STATISTICAL QUALITY CONTROL

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Preface

The objective of this book is to expose the reader to the various steps in the statistical quality control methodology. It is assumed that the reader has a basic understanding of probability and statistics taught at the junior level in colleges. The book is based on materials taught in a graduate-level course on statistical quality control in the Department of Industrial and Manufacturing Engineering at The Pennsylvania State University. The material discussed in this book can be taught in a 15-week semester and consists of nine chapters written in a logical manner. Some of the material covered in the book is adapted from journal publications. Sufficient examples are provided to illustrate the theoretical concepts covered.

I would like to thank those who have helped make this book possible. My colleague and friend, Professor Tom M. Cavalier of The Pennsylvania State University, has been encouraging me to write a textbook for the last ten years. His encouragement was a major factor in my writing this book. Many people are responsible for the successful completion of this book. I owe a lot to Professor Murray Smith of the University of Auckland, New Zealand, for his ungrudging help in generating the tables used in this book. My heartfelt thanks go to Hsu-Hua (Tim) Lee, who worked as my manager and helped me tremendously to prepare the manuscript; I would have been completely lost without his help. I would also like to thank Nicholas Smith for typing part of the manuscript and preparing the figures. Thanks are also due to Cecilia Devasagayam and Himanshu Gupta for their help in generating some of the end-of-chapter problems. I thank the numerous graduate students who took this course during the past few years, especially Daniel Finke, for their excellent suggestions for improvement.

A manuscript cannot be converted into a textbook without the help of a publisher. I would like to express my gratitude to CRC Press for agreeing to publish this book. My sincere thanks go to Cindy Renee Carelli, Engineering Acquisitions Editor at CRC Press, for her support in publishing this book. She was always willing to answer my questions and help me; publishers need persons like her to help authors. I also thank the anonymous reviewer of an earlier version of this manuscript for the excellent suggestions that led to substantial improvements of this manuscript.

Special gratitude and appreciation go to my wife, Emeline, and my children, Jean and Naveen, for the role they play in my life to make me a complete person. Finally, I thank my Lord and Savior, Jesus Christ, without whom I am nothing.

The Author

M. Jeya Chandra, Ph.D. is a professor of Industrial Engineering at The Pennsylvania State University, where he has been teaching for over 20 years. He has published over 50 papers in various journals and proceedings. In addition, he has won several teaching awards from the department, the College of Engineering, and the University. He has a B.E. in Mechanical Engineering from Madras University, India; an M.S. in Industrial Engineering from The Pennsylvania State University; and a Ph.D. in Industrial Engineering and Operations Research from Syracuse University.

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I dedicate this book to

Mr. Sudarshan K. Maini, Chairman, Maini Group, Bangalore, India,

who was a great source of encouragement during the darkest period of my professional life, and to his wonderful family.

1

Introduction

Quality can be defined in many ways, ranging from "satisfying customers' requirements" to "fitness for use" to "conformance to requirements." It is obvious that any definition of quality should include customers, satisfying whom must be the primary goal of any business. Experience during the last two decades in the U.S. and world markets has clearly demonstrated that quality is one of the most important factors for business success and growth. Businesses achieving higher quality in their products enjoy significant advantage over their competition; hence, it is important that the personnel responsible for the design, development, and manufacture of products understand properly the concepts and techniques used to improve the quality of products. Statistical quality control provides the statistical techniques necessary to assure and improve the quality of products.

Most of the statistical quality control techniques used now have been developed during the last century. One of the most commonly used statistical tools, *control charts,* was introduced by Dr. Walter Shewart in 1924 at Bell Laboratories. The *acceptance sampling* techniques were developed by Dr. H. F. Dodge and H. G. Romig in 1928, also at Bell Laboratories. The use of *design of experiments* developed by Dr. R. A. Fisher in the U.K. began in the 1930s. The end of World War II saw increased interest in quality, primarily among the industries in Japan, which were helped by Dr. W. E. Deming. Since the early 1980s, U.S. industries have strived to improve the quality of their products. They have been assisted in this endeavor by Dr. Genichi Taguchi, Philip Crosby, Dr. Deming, and Dr. Joseph M. Juran. Industry in the 1980s also benefited from the contributions of Dr. Taguchi to *design of experiments, loss function,* and *robust design*. The recent emphasis on teamwork in design has produced *concurrent engineering*. The standards for a quality system, ISO 9000, were introduced in the early 1990s. They were later modified and enhanced substantially by the U.S. automobile industries, resulting in QS-9000.

The basic steps in statistical quality control methodology are represented in Figure 1.1, which also lists the output of each step. This textbook covers most of the steps shown in the figure. It should be emphasized here that the steps given are by no means exhaustive. Also, most of the activities must be performed in a parallel, not sequential, manner. In Chapter 2, Tolerancing, assembly tolerance is allocated to the components of the assembly. Once tolerances

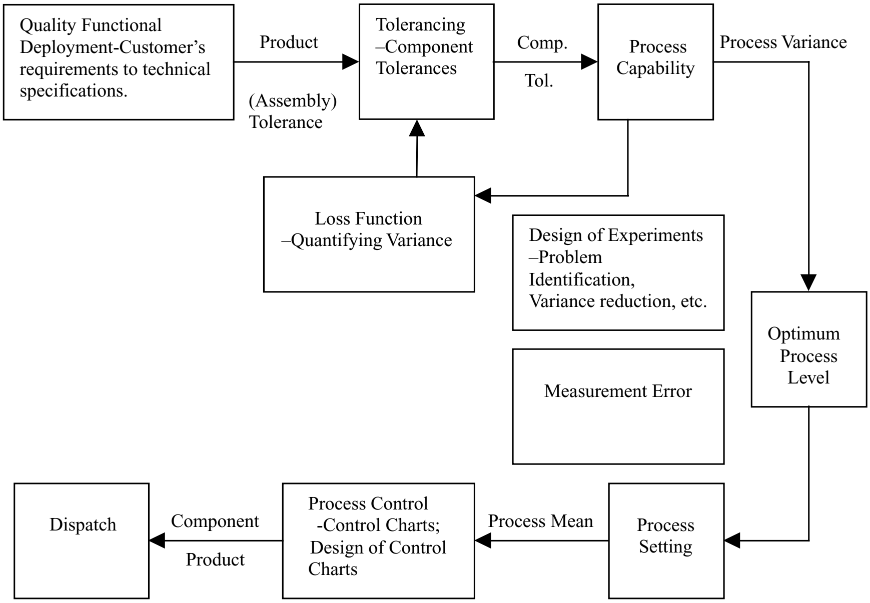

FIGURE 1.1
Quality control methodology.

on the quality characteristics of the components are determined, processes must be selected for manufacture of the components. The personnel responsible for process selection must be cognizant of the effect of quality characteristic variances on the quality of the product. This process, developed by Dr. Taguchi, is discussed in Chaper 3, Loss Function. Robust design, which is based upon loss function, is also discussed in this chapter. Process capability analysis, which is an important step for selection of processes for manufacture of the components and the product, is discussed in Chapter 4. Process capability analysis cannot be completed without ascertaining that the process is in control. Even though this is usually achieved using control charts, this topic is covered later in the book. The effect of measurement error, which is addressed in Chapter 5, should also be taken into consideration. Emphasis in the text is given to modeling of errors, estimation of error variances, and the effect of measurement errors on decisions related to quality. After process selection is completed, optimal means for obtaining the quality characteristics must be determined, and these are discussed in Chapter 6, Optimal Process Levels. The emphasis in this chapter is on the methodologies used and the development of objective functions and solution procedures used by various researchers. The next step in the methodology is process setting, as discussed in Chapter 7, in which the actual process mean is brought as close as possible to the optimal value determined earlier. Once the process setting is completed, manufacture of the components can begin. During the entire period of manufacture, the mean and variance of the process must be kept at their respective target values, which is accomplished, as described in Chapter 8, through process control, using control charts. Design of experiments, discussed in Chapter 9, can be used in any of the steps mentioned earlier. It serves as a valuable tool for identifying causes of problem areas, reducing variance, determining the levels of process parameters to achieve the target mean, and more.

Many of the steps described must be combined into one larger step. For example, concurrent engineering might combine tolerancing, process selection, robust design, and optimum process level into one step. It is emphasized again that neither the quality methodology chart in Figure 1.1 nor the treatment of topics in this book implies a sequential carrying out of the steps.

2

Tolerancing

CONTENTS

2.1 Introduction

In mass production, products are assembled using parts or components manufactured or processed on different processes or machines. This requires complete interchangeability of parts while assembling them. On the other hand, there will always be variations in the quality characteristics (length, diameter, thickness, tensile strength, etc.) because of the inherent variability introduced by the machines, tools, raw materials, and human operators. The presence of unavoidable variation and the necessity of interchangeability require that some limits be specified for the variation of any quality characteristic. These allowable variations are specified as *tolerances*. Usually, the tolerances on the quality characteristics of the final assembly/product are specified by either the customer directly or the designer based upon the functional requirements specified by the customer. The important next step is to allocate these assembly tolerances among the quality characteristics of the components of the assembly. In this chapter, we will learn some methods that have been developed for tolerance allocation among the components.

2.2 Preliminaries

We will consider assemblies consisting of k components ($k \geq 2$). The quality characteristic of component i that is of interest to the designer (user) is denoted by X_i. This characteristic is assumed to be of the Nominal-the-Better type. The upper and lower specification limits of X_i are U_i (USL_i) and L_i (LSL_i), respectively.

The assembly quality characteristic of interest to the designer (user) denoted by X is a function of X_i, $i = 1, 2, \ldots, k$. That is,

$$X = f(X_1, X_2, \ldots, X_k) \tag{2.1}$$

At first, we will consider linear functions of X_i only:

$$X = X_1 \pm X_2 \pm X_3 \pm \cdots \pm X_k \tag{2.2}$$

The upper and lower specification limits of X are U (USL) and L (LSL), respectively. These are assumed to be given by the customer or determined by the designer based on the functional requirements specified by the customer. Some examples of the assemblies with linear relationships among the assembly characteristics and component characteristics are given next.

Example 2.1

Three different assemblies are given in Figures 2.1a, b, and c. In the shaft and sleeve assembly shown in Figure 2.1a, the inside diameter of the sleeve and

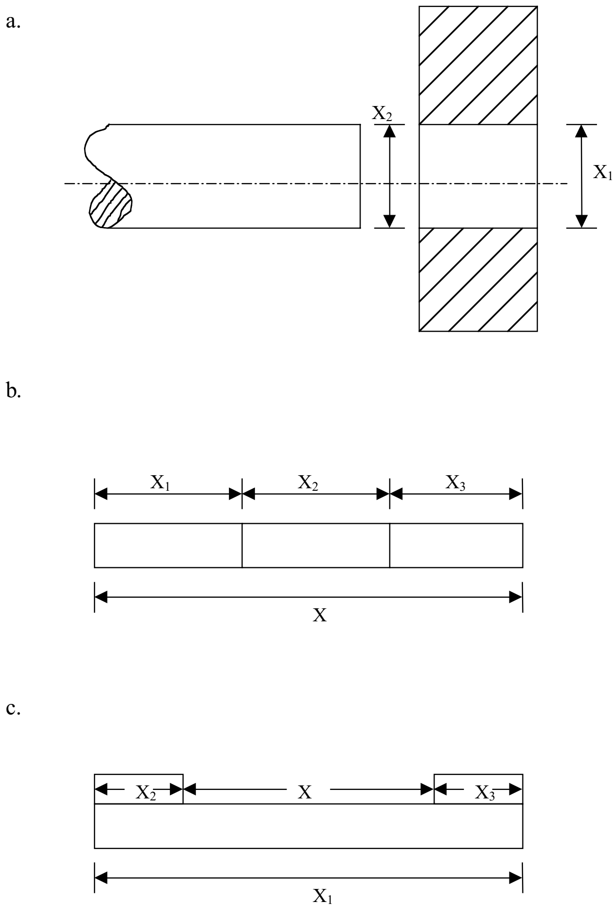

FIGURE 2.1
Some examples of assemblies.

the outside diameter of the shaft are the *component characteristics*, and the clearance between these diameters is the *assembly characteristic*. Let X_1 and X_2 represent the inside diameter of the sleeve and the outside diameter of the shaft, respectively, and let X denote the clearance between these two diameters. Then, the relationship between the assembly characteristic and the component characteristic is given by:

$$X = X_1 - X_2 \tag{2.3}$$

In the assembly given in Figure 2.1b, the component characteristics are the lengths of these components, denoted by X_1, X_2, and X_3, and X is the length of the assembly. The relationship among the tolerances in this case is given by:

$$X = X_1 + X_2 + X_3 \tag{2.4}$$

In Figure 2.1c, the assembly characteristic X is related to the component characteristics X_1, X_2, and X_3 as:

$$X = X_1 - X_2 - X_3 \tag{2.5}$$

In general, these relations can be written as in Eq. (2.2).

2.3 Additive Relationship

Tolerance is the difference between the upper and lower specification limits. Let the tolerance of X_i be T_i, $i = 1, 2, \ldots, k$, and let the tolerance of the assembly characteristic X be T. Then,

$$T_i = U_i - L_i, \quad i = 1, 2, \ldots, k \tag{2.6}$$

where L_i and U_i are the lower and upper specification limits of characteristic X_i, respectively. Similarly,

$$T = U - L, \tag{2.7}$$

where L and U are the lower and upper specification limits of X, respectively.

The relationship between T and $T_1, \ldots, T_k$ can now be derived using the assembly in Figure 2.1c as an example. The relationship among the tolerances was given in Eq. (2.5) as:

$$X = X_1 - X_2 - X_3$$

As U is the maximum allowable value of X, it is realized when X_1 is at its maximum allowable value and X_2 and X_3 are at their respective minimum allowable values. Hence,

$$U = U_1 - L_2 - L_3 \tag{2.8}$$

Similarly L, being the minimum allowable value of X, is obtained when X_1 is at its minimum allowable value and X_2 and X_3 are, respectively, at their maximum allowable values. Hence,

$$L = L_1 - U_2 - U_3 \tag{2.9}$$

Now, as per Eq. (2.7),

$$T = (U - L)$$

which can be written using Eqs.(2.8) and (2.9) as:

$$\begin{aligned} T &= (U_1 - L_2 - L_3) - (L_1 - U_2 - U_3) \\ &= (U_1 - L_1) + (U_2 - L_2) + (U_3 - L_3) \\ &= T_1 + T_2 + T_3 \end{aligned} \tag{2.10}$$

In general, for any linear function $X = X_1 \pm X_2 \pm X_3 \pm \cdots \pm X_k$,

$$T = T_1 + T_2 + T_3 + \cdots + T_k \tag{2.11}$$

This is called an *additive relationship*. The design engineer can allocate tolerances $T_1, \ldots, T_k$ among the k components, for a given (specified) T, using this additive relationship. Let us now use this relationship in an example to allocate tolerance among the components.

Example 2.2

Let us consider the assembly depicted in Figure 2.1a, having two components (sleeve and shaft) with characteristics (diameters) X_1 and X_2, respectively. The assembly characteristic is the clearance between the sleeve and the shaft, denoted by X, which is equal to:

$$X = X_1 - X_2 \tag{2.12}$$

and

$$T = T_1 + T_2 \tag{2.13}$$

Let us assume that the tolerance on X, which is T, is 0.001 in. Using Eq. (2.13), we get:

$$T_1 + T_2 = 0.001 \tag{2.14}$$

There are two unknowns, T_1 and T_2, and only one equation. In general, if the assembly has k components, there will be k unknowns and still only one equation. We need $(k-1)$ more equations or relations among the components' tolerances, T_i's, in order to solve for them. These relations usually reflect the difficulties associated with maintaining these tolerances while machining/processing the components. As we will see later, the manufacturing cost decreases when the tolerance on the quality characteristic increases. Let us assume that, in our example, the difficulty levels of maintaining both T_1 and T_2 are the same, hence the designer would like these tolerances to be equal. That is,

$$T_1 = T_2 \tag{2.15}$$

Using (2.14) and (2.15), we obtain

$$T_1 = T_2 = \frac{T}{2} = \frac{0.001}{2} = 0.0005$$

On the other hand, if it is more difficult to process component 1 than component 2, then the designer would like to have T_1 greater than T_2. For example, the following relation can be used:

$$T_1 = 2T_2 \tag{2.16}$$

In this case, using Eqs. (2.14) and (2.16), we get:

$$\begin{aligned} 2T_2 + T_2 = 0.001 \rightarrow T_2 &= 0.00033 \\ T_1 &= 0.00066 \end{aligned}$$

rounding off to five decimal places. It may be noted here that the number of decimal places carried in the tolerance values depends upon the precision of the instruments/gauges used to measure the characteristics.

2.4 Probabilistic Relationship

As this relationship depends upon the probabilistic properties of the component and assembly characteristics, it necessary to make certain *assumptions* regarding these characteristics:

1. X_i's are independent of each other.
2. Components are randomly assembled.
3. $X_i \sim N(\mu_i, \sigma_i^2)$; that is, the characteristic X_i is normally distributed with a mean μ_i and a variance σ_i^2 (this assumption will be relaxed later on).
4. The process that generates characteristic X_i is adjusted and controlled so that the mean of the distribution of X_i, μ_i, is equal to the nominal size of X_i, denoted by B_i, which is the mid-point of the tolerance region of X_i. That is,

$$\mu_i = \frac{(U_i + L_i)}{2} \tag{2.17}$$

5. The standard deviation of the distribution of the characteristic X_i, generated by the process, is such that 99.73% of the characteristic X_i

TABLE 2.1

Areas for Different Ranges Under Standard Normal Curve

Range	% Covered within the Range	% Outside the Range	Parts per million Outside the Range
$(\mu - 1\sigma)$ to $(\mu + 1\sigma)$	68.26	31.74	317,400
$(\mu - 2\sigma)$ to $(\mu + 2\sigma)$	95.44	4.56	45,600
$(\mu - 3\sigma)$ to $(\mu + 3\sigma)$	99.73	0.27	2700
$(\mu - 4\sigma)$ to $(\mu + 4\sigma)$	99.99366	0.00634	63.4
$(\mu - 5\sigma)$ to $(\mu + 5\sigma)$	99.9999426	0.0000574	0.574
$(\mu - 6\sigma)$ to $(\mu + 6\sigma)$	99.9999998	0.0000002	0.002

falls within the specification limits for X_i. Based upon the property of normal distribution, this is represented as (see Table 2.1):

$$U_i - L_i = T_i = 6\sigma_i, \quad i = 1, 2, \ldots, k \tag{2.18}$$

Let μ and σ^2 be the mean and variance, respectively, of X. As $X = X_1 \pm X_2 \pm X_3 \pm \cdots \pm X_k$,

$$\mu = \mu_1 \pm \mu_2 \pm \mu_3 \pm \cdots \pm \mu_k \tag{2.19}$$

and as the X_i's are independent of each other,

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_k^2 \tag{2.20}$$

Because of assumption 2 (above), the assembly characteristic X is also normally distributed.

Let us assume that 99.73% of all assemblies have characteristic X within the specification limits U and L. This yields a relation similar to Eq. (2.18):

$$(U - L) = T = 6\sigma \tag{2.21}$$

From Eqs. (2.18) and (2.21), we get:

$$\sigma_i^2 = \left(\frac{T_i}{6}\right)^2, \quad i = 1, 2, \ldots, k \tag{2.22}$$

and

$$\sigma^2 = \left(\frac{T}{6}\right)^2 \tag{2.23}$$

Combining Eqs. (2.20), (2.22), and (2.23) yields:

$$\left(\frac{T}{6}\right)^2 = \left(\frac{T_1}{6}\right)^2 + \left(\frac{T_2}{6}\right)^2 + \cdots + \left(\frac{T_k}{6}\right)^2 \tag{2.24}$$

or

$$T = \sqrt{T_1^2 + T_2^2 + \cdots + T_k^2} \tag{2.25}$$

The relationship given in Eq. (2.25) is called a *probabilistic relationship* and provides another means for allocating tolerances among components for a given assembly tolerance, T. Let us use this relationship to allocate tolerances among the two components of the assembly considered earlier.

Example 2.3

We may recall that by using the additive relationship (and assuming that $T_1 = T_2$), the tolerances were obtained as $T_1 = T_2 = 0.0005$ in. Now setting $T = 0.001$ in Eq. (2.25) yields:

$$\sqrt{T_1^2 + T_2^2} = 0.0005$$

We face the same problem we encountered earlier; that is, we have only one equation, whereas the number of variables is 2 (in general, it is k). If we introduce the same first relation used earlier ($T_1 = T_2$), then Eq. (2.26) gives:

$$\sqrt{2T_1^2} = 0.001 \rightarrow T_1 = \frac{0.001}{\sqrt{2}}$$

$$T_1 = T_2 = 0.00071$$

if five significant digits are kept after the decimal point.

The component tolerances T_1 and T_2 obtained using the additive and probabilistic relationships for the same assembly tolerance $T = 0.001$ and the same relationship between T_1 and T_2 ($T_1 = T_2$) are summarized in Table 2.2.

It can be seen that the probabilistic relationship yields larger values for the component tolerances compared to the additive relationship (43% more in this example). We will examine the advantages and disadvantages of this increase in component tolerances next.

Now, we have two relations between T and $(T_1,\ldots,T_k)$:

$$T = T_1 + T_2 + T_3 + \cdots + T_k \tag{2.26}$$

TABLE 2.2

Comparison of Additive and Probabilistic Relationships

	Additive	**Probabilistic**
T_1	0.0005	0.00071
T_2	0.0005	0.00071

and

$$T = \sqrt{T_1^2 + T_2^2 + \cdots + T_k^2} \tag{2.27}$$

Let us denote T in (2.26) by T_a and T in (2.27) by T_p (T_{a_i} and T_{p_i} for components); then:

$$T_a = T_{a_1} + T_{a_2} + \cdots + T_{a_k} \tag{2.28}$$

and

$$T_p = \sqrt{T_{p_1}^2 + T_{p_2}^2 + \cdots + T_{p_k}^2} \tag{2.29}$$

In Examples (2.2) and (2.3), we set $T_a = T_p = 0.001$ and solved for $T_{a_1} = T_{a_2} = 0.005$ and $T_{p_1} = T_{p_2} = 0.00071$. We saw that $T_{p_1} > T_{a_1}$ and $T_{p_2} > T_{a_2}$. Now let us examine the advantages and disadvantages of using the probabilistic relationship to allocate tolerances among the components.

2.4.1 Advantages of Using Probabilistic Relationship

It is a well-established fact that manufacturing cost decreases as the tolerance on the quality characteristic increases, as shown in Figure 2.2. Hence, the manufacturing cost of the components will decrease as a result of using the probabilistic relationship.

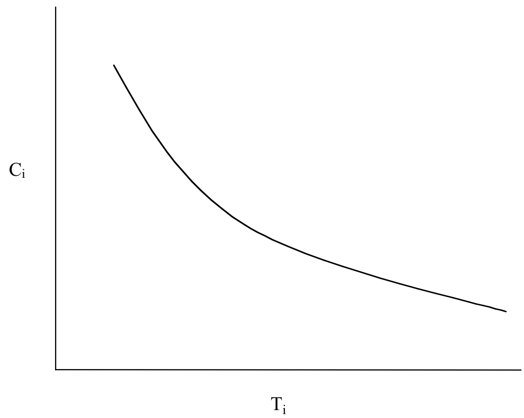

FIGURE 2.2
Curve showing cost–tolerance relationship.

2.4.2 Disadvantages of Using Probabilistic Relationship

If the probabilistic relationship is used, then the tolerance on the internal diameter of the sleeve and the outside diameter of the shaft is 0.00071. This implies that the maximum allowable range of the internal diameter of the sleeve is 0.00071. Likewise, the maximum allowable range of the outside diameter of the shaft is also 0.00071. Hence, the actual maximum range of the clearance of the assemblies assembled using these components will be

$$T_1 + T_2 = 0.00071 + 0.00071 = 0.000142$$

The allowable range of the clearance of the assemblies, T, is 0.001. This will obviously lead to rejection of the assemblies. In order to estimate the actual proportion of rejection, we need the probability distribution of the assembly characteristic, X, along with its mean and standard deviation.

If the component characteristics are normally distributed, then the assembly characteristic is also normally distributed. If the means of the component characteristics are equal to their respective nominal sizes, then the mean of the assembly characteristic is equal to the assembly nominal size. The only equation that contains the variance, σ^2, is

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_k^2 \tag{2.30}$$

The standard deviations, σ_1, and σ_2, are (per assumption (5) made earlier):

$$\sigma_1 = \frac{T_1}{6} = \frac{0.00071}{6} = 0.000118$$

and

$$\sigma_2 = \frac{T_2}{6} = 0.000118$$

Hence,

$$\sigma = \sqrt{0.000118^2 + 0.00118^2} = 0.000167$$

and

$$6\sigma = 6 \times 0.000167$$

$$= 0.001$$

Because X is normally distributed, the range 6σ contains 99.73% of the values of X, which is the assembly characteristic (see Table 2.1). Hence, the percentage rejection of the assemblies is <0.27%. This is illustrated in Figure 2.3.

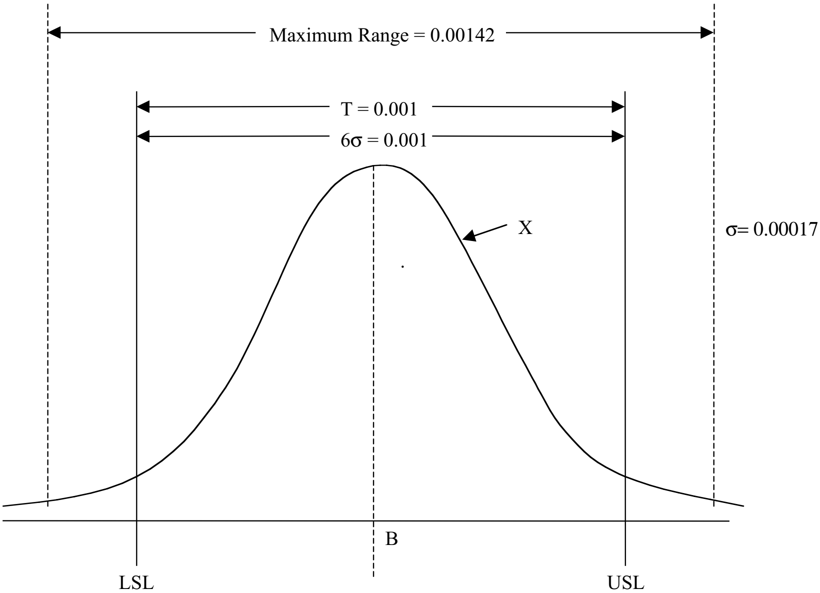

FIGURE 2.3
The result of a probabilistic relationship.

Now, let us compare the percentage of rejection of the assemblies when the component tolerances are determined using the additive relationship. Now the standard deviations, σ_1 and σ_2 are

$$\sigma_1 = \frac{0.0005}{6} = 0.0000833$$

and

$$\sigma_2 = 0.0000833$$

Hence,

$$\sigma = \sqrt{2 \times 0.0000833^2} = 0.0001179$$

and

$$6\sigma = 6 \times 0.000117 = 0.00071$$

As 6σ is less than the maximum allowable range ($T = 0.001$), the percentage rejection now is ≅0. This is illustrated in Figure 2.4.

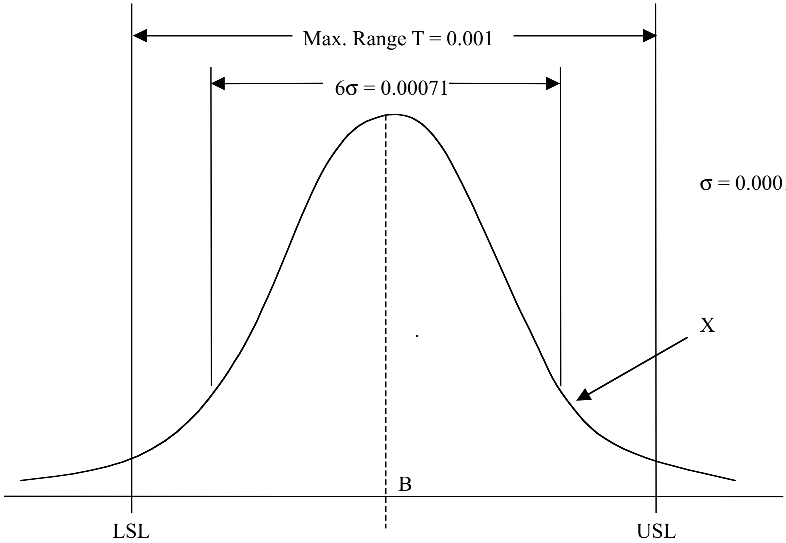

FIGURE 2.4
The result of an additive relationship.

Thus, determining component tolerances using the probabilistic relationship increases the percentage rejection of assemblies while decreasing the manufacturing cost of the components. It also increases inspection cost (100% inspection of assemblies).

2.4.3 Probabilistic Relationship for Non-Normal Component Characteristics

Let the probability density function of X_i be $f_i(x_i)$ with a mean μ_i and a variance σ_1^2. We assume that the range that contains 100% or close to 100% of all possible values of X_i is $g_i\sigma_i$. It is still assumed that:

$$T_i = g_i\sigma_i \tag{2.31}$$

(ideally $T_i >>>> g_i\sigma_i$). This can be written as:

$$\sigma_i = \frac{T_i}{g_i} \tag{2.32}$$

Now, given that $X = X_1 \pm X_2 \pm X_3 \pm \cdots \pm X_k$, the distribution of X is approximately normal, because of the Central Limit Theorem. So,

$$T_p = 6\sigma \rightarrow \sigma = \frac{T_p}{6},$$

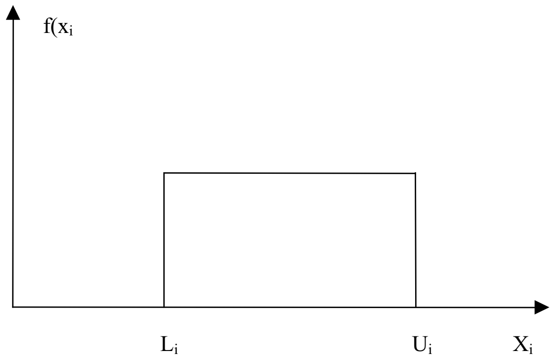

FIGURE 2.5
Uniform distribution.

assuming 99.73% coverage. Using the formula $\sigma^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_k^2$,

$$\left(\frac{T_p}{6}\right)^2 = \left(\frac{T_1}{g_1}\right)^2 + \left(\frac{T_2}{g_2}\right)^2 + \cdots + \left(\frac{T_k}{g_k}\right)^2$$

$$T_p = 6 \times \sqrt{\left(\frac{T_1}{g_1}\right)^2 + \left(\frac{T_2}{g_2}\right)^2 + \cdots + \left(\frac{T_k}{g_k}\right)^2} \tag{2.33}$$

2.4.3.1 Uniform Distribution

If $f_i(x_i)$ is a uniform distribution for all i, then (Figure 2.5):

$$T_i = U_i - L_i \tag{2.34}$$

$$\mathrm{Var}(X_i) = \frac{(\text{Range})^2}{12} = \frac{(U_i - L_i)^2}{12}$$

$$\sigma_i^2 = \frac{(T_i)^2}{12} \tag{2.35}$$

$$T_i = \sqrt{12}\ \sigma_i \quad (g_i = \sqrt{12}) \tag{2.36}$$

$$T_p = 6\sqrt{\frac{T_1^2}{12} + \frac{T_2^2}{12} + \cdots + \frac{T_k^2}{12}} = \sqrt{3}\sqrt{T_1^2 + T_2^2 + \cdots + T_k^2} \tag{2.37}$$