} = \frac{Q(\Delta p_{\text{T}})}{P_{\text{sh}}}. \tag{1.28}$$

The "T" subscript on η corresponds to the use of total pressure rise in the fluid output power calculation. Use of Δp_{s} yields the commonly used static efficiency η_{s}.

The flow rate, pressure rise, sound power level, efficiency, and input power taken together define a specific performance point for the machine being tested. This particular point of operation is obtained through the effects of a downstream throttle or an auxiliary exhaust blower, or both, as shown in Figure 1.8. Both devices allow the pressure rise and flow rate of the device to be varied by increasing or decreasing the overall resistance imposed on the fan. Reduced resistance (a more open throttle or lower back pressure) will allow more flow (Q or $\dot{m}$) at a reduced pressure rise (Δp_T or Δp_s). Successive adjustment of the throttle or back pressure provides a series of performance points that cover the full performance range of the fan for the particular speed of operation and fluid being handled. Traditionally, the results are plotted on a series of curves, as shown in Figures 1.9a–c. For pumping machinery, the flow rate, Q, is normally taken as the independent (x-axis) variable. These curves are called the machine's *performance curves.* If the fan were to be operated at a different speed, a different set of curves, similar in shape but different in magnitude, would result.

There are several important things to notice from the performance curves for this fan. First, there is a point of maximum efficiency, called the best efficiency point (BEP) defined by the η versus Q curve. The corresponding values of Q, Δp, P_{sh}, and L_W, along with the maximum value of η, define the BEP. To the right of the BEP (at higher flow rate), Δp decreases with increasing flow, yielding a negative slope that represents a usable, stable range of performance for the fan. For this fan, somewhere to the left of the BEP (at lower flow rate), the curve of Δp versus Q goes through a zero slope condition, followed by a region of positive slope. (The slope of the curve is exaggerated for effect.) In this positive slope region, the fan would operate unstably while Δp drops, η declines sharply, and L_W increases sharply. Simply put, everything goes wrong

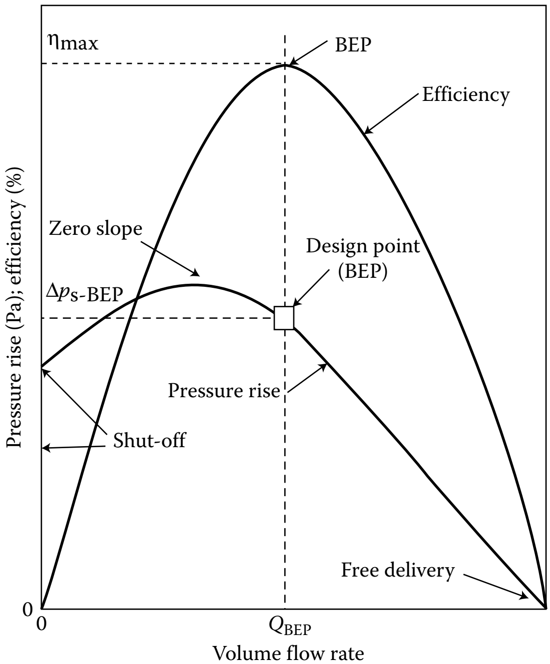

FIGURE 1.9a Efficiency and pressure rise curves with nomenclature.

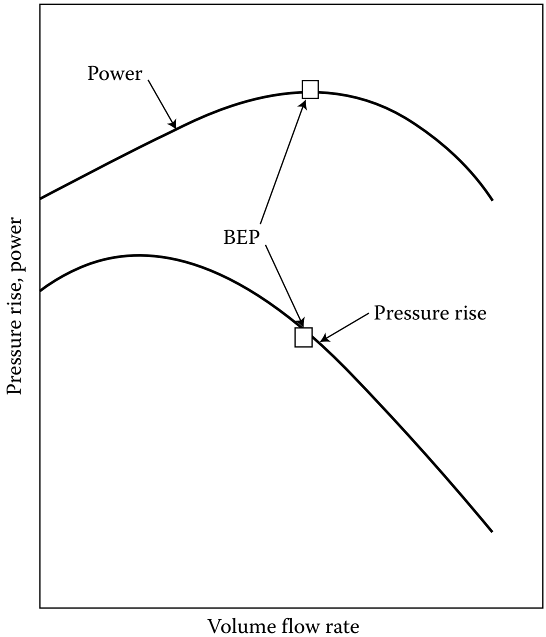

FIGURE 1.9b Input power curve (pressure rise curve repeated for reference).

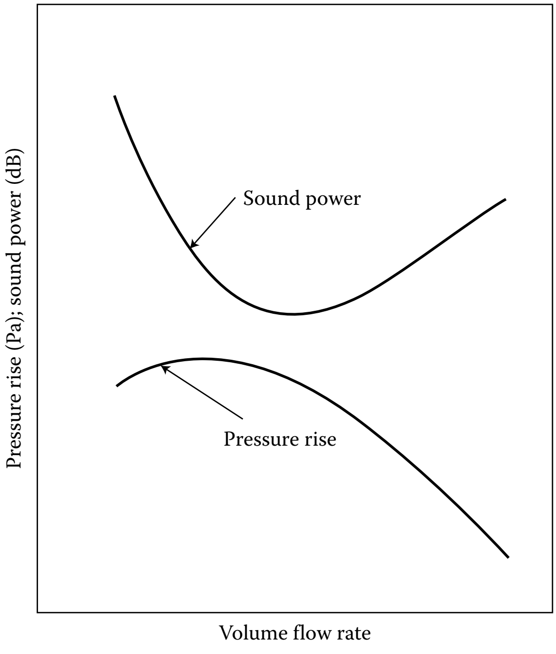

FIGURE 1.9c Sound power curve (pressure rise curve repeated for reference).

at once, and this region represents a virtually unusable regime of operation for this fan. This zone is called the *stalled region* and must be strictly avoided in selection and operation. (Stall will be discussed further in Chapter 10.) At the extreme left of the curve, when $Q = 0$, the fan is still producing some pressure rise but no flow. This limit condition is called the "blocked" or "shutoff" point. With $Q = 0$, the efficiency is zero as well. At the other extreme, $\Delta p = 0$ at the right end of the curve. This limit is called the "free-delivery" or "wide open" point and represents a maximum flow rate for the fan at the given operating speed. The efficiency is also zero at the free-delivery point because Δp is zero. It can be seen that the efficiency generally drops away from the BEP sharply on the left and more gradually on the right. The same can be said for the sound power level, L_W, with a gradual increase in noise to the right of BEP and a sharp increase to the left.

Example: Performance Test on a Fan

Consider a fan test carried out according to these procedures. The plenum chamber data gathered for a single performance point are

$$p_a = 10\,\text{in. wg}; \quad p_b = 9.8\,\text{in. wg}$$

$$p_c = 4.0\,\text{in. wg}; \quad L_W = 85\,\text{db}$$

$$P_m = 3488\,\text{W(motor power)}$$

$$\eta_e = 0.83\ \text{(motor efficiency)}.$$

Ambient air data are

$$T_{amb} = 72°\text{F}; \quad T_{wb} = 60°\text{F}; \quad p_{amb} = 29.50\,\text{in. Hg}.$$

Flow is measured by five 6-in. diameter nozzles. The total flow area is thus

$$A_{total} = 5 \times \frac{\pi}{4} \times 0.5^2 = 0.1963\,\text{ft}^2.$$

The ambient air density is calculated using the psychometric chart in Appendix A. The wet-bulb depression is

$$\Delta T_{d-w} = T_{amb} - T_{wb} = (72 - 60)°\text{F} = 12°\text{F}.$$

At the top of the psychometric chart, one enters with 12° suppression and drops vertically to the downward sloping line for $T_{dry\ bulb} = 72°\text{F}$, then horizontally to the left to intersect the upward sloping line for $p_{amb} = 29.5$ in. Hg. Finally, drops down vertically to the abscissa and reads the value for the weight density, $\rho g = 0.0731$ lb/ft^3, so that the mass density is

$$\rho = \frac{\rho g}{g} = \frac{0.0731}{32.17} = 0.00227\,\text{slug/ft}^3.$$

The dynamic viscosity at $T_{\text{amb}} = 72°\text{F}$ is $\mu = 3.68 \times 10^{-7}\ \text{slug/ft} \cdot \text{s}$, so

$$\nu = \frac{\mu}{\rho} = 1.621 \times 10^{-4}\ \text{ft}^2/\text{s}.$$

Next, calculate the nozzle pressure drop as

$$\Delta p_{\text{b-c}} = p_{\text{b}} - p_{\text{c}} = (9.8 - 4.0)\ \text{in. wg} = 5.8\ \text{in. wg}.$$

Converting this value back to basic units,

$$\Delta p_{\text{b-c}} = 5.8\ \text{in. wg} \times \left(\frac{5.204(\text{lb/ft}^2)}{\text{in. wg}}\right) = 30.18\ \text{lb/ft}^2.$$

Now the data are substituted into the equation for nozzle velocity, noting that there is no Reynolds number as yet to calculate c_{d} (since velocity is unknown). Start with $c_{\text{d}} \approx 1.0$ and make an iterative correction. A first guess is then

$$\begin{aligned} V_{\text{n}} &= c_{\text{d}} \left(\frac{2\,\Delta p_{\text{b-c}}}{\rho(1 - \beta^4)}\right)^{1/2} \\ &= 1.0 \times \left(2 \times 30.18\ \text{lb/ft}^2/0.00227\ \text{slug/ft}^3 (1 - 0^4)\right)^{1/2} \\ &= 163.1\ \text{ft/s}. \end{aligned}$$

Next, calculate the Reynolds number

$$Re_{\text{d}} = \frac{V_{\text{n}} d}{\nu} = \frac{163.1 \times 0.5}{1.621 \times 10^{-4}} = 5.09 \times 10^5.$$

Using this value estimate c_{d}:

$$\begin{aligned} c_{\text{d}} &= 0.9965 - 0.00653 \left(\frac{10^6}{Re_{\text{d}}}\right)^{1/2} \\ &= 0.9965 - 0.00653 \left(\frac{10^6}{5.09 \times 10^5}\right)^{1/2} = 0.9874. \end{aligned}$$

Adjust the velocity for $c_{\text{d}} < 1.0$ by multiplying the first guess by 0.987 to get $V_{\text{n}} = 161.0\ \text{ft/s}$. A new Reynolds number is calculated as

$$Re_{\text{d}} = 0.987 \times 5.09 \times 10^5 = 5.02 \times 10^5,$$

then, a new c_{d} is calculated as 0.9873. This change in c_{d} is negligible, so accept the value of $V_{\text{n}} = 161.0\ \text{ft/s}$. The volume flow rate becomes

$$\begin{aligned} Q &= V_{\text{n}} \times A_{\text{tot}} \\ &= 161.0\ \text{ft/s} \times 0.1963\ \text{ft}^2 \\ &= 31.57\ \text{ft}^3/\text{s}. \end{aligned}$$

The traditional unit for Q is cubic feet per minute (cfm),

$$Q = 31.57\,\text{ft}^3/\text{s} \times 60\,\text{s}/\min = 1896\,\text{cfm}.$$

The remaining item of data is the electrical power measured at the fan drive motor. The fan shaft power is obtained by multiplying by the given motor efficiency:

$$P_{\text{sh}} = \eta_{\text{e}} \times P_{\text{m}},$$

so

$$P_{\text{sh}} = 0.83 \times 3448\,\text{W} = 2862\,\text{W} = 2110\,\text{ft} \cdot \text{lb/s} = 3.84\,\text{hp}.$$

Finally, calculate the fan static pressure rise and fan static efficiency:

$$\Delta p_{\text{s}} = p_{\text{a}} - p_{\text{amb}} = 10\,\text{in. wg} - 0 = 10\,\text{in. wg},$$

$$\eta_{\text{s}} = \frac{Q\Delta p_{\text{s}}}{P_{\text{sh}}} = 31.57\,\text{ft}^3/\text{s} \times \frac{10\,\text{in. wg} \times 5.20\,\text{in. wg/psf}}{2110\,\text{ft} \cdot \text{lb/s}} = 0.778.$$

Summarizing the performance point

$$Q = 31.57\,\text{ft}^3/\text{s} = 1896\,\text{cfm},$$

$$\Delta p_{\text{s}} = 52.0\,\text{lb/ft}^2 = 10.0\,\text{in. wg},$$

$$P_{\text{sh}} = 2110\,\text{ft} \cdot \text{lb/s} = 3.84\,\text{hp},$$

$$L_{\text{W}} = 85\,\text{dB},$$

$$\eta_{\text{s}} = 0.778.$$

If the total efficiency or a correction for compressibility (discussed in Chapter 3) is desired, additional measurements of total pressure and total temperature in the discharge jet are required. Suppose these data are

$$p_{02} = 11.5\,\text{in. wg and } T_{02} = 76.7^\circ\text{F}.$$

Then

$$\Delta p_{\text{T}} = p_{02} - p_{01} = 11.5\,\text{in. wg} - 0 = 11.5\,\text{in. wg}.$$

The (incompressible) total efficiency would simply use the Δp_{T} value to yield

$$\eta_{\text{T}} = \frac{\Delta p_{\text{T}} Q}{P_{\text{sh}}} = 0.895.$$

1.10 Rating and Performance for Liquid Pumps

Liquid pumps are perhaps the most common type of turbomachine in use. Performance curves and rating methods for pumps are similar to those for fans, but there are some significant differences. A typical performance rating setup for liquid pump testing is illustrated in Figure 1.10. Standards for testing and rating pumps include ASME-PTC-8.2 *Centrifugal Pumps* (ASME, 1990) or HI-1.6 *Test Standard for Centrifugal Pumps* (HI, 1994). The test pump is installed in a piping network forming a closed loop with a pressure- and temperature-controlled reservoir. The primary performance information is based on pressure readings at the pump suction and delivery flanges, and another set of pressure readings taken upstream and downstream of a flow measurement device, typically a flow nozzle or a sharp-edged orifice plate, positioned downstream of the pump. In addition, power-measuring instrumentation is required for the pump torque and speed or, alternatively, to monitor the electrical power input to a calibrated drive motor. Usually, the reservoir temperature and pressure and the temperature and pressure at the suction flange of the pump will also be monitored. These data are useful in determining the cavitation characteristics of the pump (as will be discussed in Chapter 3).

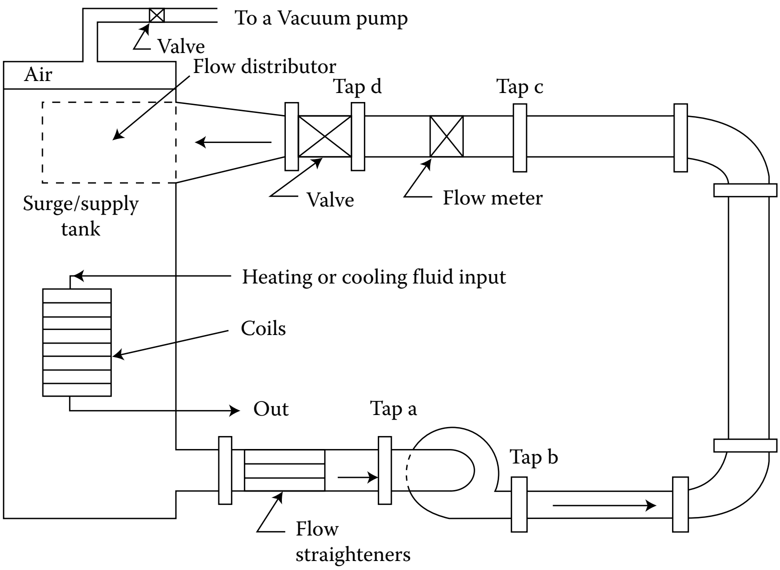

FIGURE 1.10 A liquid pump rating facility.

An important part of the flow loop is the downstream valve used to control the system resistance imposed on the pump. This valve will usually be a "noncavitating" type to avoid instabilities and noise in the pump flow.

As shown in Figure 1.10, the pressures are read from taps "a," "b," "c," and "d" on gages, transducers, or mercury manometers. For increased accuracy, the flow meter pressure drop should be read with a differential-sensing indicator. For illustration, assume that the power input to the motor, P_m, is determined with a wattmeter and the motor efficiency, η_m, is known. The pressure rise across the pump, $(p_\mathrm{b} - p_\mathrm{a})$, is used together with the flow rate and areas to calculate the pump head

$$H = \frac{p_\mathrm{b} - p_\mathrm{a}}{\rho g} + \frac{V_\mathrm{b}^2 - V_\mathrm{a}^2}{2g} + z_\mathrm{b} - z_\mathrm{a}. \tag{1.29}$$

The liquid density is evaluated from the temperature T_a.

The pressure differential across the flow meter, here assumed to be a sharp-edged orifice, is used to calculate the volume flow rate in the same manner as flow nozzles are used to determine the air flow in the fan performance test setup. The necessary characteristics for the orifice flow are the Reynolds number and the value of β for the plate. β is defined as $\beta = d/D$, the ratio of the open diameter of the orifice to the pipe's inner diameter. The discharge coefficient for the orifice is then given as

$$c_\mathrm{d} = f(Re_\mathrm{D}, \beta), \tag{1.30}$$

where the Reynolds number is defined as

$$Re_\mathrm{D} = \frac{VD}{\nu}, \tag{1.31}$$

where V is the pipe velocity, D is the pipe diameter, and ν is the kinematic viscosity of the liquid. The flow rate is calculated as

$$Q = c_\mathrm{d} A_\mathrm{t} \left(\frac{2(p_\mathrm{d} - p_\mathrm{c})}{\rho(1 - \beta^4)} \right)^{1/2}. \tag{1.32}$$

One of the several available correlations for c_d is given (ASME, 2004), for standard taps, as

$$c_\mathrm{d} = f(\beta) + 91.71\beta^{2.5}\, Re_\mathrm{D}^{-0.75}, \tag{1.33}$$

$$f(\beta) = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8, \tag{1.34}$$

$$A_\mathrm{t} = \left(\frac{\pi}{4}\right) d^2. \tag{1.35}$$

Example: Pump Rating

An example will help to illustrate the procedures for data acquisition and analysis. Sample data required to define a single point on the characteristic performance curve for a pump are

$$T_a = 25°\text{C}; \quad p_a = 2.5\,\text{kPa}; \quad P_m = 894\,\text{watts}$$
$$p_d = 155.0\,\text{kPa}; \quad p_c = 149.0\,\text{kPa}; \quad \eta_e = 0.890$$
$$D = 10\,\text{cm}; \quad d = 5\,\text{cm}.$$

The inlet temperature yields a water density of 997 kg/m^3 and a kinematic viscosity of 0.904×10^{-6} m^2/s. As usual, one cannot calculate an *a priori* Reynolds number to establish c_d, so an initial value of c_d is estimated and then iterated to a final result. If one assumes that Re_d is very large, then

$$c_d \cong 0.5959 + 0.0312(0.5)^{2.1} - 0.184(0.5)^8 = 0.6025.$$

The first estimate for volume flow rate is

$$Q = c_d A_t \left(\frac{2(p_d - p_c)}{\rho(1-\beta^4)}\right)^{1/2}$$
$$= (0.6025)(0.001963\,\text{m}^2)\left[\frac{(6000\,\text{N/m}^2)}{(997\,\text{kg/m}^3)(1-0.5^4)}\right]^{1/2}$$

or

$$Q = 0.004238\,\text{m}^3/\text{s},$$

so that $V_t = 2.16$ m/s. As the first refinement, the Reynolds number is

$$Re_D = \beta\, Re_d = \beta\left(\frac{V_t d}{\nu}\right) = 0.5\left(\frac{2.16 \times 0.05}{0.904 \times 10^{-6}}\right) = 5.973 \times 10^4.$$

Recalculating c_d yields

$$c_d = 0.6025 + \frac{91.7(0.5)^{2.5}}{(5.974 \times 10^4)^{0.75}}$$
$$= 0.6025 + 0.0042 = 0.6067.$$

This 0.6% change is small, so evaluate Q as

$$Q = 1.006 \times 0.00424\,\text{m}^3/\text{s} = 4.26\,\text{l/s}.$$

Volume flow rate in liters per second, while not in fundamental SI units, is common practice in metric usage.

Figure 1.10 suggests that the pipe size (and hence fluid velocity) and centerline elevation change across the pump, but no data are available, so the pump head is estimated from the pressure differential measured across the pump

$$H = \frac{p_b - p_a}{\rho g} + \frac{V_b^2 - V_a^2}{2g} + z_b - z_a \approx \frac{p_b - p_a}{\rho g}$$

$$= \frac{(155{,}000 - 2500)(\mathrm{N/m^2})}{997 \times 9.81\ \mathrm{N/m^3}} = 15.59\ \mathrm{m}.$$

The fluid power is

$$P_{fl} = \rho g Q H = (9.807\ \mathrm{m/s^2})(997\ \mathrm{kg/m^3})(0.00426\ \mathrm{m^3/s})(15.59\ \mathrm{m}),$$

$$P_{fl} = 649\ \mathrm{W}.$$

Calculating the shaft power from the motor power and efficiency

$$P_{sh} = P_m \eta_e = 796\ \mathrm{W},$$

allows calculation of the pump efficiency according to

$$\eta = \frac{P_{fl}}{P_{sh}} = \frac{649\ \mathrm{W}}{796\ \mathrm{W}} = 0.815.$$

This is once again an overall efficiency, accounting for both mechanical and hydraulic (flow path) losses. The overall efficiency is related to the mechanical efficiency, η_M, and the hydraulic efficiency, η_H (defined in Section 1.5), by

$$\eta = \eta_M \times \eta_H. \tag{1.36}$$

In summary, the test point for the pump is defined by

$$Q = 0.00426\ \mathrm{m^3/s},$$

$$H = 15.59\ \mathrm{m},$$

$$P_{sh} = 796\ \mathrm{W},$$

$$\eta = 0.815.$$

As in the fan test, the valve downstream of the pump can be opened or closed to decrease or increase the flow resistance and provides a range of points to create the performance curves (also called the *characteristic curves*) for the pump. The characteristic curves for a pump are similar to those for fans (Figure 1.9a–c) with a few differences. The fluid energy parameter is usually the pump head, H, as opposed to the pressure rise. It is not typical to present sound power as a pump performance parameter. Finally, the units for flow Q are more typically gpm or l/min for pumps.

1.11 Compressible Flow Machines

Liquid pumps and hydraulic turbines are essentially incompressible flow machines; a pressure change of ~ 21 MPa (3000 psi) would be required to produce a 1% density change in water. Fans that handle a highly compressible fluid often create a pressure change large enough to cause a small density change (a pressure change of 1400 Pa [5.6 in. wg] will produce a density change of 1% in air); however, the effects of compressibility in fans can be handled with a "correction factor" (discussed in Chapter 3). For machines such as compressors and steam and gas turbines, significant fluid density changes occur between the inlet and the outlet. There are several differences between performance and rating of incompressible flow machines and these compressible flow machines.

Probably the most obvious change in performance variables is to use the mass flow rate, $\dot{m}$ in place of the volume flow rate. If a volume flow rate is desired, it is customary to base the volume flow rate on the machine inlet stagnation density ($Q_{\text{in}} = \dot{m}/\rho_{01}$). Another obvious performance parameter is the shaft power, P_{sh}, which clearly has the same significance for both compressible and incompressible flow machines.

The most significant changes are in the fluid specific energy and in the fluid input or output power. First, consider the pressure and temperature. When dealing with compressible flow machines, it is customary to combine the fluid properties with the fluid velocity/kinetic energy by using the stagnation properties (also called total properties) (see White, 2008; Gerhart et al., 1992):

$$T_0 = T + \frac{V^2}{2c_{\text{p}}} = T\left(1 + \frac{V^2}{2c_{\text{p}}T}\right) = T\left(1 + \frac{\gamma - 1}{2}Ma^2\right), \tag{1.37}$$

$$p_0 = p\left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)} = p\left(1 + \frac{V^2}{2c_{\text{p}}T}\right)^{\gamma/(\gamma-1)} = p\left(1 + \frac{\gamma - 1}{2}Ma^2\right)^{\gamma/(\gamma-1)}, \tag{1.38}$$

$$\rho_0 = \frac{p_0}{RT_0}, \tag{1.39}$$

where V is the fluid velocity and Ma is the Mach number. It is assumed that the fluid can be modeled as a perfect gas.

Using stagnation properties, the conservation of energy equation (Equation 1.19) becomes

$$\dot{Q} - P_{\text{sh}} = \dot{m}c_{\text{p}}(T_{02} - T_{01}) \tag{1.40}$$

and the mechanical energy equation (Equation 1.20) becomes

$$-P_{\mathrm{sh}} - \dot{\Phi} = \dot{m}\left(\int_1^2 \frac{\mathrm{d}p_0}{\rho_0}\right). \tag{1.41}$$

(*Note*: In these equations, the "thermodynamic" symbol for work rate, $\dot{W}$, has been replaced by P for "power," the symbol used in this book.) For all practical purposes, turbomachines are adiabatic ($\dot{Q} = 0$), so the specific work on/by the fluid, from Equation 1.40 is

$$-w = c_{\mathrm{p}}(T_{02} - T_{01}). \tag{1.42}$$

The ideal process would be frictionless, as well as adiabatic, and the fluid power is obtained by putting $\dot{\Phi} = 0$ and using the isentropic process equation ($\rho_0 = \text{constant} \times p_0^{1/\gamma}$) to perform the integral in Equation 1.41, giving

$$-P_{\mathrm{fl}} = \dot{m}\left(\frac{\gamma}{\gamma - 1}\frac{p_{01}}{\rho_{01}}\left[\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1\right]\right) = \dot{m}c_{\mathrm{p}}T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1\right], \tag{1.43}$$

where perfect gas relations from Section 1.4 have been used. The fluid specific energy follows by dividing by the mass flow rate

$$-\frac{P_{\mathrm{fl}}}{\dot{m}} = c_{\mathrm{p}}T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1\right]. \tag{1.44}$$

This parameter corresponds to gH for a pump and $\Delta p/\rho$ for a fan. The specific energy for a compressible fluid depends on the initial temperature of the fluid and on the pressure ratio across the machine. It is convenient to define an *isentropic head*, H_{s}, by

$$gH_{\mathrm{s}} \equiv c_{\mathrm{p}}T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1\right]. \tag{1.45}$$

The (isentropic) efficiency of a compressor is the ratio of the fluid-specific energy to the work done on the fluid. Using Equations 1.42 and 1.44

$$\eta_{\mathrm{c}} = \frac{(p_{02}/p_{01})^{(\gamma-1)/\gamma} - 1}{(T_{02}/T_{01}) - 1}, \tag{1.46}$$

which is identical to Equation 1.7, except that stagnation pressures and temperatures are used so that a "total" efficiency results.