Risk Modeling for Determining Value and Decision Making

Glenn Koller

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I express my deepest appreciation to my wife Karen, my daughters Stephanie and Kathy, and my son Matthew. Writing a book is an exceedingly time-consuming undertaking. Every hour spent on the book-writing effort is an hour not spent with the family. I truly appreciate their love, patience, and understanding.

To Karen, Stephanie, Kathy, and Matthew —
my reasons for living.
Thank you for your love, patience, and understanding.

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Introduction

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SCOPE

This book is a follow-on to an initial volume entitled *Risk Assessment and Decision Making in Business and Industry: A Practical Guide* published by CRC Press (ISBN 0-8493-0268-4) which henceforth will be referred to as "the first book." In the first book were delineated in narrative style and in plain English the fundamental processes and technologies that compose the art and science of risk assessment and decision making and a few examples thereof. It is assumed that the reader of this volume is familiar with those processes and principles, and much of the material presented in the first book will not be repeated or reviewed in this treatise. Topics covered in the first book include

- Risk assessment process
- Organizational/cultural issues
- Risk communication
- Education
- Risk vision
- Building a consensus model
- Consistency
- Building a contributing-factor diagram
- Double dipping
- Bayesian analysis
- Decision trees
- Factor analysis
- Neural nets
- Monte Carlo analysis
- Distributions
- Decisions
- Chance of failure (abject and financial)
- Time-series analysis
- Dependence
- Risk-weighted values

- Sensitivity analysis
- A host of risk assessment examples

To gain a more complete and comprehensive understanding of the many processes and techniques alluded to in this book (and much more), it is recommended that the reader review the information in the first book. Chapters 5, 6, and 9 through 13 of the first book are, with some modification, repeated in this volume as Chapters 12 through 18 in the Fundamentals of Risk Assessment section. The publisher felt that these chapters were essential to the understanding of the precepts and technologies utilized in the examples of risk and uncertainty assessment presented in the first 11 chapters of this book.

The first book included a small number of examples of application of risk assessment (legal, fate-transport, qualitative, time series, financial, plant construction, and others). Since its publication, there has been a clamor from numerous business, legal, and academic venues for a greater number of more detailed, real-life examples of the principles and techniques set out in that volume. The purpose of this book is to quell that din.

Risk and uncertainty examples outlined in this treatise are more comprehensive and meticulous than those delineated in the first book. Even so, risk assessment renditions in this volume still are generally elementary relative to most real-world applications of risk technology and processes. I had to strike a balance. The models described herein had to be detailed enough to have credibility and ring true with practitioners in the various fields represented by the examples. However, exceedingly long, complex, and minutely detailed risk models do not make stellar examples. Such complex applications generally are too "clumsy" to present, too lengthy, and numbingly esoteric. A point that is attempting to be made in such a model typically is lost because the logic thread is of too great a length and tied in far too great a number of cognitive knots. Therefore, the examples given here are just detailed and comprehensive enough to have credibility while hopefully retaining the quality of being eminently readable.

REALISM

I also have tried to inject an element of business-life realism into each narrative. Have you ever been put into the position of needing to accomplish a goal or deliver a product without having been afforded sufficient human and/or financial resources? Have you required crucial input from individuals who are not obligated to comply with your request for help? Have you ever been compelled to make a decision with far fewer facts than you think are required? Have you ever been given all the responsibility to make something happen but no authority to do so? Yes? Well, me too!

Each example is presented as a narrative. In most narratives, I follow a fictitious individual or set of individuals through the process of designing and enacting a risk model and interpreting the output. The narrative approach allows me to present serious subject matter in a light-hearted and readable manner. In addition, the narrative format facilitates the injection of real-world scenarios (such as those mentioned in the preceding paragraph) and to detail how such situations are handled in a business and risk assessment context. Although relatively simple, the presented risk models are real and the business and human-interaction discourses are true to life.

MODELS, VALIDATION, AND PRECISION

Most examples presented in this book include at least some computer code that encapsulates the logic of the risk model or models. The "language" in which these computer programs are presented is that of a risk assessment system devised by myself and my esteemed colleagues (Mike A. Long and Phil Hammond). Although the risk/uncertainty models presented here were generated using a software system not generally available, the models and the logic therein can be easily reproduced, for the most part, using any of a number of commercially available risk-related software systems.

The language in which the programs are presented is irrelevant. If presented in C++, undoubtedly someone would want them converted to FORTRAN (imagine that!). If presented in BASIC, a cadre of readers would prefer to see them described in JAVA. And so it goes.

The computer programs have been written using variable names that are explicit and understandable. The logic is laid out "from top to bottom," including comments in the code and a list of detailed variable definitions. This has been done to facilitate the translation of the model logic into any language. It is the logic, not the language, that is paramount. Admittedly, some user-written macros or subroutines might have to be composed to emulate certain system functions in the code presented (for example, the resampling of a distribution across time periods within a single Monte Carlo iteration).

A related point is that *there is nothing special about the risk models presented here*. The variables used and the codes displayed are but one set of variables and programs that might have been used to attack a given problem. I do not put forward these models as "the way to do it"; rather, these models are meant to instill in the reader that "it can be done." This is just one way, and not necessarily the best way, to accomplish the task.

Validation of the models can, in most instances, be done only over time. Attempting to validate the model relative to some other path that might have been taken or another model that might have been used is folly. You rarely, if ever, have the opportunity to know the result(s) of the "road not taken." In the case of many of the scenarios described in this book, the only validation possible is to deem whether the decisions made using model output were, in the end, good decisions for the company. If the majority of the courses taken result in economic prosperity for the company, then the models are vindicated on those grounds. You may argue that the company might have done just as well or better had they not even considered undertaking the risk-model route. You are welcome to do that.

Some of the model-output parameters are represented by numbers that have more "precision" than the numbers that went into creating them. This is a function of the software utilized to represent the output parameters. It allows the specification of X significant digits, but not how many are on either side of the decimal point. Therefore, to accommodate coefficients of relatively great magnitude, many significant figures had to be specified. This has the unfortunate and unintended effect for coefficients of relatively small magnitud, to have many more digits to the right of the decimal place than are warranted. The "significant figure" problem is not an easy one to dynamically adjust in the off-the-shelf graphics package employed here. Therefore, like so many other things in life, I decided not to like it, but just to live with it. The reader should know that I am aware of this problem but decided to chance the wrath of the readers rather than extend by a year the production of this volume. Your patience and understanding are requested and appreciated.

VALUE

As the reader, you will note that in each chapter there is an emphasis on the value of the opportunity that is the subject of the example. Value should be, but too often is not, the focus of a risk assessment model. Most models I have seen that are created by others tend to stop short of the calculation-of-value step. That is, if a risk model starts out as an environmental assessment, the answer (output) variables tend to be dosages of toxins, concentrations of an element in the environment, and so on. Likewise, construction models typically result in variables that express the cost and scheduling aspects of a project. Legal assessments tend to culminate in assessments of damages. Marketing models generate results that speak to the demographics, sales volumes, and margins related to a given scenario. The point is, businesses generally wish to make (and should make) tactical and strategic decisions based on the value of the opportunity to the corporation.

In this book I tend to focus on the net present value (NPV) of opportunities as the indicator of value. Measures of value such as internal rate of return (IRR), discounted return on investment (DROI), and return on capital employed (ROCE) can be equally valid in a given situation. The point is that nearly any risk assessment—whether it begins as a legal, technical, environmental, business, construction/manufacturing, or some other type—can and should generate a measure of value as an output parameter. This not only gives a corporation a true measure of the worth of an opportunity over time, but also affords a common basis upon which the corporation can compare, rank, and manage a portfolio of diverse entities. I am a firm believer in the philosophy of measuring the value of projects and opportunities, and the risk assessment examples in this book reflect that bent.

Section 1

Examples

1 Two Approaches to Solving Decision Trees — Class-Action Suit Example

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INTRODUCTION

Classical solution of a decision tree involves beginning at the "end" leaf nodes and multiplying leaf-node values by branch probabilities. The products of such multiplications are summed until we reach the "root" of the tree where the sum of the products results in an expected value for the tree.

Use of single-valued (i.e., deterministic) decision trees, especially for legal analyses and decisions, is a less-than-optimal approach for at least three reasons. Other concerns such as the inability of deterministic decision trees to consider the effects of "soft" issues through the use of chance of failure (see Chapter 16) also lend to the inadequacy in this arena. However, only three major drawbacks will be discussed briefly here.

The first foible relates to how a single erroneous assumption as a leaf-node value can render useless the calculated decision tree result. For example, most legal logic trains are lengthy. Any decision tree for a real-life legal analysis is really a "decision bush" with many (sometimes hundreds) of branches and associated leaf nodes. When using a deterministic tree, attorneys and clients must discuss and settle upon a single value at each leaf node. Because in the classical solution of the decision tree all leaf-node values and probabilities are multiplied together, a single incorrect value can invalidate the calculated result. In many instances, this error is difficult to detect and the problem goes unresolved. Use of ranges for values and accounting for dependence between parameters (see below) significantly alleviates this problem.

A second impediment to the use of deterministic decision trees is the inability to practically determine a reasonable range of results. That is, each solution of the decision tree results in a single calculated answer. In a tree that might contain hundreds of branches, it is an impractical task to change a single value at a given node and rerun the model to determine the impact of the change on the calculated answer. In addition, it often is necessary to change more than one leaf-node value for a given solution of the tree. It should be obvious that, in a tree that contains tens or hundreds of branches and nodes, the number of possible permutations can be in the thousands (or even millions, depending on the size of the tree). Attempting to generate a complete range of possible outcomes in this manner simply is not practical.

A third, but by no means final, glitch associated with the use of deterministic decision trees in legal analyses relates to the dependence between variables. In a deterministic tree, each value must be arrived at by employment of in-depth conversations. This mechanism is no different when distributions are used. When discussing the value for each node, humans must keep in mind the values they have entered for all other related nodes. For example, if we at some point in the decision tree have to enter the possible penalties associated with a judgment, we might have to relate that number to a previously entered leaf-node value (somewhere else in the tree) that indicates upon how many counts the client was deemed guilty. Typically, when approaching a decision tree in a deterministic manner, it is up to the humans, when changing values at leaf nodes, to keep in mind the relationship of those values to other values in the tree.

Using a probabilistic approach to solve decision trees does not absolve the tree makers from such considerations. However, most probabilistic software packages (those worth their salt, so to speak) contain a mechanism by which the tree builders can assign dependencies between variables. The tree can be instructed to select a value for a given leaf node that is reasonable with respect to other leaf-node values already selected. This means that consideration of dependencies needs to be addressed only once — when initially setting up the tree. Subsequent runs (or, in the case of Monte Carlo analysis, subsequent iterations) will honor the dependencies established. This is a tremendous benefit and time saver.

Solving decision trees probabilistically simply replaces the leaf-node deterministic values (and sometimes the branch probabilities, but this is a more intractable problem) with distributions, and the tree is solved many times. On each solution of the tree, a random grab is made from each leaf-node distribution, and the expected value is calculated in the usual way. Repeated random grabs and solutions of the tree result in a distribution of expected values.

The range of the expected values that results from this process is not representative of the full range of possibilities that one might encounter employing the decision tree in real life. An alternative method of probabilistic branching without using the branch probabilities as multipliers yields much more realistic results for a class of problems.

Decision trees are an often-used vehicle for analysis of legal strategies and cases. For example, an attorney might consider the following logic for a possible litigation:

> "We might settle the case. If we don't settle, we would take the case to court where we have a pretty fair chance of winning. However, if we lose the case, we will be subject to damages which could be of two kinds …."

This type of logic often is diagramed as a decision tree. Even when probabilistic techniques are employed, solving such a decision tree in the classical manner results in a range of solution values that is not representative of real life. Neither the range of answers nor the associated probabilities are what the attorney really wants. In the following example I will demonstrate the difference between the classical approach and the probabilistic branching method for solving decision trees. A class-action-suit scenario will be used, but this logic holds true for decision trees that are applied to almost any problem.

BUILDING THE DECISION TREE

Perry is an attorney for the law firm of Lamb, Curry, and Rice. Regarding a major class action suit, Perry has been assigned the task of analyzing the situation and is to determine whether it is economically more desirable to settle the case or to take it to court. Perry has done his homework and has determined that the major aspects of the case are thus:

- We could settle the case for a large sum of cash.
- We could fight the case in court.
- If we go to court, we could win and our damages would be minimized.
- If we lose the case in court, we might be held responsible for physical damages only.
- If we lose the case, we might be held responsible for physical damages and, in addition, be made to pay punitive damages.
- If we are made to pay punitive damages, they may relate only to the contemporary physical damages.
- If, however, the jury decides that we are culpable for past damages, we might be made to pay punitive damages of a retroactive nature. Such punitive damages could be of much greater magnitude.

At first blush, this seems simple to Perry and he quickly generates the diagram shown in Figure 1.1. Now comes the more arduous and subjective task of populating the tree with probabilities and leaf-node consequences. To do this, Perry realizes that he will need the help of others. Perry convenes a meeting of his fellow attorneys.

At the meeting the group agrees on the branch probabilities and deterministic leaf-node values shown in Figure 1.2. Recalling from his college days the proper method for solving a decision tree, Perry quickly calculates the tree's expected value (EV) by applying the following calculation:

$$(0.3 \times 1.5) + 0.7 \times ((0.2 \times 3.5) + (0.8 \text{ x } ((0.7 \times 3.3) + (0.3 \times 6.5)))) = 3.326 \quad (1.1)$$

The only thing you can know for sure about the single-valued answer of a deterministic result (especially one carried out to three decimal places) is that it is not the value that real life will yield. Perry surmises that what he really wants is a probabilistic analysis of this case. To get such an analysis, Perry approaches Mason, the corporate risk expert. Mason advises Perry that he can easily convert the decision tree to a probabilistic model, but that Perry will have to replace most or all of the deterministic leaf-node values with ranges of values (i.e., distributions).

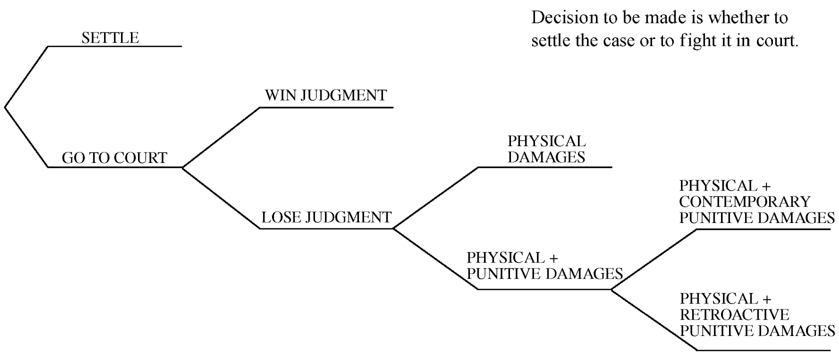

FIGURE 1.1 Basic decision tree for class-action suit.

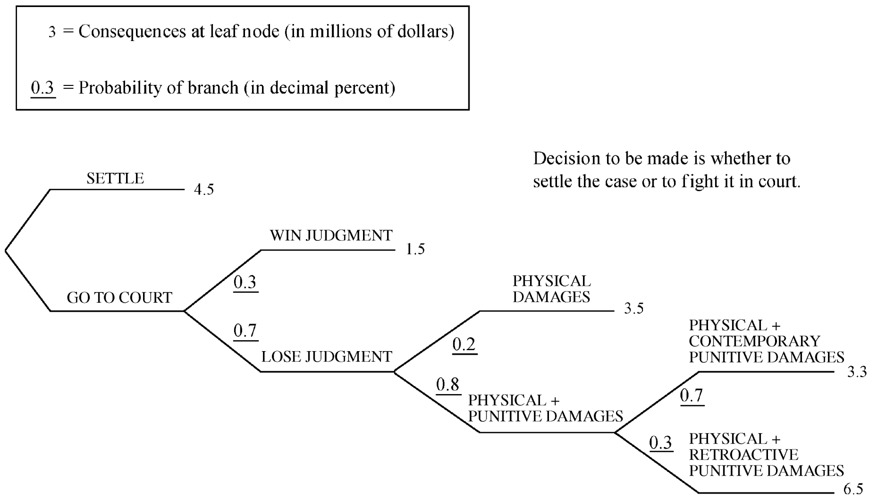

FIGURE 1.2 Class-action-suit decision tree with deterministic probabilities and leaf-node values.

After a week of meetings and data gathering, Perry returns to Mason with minimum, most likely, and maximum values for each leaf node in his decision tree. Mason will use the minimum, most likely, and maximum values in his proprietary software to build a distribution at each node. The resulting decision tree appears in Figure 1.3. The agreed-upon distributions are shown in Figures 1.4 through 1.8.

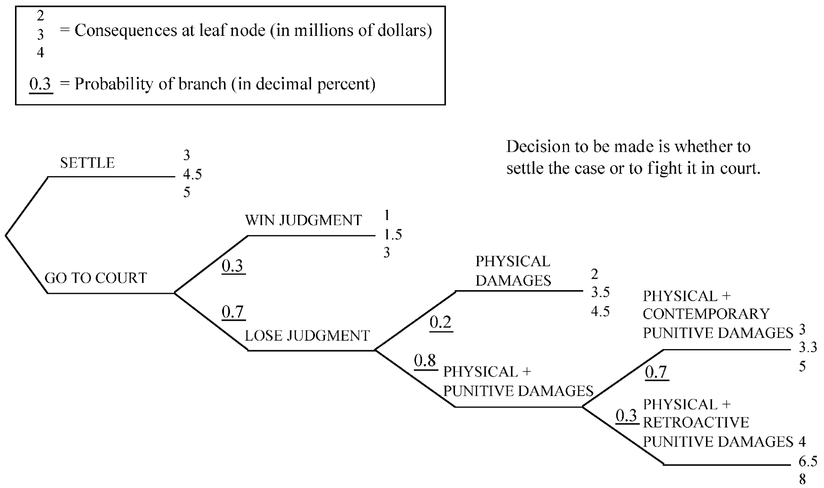

FIGURE 1.3 Class-action-suit decision tree with deterministic probabilities and leaf-node minimum, most likely, and maximum values for building distributions.

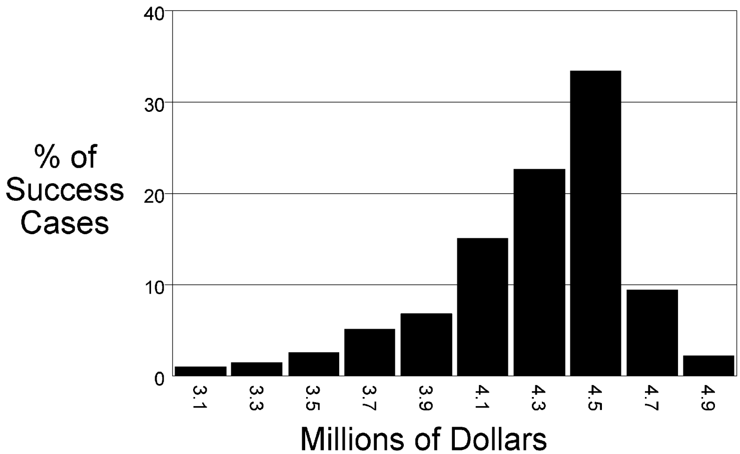

FIGURE 1.4 Distribution of settlement damages values.

The simple computer code generated by Mason for the probabilistic model is shown below.

```
PunDam = PhysPunProb * ((PhysConPunProb * PhysConPunDam) +
   (PhysRetPunProb * PhysRetPunDam));
```

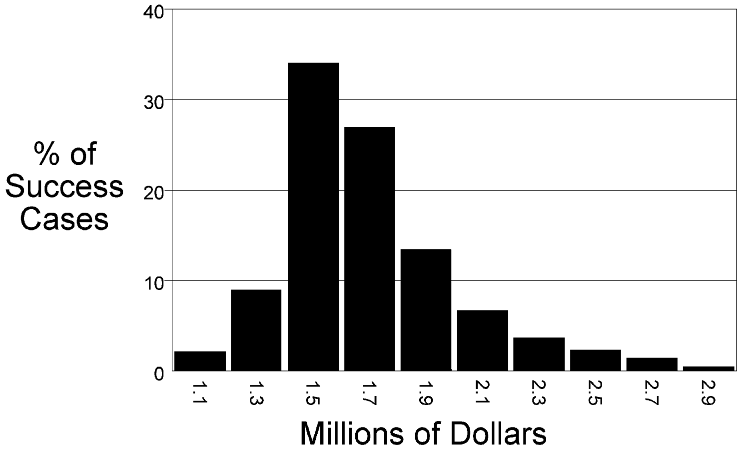

FIGURE 1.5 Leaf-node distribution for "win judgment" branch.

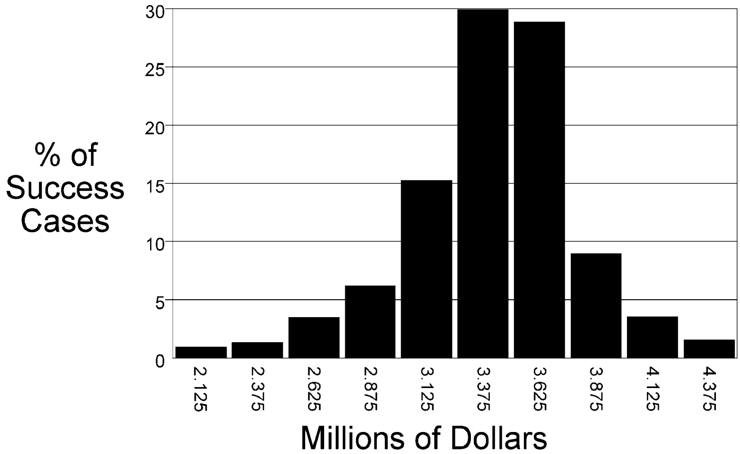

FIGURE 1.6 Leaf-node distribution for "physical damages" branch

PhysPunDam = LoseProb * (PunDam + (PhysProb * PhysDam));
Fight = (WinProb * WinDam) + PhysPunDam;
Delta = Fight – Settle;

where

PunDam is the amount the firm will have to pay for punitive damages
PhysPunProb is the probability that the firm will be held responsible for physical and punitive damages
PhysConPunProb is the probability that the firm will be held responsible for payment of physical and contemporary punitive damages

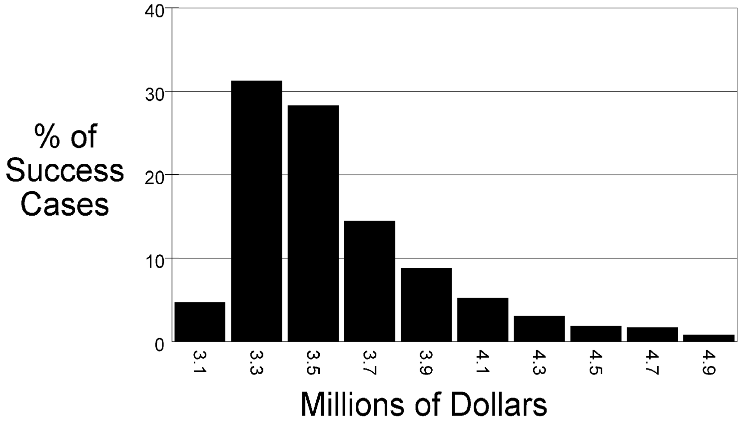

FIGURE 1.7 Leaf-node distribution for "physical and contemporary punitive damages" branch.

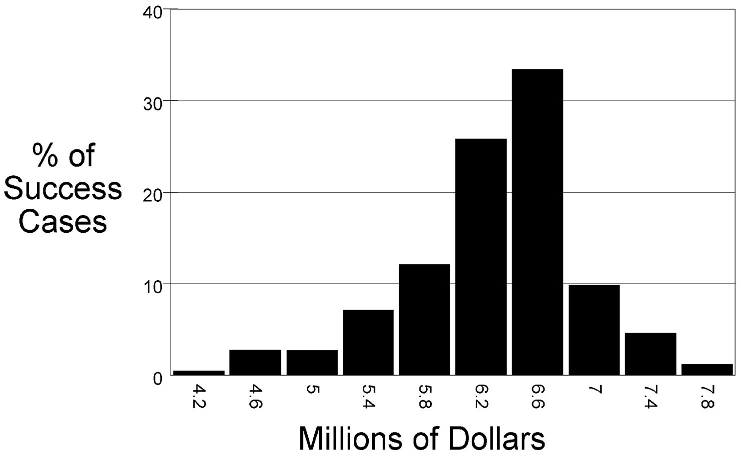

FIGURE 1.8 Leaf-node distribution for "physical and retroactive punitive damages" branch.

PhysConPunDam is the amount of physical and contemporary punitive damages the firm might have to pay
PhysRetPunProb is the probability that the firm will be held responsible for payment of physical and retroactive punitive damages
PhysRetPunDam is the amount of physical and retroactive punitive damages the firm might have to pay
PhysPunDam is the total amount of physical and punitive damages for which the firm might be held responsible
LoseProb is the probability of losing the judgment
PhysProb is the probability of having to pay only physical damages

PhysDam is the amount of physical damages alone
Fight is the total cost of fighting the case in court
WinProb is the probability that the firm will win the judgment
WinDam is the amount it will cost the firm if they win the judgment
Delta is the difference between the damages related to fighting the case relative to settlement
Settle is the cost of settling the case

This model iteratively solves the above system of equations a total of 1500 times. On each of the 1500 iterations, a value is randomly selected from each leaf-node distribution and plugged into the equations. This Monte Carlo process, then, yields 1500 answers. The frequency and cumulative frequency plots for the 1500 results are shown in Figure 1.9.

At first, Perry is happy with the plots shown in Figure 1.9. However, after staring at the input distributions, the decision tree, and the output plots, he begins to feel a bit uneasy.

Perry notes that the entire range of answers on, for example, the cumulative frequency plot is from about 2.8 to about 4.4. These values do not even come close to the lowest value on the decision tree ($1 million if Lamb, Curry, and Rice wins the case) or the highest value on the tree ($8 million if the firm has to pay retroactive punitive damages).

This is a troubling result, but Perry can't quite put his finger on the problem. To help resolve the dilemma, Perry creates a simple two-branch decision tree with deterministic (i.e., single-valued) leaf-node values. This simple tree is shown in Figure 1.10.

WHAT IS THE QUESTION?

Using the conventional decision-tree logic put together by Mason, the probabilities on the branches are used as multipliers for leaf-node values. Solution of the simple tree shown in Figure 1.10 would be:

$$\text{E.V.} = (0.6 \times 1) + (0.4 \times 10) = 4.6 \tag{1.2}$$

In this equation, E.V. is the expected value for the tree which is 4.6. Perry notes that if the leaf-node values of 1 and 10 represented real-life consequences, a result of 4.6 is not a possibility. The real-life answers will either be 1 or 10. He also notes that this will be true regardless of the nonzero values assigned as probabilities for the decision-tree branches.

Perry thinks that the cause of the problem might lie in the fact that he is treating the problem in a deterministic manner. To test this, Perry asks Mason to build a new model for the simple decision tree shown in Figure 1.11 and to use the values minimum = 1, most likely = 2, maximum = 3 for the top-branch leaf-node distribution. For the bottom-branch leaf node, Perry gives Mason the distribution-building values of minimum = 8, most likely = 9, and maximum = 10. Mason builds and runs the model. Plots resulting from the model are shown in Figure 1.12.

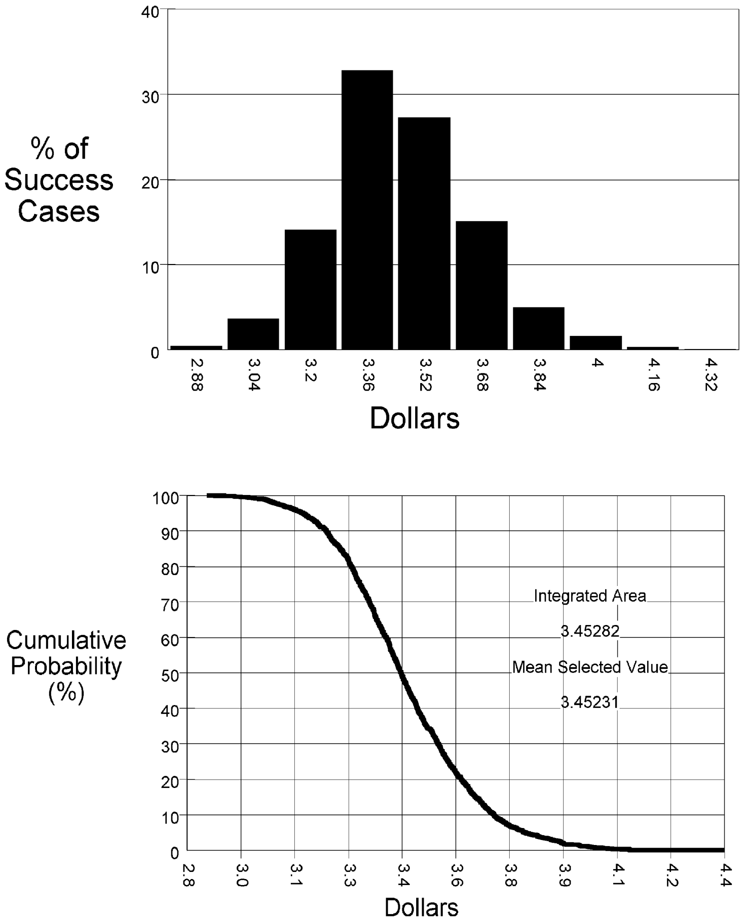

FIGURE 1.9 Frequency and cumulative frequency plots resulting from solving the decision tree using the Monte Carlo method and conventional decision-tree solving logic for damages for go to trial.

Inspection of the plots in Figure 1.12 quickly tells Perry that treating the problem probabilistically is also not the solution. Even though distributions were used for the two leaf nodes, the X-axis values in Figure 1.12 do not come close to the values used in the two distributions. In real life, taking the top branch of the decision tree (i.e., winning), regardless of the probability of taking that branch, a consequence between 1 and 3 would be realized. Likewise, regardless of the probability of taking the bottom branch, the real-life consequence of going down that path (i.e., losing) would be a value between 8 and 10. Conventional decision-tree logic that utilizes the probabilities as multipliers will not yield such results.

Perry now realizes that the conventional method for solving decision trees results in an expected value for the tree. Perry has to wrestle with the dilemma of just what question he really wants to have answered.

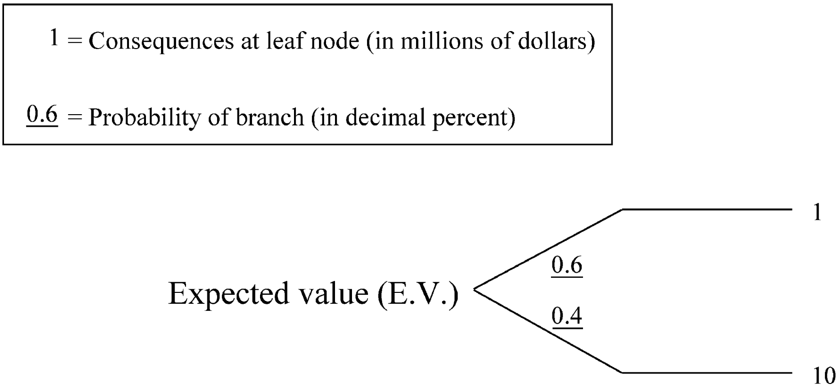

FIGURE 1.10 Simple two-branch decision tree with deterministic probabilities and leaf-node values.

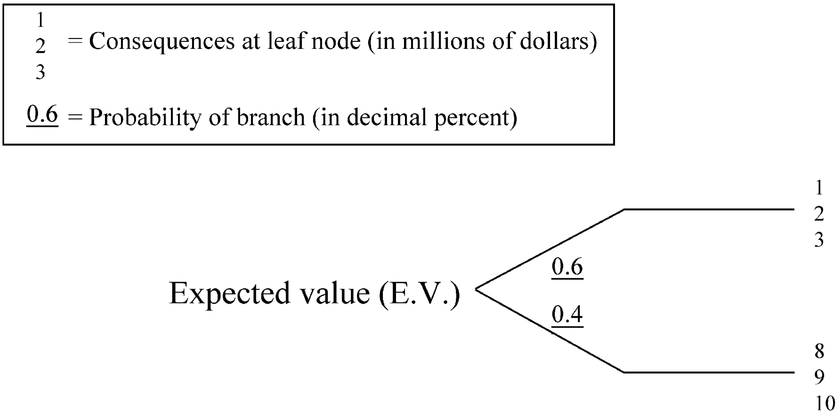

FIGURE 1.11 Simple two-branch decision tree with deterministic probabilities and distributions at leaf-node values.

After consulting with other attorneys and clients, Perry comes to the conclusion that the traditional method for solving decision trees — one that yields expected values — is not the answer to the actual question posed by a class-action-suit legal decision. All parties agree that for this application, the probabilities on the tree branches should not be used as multipliers for leaf-node values. Rather, a branch probability should be viewed as the chance that a branch will be taken. The decision to take a branch will be decided by, at each decision node, the generation of a random number. For example, in the decision tree shown in Figure 1.13, a random number between 0 and 1 would be generated at the decision point (point at which the branches

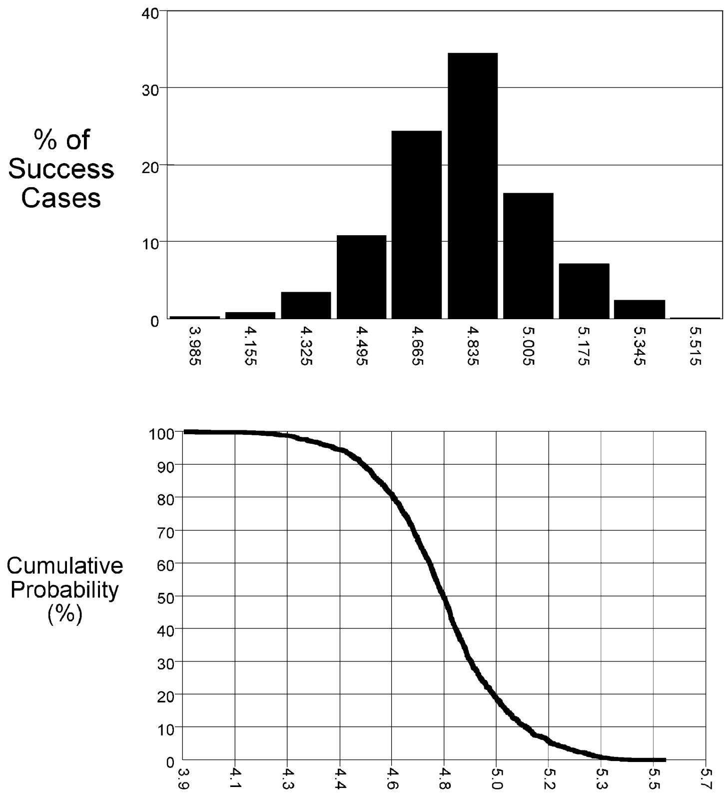

FIGURE 1.12 Frequency and cumulative frequency plots resulting from solving the simple two-branch decision tree for expected value using the Monte Carlo method and conventional decision-tree solving logic.

join — in this case, the E.V. position). If the random number generated is less than or equal to 0.6, then we would take the top branch and our consequence would be a number between 1 and 3. If the random number were greater than 0.6, then our consequence would be a value between 8 and 10. This process might be repeated hundreds or thousands of times in a Monte Carlo model resulting, in this case, in a bimodal distribution.

Since Perry now believes he knows what he wants, he approaches Mason with the concept and convinces him to write yet another program that captures the logic of probabilistic branching. Mason, too, is convinced that this is the right solution to the problem and produces the following program.

```
PunDam = if(rand() <= PhysRetPunProb, PhysRetPunDam,
   PhysConPunDam);
PhysPunDam = if(rand() <= PhysPunProb, PunDam, PhysDam);
```

Fight = if(rand() <= LoseProb, PhysPunDam, WinDam);
Delta = Fight – Settle;

where, in addition to the variables in the previous model,

rand is a random number between 0 and 1 that is generated at each decision point to determine which branch of the decision tree that is taken.

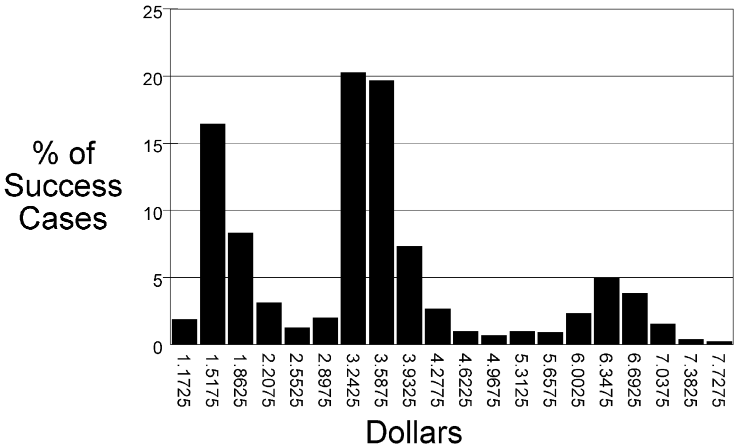

FIGURE 1.13 Frequency plot for go to trial damages resulting from using the Monte Carlo method and probabilistic-branching method to solve the decision tree shown in Figure 1.3.

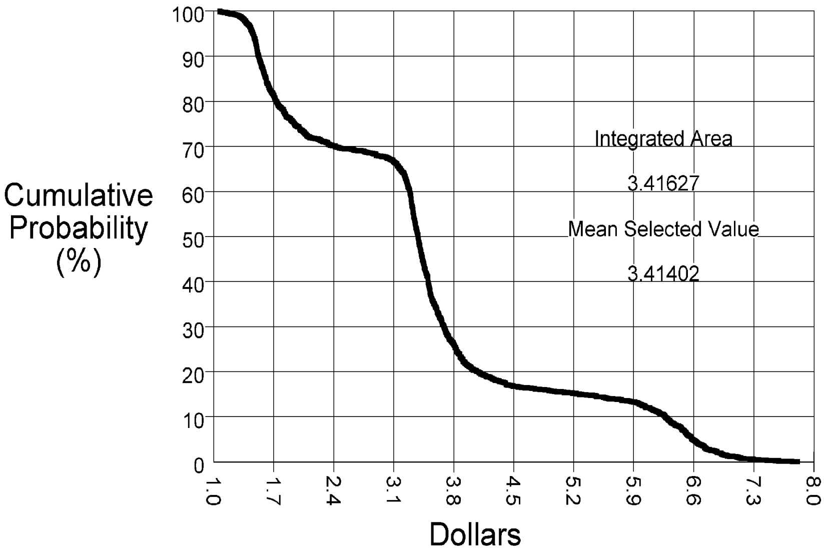

FIGURE 1.14 Cumulative frequency go to trial damages plot resulting from using the Monte Carlo method and probabilistic-branching method to solve the decision tree shown in Figure 1.3.

INTERPRETATION OF THE PROBABILISTIC-BRANCHING MODEL

Output plots from the probabilistic-branching model can be seen in Figures 1.13 and 1.14. After viewing and considering the results from this model, Perry is far more satisfied. It can be seen from Figures 1.13 and 1.14 that the extremes of the output distribution approach the extremes represented by the distributions of the leaf nodes. For example, the output-distribution values of greatest magnitude are near 10. This corresponds to the "physical and retroactive punitive damages" leaf-node-consequence distribution. It can be seen from the cumulative frequency plot that there is a very small probability of realizing a consequence near 10, but it is, nonetheless, a possibility. This small possibility corresponds to the relatively unlikely event that as decisions are made at each of the tree's decision nodes (moving from left to right), we will end up on this branch. The small likelihood of a value near 10 also reflects the relatively minute chance that even if we do end up on this branch, a random draw from the leaf-node distribution will result in selection of a value near 10 from that distribution. This emulates real life.

Cumulative frequency curves resulting from the classical method for solving decision trees tend to be smooth and "well behaved" (Figure 1.9). This is a natural consequence of calculating a "blended" expected value on each iterative solution of the decision tree. A cumulative frequency plot resulting from the probabilistic-branching method, however, is more likely to appear multi-modal. This results from the disparity in the magnitude of values at the various leaf notes (see Figure 1.3).

Figure 1.13 shows the frequency-plot equivalent of the cumulative frequency plot in Figure 1.14. Note that there are essentially three "modes," i.e., "peaks" in the distribution. The peak representing values of lowest magnitude is intermediate in height (frequency). This peak's X-axis position and relative frequency are mainly the results of the 30% chance of winning the judgment (see Figure 1.3). The peak of greatest frequency near the middle of the X-axis is of relatively great frequency because all but one branch of the decision tree has leaf-node values that could result in a value in the range represented by this peak. Similarly, the right-most peak in Figure 1.13 indicates that there is a relatively small chance that a value near the maximum of $8 million will be realized.

Perry is much more satisfied with his interpretation of the cumulative frequency curve resulting from the probabilistic-branching model. Interpretation of the curve indicates that there is a 100% chance of the case resulting in a cost of $1 million or more. The $1 million figure represents a real-life possibility if the firm wins the judgment. There exists about a 70% chance that damages from the case will be about $2 million or greater. There is about a 20% chance that damages will be $4 million or more. A small chance exists that a value near the maximum $8 million figure will be realized.

"Horizontal" sections of the cumulative frequency curve correspond to "low spots" in the frequency plot. These parts of the curve represent dollar ranges that are less likely to occur. Conversely, steeper "vertical" sections of the cumulative frequency curve correspond to "peaks" in the frequency display and indicate dollar