# Uncertainty Modeling and Analysis in Engineering and the Sciences

Bilal M. Ayyub George J. Klir



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To my wife, Deena, and our children, Omar, Rami, Samar, and Ziad.

Bilal M. Ayyub

To my wife, Milena, and our daughter, Jane.

George J. Klir

### Preface

The treatment of uncertainty in analysis, design, and decision making is going through a paradigm shift from a probabilistic framework to a generalized framework that includes both probabilistic and nonprobabilistic methods. Presently, analysts, including engineers and scientists, recognize the presence of uncertainty and treat it formally. For example, engineers analyze and model uncertainty in many of their specialty fields, such as the development of building codes, analysis of natural hazards (e.g., floods, wind, and earthquakes), decision making in infrastructure maintenance expenditure, homeland security and protection of assets, and environmental risks. Similarly, scientists analyze and model uncertainty in many of their specialty fields, such as the diagnostics of diseases, health effects of food additives and toxins, pharmaceutical research for developing new drugs, understanding of physical phenomena, prediction and forecasting in economy and weather, and sociopolitical changes, trends, and evolutions. The interest in uncertainty will continue to increase as we continue to design complex systems and deal with new technologies, systems, and materials, and are increasingly required to make critical decisions with potentially high adverse consequences. Also, political, societal, and financial requirements are increasing, thereby adding new dimensions of complexity in meeting the societal demands. The expectations of society are becoming larger than ever, and its tolerance to errors is diminishing. The aggregate of these factors produces an environment that requires the formal consideration of uncertainty in decision making at all levels in a systems framework.

Problems that are commonly encountered by engineers and scientists require decision making under conditions of uncertainty, lack of knowledge, and ignorance. The lack of knowledge and ignorance can be related to the definition of a problem, the alternative solution methodologies and their results, and the nature of the solution outcomes. Based on present trends, analysts will need to solve complex problems with decisions made under conditions of limited resources, thus necessitating increased reliance on the proper treatment of uncertainty and the use of expert opinions. This book is therefore intended to better prepare future analysts, as well as assist practitioners in understanding the fundamentals of knowledge and ignorance, how to model and analyze uncertainty, and how to select appropriate analytical tools for a particular problem.

Traditionally, intelligence is defined as the ability to understand and adapt to the environment by using a combination of inherited abilities and learning experiences. This ability certainly includes the analysis of uncertainty and making decisions under conditions of uncertainty. This is true of many organisms — from ants to aardvarks to humans. Any organism that survives the remorseless rigors of evolution is sufficiently intelligent for its role in life. Likewise, machines need to be sufficiently intelligent to make decisions suitable for their functions and adapt to and deal with the presence of uncertainty. Any collectives of human decision makers and their decision-aiding machines must make, in the aggregate, good decisions. But these decisions are almost always made under conditions of uncertainty.

The term *uncertainty* can be viewed as a component of ignorance. A taxonomic breakdown of ignorance can reveal many components having a strong association with human cognition of information and knowledge construction philosophies and practices. Uncertainty and information as a pair, and ignorance and knowledge as another pair, are studied in this book since they are tightly interconnected, as the former component of each pair describes a deficiency in the respective latter component, while the latter component of a pair can be viewed as the respective capacity available to reduce the respective former component. The identification and treatment of this duality of the respective components of a pair offer opportunities to enhance understanding of underlying problems or issues and our ability to make decisions. This book covers primary components of ignorance and their impact on our practice and our ability to make decisions. This book gives an overview of the current state of uncertainty modeling and analysis, and covers emerging theories with emphasis on practical applications in engineering and the sciences.

The complexity of a particular decision situation could increase substantially by the inclusion of uncertainties, thus requiring, in many cases, the reliance on experts to shed light on the situation. The complexity of our society and its knowledge base requires its members to specialize and become experts to attain recognition and reap rewards to the society and themselves. We commonly deal with or listen to experts on a regular basis, such as weather forecasts by weather experts, stock and financial reports by seasoned analysts, suggested medication or procedures by medical professionals, policies by politicians, and analyses by world affairs experts. We know from our own experiences that experts are valuable sources of information and knowledge, and can also be wrong in their views rendered to us. Expert opinions, therefore, can be considered to include or constitute nonfactual information. The fallacy of these opinions might disappoint us, but does not surprise us since issues that require experts tend to be difficult or complex, with a lot of uncertainty, and sometimes with divergent views. The nature of some of these complex issues could only yield views that have subjective truth levels; therefore, they allow for contradictory views that might all be somewhat credible. In political and economic world affairs and international conflicts, such issues are of common occurrence. For example, we have recently witnessed the debates that surrounded the membership of the People's Republic of China to the World Trade Organization in 1999, or experts airing their views on the insoluble Arab-Israeli affairs for the last century, or analysts' views on the war in Iraq in 2003, or future oil prices in 2005. These issues have a common feature of the presence of complexity and uncertainties requiring the use of expert opinions. Such issues and situations are also encountered in engineering, the sciences, medical fields, social research, stock and financial markets, and the legal practice.

Experts, with all their importance and value, can be viewed as double-edged swords. Not only do they bring in a deep knowledge base and thoughts, but also they could infuse biases and pet theories. The selection of experts, elicitation, and aggregation of their opinions should be performed and handled carefully by recognizing uncertainties associated with this type of information, and sometimes with skepticism. A primary reason for using expert opinions is to deal with uncertainty in selected technical issues related to a system of interest. Issues with significant uncertainty, issues that are controversial or contentious, issues that are complex, issues with limited objective information, or issues that can have a significant effect on risk are most suited for expert opinion elicitation. The value of the expert opinion elicitation comes from its initial intended uses as a heuristic tool, not a scientific tool, for exploring vague and unknowable issues that are otherwise inaccessible. It is not a substitute to scientific, rigorous research.

Current techniques for visualizing information commonly do not include degrees of certainty (or the degrees and types of ignorance) associated with individual or aggregated information. For example, for a commander in a battlefield to command, she or he needs to choose. To choose is to decide — almost always on the basis of imperfect information — and momentous decisions require knowledge of threats with a degree of certainty that might not be a requisite for decisions less momentous than waging war. Battlespace visualization techniques should allow both information and uncertainty to be portrayed effectively and grouped intuitively. Intelligent agents are promising technologies that may facilitate visualization of data and information uncertainty. Civilian applications can also be constructed to meet societal needs, such as Internet information metatagging for uncertainty and uncertainty visualization of search results.

In preparing this book, the authors strove to achieve the following objectives:

- 1. To develop a philosophical foundation for the meaning, nature, and hierarchy of knowledge and ignorance
- 2. To provide background information and historical developments related to knowledge, ignorance, and the elicitation of expert opinions
- 3. To provide a systems framework for the analysis and modeling of uncertainty
- 4. To summarize and illustrate methods for encoding data and expressing information
- 5. To provide and illustrate methods for uncertainty and information synthesis
- 6. To develop and illustrate methods for uncertainty measures and related criteria for knowledge construction
- 7. To examine and illustrate methods for uncertainty propagation in input–output systems
- 8. To guide the readers of the book on how to effectively elicit opinions from experts in such a way that would increase the truthfulness of the outcomes
- 9. To provide methods for visualizing uncertainty
- 10. To provide practical applications in these areas based on recent studies

The book introduces fundamental concepts of classical sets, fuzzy sets, rough sets, probability, Bayesian methods, interval analysis, fuzzy arithmetic, interval probabilities, evidence theory, open-world models, sequences, and possibility theory. These methods are presented in a style tailored to meet the needs of practitioners in many specialty fields, such as engineering, physical and social sciences, economics, law, and medicine. The book emphasizes the practical use of these methods, and establishes their limitations, advantages, and disadvantages. Although the applications at the end of the book were developed with emphasis on engineering, technological, and economics problems, the methods can also be used to solve problems in other fields, such as social sciences, law, insurance, business, and management.

#### STRUCTURE, FORMAT, AND MAIN FEATURES

This book was written with a dual use in mind, as both a self-learning guidebook and a required textbook for a course. In either case, the text has been designed to achieve important educational objectives of introducing theoretical bases, guidance and applications of the analysis, and modeling of uncertainty.

The eight chapters of the book lead the readers from the definition of needs, to the foundations of the concepts covered in the book, to theory and guidance and applications. The first chapter provides an introduction that discusses systems, knowledge (its sources and acquisition), and ignorance (its categories as bases for modeling and analyzing uncertainty). The practical use of concepts and tools presented in the book requires a framework and a frame of thinking that deals holistically with problems and issues as systems. Background information on system modeling is provided also in Chapter 1. Appendix A is called out in Chapter 1 to offer a historical perspective on knowledge.

Chapter 2 presents the fundamentals of encoding data and expressing information using classical set theory, fuzzy sets, and rough sets. Basic operations for these sets are defined and demonstrated. Fuzzy relations and fuzzy arithmetic can be used to express and combine collected information. The fundamentals of probability theory, possibility theory, interval probabilities, and monotone measures are summarized as they relate to uncertainty analysis. Examples are used in this chapter to demonstrate the various methods and concepts.

Chapter 3 covers uncertainty and information synthesis based on a missionbased system definition. The chapter starts by introducing measure theory and monotone measures and includes possibility theory and Dempster–Shafer theory of evidence, and then compares and contrasts them with probability theory with some of its variations and special applications, including linguistic probabilities, Bayesian probabilities, imprecise probabilities (including interval probabilities), interval cumulative distribution functions, and probability bounds. This chapter also discusses various multivariate dependence types and their models and describes fuzzy measures and fuzzy integrals.

Chapter 4 provides definitions and classification of uncertainty measures, including nonspecificity measures, such as the Hartley, evidence, possibility, and fuzzy sets' nonspecificity measures; entropy-like measures, such as Shannon entropy, discrepancy measure, and entropy measures for evidence theory of dissonance and confusion; and fuzziness measure. The chapter also includes applications relating to combining expert opinions.

Chapter 5 introduces uncertainty-based criteria for the construction of knowledge that include a minimum uncertainty criterion, maximum uncertainty criterion, and uncertainty invariance criterion, with demonstrative examples of aggregating expert opinions. The chapter also introduces methods for open-world analysis, including statistical estimators for sequences and patterns, such as the Laplace model, add-c model, and Witten–Bell model, and an analytical estimator based on the theory of evidence, i.e., the transferable belief model for evidential reasoning and belief revision. Applications to diagnostics are discussed.

Chapter 6 focuses on a class of models in engineering and the sciences of relating input variables to output variables for a system, building on knowing the underlying physical laws, such as material mechanics, and utilizing constraints, such as boundary conditions. The numerical computations might be based on finite element methods that are used to model the entire system. The model complexity can be increased by considering nonlinearity in behavior and other special considerations, such as bifurcation, instability, logic rules, and across-discipline or across-physics interactions. This chapter also presents methods for propagating uncertainty in input–output systems. The methods presented in this chapter are illustrated using simple linear systems. These methods form the basis for potential extensions to complex cases.

Chapter 7 provides guidance on using expert opinion elicitation processes. These processes can be viewed as variations of the Delphi technique, with scenario analysis based on uncertainty models, ignorance, knowledge, information and uncertainty modeling related to experts and opinions, and nuclear industry experiences and recommendations. This chapter also demonstrates the applications of expert opinion elicitation by summarizing results from practical examples.

Chapter 8 provides techniques for visualizing uncertainty in information. Visualization techniques are needed to allow both information and uncertainty to be portrayed effectively and grouped intuitively. This need is demonstrated, and icons are introduced for uncertainty and ignorance that are called uncerticons and ignoricons, respectively.

In each chapter of the book, computational examples are given in the individual sections of the chapter, with more detailed engineering applications provided in some of the key chapters. Also, each chapter includes a set of exercise problems that cover topics discussed in the chapter. The problems were carefully designed to meet the needs of instructors in assigning homework and the readers in practicing the fundamental concepts.

For the purposes of teaching, the book can be covered in one semester. The chapter sequence can be followed as a recommended sequence. However, if needed, instructors can choose a subset of the chapters for courses that do not permit a complete coverage of all chapters, or a coverage that cannot follow the order presented. In addition, selected chapters can be used to supplement courses that do not deal directly with uncertainty modeling and analysis, such as risk analysis, reliability assessment, expert opinion elicitation, economic analysis, systems analysis, litigation analysis, and social research courses. Chapters 1 and 2 can be covered concurrently, or preferably, Chapter 2 covered after Chapter 1. Appendix A is called out in Chapter 1 to offer historical perspective on knowledge. Chapter 3 builds on some of the materials covered in Chapter 2. Chapter 4 builds on some of the materials covered in Chapter 5 builds on Chapters 3 and 4 and should be covered after completing Chapter 4. Chapter 6 requires knowledge of materials



FIGURE 1 Sequence of chapters.

covered in Chapters 2 and 3. Chapter 7 provides guidance on using expert opinion elicitation and can be introduced independently. Chapter 8 also can be introduced after Chapter 1. The book also contains an extensive bibliography at its end. The accompanying schematic diagram (Figure 1) illustrates possible sequences of these chapters in terms of their interdependencies.

The authors invite users of the book to send any comments on its structure or content to the e-mail address ba@umd.edu. These comments will be used in developing future editions of the book. Also, users of the book are invited to visit the website of the Center for Technology and System Management at the University of Maryland, College Park, to find information posted on various projects and publications that can be related to uncertainty and risk analysis. The URL address is http://www.ctsm.umd.edu.

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### About the Authors

**Bilal M. Ayyub** is a professor of civil and environmental engineering and the director of the Center for Technology and Systems Management at the University of Maryland, College Park. He has extensive expertise in developing and using uncertainty and risk analysis methods, with over 450 professional papers and publications. Dr. Ayyub is the author and co-author of six books, with one of them as a second edition, and the editor and co-editor of seven books. His research received support from the Air Force, Navy, Coast Guard, Army Corps of Engineers, National Science Foundation, Department of Homeland Security, Homeland Security Institute, Ford Motor Company, Maryland Emergency Management Agency, Maryland State Highway Administration, American Society of Mechanical Engineers, and several engineering companies. Dr. Ayyub has served the engineering community in various capacities through societies that include ASNE (life member), ASCE (fellow), ASME (fellow), SNAME (fellow), IEEE (senior member), NAFIPS, Society for Risk Analysis, and World Future Society, among others. He chaired the ASCE Committee on the Reliability of Offshore Structures, the design philosophy panel of the SNAME Ship Structures Committee, and the Naval Engineers Journal Committee of ASNE. Presently, he is chairman of the Safety Engineering and Risk Analysis (SERAD) Division of ASME. He also was the general chairman of the first, second, third, and fourth International Symposia on Uncertainty Modeling and Analysis that were held in 1990, 1993, 1995, and 2003. Currently, he is member of the Journal of Ship Research Committee of SNAME. Dr. Ayyub is the recipient of the ASNE "Jimmie" Hamilton Award for the best papers in the Naval Engineers Journal in 1985, 1992, 2000, 2002, and 2004. Also, he received the ASCE "Outstanding Research Oriented Paper" in the Journal of Water Resources Planning and Management for 1987, the NAFIPS K.S. Fu Award for Professional Service in 1995, the ASCE Edmund Friedman Award in 1989, and the ASCE Walter L. Huber Civil Engineering Research Prize in 1997, among other awards and certificates of appreciation. Also, he was awarded the State of Maryland Governor's Citation for "positive contributions, leadership and distinguished service ... in honor and appreciation of your selfless efforts on behalf of the community." His biography is included in many biographical sources, such as Who's Who in America and Who's Who in the World.

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has been editor of the International Journal of General Systems since 1974 and the International Book Series on Systems Science and Systems Engineering since 1985. He was president of SGSR (1981 to 1982), IFSR (1980 to 1984), NAFIPS (1988 to 1991), and IFSA (1993 to 1995). He is a fellow of IEEE and IFSA and has received numerous awards and honors, including five honorary doctoral degrees, the Gold Medal of Bernard Bolzano, the Lotfi A. Zadeh Best Paper Award, the Kaufmann's Gold Medal, SUNY Chancellor's Award for Excellence in Research, and the IFSA Award for Outstanding Achievement. His biography is included in many biographical sources, including *Who's Who in America, Who's Who in the World, American Men and Women of Science, Outstanding Educators of America, Contemporary Authors*, etc. His research has been supported for more than 20 years by grants from NSF, ONR, the U.S. Air Force, NASA, NATO, Sandia National Laboratories, and a number of industries.

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# 1 Systems, Knowledge, and Ignorance

The greatest enemy of knowledge is not IGNORANCE, it is the ILLUSION of knowledge.

- Stephen Hawking

#### **1.1 DATA ABUNDANCE AND UNCERTAINTY**

Intelligence is defined as the ability to understand and adapt to the environment by using a combination of inherited abilities and learning experiences. This ability certainly includes the analysis of uncertainty and making decisions under conditions of uncertainty. The definition of intelligence is applicable to living systems — from ants to aardvarks to humans — as well as machines. Any organism that survives the remorseless rigors of evolution is sufficiently intelligent for its role in life. Likewise, machines need to be sufficiently intelligent to make decisions suitable for their functions and adapt to and deal with the presence of uncertainty. Any collectives of human decision makers and their decision-aiding machines must make, in the aggregate, good decisions.

The ability of a living system or machine to make appropriate decisions can be taken as a measure of intelligence. This decision-making ability requires the processing of data and information, construction of knowledge, and assessment of associated uncertainties and risks. The analysis and modeling of uncertainty enhances this ability of making appropriate decisions, thereby increasing intelligence. This need to model and analyze uncertainties also stems from the awareness that data abundance does not necessarily give us certainty, and sometimes can lead to error in decision making, with undesirable outcomes due to either overwhelming, confusing situations or a sense of overconfidence leading to an improper information use. The former situations can be an outcome of the limited capacity of a human mind in some situations to deal with complexity and data abundance, whereas the latter can be attributed to a higher order of ignorance, called the ignorance of self-ignorance.

As our society advances in many scientific dimensions and invents new technologies, human knowledge is being expanded through observation, discovery, information gathering, and propositional logic. Also, the access to newly generated information is becoming easier than ever as a result of computers and the Internet. We have entered an exciting era where electronic libraries, online databases, and information on every aspect of our civilization, such as patents, engineering products, literature, mathematics, physics, medicine, philosophy, and public opinions, are becoming a mouse-click or a few clicks away. In this era, computers can generate even more information from abundantly available online data. Society can act or react based on this information at the speed of its generation, creating sometimes nondesirable situations, for example, price or political volatilities. There is a great need to assess uncertainties associated with information and quantify our state of knowledge or ignorance. The accuracy, quality, and incorrectness of such information, and knowledge incoherence are coming under focus by philosophers, scientists, engineers, technologists, decision and policy makers, regulators and lawmakers, and our society as a whole. As a result, uncertainty and ignorance analyses are receiving a lot of attention by our society. We are moving from emphasizing the state of knowledge expansion and creation of information to a state that includes knowledge and information assessment by critically evaluating them in terms of relevance, completeness, nondistortion, coherence, and other key measures.

Our society is becoming less forgiving and more demanding from our knowledge base. The use of noncredible information, leading to questionable decisions, could place decision makers on the defensive. On the other hand, untimely processing and use of any available information, even if it might be inconclusive, can be treated worse than a lack of knowledge and ignorance. In the January 2003 State of the Union address, U.S. President George W. Bush stated, "The British government has learned that Saddam Hussein recently sought significant quantities of uranium from Africa." A few months later, after the conclusion of the war on Iraq in May 2003, senior White House officials conceded the information that former Iraqi president Hussein tried to buy uranium from Niger was inaccurate, but they said Bush's State of the Union speech was based on a broader range of intelligence. The argument that Iraq was trying to reconstitute its nuclear weapons program was a key point in the administration's rationale for war. These statements and decisions were made despite the March 2003 International Atomic Energy Agency dismissal as forgeries documents that alleged Iraq may have tried to buy 500 tons of uranium from Niger. The news elevated the problem to scandalous levels for this action on uncertain information, although inaction on uncertain intelligence, such as the "intelligence failure" in the case of the 2001 World Trade Center attacks, was treated as scandalous and was investigated due to its unacceptability. Any inaction due to noncredible information can be easily taken by our demanding society to be as erroneous as an action based on noncredible information — hence the need for uncertainty assessment, modeling, and analysis.

Making appropriate decisions commonly entails risks requiring risk control and management. Although people have control over the levels of some technologycaused risks to which they are exposed, reduction of risk needs to be pursued by governments and corporations in response to increasing demands by our society. Risk reduction generally entails a reduction of benefits to people, thus posing a serious dilemma. Moreover, the public and policy makers are required, with increasing frequency, to subjectively weigh benefits against risks and assess associated uncertainties when making decisions. Not using a systems or holistic approach, vulnerability exists for overpaying to reduce one set of risks that may introduce offsetting or larger risks of another kind. Such risk-based decisions require uncertainty modeling and analysis. The objective of this chapter is to present a systems framework for uncertainty modeling and analysis, and to discuss knowledge, its sources and acquisition, and ignorance and its categories. The practical use of concepts and tools presented in the book requires a framework and a frame of thinking that deals holistically with problems and issues as systems.

#### **1.2 SYSTEMS FRAMEWORK**

#### **1.2.1** Systems Definitions and Modeling

The definition and articulation of problems in engineering and the sciences is a critical task in the processes of analysis and design, and can be systematically performed within a systems framework. "The mere formulation of a problem is often far more essential than its solution," Albert Einstein said. "What we observe is not nature itself, but nature exposed to our method of questioning," Werner Karl Heisenberg said. Commonly, an engineering project can be modeled to include a segment of its environment that interacts significantly with it to define an underlying system. The boundaries of the system are drawn based on the mission, goals, and objectives of the analysis, and the class of performances (including failures) under consideration.

A generalized systems formulation allows scientists and engineers to develop a complete and comprehensive understanding of the nature of a problem, and underlying physical phenomena, processes, and activities. In a system formulation, an image or a model of an object that emphasizes some important and critical properties is defined. System definition is usually the first step in an overall methodology formulated for achieving a set of objectives. This definition can be based on observations at different system levels that are established based on these objectives. The observations can be about the different elements (or components) of the system, interactions among these elements, and the expected behavior of the system. Each level of knowledge that is obtained about an engineering problem defines a system to represent the project or the problem. As additional levels of knowledge are added to previous ones, higher epistemological levels of system definition and description are attained that, taken together, form a hierarchy of the system descriptions.

Informally, what is a system? The term *system* originates from the Greek word *systma*, which means an organized whole. According to *Webster's Dictionary*, a *system* is defined as "a regularly interacting or interdependent group of items forming a unified whole," such as a solar system, school system, or system of highways. For scientists and engineers, the definition can be stated as "a regularly interacting or interdependent group of items forming a unified whole that has some attributes of interest." Alternatively, a system can be defined as a group of interacting, interrelated, or interdependent elements that together form a complex whole that can be a complex physical structure, process, or procedure of some attributes of interest. All parts of a system are related to the same overall process, procedure, or structure, yet they are different from one another and often perform completely different functions. It follows from these definitions that the term *system* stands, in general, for a *set of things* and a *relation among the things*. It can be formally stated as

$$S = (T, R) \tag{1.1}$$

where S, T, and R denote, respectively, a system, a set of things, and a relation (or possibly a set of relations) defined on T. This commonsense expression by the pair (T, R) seems overly simple. Its simplicity, however, is only on the surface. While the definition is very simple in its form, it contains symbols, T and R, that are extremely rich in content. T stands not only for a single set with arbitrary elements, finite or infinite, but also, for example, for a power set, a power set of a power set, etc., or any arbitrary set of sets. Furthermore, things in T may have special properties by which systems are distinguished from one another. These properties can be referred to as *thinghood properties*. The content of symbol R is even richer. For each set T, with its special characteristics, the symbol stands for any conceivable relation defined on T. Formally, a relation is a subset of some Cartesian product of given sets. Even if T is only a single set, R stands for a relation from a family of distinct types of relations:  $R \subset T \times T$  (binary relations),  $R \subset T \times T \times T$  (ternary relations), etc. When T is a set of sets, the variety of distinct types of relations virtually explodes. For example, when T consists of just two sets, say  $T = \{X, Y\}$ , the number of types of relations grows quite rapidly, including, for example, the following types:

$$R \subset X \times Y \tag{1.2a}$$

$$R \subset (X \times X) \times Y \tag{1.2b}$$

$$R \subset (X \times X) \times (Y \times Y) \tag{1.2c}$$

$$R \subset (X \times X) \times (X \times Y) \tag{1.2d}$$

$$R \subset (X \times Y) \times (X \times Y) \tag{1.2e}$$

$$R \subset (X \times X \times X) \times (Y \times Y \times Y) \tag{1.2f}$$

$$R \subset (X \times Y) \times (X \times Y) \times (X \times Y)$$
(1.2g)

Although these few examples illustrate the great variety of possibilities represented by the single symbol R, they still do not capture the full richness of this symbol. The form of the Cartesian product on which a relation is defined is only one property of the relation. Other properties depend on the nature of elements of the relevant Cartesian product that are included in the relation. All these properties of relations can be subsumed under the suggestive name *systemhood properties*.

The simplicity of the commonsense expression of a system is, paradoxically, its weakness as well as its strength. The definition is weak because it is too general and, consequently, of little pragmatic value. It is strong because it encompasses all other, more specific definitions of systems. Due to its full generality, the commonsense expression qualifies for a criterion by which we can determine whether any given object is a system or not: an object is a system if and only if it can be described in the form that conforms to Equation 1.1.

Once we have the capability of distinguishing objects that are systems from those that are not, it is natural to define systems science as a *science whose objects of study are systems*. It is significant that this definition refers to systems, but not to any particular types of systems, such as physical systems, biological systems, social systems, or economic systems. This implies that these distinctions of systems, which are expressed solely in terms of the things involved, are not significant in systems science. This means, in turn, that systems science is concerned with systemhood properties of systems rather than their thinghood properties.

Classical science, which is predominately oriented to thinghood properties, and systems science, which is predominately oriented to systemhood properties, are two distinct perspectives from which scientific inquiry can be approached. These perspectives are complementary. Although classical scientific inquiries are almost never devoid of issues involving systemhood properties, these issues are not of primary interest in classical science and have been handled in an opportunistic, *ad hoc* fashion. There is no place in classical science for a comprehensive and thorough study of the various properties of systemhood. The systems perspective thus cannot be fully developed within the confines of classical science. It was liberated only through the emergence of systems science. While the systems perspective was not essential when science dealt with simple systems, its significance increases with the growing complexity of systems of our current interest. From the standpoint of the disciplinary classification of classical science, systems science is clearly cross-disciplinary.

Systems are traditionally grouped in various overlapping categories, such as:

- 1. Natural systems, e.g., river systems and energy systems
- 2. Human-made systems that can be embedded in the natural systems, e.g., hydroelectric power systems and navigation systems
- 3. Physical systems that are made of real components occupying space, e.g., automobiles and computers
- 4. Conceptual systems that could lead to physical systems
- 5. Static systems that are without any activity, e.g., bridges subjected to dead loads
- 6. Dynamic systems, e.g., transportation systems
- 7. Closed- or open-loop systems, e.g., a chemical equilibrium process and logistic systems, respectively.

Blanchard (1998) provides additional information on these categories.

#### 1.2.2 Realism and Constructivism in Systems Thinking

The emergence of systems science is from two different views about the nature of knowledge: *realism* and *constructivism*. According to realism, a system that is obtained by applying correctly the principles and methods of science *represents* some aspect of the real world. This representation is only approximate, due to limited resolution of our sensors and measuring instruments, but the relation comprising the system is a *homomorphic image* of its counterpart in the real world. Using more

refined instruments, the homomorphic mapping between entities of the system of concern and those of its real-world counterpart (the corresponding real system) becomes also more refined, and the system becomes a better representation of its real-world counterpart. Realism thus assumes the existence of systems in the real world, which are usually referred to as *real systems*. It claims that any system obtained by sound scientific inquiry is an approximate (simplified) representation of a real system via an appropriate homomorphic mapping.

According to constructivism, all systems are artificial abstractions. They are not made by nature and presented to us to be discovered, but we construct them by our perceptual and mental capabilities within the domain of our experiences. The concept of a system that requires correspondence to the real world is illusory because there is no way of checking such correspondence. We have no access to the real world except through experience. It seems that the constructivist view has become predominant, at least in systems science, particularly in the way formulated by von Glasersfeld (1995). According to this formulation, constructivism does not deal with ontological questions regarding the real world. It is intended as a theory of knowing, not a theory of being. It does not require the denial of ontological reality. Moreover, the constructed systems are not arbitrary: they must not collide with the constraints of the experiential domain. The aim of constructing systems is to organize our experiences in useful ways. A system is useful if it helps us to achieve some aims, for example, to predict, retrodict, control, make proper decisions, etc.

#### **1.2.3 TAXONOMY OF SYSTEMS**

Since systems science is oriented to the study of systemhood properties, its aim is to understand these properties as completely as possible. The following are key steps in pursuing this aim:

- 1. Dividing the spectrum of conceivable systems into significant categories defined in terms of systemhood properties
- 2. Studying individual categories of systems and their relationship
- 3. Organizing these categories into a coherent whole
- 4. Studying systems problems that emerge from the underlying set of organized systems categories
- 5. Studying methodological issues regarding the various types of systems problems
- 6. Studying metamethodological issues emerging from systems methodology

A prerequisite for dividing systems by their systemhood properties into significant categories is developing a conceptual framework within which these properties can properly be codified. Each framework determines the scope of systems conceived. It captures some basic categories of systems, each of which characterizes a certain type of knowledge representation, and provides a basis for further classification of systems within each category. To establish firm foundations of systems science, a comprehensive framework is needed to capture the full scope of systemhood properties.

#### 1.2.3.1 Epistemological Categories of Systems

Several conceptual frameworks that attempt to capture the full scope of systems currently conceived have been proposed by Klir (1985), Mesarovic and Takahara (1975, 1988), Wymore (1976), and Zeigler (1976). In spite of differences in terminology and in the way in which these frameworks evolved, they have essentially the same expressive power. As an example, a particular framework developed by Klir (1985) is described here, which is known in the literature as the *general systems problem solver* (GSPS). The kernel of the GSPS is a hierarchy of *epistemological categories of systems*, which represents the most fundamental taxonomy of systems. The following is a brief outline of the basic levels in this hierarchy.

At the lowest level of the epistemological hierarchy, an *experimental frame* is defined in terms of appropriate variables and their state sets (value sets). In addition, some supporting medium (such as time, space, or population) within which the variables change their states is also specified. Furthermore, variables may be classified as input and output variables.

An experimental frame (also called a *source system*) may be viewed as a *data description language*. When actual data described in this language become available, we move to the next level in the hierarchy. Systems on this level are called *data systems*.

When variables of an experimental frame are characterized by a relationship among them, we move to a level that is still higher in the hierarchy. It is assumed on this level that the relationship among the variables is invariant with respect to the supporting medium involved. That is, it is time invariant, space invariant, spacetime invariant, population invariant, etc. The relationship may involve not only variables contained in the experimental frame, but also additional variables defined in terms of the former by specific translation rules in the supporting medium. When the supporting medium is time, for example, we obtain lagged variables. Systems on this level are called *behavior systems*. Some of these systems can also be characterized conveniently as *state transition systems*.

A data system is represented by a behavior system if, under appropriate initial or boundary conditions, the support-invariant relation of the latter can be utilized for generating the data of the former. The generative capability of a behavior system extends, of course, beyond any given data. That is, a behavior system is capable to generate, for example, predictions or retrodictions of the variables involved. Moreover, it provides us with an explanation of the behavior of the variables within the given supporting medium.

Climbing further up the hierarchy involves two principles of integrating systems as components in larger systems. According to the first principle, several behavior systems (or sometimes lower-level systems) that may share some variables or interact in some other way are viewed as subsystems integrated into one overall system. Overall systems of this sort are called *structure systems*. The subsystems forming a structure system are often called its *elements*.

When elements of a structure system are themselves structure systems, this overall system is called a *second-order structure system*. *Higher-order structure systems* are defined recursively in the same way.

According to the second integrating principle, an overall system is viewed as varying (in time, space, etc.) within a class of systems of any of the other types. The change from one system to another in the delimited class is described by a replacement procedure that is invariant with respect to the supporting medium involved (time, space, etc.). Overall systems of this type are called *metasystems*.

In principle, the replacement procedure of a metasystem may also change. Then, an invariant (changeless) higher-level procedure is needed to describe the change. Systems of this sort, with two levels of replacement procedures, are called *metasystems of second order*. *Higher-order metasystems* are then defined recursively in the same way. Structure systems whose elements are metasystems are also allowed by the framework, similarly as metasystems defined in terms of structure systems.

The key feature of the epistemological hierarchy is that every system defined on some level in the hierarchy entails knowledge associated with all corresponding systems on lower levels and, at the same time, contains some knowledge that is not available in any of these lower-level systems.

The number of levels in the epistemological hierarchy is potentially infinite. In practice, however, only a small number of levels is considered. For each particular number of levels, the hierarchy is a semilattice. For five levels, for example, a part of the semilattice is expressed by the Hasse diagram in Figure 1.1. The circles represent the various epistemological categories of systems, and the arrows indicate the ordering from lower to higher categories. Symbols E, D, and B denote experimental frames (source systems), data systems, and behavior systems, respectively. Symbol S, used as a prefix, stands for structure systems. For example, SD denotes structure systems whose elements are data systems. Symbol S<sup>2</sup> denotes structure systems of second order. For example, S<sup>2</sup>B denotes structure systems of structure systems whose elements are behavior systems. Symbols M and M<sup>2</sup> denote metasystems and metametasystems, respectively. The combination SM and MS denotes structure systems whose elements are metasystems and metasystems whose elements are structure systems, respectively. The diagram in Figure 1.1 describes only a part of the first five levels in the epistemological hierarchy; it can be extended in an obvious way to combinations such as S3B, S2MB, SMSB, M2SB, S2MB, etc.

Categories of systems captured by the epistemological hierarchy are actually categories in the strong sense of mathematical category theory. It is useful to further classify systems subsumed under each epistemological category by relevant methodological distinctions. The aim of this classification is to capture the relationship between classes of systems and methods applicable to problems associated with the systems. Examples of methodological distinctions are those between systems based on discrete variables and systems based on continuous variables, between deterministic and nondeterministic systems, and between dynamic and spatial systems.

In the subsequent sections, the source, data, generative, structure, and metasystems are described and illustrated in Examples 1.1 and 1.2.

#### 1.2.3.2 Source (or Experimental Frame) Systems

At the first level of knowledge, which is usually referred to as level 0, the system is known as a *source system*. Source systems comprise three different components,



**FIGURE 1.1** Epistemological hierarchy of systems categories. E = experimental frame or source system; D = data system; B = behavior system; SE, SD, SB = structure systems based on source, data, and behavior systems, respectively; S<sup>2</sup>E, S<sup>2</sup>D, S<sup>2</sup>B = second-order structure systems of the three types; ME, MD, MB = metasystems based on source, data, and behavior systems, respectively; M<sup>2</sup>E, M<sup>2</sup>D, M<sup>2</sup>B = second-order metasystems of the three types; SME, SMD, SMB = structure systems based on metasystems of the three types; MSE, MSD, MSB = metasystems based on structure systems based on structure systems of the three types.

namely, object systems, specific image systems, and general image systems, as shown in Figure 1.2. The object system constitutes a model of the original object. It is composed of an object, attributes, and a backdrop. The object represents the specific problem under consideration. The attributes are the important and critical properties or variables selected for measurement or observation as a model of the original object. The backdrop is the domain or space within which the attributes are observed. The specific image system is developed based on the object. This image is built through observation channels that measure the attribute variation within the backdrop. The attributes when measured by these channels correspond to the variables in the specific image system. The attributes are measured within a support set that corresponds to the backdrop. The support can be either time or space, or can be population. Combinations of two or more of these supports are also possible. Before upgrading the system to a higher knowledge level, the specific image system can be abstracted into a general format. A mapping function is utilized for this purpose among the different states of the variables to a set of generals that is used for all the variables.

There are some methodological distinctions that could be defined in this level. Ordering is one of these distinctions that is realized within state or support sets. Any



FIGURE 1.2 Source system components.

set can be either ordered or not ordered, and those that are ordered may be partially ordered or linearly ordered. An ordered set has elements that can take real values, or values on an interval or ratio scale. A partially ordered set has elements that take values on an ordinal scale; for example, military ranks are partially ordered. A nonordered set has components that take values on a nominal scale, such as gender classification of people or political party affiliations of people. Distance is another form of distinction, where the distance is a measure between pairs of elements of an underlying set. It is obvious that if the set is not ordered, the concept of distance is not valid. Continuity is another form of distinction, where variables or support sets could be discrete or continuous. The classification of the variables as input or output variables forms another distinction. Those systems that have classified input-output variables are referred to as directed systems; otherwise, they are referred to as neutral systems. The last distinctions that could be realized in this level are related to the observation channels, which could be classified as crisp or fuzzy, corresponding to nonvague and vague information channels, respectively. For example, the number of hurricanes in a year hitting a region is uncertain, but takes on discrete crisp counts, whereas the fit or comfort level associated with wearing a piece of garment can only be measured in vague terms using linguistic terms such as comfortable, not comfortable, or partly comfortable. Figure 1.3 summarizes methodological distinctions realized in the first level of knowledge.



FIGURE 1.3 Methodological distinctions of source systems.

#### 1.2.3.3 Data Systems

The second level of a hierarchical system classification is the data system. The data system includes a source system together with actual data used for the states of variables for each attribute. The actual states of the variables at the different support instances yield the overall states of the attributes. Special functions and techniques are used to infer information regarding an attribute, based on the states of the variables representing it. A formal definition of a data system could be expressed as follows:

$$D = \{S, a\} \tag{1.3}$$

where D = data system, S = the corresponding source system, and a = observed data that specify the actual states of the variables at different support instances.

#### 1.2.3.4 Generative Systems

At the generative knowledge level, support-independent relations are defined to describe the constraints among the variables. These relations could be utilized in generating states of the basic variables for a prescribed initial or boundary condition. The set of basic variables includes those defined by the source system and possibly some additional variables that are defined in terms of the basic variables. There are two main approaches for expressing these constraints. The first approach consists of a support-independent function that describes the *behavior* of the system. A function defined as such is known as a *behavior function*. The second approach consists of relating successive *states* of the different variables. In other words, this function describes a relationship between the current overall state of the basic variables and the next overall state of the same variables. A function defined as such is known as a *state transition function*. For example, a state transition function can



FIGURE 1.4 A Markov transition diagram for repairable systems.

be used to model repairable systems. Such systems can be assumed for the purpose of demonstration to exit in either a normal, i.e., operating, state or failed state, as shown in Figure 1.4. A system in a normal state makes transitions to either normal states that are governed by its reliability level (i.e., it continues to be normal) or failed states through failure. Once it is in a failed state, the system makes transitions to either failed states that are governed by its repairable-ease level (i.e., it continues to be failed) or normal states through repair. Therefore, four transition probabilities are needed for the following cases:

- Normal-to-normal state transition
- Normal-to-failed state transition
- Failed-to-failed state transition
- Failed-to-normal state transition

The sum of probabilities for transitions originating from the same state must add up to 1. These probabilities can be determined by testing the system or based on analytical modeling of the physics of failure and repair logistics as provided by Kumamoto and Henley (1996).

A generative system defined by a behavior function is referred to as a *behavior system*, whereas if it is defined by a state transition function, it is known as a *state transition system*. State transition systems can always be converted into equivalent behavior systems, which makes the behavior systems more general.

The constraints among the variables at this level can be represented using many possible views or perspectives that are known as *masks*. A mask represents the pattern in the support set that defines sampling variables that should be considered. The sampling variables are related to the basic variables through translation rules that depend on the ordering of the support set. A formal definition of a behavior system could be expressed as

$$E_B = (I, K, f_B) \tag{1.4a}$$

where  $E_B$  = the behavior system defined as the triplet of three items, I = the corresponding general image system or the source system as a whole, K = the chosen mask, and  $f_B$  = the behavior function. If the behavior function is used to generate

data or states of the different variables, the sampling variables should be partitioned into generating and generated variables. The generating variables represent initial conditions for a specific generating scheme. The system in this form is referred to as a *generative behavior system*. The formal definition for such a system could be expressed as

$$E_{GB} = (I, K_G, f_{GB})$$
 (1.4b)

where  $E_{GB}$  = the generative behavior system defined as the triplet of three items; I = the corresponding general image system or the source system as a whole;  $K_G$  = the chosen mask partitioned into submasks, namely, a generating submask that defines the generating sampling variables and a generated submask that defines the generated variables; and  $f_{GB}$  = the generative behavior function, which should relate the occurrence of the general variables to that of the generating variables in a conditional format.

Most engineering and scientific models, such as the basic Newton's law of force computed as the product of mass of an object and its acceleration, or computing the stress in a rod under axial loading as the applied force divided by the cross-sectional area of the rod, can be considered generative systems that relate basic variables such as mass and acceleration to force, or axial force and area to stress, respectively. In these examples, these models can be considered behavior systems.

Several methodological distinctions can be identified in this level. One of these distinctions is the type of behavior function used. For nondeterministic systems where variables have more than one potential state for the same support instant, a degree of belief or a likelihood measure to each potential state in the overall state set of the sampling variables should be assigned. They can be used to quantify uncertainty using uncertainty measures, discussed in detail in Chapter 4. Each one of these measures is considered to form a certain distinction within the generative system. Probability distribution functions and possibility distribution functions are widely used to construct behavior functions as introduced in Chapters 3 and 4. The determination of a suitable behavior function for a given source system, mask, and data is not an easy task. Potential behavior functions should meet a set of conditions to be satisfactorily accepted. These conditions should be based on the actual constraints among the variables. They also relate to the degree of generative uncertainty and complexity of the behavior system. Another distinction at this level could be identified in relation to the mask used. If the support set is ordered, the mask is known as *memory dependent*; otherwise, the mask is referred to as *memoryless*. Figure 1.5 summarizes the different distinctions identified in this knowledge level.

#### 1.2.3.5 Structure Systems

Structure systems are sets of smaller systems or subsystems, as previously discussed. The subsystems could be source, data, or generative systems. These subsystems may be coupled due to having common variables or due to interaction in some other form. A formal definition of a structure system could be expressed as follows:



FIGURE 1.5 Example methodological distinctions for generative systems.

$$SE_B = \{ (V_i, E_B^i), \text{ for all } i \in e \}$$

$$(1.5)$$

where  $SE_B = a$  structure system whose elements are behavior systems,  $V_i =$  the set of sampling variables for the element of the behavior system,  $E_B^{\ i} = i^{\text{th}}$  behavior system, and e = the total number of elements or subsystems in the structure system with all *i* that belong to *e*, i.e., for all  $i \in e$ .

#### 1.2.3.6 Metasystems

Metasystems are introduced for the purpose of describing changes within a given support set. The metasystem consists of a set of systems defined at some lower knowledge level and some support-independent relation. Referred to as a replacement procedure, this relation defines the changes in the lower-level systems. All the lower-level systems should share the same source system. There are two different approaches whereby a metasystem could be viewed in relation to the structure system. The first approach is introduced by defining the system as a structure metasystem. The second approach consists of defining a metasystem of a structure system whose elements are behavior systems.

#### **EXAMPLE 1.1 SYSTEM DEFINITION OF CONSTRUCTION OPERATIONS**

Construction management concerns itself, among other things, with the real-time control of construction or production activities. However, in order to develop a control system for a construction activity, this activity has to be suitably defined depending on its nature and methods of control using a *hierarchical control system* (Abraham et al., 1989; Ayyub and Hassan, 1992a, 1992b, 1992c). The hierarchical system classification enables the decomposition of the overall construction activity into subsystems that represent the different processes involved in each activity. Then each process could be decomposed into tasks that are involved in performing the process. For construction activities, a set theory framework is suitable for representing the variables of the problem. The ability to infer information about the overall system prediction techniques (Chestnut, 1965; Hall, 1962, 1989; Klir, 1969, 1985; Wilson, 1984). In this example, levels of an epistemological hierarchy are defined for the purpose of real-time control.

#### Source Systems

For the purpose of illustration, the construction activities of concrete placement are considered and their knowledge level upgraded throughout the course of this example. The first step in defining the system for these construction activities is to identify a goal, in this case construction control by safely placing high-quality concrete efficiently and precisely. This goal can be defined through some properties or attributes of interest that can include safety, quality, productivity, and precision. Considering only two attributes of construction activities, i.e., safety and quality, the variables or factors that affect those attributes should be identified. Only two variables are assumed to affect the safety attribute. These variables could be quantitatively or qualitatively defined depending on their nature. For qualitative variables, linguistic terms are used and can be modeled using fuzzy set theory (which is formally introduced in Chapter 2) to define the potential states, together with a suitable observation channel that yields a quantitative equivalent for each state (Klir, 1985; Klir and Folger, 1988; Zimmerman, 1985). An example of this variable type is labor experience  $(v_1)$ , which is used herein. This variable is assumed to have four potential states: fair, good, moderate, and excellent. These linguistic measures can be defined using fuzzy sets. Using a scale of 0 to 10 for the level of experience, these measures can be defined as shown in Figure 1.6. The vertical axis in the figure represents the degree of belief that the corresponding experience value belongs to the fuzzy sets of fair, good, moderate, or excellent experience, where experience is on a scale of 0 to 10 (0 = absolutely no experience)and 10 = the absolute highest experience). A mathematical operator can then be defined in order to get a quantitative equivalent for each state. A one-to-one mapping function is used in order to define the corresponding general states of the variable  $(v_1)$ . The second variable  $(v_2)$  is the method of construction. This variable could have three potential states, e.g., a traditional method, slip form method, and precast element method. This is a crisp variable, and its observation channel is represented by an engineer who decides which method should be used. A similar one-to-one mapping function is used to relate the different construction methods to the corresponding general states of the variable  $(v_2)$ .



FIGURE 1.6 Fuzzy definitions of experience.

The next step in the definition of this system is the identification of the different supports, i.e., backdrops. In this example, the supports include time, space, and population. The time support is needed in measuring the progress of the different variables during the construction period. Assuming a construction period of 2 months with weekly observations, the time support set has eight elements that correspond to the weeks during the construction period. In other words, the elements are week 1, week 2, ..., week 8. The space support is used in relating the current state of each variable at a specific time support instant to a specific location in space within the system. As an example, a space support set with elements that represent the type of structural element under construction is considered. These elements are columns, beams, slabs, and footings. Such a classification constitutes a space support set with four potential elements. The population support is used to represent the performance of units having the same structure with respect to the same variables. The population support set in this example can represent the set of different crews involved in the construction activity. This support set could have four potential elements: a falsework crew, a rebar crew, a concreting crew, and a finishing crew. The overall support set, which represents the domain within which any of the defined variables can change, is defined by the Cartesian product of the three support sets. In other words, each variable is measured at a specific time instant in a specific location for a specific working crew. Therefore, the overall state of the attribute at a specific time instant is related to the performance and location of the working crew at that time. This fine classification allows for a complete identification of the reasons and factors that are responsible for a measured state of an attribute. This process enables construction control, and results in much more precise and accurate corrective actions. Table 1.1 summarizes different potential states for each variable together with observation channels  $(o_i)$ , a specific variable  $(v_i)$ , and corresponding general variables  $(v_i)$ . This example is based on the assumption that personnel with poor experience are not used in the construction activities. The observation channel is taken as a maximum operator to obtain the specific variable  $(v_i)$ . For example, using the maximum operator on poor produces 2 from Figure 1.6. The mapping from  $v_i$  to  $v'_i$  is a one-to-one mapping that can be made for abstraction purposes to some generalized states. The tabulated values under  $v_i'$  in Table 1.1 were selected arbitrarily for demonstration purposes of such a mapping. Table 1.2 summarizes the different elements for each support set. Table 1.3 shows the overall support set for a combination of two of the supports considered in this example of time and space. For

Variable	States	Observation Channel o <sub>i</sub>	Specific Variable <i>v<sub>i</sub></i>	Mapping Type	General Variable <i>v</i> <sub>i</sub> ′
$v_1$	Poor	Maximum	2	One-to-one	0
	Fair	Maximum	5	One-to-one	1
	Good	Maximum	8	One-to-one	2
	Moderate	Maximum	9	One-to-one	3
	Excellent	Maximum	10	One-to-one	4
$v_2$	Traditional method	One-to-one	Method 1	One-to-one	10
	Slip form method	One-to-one	Method 2	One-to-one	20
	Precast method	One-to-one	Method 3	One-to-one	30

#### TABLE 1.1 States of Variables

### TABLE 1.2Elements of the Different Support Sets

Support	Specific Element	Mapping Type	General Element
Time	Week 1	One-to-one	11
	Week 2	One-to-one	21
	Week 3	One-to-one	31
	Week 4	One-to-one	41
	Week 5	One-to-one	51
	Week 6	One-to-one	61
	Week 7	One-to-one	71
	Week 8	One-to-one	81
Space	Columns	One-to-one	12
	Beams	One-to-one	22
	Slabs	One-to-one	32
	Footings	One-to-one	42
Population	Falsework crew	One-to-one	13
	Rebar crew	One-to-one	23
	Concreting crew	One-to-one	33
	Finishing crew	One-to-one	43

example, the pair [12, 11] in Table 1.3 indicates columns (i.e., general element 12 according to Table 1.2) and week 1 (i.e., general element 11 according to Table 1.2).

The source system defined as such is classified as neutral since an input–output identification was not considered. The variables used herein are discrete. The time support set is linearly ordered, while the space and population support sets are not ordered. Observation channels for variable  $v_1$  are linearly ordered, while those for variable  $v_2$  are not ordered. Observation channels for variable  $v_1$  are fuzzy, while those for variable  $v_2$  are crisp. Figure 1.7 shows a procedure diagram of the source system for this example.

			Time (Week)						
Space	11	21	31	41	51	61	71	81	
Columns (12)	[12, 11]	[12, 21]	[12, 31]	[12, 41]	[12, 51]	[12, 61]	[12, 71]	[12, 81]	
Beams (22)	[22, 11]	[22, 21]	[22, 31]	[22, 41]	[22, 51]	[22, 61]	[22, 71]	[22, 81]	
Slabs (32)	[32, 11]	[32, 21]	[32, 31]	[32, 41]	[32, 51]	[32, 61]	[32, 71]	[32, 81]	
rooungs (42)	[42, 11]	[42, 31]	[42, 41]	[42, 41]	[42, 51]	[42, 01]	[42, 71]	[42, 81]	





FIGURE 1.7 A source system of a construction activity.

#### Data Systems

Considering the two variables previously defined,  $v_1$  for labor experience and  $v_2$  for method of construction, example data are introduced to illustrate the formulation of the data system. Variable  $v_1$  was defined as a fuzzy variable with fuzzy observation channels. This variable can transition to potential states at any support instant with some degrees of belief. Considering the combination of time and space supports, this formulation results in a three-dimensional data matrix for variable  $v_1$ . Any two-dimensional data matrix has the degrees of belief of each potential state as its entries. Variable  $v_2$  was defined as a crisp variable with crisp observation channels. As a result, the