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Preface

Gravitational waves today represent a hot topic, which promises to play a central role in astrophysics, cosmology and theoretical physics.

Technological developments have led us to the brink of their direct observation, which could become a reality in the coming years.

The direct observation of gravitational waves will open an entirely new field; gravitational wave astronomy. This is expected to bring a revolution in our knowledge of the universe by allowing the observation of hitherto unseen phenomena such as coalescence of compact objects (neutron stars and black holes), fall of stars into supermassive black holes, stellar core collapses, big-bang relics and the new and unexpected.

During Spring 1999, the SIGRAV—Società Italiana di Relatività e Gravitazione (Italian Society of Relativity and Gravitation) sponsored the organization of a doctoral school on ‘Gravitational Waves in Astrophysics, Cosmology and String Theory’, which took place at the Center for Scientific Culture ‘Alessandro Volta’ located in the beautiful environment of Villa Olmo in Como, Italy.

This book brings together the courses given at the school and provides a comprehensive review of gravitational waves. It includes a wide range of contributions by leading scientists in the field. Topics covered are: the basics of GW with some recent advanced topics, GW detectors, the astrophysics of GW sources, numerical applications and several recent theoretical developments. The material is written at a level suitable for postgraduate students entering the field.

The main financial support for the School came from the University of Insubria at Como-Varese. Other contributors were the Department of Chemical, Physical and Mathematical Sciences of the same University, the Physics Departments of the Universities of Milan and Turin, and the Institute of Physics of Interplanetary Space—CNR, Frascati.

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I Ciufolini, V Gorini, U Moschella and P Fré
Como
12 June 2000

Chapter 1

Gravitational waves, theory and experiment (an overview)

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General relativity and electrodynamics display profound similarities and yet fundamental differences [1, 2]. In this connection, it may be interesting to point out some historical analogies between the two fields.

The enormous success of Maxwell's equations did not rest only in the fact that they incorporated, together with the Lorentz force equation, all the laws of electricity and magnetism, but also that on their basis James Clerk Maxwell (1831–1879) was able (in 1873) to predict the existence of a solution consisting of electric and magnetic fields changing in time, carrying energy and propagating with speed c in vacuum: the electromagnetic waves. Nevertheless, some distinguished physicists, such as Lord Kelvin, had serious doubts about the existence of such waves: 'The so-called "electromagnetic theory of light" has not helped us hitherto ... it seems to me that it is rather a backward step ... the one thing about it that seems intelligible to me, I do not think is admissible ... that there should be an electric displacement perpendicular to the line of propagation'. However, in 1887, eight years after Maxwell's death, electromagnetic waves were both generated and detected by Heinrich Hertz (1857–1894); then, in 1901, Guglielmo Marconi transmitted and received signals across the Atlantic Ocean.

In the twentieth century the detection and study of electromagnetic waves, other than visible light, opened a new era of dramatic changes in the knowledge of our universe: cosmic radio waves, discovered in the 1930s, revealed in subsequent decades colliding galaxies; quasars with dimensions of the order of the solar system but having luminosities orders of magnitude larger than our galaxy; enormous jets from galactic nuclei and quasars reaching lengths of hundreds of thousands of light years, rapidly rotating pulsars with rotational periods of a few milliseconds and, not least, the cosmic microwave background, a relic of the hot big bang. X-rays revealed accretion disks about black holes and neutron stars. Similarly, millimetre, infrared and ultraviolet radiation, and gamma rays opened other dramatic windows of knowledge on our universe.

In the same way, in general relativity [1, 2], Einstein's field equations (1915) not only described the gravitational interaction via the spacetime curvature generated by mass-energy, but also contained, through the Bianchi identities, the equations of motion of matter and fields, and on their basis Albert Einstein, in 1916, a few months after the formulation of the theory, predicted the existence of curvature perturbations propagating with speed c on a flat and empty spacetime; the gravitational waves [4]. Einstein's gravitational-wave theory was a linearized theory treating weak waves as weak perturbations of a flat background [1, 3, 5]. Similarly to what happened when electromagnetic waves were first predicted, some distinguished physicists had serious doubts about their existence. Arthur Eddington thought that these weak-field solutions of the wave equation obtained from Einstein's field equations were just coordinate changes which were 'propagating . . . with the speed of thought' [6].

The linearized theory of gravitational waves had its limits because the linear approximation is not valid for sources where gravitational self-energy is not negligible. It was only in 1941 that Landau and Lifshitz [7] described the emission of gravitational waves by a self-gravitating system of slowly moving bodies. However, in the following years there were serious doubts about the reality of gravitational waves and not until 1957 did a gedanken experiment by Hermann Bondi show that gravitational waves do indeed carry energy [8].

This thought experiment was based on a system of two beads sliding on a stick with only a slight friction opposing their motion. If a plane gravitational wave impinges on this system, the beads move back and forth on the stick because of the change in the proper distance between them due to the change of the metric, i.e. to the gravitational-wave perturbation; this change is governed by the geodesic deviation equation and the proper dispacement between the two beads is a function of the gravitational-wave metric perturbation. Thus, the friction between beads and stick heats the system and thus increases the temperature of the stick. Therefore, since there is an energy transfer from gravitational waves to the system in the form of increased temperature of the system, this thought experiment showed that gravitational waves do indeed carry energy and are a real physical entity [1, 8].

It is interesting to note that in 1955 John Archibald Wheeler had devised the conceivable existence of a body with no 'mass' built up by gravitational or electromagnetic, radiation alone [9]. Indeed, an object can, in principle, be constructed out of gravitational radiation or electromagnetic radiation, or a mixture of the two, and may hold itself together by its own gravitational attraction. A collection of radiation held together in this way, is called a geon (gravitational electromagnetic entity) and studied from a distance, such an object would present the same kind of gravitational attraction as any other mass. Yet, nowhere inside the geon is there a place where there is 'mass' in the conventional sense of the term. In particular, for a geon made of pure gravitational radiation—a gravitational geon—there is no local measure of energy, yet there is global energy. The gravitational geon owes its existence to a dynamical localized—but everywhere regular—curvature of spacetime, and to nothing more. Thus, a geon is a collection of electromagnetic or gravitational-wave energy, or a mixture of the two, held together by its own gravitational attraction, that was described by Wheeler as 'mass without mass'.

In the 1960s, Joseph Weber began the experimental work to detect gravitational waves. He was essentially alone in this field of research [10]. Then, the theoretical work of Wheeler, Bondi, Landau and Lifshitz, Isaacson, Thorne and others and the experimental work of Weber, Braginski, Amaldi and others opened a new era of research in this field. In 1972 Steven Weinberg wrote '... gravitational radiation would be interesting even if there were no chance of ever detecting any, for the theory of gravitational radiation provides a crucial link between general relativity and the microscopic frontiers of physics' [11].

Today gravitational waves, both theory and experiment, are one of the main topics of research in general relativity and gravitation [3].

In the same way as electromagnetic waves other than visible light, that is radio, millimetre, infrared, ultraviolet, x-ray and gamma-ray astronomy opened new windows and brought radical changes in our knowledge of the universe, gravitational-wave astronomy is expected to bring a revolution in our knowledge of the universe by observing new exotic phenomena such as formation and collision of black holes, fall of stars into supermassive black holes, primordial gravitational waves emitted just after the big bang .... Nevertheless, today, about 85 years after the prediction of gravitational waves by Einstein, the only evidence for their actual existence is indirect and comes from the observation of the energy loss from the binary pulsar system PSR 1913+16, discovered in 1974 by Hulse and Taylor [12]. Quite remarkably, though of no surprise, the observed energy loss of the binary pulsar is in agreement with the theoretical prediction by general relativity for the energy loss by gravitational radiation emitted by a binary system, to within less than 0.3% error (in this respect, it might be interesting to note here that, in regard to the field that in general relativity is formally analogous to the magnetic field in electrodynamics, i.e. the so-called gravitomagnetic field, predicted by Lense and Thirring in 1916, the first evidence and measurement of the existence of such an effect on Earth's satellites, due to the Earth's rotation, was published only in 1996, that is 80 years after the derivation of the effect [13]). Thus, today, together with the enormous experimental efforts to detect gravitational waves, from bar detectors to laser interferometers on Earth, GEO-600, LIGO, VIRGO, ..., and from laser interferometers in space, LISA, to Doppler tracking of interplanetary spacecrafts, there is, aimed at increasing the chances of future detections, a strongly related theoretical and computational work to understand and predict the emission or gravitational waves from astrophysical systems in strong field conditions [3]. In this book contributions of leading experts in the field of gravitational waves, both theoretical and experimental, are presented.

The basic contribution by Bernard Schutz and Franco Ricci deals with the main features of gravitational waves, sources and detectors. The contribution is divided into six chapters and some chapters are followed by a few exercises. The first chapter describes the linearized theory and the fundamental properties of weak gravitational waves, perturbations of a flat background, analysed in the so-called transverse-traceless gauge. The second and third chapters deal with detectors and astrophysical sources; in particular an overview is presented of the most important detectors under construction (their physics, sensitivity and opportunity for the future) and the main expected sources of gravitational waves, such as binary systems, neutron stars, pulsars, γ-ray bursts, etc. The fourth chapter deals with the mathematical theory of waves in general, stress-energy tensor and energy carried by gravitational waves. The subsequent chapter describes radiation generation in linearized theory: mass- and current-quadrupole radiation, i.e. the quadrupole formulae for the outgoing flux of gravitational-wave energy emitted by a system characterized by slow motion. Finally, the last chapter describes some applications of radiation theory to some sources: binary systems and especially *r-modes* of neutron stars.

The contribution by Guido Pizzella deals with bar detectors of gravitational waves. A gravitational-wave resonant detector is usually a cylindrical bar of length L. The small change δL in the length of the whole bar at the fundamental resonance angular frequency, ω_0, can be described by the solution of the equation of a harmonic oscillator, with resonance angular frequency ω_0 (with a supplementary $4/\pi^2$ factor obtained by solving the problem of a continuous bar). In a gravitational-wave resonant detector the mechanical oscillations of the bar induced by a gravitational wave are converted by an electromechanical transducer into electric signals which are amplified with a low noise amplifier, such as a dc SQUID. Then the data analysis is performed. Using a resonant antenna one measures the Fourier component of the metric perturbation near the antenna resonance frequency ω_0. The typical damping time of the resonant detector is $2Q/\omega_0$, where Q is the so-called quality factor of the resonant detector. The ultimate sensitivity of bar antennae to a fractional change in dimension due to a short burst of gravitational radiation has been estimated to be of the order of 10^{-20} or 10^{-21}. Bar detectors, usually 3 m long aluminum bars, work at a typical frequency of about 10^3 Hz. Resonant antennae were first built by J Weber, around 1960, at the University of Maryland. Subsequently, gravitational-wave resonant detectors have been operated by the following universities: Beijing, Guangzhou, Louisiana, Maryland, Moscow, Rome, Padua, Stanford, Tokyo and Western Australia at Perth. The contribution of Pizzella deals with the bandwidth and the sensitivities of resonant detectors. It is shown that it might be possible to reach a frequency bandwidth up to 50 Hz. The sensitivity of five cryogenic bar detectors in operation, ALLEGRO, AURIGA, EXPLORER, NAUTILUS AND NIOBE is then discussed.

The paper by Angela Di Virgilio treats laser interferometers on Earth and in particular the Italian–French antenna VIRGO. Gravitational-wave laser-interferometers on Earth will operate in the frequency range between 10^{4} Hz and a few tens of hertz. Various types of gravitational-wave laser interferometers have been proposed, among which are the standard Michelson and Fabry–Perot types. A Michelson-type gravitational-wave laser interferometer is essentially made of three masses suspended with wires at the ends of two orthogonal arms of length l. When a gravitational-wave with reduced wavelength $\bar{\lambda}_{\mathrm{GW}} \gg l$ is impinging, for example, perpendicularly to this system, variations in the metric perturbation h due to the gravitational wave will, in turn, produce oscillations in the difference between the proper lengths of the two arms $\delta l(t)$ and therefore oscillations in the relative phase of the laser light at the beamsplitter; thus, they will finally produce oscillations in the intensity of the laser light measured by the photodetector. If the laser light will travel back and forth between the test masses $2N$ times ($N =$ number of round trips), then the variation of the difference between the proper lengths of the two arms will be (assuming $Nl \ll \bar{\lambda}_{\mathrm{GW}}$): $\Delta l = 2Nlh(t)$, and therefore, the relative phase delay due to the variations in δl will be:

$$\Delta\phi = \frac{\Delta l}{\bar{\lambda}_{\mathrm{L}}} = \frac{2Nl}{\bar{\lambda}_{\mathrm{L}}} h(t),$$

where $\bar{\lambda}_{\mathrm{L}}$ is the reduced wavelength of the laser light.

For most of the fundamental limiting factors of these Earth-based detectors, such as seismic noise, photon shot noise, etc ..., the displacement noise is essentially independent from the arms length l. Therefore, by increasing l one increases the sensitivity of the detectors.

Two antennae with 4 km arm lengths in the USA, the MIT and Caltech LIGOs, should reach sensitivities to bursts of gravitational radiation of the order of $h \sim 10^{-20}$–10^{-21} between 1000 and 100 Hz. GEO-600 is an underground 600 m laser interferometer built by the University of Glasgow and the Max-Planck-Institutes for Quantum Optics and for Astrophysics at Garching. TAMA is a 300 m antenna in Japan and ACIGA is a 3 km antenna planned in Australia. The paper of Di Virgilio describes the 3 km laser interferometer VIRGO, built by INFN of Pisa together with the University of Paris-Sud at Orsay, that should reach frequencies of operation as low as a few tens of hertz, using special filters to eliminate the seismic noise at these lower frequencies. The ultimate burst sensitivity for all of the above large interferometers is currently estimated to be of the order of 10^{-22} or 10^{-23} at frequencies near 100 Hz.

The paper of Peter Bender describes the space gravitational-wave detector LISA (Laser Interferometer Space Antenna). Below about 10 Hz the sensitivity of Earth-based gravitational-wave detectors is limited by gravity gradients variations. Even for perfect isolation of a detector from seismic and ground noise, an Earth-detector would still be affected by the time changes in the gravity field due to density variations in the Earth and its atmosphere. Due to this source of noise the sensitivity has been calculated to worsen as roughly the inverse fourth power of the frequency.

Therefore, to avoid this type of noise and to reduce noise from other sources, one should use an interferometer far from Earth and with very long arms. Indeed, to detect gravitational waves in the range of frequencies between about 10^{-4} and 1 Hz, Bender proposes to orbit in the solar system a space interferometer made of three spacecraft at a typical distance from each other of 5000 000 km.

Although the phase measurement system and the thermal stability are essential requirements, it is the main technological challenge of this experiment to keep very small the spurious accelerations of the test masses. A drag compensating system will be able to largely reduce these spurious accelerations.

Considering all the error sources, it has been calculated that, for periodic gravitational waves, with an integration time of about one year, LISA should reach a sensitivity able to detect amplitudes of $h \sim 10^{-23}$, in the range of frequencies between 10^{-3} and 10^{-2} Hz, amplitudes from $h \sim 10^{-20}$ to about 10^{-23} between 10^{-4} and 10^{-3} Hz, and amplitudes from $h \sim 10^{-22}$ to about 10^{-23} between 1 and 10^{-2} Hz.

Therefore, comparing the LISA sensitivity to the predicted theoretical amplitudes of gravitational radiation at these frequencies, LISA should be able to detect gravitational waves from galactic binaries, including ordinary main-sequence binaries, contact binaries, cataclysmic variables, close white dwarf binaries, neutron star and black hole binaries. The LISA sensitivity should also allow detection of possible gravitational pulses from distant galaxies from the inspiral of compact objects into supermassive black holes in Active Galactic Nuclei and from collapse of very massive objects to form black holes. LISA should also allow us to detect the stochastic background due to unresolved binary systems.

The contribution by Francesco Fucito treats spherical shape antennae and the detection of scalar gravitational waves. General relativity predicts only two independent states of polarization of a weak gravitational wave, the so-called '$\times$' and '+' ones. Nevertheless, metric theories of gravity alternative to general relativity and non-metric theories of gravity predict different polarization states (up to six components in metric theories [14]). For example, the Jordan–Brans–Dicke theory predicts also an additional scalar component of a gravitational wave and, as the author explains, string theory could also imply the existence of other components. In this paper the possibility is described of placing limits on, or detecting, these additional polarizations of a gravitational wave, thus testing theories of gravity alternative to general relativity, by using spherical shape detectors. Spheroidal detectors of gravitational waves of two types are discussed, standard and hollow spherical ones.

The paper by Babusci, Foffa, Losurdo, Maggiore, Matone and Sturani treats stochastic gravitational waves. As the authors explain, the stochastic gravitational-wave background (SGWB) is a random background of gravitational waves without any specific sharp frequency component that might give information about the very early stages of our universe. It is important to note that relic cosmological gravitational waves emitted near the big bang might provide unique information on our universe at a very early stage. Indeed, as regards the cosmic microwave background radiation, electromagnetic waves decoupled a few 10^5 years after the big bang, whereas relic cosmological gravitational waves, the authors explain, might come from times as early as a few 10^{-44} s. The authors discuss that, in order to increase the chances of detecting a stochastic background of gravitational waves, the correlation of the outputs between two, or more, detectors would be convenient. Thus, after discussing three different detectors: laser interferometers, cylindrical bars and spherical antennae, the authors present various possibilities of correlation, between two laser interferometers (VIRGO, LIGOs, GEO-600 and TAMA-300), and between a laser interferometer and a cylindrical bar (AURIGA, NAUTILUS, EXPLORER) or a spherical antenna; they also discuss correlation between more than two detectors.

In the second part of this paper they discuss sources of the background of stochastic gravitational waves: topological defects in the form of points, lines or surfaces, called monopoles, cosmic strings and domain walls. In particular, they discuss cosmic strings and hybrid defects; inflationary cosmological models; string cosmology; and first-order phase transitions which occurred in the early stage of the expansion of the universe, for example in GUT-symmetry breaking and electroweak-symmetry breaking. Finally, they discuss astrophysical sources of stochastic gravitational waves. The conclusion is that the frequency domain of cosmological and astrophysical sources of stochastic gravitational waves might be very different and thus, the authors conclude, the astrophysical backgrounds might not mask the detection of a relic cosmological gravitational-wave background at the frequencies of the laser interferometers on Earth.

The contribution by Nicolai and Nagar deals with the symmetry properties of Einstein's vacuum field equations when the theory is reduced from four to two dimensions, namely in the presence of two independent spacelike commuting Killing vectors. Under these conditions, and using the vierbein formalism, the authors show that one can use a Kaluza–Klein ansatz to rewrite the Einstein–Hilbert Lagrangian in the form of two different two-dimensionally reduced Lagrangians named the Ehlers and Matzner–Misner ones, respectively, after the people who first introduced them. Each of these two Lagrangians represents two-dimensional reduced gravity in the conformal gauge as given by a part of pure two-dimensional gravity, characterized by a conformal factor and a dilaton field plus a 'matter part' given by two suitable bosonic fields. In either case, the matter part has a structure of a nonlinear sigma model with an SL(2,R)/SO(2) symmetry. These two different nonlinear symmetries can be combined into a unified infinite-dimensional symmetry group of the theory, called the Geroch group, whose Lie algebra is an affine Kac–Moody algebra, and whose action on the matter fields is both nonlinear and non-local. The existence of such an infinite-dimensional symmetry guarantees that the two-dimensionally reduced nonlinear field equations are integrable. This can be shown in a standard way by exploiting the symmetry to prove the equivalence of the theory to a system of linear differential equations whose compatibility conditions yield just the nonlinear equations that one wants to solve. As an example of the application of the method to the construction of exact solutions of the two-dimensionally reduced Einstein's equations, the results are employed to derive the exact expression of the metric which describe colliding plane gravitational waves with collinear and non-collinear polarization.

Gasperini's contribution deals with string cosmology and with the basic ideas of the so-called pre-big bang scenario of string cosmology. Then it treats the interesting problem of observable effects in different cosmological models, and in particular the so-called background of relic gravitational waves, comparing it with the expected sensitivities of the gravitational-wave detectors. The conclusion is that the sensitivity of the future advanced detectors of gravitational waves may be capable of detecting the background of gravitational waves predicted in the pre-big bang scenario of string cosmology and thus these detectors might test different cosmological models and also string theory models.

The paper by Bini and De Felice studies the problem of the behaviour of a test gyroscope on which a plane gravitational wave is impinging. The authors analyse whether there might be observable effects, i.e. a precession of the gyroscope with respect to a suitably defined frame of reference that is not Fermi–Walker transported.

The contribution by Luc Blanchet deals with the post-Newtonian computation of binary inspiral waveforms. In general relativity, the orbital phase of compact binaries, when gravitational radiation emitted is considered, is not constant as it is in the Newtonian calculation, but is a complex, nonlinear function of time, depending on small post-Newtonian corrections. For the data analysis on detectors, a formula containing at least the 3PN (third-post-Newtonian) order beyond the quadrupole formalism (see the contribution by Schutz and Ricci) is needed, that is a formula including terms of the order of $(v/c)^6$ (where v is a typical velocity in the source and c is the speed of light). Blanchet's paper thus treats the derivation of the third-post-Newtonian formula for the emission of gravitational radiation from a self-gravitating binary system.

The paper by Ed Seidel deals with numerical relativity. Among the astrophysical sources of gravitational radiation that might be detected by laser interferometers on Earth there is the spiralling coalescence of two black holes or neutron stars. However, gravitational waves are so weak at the detectors on Earth that, as Seidel explains in his paper, one needs to know the waveform in order to reliably detect them, in other words gravitational-wave signals can be interpreted and detected only by comparing the observational data with a set of theoretically determined 'waveform templates'. Unfortunately, we can solve the Einstein's field equations (coupled, nonlinear partial differential equations) only in especially simple cases. Thus, to find solutions of the Einstein's equations, for example in a system with emission of gravitational radiation, we need to find numerical solutions of these field equations, i.e. we need *numerical relativity*. Nevertheless, even the numerical approach to the emission of gravitational waves in strong field is extremely difficult and computer-time consuming. For example, as Seidel explains, the computer simulation of the coalescence of a compact object binary will require several years of super-computer time. However, special codes to solve the complete set of Einstein's equations have been designed that run very efficiently on large-scale parallel computers, in particular, one of these codes, the Cactus Computational Toolkit is presented in this paper. Then, after a description of the numerical formulation of the theory of general relativity, constraint equations and evolution equations, the numerical techniques for solving the evolution equations are reported and finally some recent applications, including gravitational waves and the evolution and collisions of black holes, are presented. It is important to note that there have been and there are large collaborations in numerical relativity, including: the NSF Black Hole Grand Challenge Project, the NASA Neutron Star Grand Challenge Project, the NCSA/Potsdam/Washington University numerical relativity collaboration and a EU European collaboration of ten institutions.

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PART 1

GRAVITATIONAL WAVES, SOURCES AND DETECTORS

Bernard F Schutz[1] *and Franco Ricci*[2]

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Synopsis

Gravitational waves and their detection are becoming increasingly important both for the theoretical physicist and the astrophysicist. In fact, technological developments have enabled the construction of such sensitive detectors (bars and interferometers) that the detection of gravitational radiation could become a reality during the next few years. In these lectures we give a brief overview of this interesting and challenging field of modern physics.

The topics covered are divided into six lectures. We begin (chapter 2) by describing gravitational waves in linearized general relativity, where one can examine most of the basic properties of gravitational radiation itself; propagation, gauge invariance and interactions with matter (and in particular with detectors).

The second lecture (chapter 3) deals with gravitational-wave detectors: how they operate, what their most important sources of noise are, and what mechanisms are used to overcome noise. We report here on the most important detectors planned or under construction (both ground-based and space-based ones), their likely sensitivity and their prospects for making detections. Other speakers will go into much more detail on specific detectors, such as LISA.

The third lecture (chapter 4) deals with the astrophysics of likely sources of gravitational waves: binary systems, neutron stars, pulsars, x-ray sources, supernovae/hypernovae, γ-ray bursts and the big bang. We estimate the expected wave amplitude h and the suitability of specific detectors for seeing waves from each source.

The fourth lecture (chapter 5) is much more theoretical. Here we develop the mathematical theory of gravitational waves in general, their effective stress-energy tensor, the energy carried by gravitational waves, and the energy in a random wave field (gravitational background generated by the big bang).

The fifth lecture (chapter 6) takes the theory further and examines the generation of gravitational radiation in linearized theory. We show in some detail how both mass-quadrupole and current-quadrupole radiation is generated, including how characteristics of the radiation such as its polarization are related to the motion of the source. Current-quadrupole radiation has become important very recently and may indeed be one of the first forms of gravitational radiation to be detected. We attempt to give a physical description of the way it is generated.

The final lecture (chapter 7) explores applications of the theory we have developed to various sources. We calculate the quadrupole moment of a binary system, the energy radiated in the Newtonian approximation and the back-reaction on the orbit. We conclude with a brief introduction to the current-quadrupole-driven instability in the r-modes of neutron stars.

Chapters 2 and 5 are followed by a few exercises to assist students. We presume the reader has some background in general relativity and its mathematical tools in differential geometry, at the level of the introductory chapters of Schutz (1985). A list of references is presented at the end of these lectures of sources suitable for further and background reading.

Chapter 2

Elements of gravitational waves

General relativity is a theory of gravity that is consistent with special relativity in many respects, and in particular with the principle that nothing travels faster than light. This means that changes in the gravitational field cannot be felt everywhere instantaneously: they must propagate. In general relativity they propagate at exactly the same speed as vacuum electromagnetic waves: the speed of light. These propagating changes are called gravitational waves.

However, general relativity is a nonlinear theory and there is, in general, no sharp distinction between the part of the metric that represents the waves and the rest of the metric. Only in certain approximations can we clearly define gravitational radiation. Three interesting approximations in which it is possible to make this distinction are:

- linearized theory;
- small perturbations of a smooth, time-independent background metric;
- post-Newtonian theory.

The simplest starting point for our discussion is certainly linearized theory, which is a weak-field approximation to general relativity, where the equations are written and solved in a nearly flat spacetime. The static and wave parts of the field cleanly separate. We idealize gravitational waves as a 'ripple' propagating through a flat and empty universe.

This picture is a simple case of the more general 'short-wave approximation', in which waves appear as small perturbations of a smooth background that is time dependent and whose radius of curvature is much larger than the wavelength of the waves. We will describe this in detail in chapter 5. This approximation describes wave propagation well, but it is inadequate for wave generation. The most useful approximation for sources is the post-Newtonian approximation, where waves arise at a high order in corrections that carry general relativity away from its Newtonian limit; we treat these in chapters 6 and 7.

For now we concentrate our attention on linearized theory. We follow the notation and conventions of Misner *et al* (1973) and Schutz (1985). In particular we choose units in which $c = G = 1$; Greek indices run from 0 to 3; Latin indices run from 1 to 3; repeated indices are summed; commas in subscripts or superscripts denote partial derivatives; and semicolons denote covariant derivatives. The metric has positive signature. These above two textbooks and others referred to at the end of these chapters give more details on the theory that we outline here. For an even simpler introduction, based on a scalar analogy to general relativity, see [1].

2.1 Mathematics of linearized theory

Consider a perturbed flat spacetime. Its metric tensor can be written as

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}, \quad |h_{\alpha\beta}| \ll 1, \quad \alpha, \beta = 0, \ldots, 3 \tag{2.1}$$

where $\eta_{\alpha\beta}$ is the Minkowski metric $(-1, 1, 1, 1)$ and $h_{\alpha\beta}$ is a very small perturbation of the flat spacetime metric. Linearized theory is an approximation to general relativity that is correct to first order in the size of this perturbation. Since the size of tensor components depends on coordinates, one must be careful with such a definition. What we require for linearized theory to be valid is that there should exist a coordinate system in which equation (2.1) holds in a suitably large region of spacetime. Even though $\eta_{\alpha\beta}$ is not the true metric tensor, we are free to *define* raising and lowering indices of the perturbation with $\eta_{\alpha\beta}$, as if it were a tensor on flat spacetime. We write

$$h^{\alpha\beta} := \eta^{\alpha\gamma}\eta^{\beta\delta}h_{\gamma\delta}.$$

This leads to the following equation for the inverse metric, correct to first order (all we want in linearized theory):

$$g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta}. \tag{2.2}$$

The mathematics is simpler if we define the *trace-reversed* metric perturbation:

$$\bar{h}_{\alpha\beta} := h_{\alpha\beta} - \tfrac{1}{2}\eta_{\alpha\beta}h, \tag{2.3}$$

where $h := \eta_{\alpha\beta}h^{\alpha\beta}$. There is considerable coordinate freedom in the components $h_{\alpha\beta}$, since we can wiggle and stretch the coordinate system with a comparable amplitude and change the components. This coordinate freedom is called *gauge freedom*, by analogy with electromagnetism. We use this freedom to enforce the *Lorentz (or Hilbert)* gauge:

$$\bar{h}^{\alpha\beta}{}_{,\beta} = 0. \tag{2.4}$$

In this gauge the Einstein field equations (neglecting the quadratic and higher terms in $h^{\alpha\beta}$) are just a set of decoupled linear wave equations:

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}^{\alpha\beta} = -16\pi T^{\alpha\beta}. \tag{2.5}$$

To understand wave propagation we look for the easiest solution of the vacuum gravitational field equations:

$$\Box \bar{h}^{\alpha\beta} \equiv \left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}^{\alpha\beta} = 0. \tag{2.6}$$

Plane waves have the form:

$$\bar{h}_{\alpha\beta} = \mathcal{A}\boldsymbol{e}_{\alpha\beta}\exp(\mathrm{i}k_\gamma x^\gamma) \tag{2.7}$$

where the amplitude $\mathcal{A}$, polarization tensor $\boldsymbol{e}^{\alpha\beta}$ and wavevector k^γ are all constants. (As usual one has to take the real part of this expression.)

The Einstein equations imply that the wavevector is 'light-like', $k^\gamma k_\gamma = 0$, and the gauge condition implies that the amplitude and the wavevector are orthogonal: $\boldsymbol{e}^{\alpha\beta}k_\beta = 0$.

Linearized theory describes a classical gravitational field whose quantum description would be a massless spin 2 field that propagates at the speed of light. We expect from this that such a field will have only two independent degrees of freedom (helicities in quantum language, polarizations in classical terms). To show this classically we remember that $h_{\alpha\beta}$ is symmetric, so it has ten independent components, and that the Lorentz gauge applies four independent conditions to these, reducing the freedom to six. However, the Lorentz gauge does not fully fix the coordinates. In fact if we perform another infinitesimal coordinate transformation ($x^\mu \to x^\mu + \xi^\mu$ with $\xi^\mu{}_{,\nu} = O(h)$) and impose $\Box\xi^\mu = 0$, we remain in Lorentz gauge. We can use this freedom to demand:

$$\boldsymbol{e}^{0\alpha} = 0 \Longrightarrow \boldsymbol{e}^{ij}k_j = 0 \quad \text{(transverse wave)}, \tag{2.8}$$

$$\boldsymbol{e}^i{}_i = 0 \quad \text{(traceless wave)}. \tag{2.9}$$

These conditions can only be applied outside a sphere surrounding the source. Together they put the metric into the *transverse-traceless* (TT) gauge. We will explicitly construct this gauge in chapter 5.

2.2 Using the TT gauge to understand gravitational waves

The TT gauge leaves only *two independent polarizations* out of the original ten, and it ensures that $\bar{h}_{\alpha\beta} = h_{\alpha\beta}$. In order to understand the polarization degrees of freedom, let us take the wave to move in the z-direction, so that $k_z = \omega$, $k^0 = \omega$, $k_x = 0$, $k_y = 0$; the TT gauge conditions in equations (2.8) and (2.9) lead to $\boldsymbol{e}^{0\alpha} = \boldsymbol{e}^{z\alpha} = 0$ and $\boldsymbol{e}^{xx} = -\boldsymbol{e}^{yy}$. This leaves only two independent components of the polarization tensor, say $\boldsymbol{e}^{xx}$ and $\boldsymbol{e}^{xy}$ (which we denote by the symbols $\oplus, \otimes$).

A wave for which $\boldsymbol{e}^{xy} = 0$ (pure $\oplus$ polarization) produces a metric of the form:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + (1 + h_+)\,\mathrm{d}x^2 + (1 - h_+)\,\mathrm{d}y^2 + \mathrm{d}z^2, \tag{2.10}$$

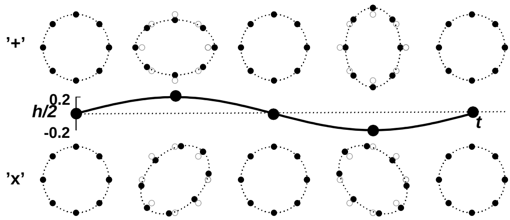

Figure 2.1. Illustration of two linear polarizations and the associated wave amplitude.

where $h_+ = \mathcal{A}\boldsymbol{e}^{xx}\exp[-\mathrm{i}\omega(t-z)]$. Such a metric produces opposite effects on proper distance at the two transverse axes, contracting one while expanding the other.

If $\boldsymbol{e}^{xx} = 0$ we have pure $\otimes$ polarization $h_\times$ which can be obtained from the previous case by a simple 45° rotation, as in figure 2.1. Since the wave equation and TT conditions are linear, a general wave will be a linear combination of these two polarization tensors. A circular polarization basis would be:

$$\boldsymbol{e}_\mathrm{R} = \frac{1}{\sqrt{2}}(\boldsymbol{e}_+ + \mathrm{i}\boldsymbol{e}_\times), \quad \boldsymbol{e}_\mathrm{L} = \frac{1}{\sqrt{2}}(\boldsymbol{e}_+ - \mathrm{i}\boldsymbol{e}_\times), \tag{2.11}$$

where $\boldsymbol{e}_+$, $\boldsymbol{e}_\times$ are the two linear polarization tensors and $\boldsymbol{e}_\mathrm{R}$ and $\boldsymbol{e}_\mathrm{L}$ are polarizations that rotate in the right-handed and left-handed directions, respectively. It is important to understand that, for circular polarization, the polarization pattern rotates around the central position, but test particles themselves rotate only in small circles relative to the central position.

Now we compute the effects of a wave in the TT gauge on a particle at rest in the flat background metric $\eta_{\alpha\beta}$ before the passage of the gravitational wave. The geodesic equation

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu{}_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0$$

implies in this case:

$$\frac{\mathrm{d}^2x^i}{\mathrm{d}\tau^2} = -\Gamma^i{}_{00} = -\frac{1}{2}(2h_{i0,0} - h_{00,i}) = 0, \tag{2.12}$$

so that the particle *does not move*. The TT gauge, to first order in $h_{\alpha\beta}$, represents a coordinate system that is comoving with freely-falling particles. Because $h_{0\alpha} = 0$, TT time is proper time on the clock of freely-falling particles at rest.

Tidal forces show the action of the wave independently of the coordinates. Let us consider the equation of geodesic deviation, which governs the separation of two neighbouring freely-falling test particles A and B. If the particles are initially at rest, then as the wave passes it produces an oscillating curvature tensor, and the separation ξ of the two particles is:

$$\frac{\mathrm{d}^2\xi^i}{\mathrm{d}t^2} = R^i{}_{0j0}\xi^j. \tag{2.13}$$

To calculate the component $R^i{}_{0j0}$ of the Riemann tensor in equation (2.13), we can use the metric in the TT gauge, because the Riemann tensor is gauge-invariant at linear order (see exercise (d) at the end of this chapter). Therefore, we can replace $R^i{}_{0j0}$ by $R^i{}_{0j0} = \frac{1}{2}h^{\mathrm{TT}i}{}_{j,00}$ and write:

$$\frac{\mathrm{d}^2\xi^i}{\mathrm{d}t^2} = \frac{1}{2}h^{\mathrm{TT}i}{}_{j,00}\xi^j. \tag{2.14}$$

This equation, with an initial condition $\xi^j_{(0)} =$ constant, describes the oscillations of Bs location as measured in the proper reference frame of A. The validity of equation (2.14) is the same as that of the geodesic deviation equation: geodesics have to be close to one another, in a neighbourhood where the change in curvature is small. In this approximation a gravitational wave is like an extra force, called a *tidal force*, perturbing the proper distance between two test particles. If there are other forces on the particles, so that they are not free, then as long as the gravitational field is weak, one can just add the tidal forces to the other forces and work as if the particle were in special relativity.

2.3 Interaction of gravitational waves with detectors

We have shown above that the TT gauge is a particular coordinate system in which the polarization tensor of a plane gravitational wave assumes a very simple form. This gauge is comoving for freely-falling particles and so it is not the locally Minkowskian coordinate system that would be used by an experimenter to analyse an experiment. In general relativity one must always be aware of how one's coordinate system is defined.

We shall analyse two typical situations:

- the detector is small compared to the wavelength of the gravitational waves it is measuring; and
- the detector is comparable to or larger than that wavelength.

In the first case we can use the geodesic deviation equation above to represent the wave as a simple extra force on the equipment. Bars detectors can always be analysed in this way. Laser interferometers on the Earth can be treated this way too. In these cases a gravitational wave simply produces a force to be measured. There is no more to say from the relativity point of view. The rest of the detection story is the physics of the detectors. Sadly, this is not as simple as gravitational wave physics!

In the second case, the geodesic deviation equation is not useful because we have to abandon the 'local mathematics' of geodesic deviation and return to the 'global mathematics' of the TT gauge and metric components $h^{\mathrm{TT}}{}_{\alpha\beta}$. Space-based interferometers like LISA, accurate ranging to solar-system spacecraft and pulsar timing are all in this class. Together with ground interferometers, these are *beam detectors*: they use light (or radio waves) to register the waves.

To study these detectors, it is easiest to remain in the TT gauge and to calculate the effect of the waves on the (coordinate) speed of light. Let us consider, for example, the $\oplus$ metric from equation (2.10) and examine a null geodesic moving in the x-direction. The speed along this curve is:

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 = \frac{1}{1+h_+}. \tag{2.15}$$

This is only a *coordinate speed*, not a contradiction to special relativity.

To analyse the way in which detectors work, suppose one arm of an interferometer lies along the x-direction and the wave, for simplicity, is moving in the z-direction with a $\oplus$ polarization of *any* waveform $h_+(t)$ along this axis (it is a plane wave, so its waveform does not depend on x). Then a photon emitted at time t from the origin reaches the other end, at a fixed coordinate position $x = L$, at the coordinate time

$$t_{\mathrm{far}} = t + \int_0^L \sqrt{1 + h_+(t(x))}\,\mathrm{d}x, \tag{2.16}$$

where the argument $t(x)$ denotes the fact that one must know the time to reach position x in order to calculate the wave field. This implicit equation can be solved in linearized theory by using the fact that h_+ is small, so we can use the first-order solution of equation (2.15) to calculate $h_+(t)$ to sufficient accuracy.

To do this we expand the square root in powers of h_+, and consider as a zero-order solution a photon travelling at the speed of light in the x-direction of a flat spacetime. We can set $t(x) = t + x$. The result is:

$$t_{\mathrm{out}} = t + L + \tfrac{1}{2}\int_0^L h_+(t+x)\,\mathrm{d}x. \tag{2.17}$$

In an interferometer, the light is reflected back, so the return trip takes

$$t_{\mathrm{return}} = t + L + \tfrac{1}{2}\left[\int_0^L h_+(t+x)\,\mathrm{d}x + \int_0^L h_+(t+x+L)\,\mathrm{d}x\right]. \tag{2.18}$$

What one monitors is changes in the time taken by a return trip as a function of time at the origin. If there were no gravitational waves t_{return} would be constant because L is fixed, so changes indicate a gravitational wave.

The rate of variation of the return time as a function of the start time t is

$$\frac{\mathrm{d}t_{\mathrm{return}}}{\mathrm{d}t} = 1 + \frac{1}{2}[h_+(t+2L) - h_+(t)]. \tag{2.19}$$

This depends only on the wave amplitude when the beam leaves and when it returns.

Let us consider now a more realistic geometry than the previous one, and in particular suppose that the wave travels at an angle θ to the z-axis in the x–z plane. If we redo this calculation, allowing the phase of the wave to depend on x in an appropriate way, and taking into account the fact that $h_+^{\mathrm{TT}xx}$ is reduced if the wave is not moving in a direction perpendicular to x, we find (see exercise (a) at the end of this chapter for the details of the calculation)

$$\begin{aligned}\frac{\mathrm{d}t_{\text{return}}}{\mathrm{d}t} = \frac{1}{2}\{&(1-\sin\theta)h_+^{xx}(t+2L) - (1+\sin\theta)h_+^{xx}(t) \\ &+ 2\sin\theta h_+^{xx}[t + L(1-\sin\theta)]\}.\end{aligned} \tag{2.20}$$

This three-term relation is the starting point for analysing the response of all beam detectors. This is directly what happens in radar ranging or in transponding to spacecraft, where a beam in only one direction is used. In long-baseline interferometry, one must analyse the second beam as well. We shall discuss these cases in turn.

2.4 Analysis of beam detectors

2.4.1 Ranging to spacecraft

Both NASA and ESA perform experiments in which they monitor the return time of communication signals with interplanetary spacecraft for the characteristic effect of gravitational waves. For missions to Jupiter and Saturn, the return times are of the order 2–4 $\times$ 10^3 s. Any gravitational wave event shorter than this will leave an imprint on the delay time three times: once when the wave passes the Earth-based transmitter, once when it passes the spacecraft, and once when it passes the Earth-based receiver. Searches use a form of pattern matching to look for this characteristic imprint. There are two dominant sources of noise: propagation-time irregularities caused by fluctuations in the solar wind plasma, and timing noise in the clocks used to measure the signals. The plasma delays depend on the radio-wave frequency, so by using two transmission frequencies one can model and subtract the plasma noise. Then if one uses the most stable atomic clocks, it is possible to achieve sensitivities for h of the order 10^{-13}. In the future, using higher radio frequencies, such experiments may reach 10^{-15}. No positive detections have yet been made, but the chances are not zero. For example, if a small black hole fell into a massive black hole in the centre of the Galaxy, it would produce a signal with a frequency of about 10 mHz and an amplitude significantly bigger than 10^{-15}. Rare as this might be, it would be a dramatic event to observe.

2.4.2 Pulsar timing

Many pulsars, in particular old millisecond pulsars, are extraordinarily regular clocks, whose random timing irregularities are too small for even the best atomic clocks to measure. Other pulsars have weak but observable irregularities. Measurements of or even upper limits on any of these timing irregularities for single pulsars can be used to set *upper limits* on any background gravitational wave field with periods comparable to or shorter than the observing time. Here the three-term formula is replaced by a simpler two-term expression (see exercise (b) at the end of this chapter), because we only have a one-way transmission from the pulsar to Earth. Moreover, the transit time of a signal to Earth from the pulsar may be thousands of years, so we cannot look for correlations between the two terms in a given signal. Instead, the delay time is a combination of the effects of uncorrelated waves at the pulsar when the signal was emitted and at the Earth when it is received.

If one simultaneously observes two or more pulsars, the Earth-based part of the delay is correlated between them, and this offers a means of actually detecting long-period gravitational waves. Observations require a timescale of several years in order to achieve the long-period stability of pulse arrival times, so this method is suited to looking for strong gravitational waves with periods of several years.

2.4.3 Interferometry

An interferometer essentially measures changes in the difference in the return times along two different arms. It does this by looking for changes in the interference pattern formed when the returning light beams are superimposed on one another. The response of each arm will follow the three-term formula in equation (2.20), but with a different value of θ for each arm, depending in a complicated way on the orientation of the arms relative to the direction of travel and the polarization of the wave. Ground-based interferometers are small enough to use the small-L formulae we derived earlier. However, LISA, the space-based interferometer that is described by Bender in this book, is larger than a wavelength of gravitational waves for frequencies above 10 mHz, so a detailed analysis of its sensitivity requires the full three-term formula.

2.5 Exercises for chapter 2

Suggested solutions for these exercises are at the end of chapter 7.

(a) 1. *Derive the full three-term return equation, reproduced here:*

$$\frac{\mathrm{d}t_{\mathrm{return}}}{\mathrm{d}t} = \frac{1}{2}\{(1-\sin\theta)h_+^{xx}(t+2L) - (1+\sin\theta)h_+^{xx}(t) \\ + 2\sin\theta h_+^{xx}[t+L(1-\sin\theta)]\}. \tag{2.21}$$

2. *Show that, in the limit where L is small compared to the wavelength of the gravitational wave, the derivative of the return time is the derivative of the excess proper distance $\delta L = L h_+^{xx}(t)\cos^2\theta$ for small L. Make sure you know how to interpret the factor of $\cos^2\theta$.*
3. *Examine the limit of the three-term formula when the gravitational wave is travelling along the x-axis too ($\theta = \pm\frac{\pi}{2}$): what happens to light going parallel to a gravitational wave?*

(b) *Derive the two-term formula governing the delays induced by gravitational waves on a signal transmitted only one-way, for example from a pulsar to Earth.*

(c) *A frequently asked question is: if gravitational waves alter the speed of light, as we seem to have used here, and if they move the ends of an interferometer closer and further apart, might these effects not cancel, so that there would be no measurable effects on light? Answer this question. You may want to examine the calculation above: did we make use of the changing distance between the ends, and why or why not?*

(d) *Show that the Riemann tensor is gauge-invariant in linearized theory.*

Chapter 3

Gravitational-wave detectors

Gravitational radiation is a central prediction of general relativity and its detection is a key test of the integrity of the theoretical structure of Einstein's work. However, in the long run, its importance as a tool for observational astronomy is likely to be even more important. We have excellent observational evidence from the Hulse–Taylor binary pulsar system (described in chapter 4) that the predictions of general relativity concerning gravitational radiation are quantitatively correct. However, we have incomplete information from astronomy today about the likely sources of detectable radiation.

The gravitational wave spectrum is completely unexplored, and whenever a new electromagnetic waveband has been opened to astronomy, astronomers have discovered completely unexpected phenomena. This seems to me just as likely to happen again with gravitational waves, especially because gravitational waves carry some kinds of information that electromagnetic radiation cannot convey. Gravitational waves are generated by bulk motions of masses, and they encode the mass distributions and speeds. They are coherent and their low frequencies reflect the dynamical timescales of their sources.

In contrast, electromagnetic waves come from individual electrons executing complex and partly random motions inside their sources. They are incoherent, and individual photons must be interpreted as samples of the large statistical ensemble of photons being emitted. Their frequencies are determined by microphysics on length scales much smaller than the structure of the astronomical system emitting them. From electromagnetic observations we can make inferences about this structure only through careful modelling of the source. Gravitational waves, by contrast, carry information whose connection to the source structure and motion is fairly direct.

A good example is that of massive black holes in galactic nuclei. From observations that span the electromagnetic spectrum from radio waves to x-rays, astrophysicists have inferred that black holes of masses up to $10^9 M_\odot$ are responsible for quasar emissions and control the jets that power the giant radio emission regions. The evidence for the black hole is very strong but indirect: no other known object can contain so much mass in such a small volume. Gravitational wave observations will tell us about the dynamics of the holes themselves, providing unique signatures from which they can be identified, measuring their masses and spins directly from their vibrational frequencies. The interplay of electromagnetic and gravitational observations will enrich many branches of astronomy.

The history of gravitational-wave detection started in the 1960s with J Weber at the University of Maryland. He built the first *bar detector*: it was a massive cylinder of aluminium ($\sim 2 \times 10^3$ kg) operating at room temperature (300 K) with a resonant frequency of about 1600 Hz. This early prototype had a modest sensitivity, around 10^{-13} or 10^{-14}.

Despite this poor sensitivity, in the late 1960s Weber announced the detection of a population of coincident events between two similar bars at a rate far higher than expected from instrumental noise. This news stimulated a number of other groups (at Glasgow, Munich, Paris, Rome, Bell Laboratories, Stanford, Rochester, LSU, MIT, Beijing, Tokyo) to build and develop bar detectors to check Weber's results. Unfortunately for Weber and for the idea that gravitational waves were easy to detect, none of these other detectors found anything, even at times when Weber continued to find coincidences. Weber's observations remain unexplained even today. However, the failure to confirm Weber was in a real sense a confirmation of general relativity, because theoretical calculations had never predicted that reasonable signals would be strong enough to be seen by Weber's bars.

Weber's announcements have had a mixed effect on gravitational-wave research. On the one hand, they have created a cloud under which the field has laboured hard to re-establish its respectability in the eyes of many physicists. Even today the legacy of this is an extreme cautiousness among the major projects, a conservatism that will ensure that the next claim of a detection will be ironclad. On the other hand, the stimulus that Weber gave to other groups to build detectors has directly led to the present advanced state of detector development.

From 1980 to 1994 groups developed detectors in two different directions:

- *Cryogenic bar detectors*, developed primarily at Rome/Frascati, Stanford, LSU and Perth (Australia). The best of these detectors reach below 10^{-19}. They are the only detectors operating continuously today and they have performed a number of joint coincidence searches, leading to upper limits but no detections.
- *Interferometers*, developed at MIT, Garching (where the Munich group moved), Glasgow, Caltech and Tokyo. The typical sensitivity of these prototypes was 10^{-18}. The first long coincidence observation with interferometers was the Glasgow/Garching 100 hr experiment in 1989 [2].

In fact, interferometers had apparently been considered by Weber, but at that time the technology was not good enough for this kind of detector. Only 10–15 years later, technology had progressed. Lasers, mirror coating and polishing techniques and materials science had advanced far enough to allow the first practical interferometers, and it was clear that further progress would continue unabated. Soon afterwards several major collaborations were formed to build large-scale interferometric detectors:

- LIGO: Caltech and MIT (NSF) LIGO;
- VIRGO: France (CNRS) and Italy (INFN)
- GEO600: Germany (Max Planck) and UK (PPARC).

Later, other collaborations were formed in Australia (AIGO) and Japan (TAMA and JGWO). At present there is still considerable effort in building successors to Weber's original resonant-mass detector: ultra-cryogenic bars are in operation in Frascati and Padua, and they are expected to reach below 10^{-20}. Further, there are proposals for a new generation of spherical or icosahedral solid-mass detectors from the USA (LSU), Brazil, the Netherlands and Italy. Arrays of smaller bars have been proposed for observing the highest frequencies, where neutron star normal modes lie.

However, the real goal for the near future is to break through the 10^{-21} level, which is where theory predicts that it is not unreasonable to expect gravitational waves of the order of once per year (see the discussion in chapter 4 later). The first detectors to reach this level will be the large-scale interferometers that are now under construction. They have very long arms: LIGO, Hanford (WA) and Livingstone (LA), 4 km; VIRGO: Pisa, 3 km; GEO600: Hannover, 600 m; TAMA300: Tokyo, 300 m.

The most spectacular detector in the near future is the space-based detector LISA, which has been adopted by ESA (European Space Agency) as a Cornerstone mission for the twenty-first century. The project is now gaining a considerable amount of momentum in the USA, and a collaboration between ESA and NASA seems likely. This mission could be launched around 2010.

3.1 Gravitational-wave observables

We have described earlier how different gravitational-wave observables are from electromagnetic observables. Here are the things that we want to measure when we detect gravitational waves:

- $h_+(t)$, $h_\times(t)$, $phase(t)$: the amplitude and polarization of the wave, and the phase of polarization, as functions of time. These contain most of the information about gravitational waves.
- θ, ϕ: the direction on the sky of the source (except for observations of a stochastic background).

From this it is clear that gravitational-wave detection is not the same as electromagnetic-radiation detection. In electromagnetic astronomy one almost always rectifies the electromagnetic wave, while we can follow the oscillations of the gravitational wave. Essentially in electromagnetism one detects the power in the radiation, while for gravitational radiation, as we have said before, one detects the wave coherently.

Let us consider now what we can infer from a detection. If the gravitational wave has a short duration, of the order of the sampling time of the signal stream, then each detector will usually give just a single number, which is the amplitude of the wave projected on the detector (a projection of the two polarizations h_+ and $h_\times$). If the wave lasts more than one sampling time, then this information is a function of time.

If the signal lasts for a sufficiently long time, then both the amplitude and the phase of the wave can be affected by the motion of the detector, which moves and turns with the motion of the Earth. This produces an amplitude and phase modulation which is not intrinsic to the signal. If the signal's intrinsic form is understood, then this modulation can be used to determine the location of the source. We distinguish three distinct kinds of signals, from the point of view of observations.

Bursts have a duration so short that modulation due to detector motion is not observable. During the detection, the detector is effectively stationary. In this case we need at least three, and preferably four, interferometers to triangulate the positions of bursts on the sky and to find the two polarizations h_+ and $h_\times$. (See discussions in Schutz 1989.) A network of detectors is essential to extract all the information in this case.

Continuous waves by definition last long enough for the motion of the detector to induce amplitude and phase modulation. In this case, assuming a simple model for the intrinsic signal, we can use the information imprinted on the signal (the amplitude modulation and phase modulation) to infer the position and polarization amplitude of the source on the sky. A single detector, effectively, performs aperture synthesis, finding the position of the source and the amplitude of the wave entirely by itself. However, in order to be sure that the signal is not an artefact, it will be important that the signal is seen by a second or third detector.

Stochastic backgrounds can be detected just like noise in a single detector. If the detector noise is well understood, this excess noise may be detected as a stochastic background. This is closely analogous to the way the original microwave background detection was discovered.

A more reliable method for detecting stochastic radiation is the cross-correlation between two detectors, which experience the same cosmological noise but have a different intrinsic noise. Coherent cross-correlation between two detectors eliminates much detector noise and works best when detectors are closer than a wavelength.

In general, detection of gravitational waves requires joint observing by a network of detectors, both to increase the confidence of the detection and to provide accurate information on other physical observables (direction, amplitude and so on). Networks can be assembled from interferometers, bars, or both.

3.2 The physics of interferometers

Interferometric gravitational-wave detectors are the most sensitive instruments, and among the most complex, that have ever been constructed. They are remarkable for the range of physics that is important for their construction. Interferometer groups work at the forefront of the development in lasers, mirror polishing and coating, quantum measurement, materials science, mechanical isolation, optical system design and thermal science. In this section we shall only be able to take a fairly superficial look at one of the most fascinating instrumentation stories of our age. A good introduction to interferometer design is Saulson (1994).

Interferometers use laser light to compare the lengths of two perpendicular arms. The simplest design, originated by Michelson for his famous experiment on the velocity of light, uses light that passes up and down each arm once, as in the first panel in figure 3.1. Imagine such an instrument with identical arms defined by mirrors that hang from supports, so they are free to move horizontally in response to a gravitational wave. If there is no wave, the arms have the same length, and the light from one arm returns exactly in phase with that from the other. When the wave arrives, the two arms typically respond differently. The arms are no longer the same length, and so the light that arrives back at the centre from one arm will no longer be in phase with that arriving back from the other arm. This will produce a shift in the interference fringes between the two beams. This is the principle of detection.

Real detectors are designed to store the light in each arm for longer than just one reflection (see figure 3.1(*b*)). It is optimum to store the light for half of the period of the gravitational wave, so that on each reflection the light gains an added phase shift. Michelson-type *delay-line* interferometers store the light by arranging multiple reflections. *Fabry–Perot* interferometers store the light in cavities in each arm, allowing only a small fraction to escape for the interference measurement (figure 3.1(*e*)).

An advantage of interferometers as detectors is that the gravitational-wave-induced phase shift of the light can be made larger simply by making the arm length larger, since gravitational waves act by tidal forces. A detector with an arm length $l = 4$ km responds to a gravitational wave with an amplitude of 10^{-21} with

$$\delta l_{\mathrm{gw}} \sim \tfrac{1}{2} hl \sim 2 \times 10^{-18}\ \mathrm{m} \tag{3.1}$$

where δl_{gw} is the change in the length of one arm. If the orientation of the interferometer is optimum, then the other arm will change by the same amount in the opposite direction, so that the interference fringe will shift by twice this length.

If the light path is folded or resonated, as in figure 3.1(*b*) and (*d*), then the effective number of bounces can be traded off against overall length to achieve a given desired total path length, or storage time. Shorter interferometers with many bounces have a disadvantage, however: even though they can achieve the