

**Volume 249**

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# **Abelian Groups, Rings, Modules, and Homological Algebra**

Edited by

**Pat Goeters**

**Overtoun M. G. Jenda**



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# **Abelian Groups, Rings, Modules, and Homological Algebra**

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# **Abelian Groups, Rings, Modules, and Homological Algebra**

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The 2004 Abelian Groups Rings and Modules Conference Participants and Contributors  
Honor

EDGAR EARLE ENOCHS

For his dedicated mentoring and contributions to Algebra

---

## *Biography of Professor Edgar Enochs*

Edgar Earle Enochs was born on September 13, 1932 in Pike County, Mississippi. He obtained his bachelor's degree in 1958 from Louisiana State University and his Ph.D. degree from the University of Notre Dame, also in 1958, under the supervision of Professor Donald John Lewis. His Ph.D. thesis was titled "Infinite Abelian Groups." In the same year, on June 21, 1958 he married Louise Smith of Baton Rouge, Louisiana. He has seven children: Corinne, Mary Jane, Kathryn, Maureen, Madelaine, Anne, and John, and thirteen grandchildren.

Professor Enochs started his academic career as an instructor at the University of Chicago (1958 - 1960). In 1960, he joined the University of South Carolina as an assistant professor. In 1962 he was promoted to associate professor, and became full professor in 1966. In 1967, he moved to the University of Kentucky, where he has remained since.

Professor Enochs has had an illustrious and prolific career. Having started his research in infinite abelian groups, he has expanded his research interest to a wide range of other areas such as group theory, commutative and non-commutative algebra, modules, category theory, algebraic geometry, homological algebra, and representation theory just to name a few. Most of his papers have resulted in creating and growing new areas of research in Algebra. In particular, his 1963 and 1971 papers on "torsion free covering modules" formed a basis of the work on covers (right approximations) that is still being done today. Another paper that has had a major impact is his 1981 paper on "injective and flat covers and resolvents," which is the foundation of the relative homological algebra research being done today by researchers in the Enochs School. This remarkable paper was followed by the 1985 paper that he co-wrote with one of his students on "balanced functors" that formed a basis for what is now known as Gorenstein relative homological algebra. Professor Enochs has traveled all over the world giving lectures and talks and has continuously hosted research visitors at the University of Kentucky to work on the above research topics (and others) and their connections to commutative and non-commutative algebra, representation theory, sheaves, etc. In many cases, he has single handedly jump-started the visitors' research careers.

Professor Enochs has had a profound impact on mathematics education in the U.S., having supervised over 44 Ph.D. theses, including one of the editors of this book. He is an outstanding teacher and is a recipient of the University of Kentucky's teaching excellence awards: Alumni Association Great Teacher Award and the Sturgill Award for Contributions to Graduate Education.

Even with such stellar accomplishments, Professor Enochs is still the nicest, kindest, and most helpful person, and he is a pure joy to meet and work with.



---

## **Publications of Professor Edgar Enochs**

1. Gorenstein categories Tate cohomology on projective schemes (with Sergio Estrada and Juan Ramon Garcia Rozas), submitted.
2. The  $\aleph_1$ -product of DG-injective complexes (with Alina Iacob), to appear in Proc. Edinburgh Math. Soc.
3. The structure of compact co-Galois groups (with Sergio Estrada, Juan Ramon Garcia Rozas and Luis Oyonarte), to appear in Houston J. Math.
4. Gorenstein flat covers and cotorsion envelopes (with Sergio Estrada and Blas Torrecillas), to appear in J. Algebras Represent. Theory.
5. Covers and envelopes by V-Gorenstein modules (with Juan Antonio Lopez Ramos and Overtoun M.G. Jenda), to appear in Comm. Algebra.
6. A non-commutative generalization of Auslander's last theorem (with Overtoun M.G. Jenda and Juan Antonio Lopez Ramos), to appear in the International Journal of Math. and Math. Sciences.
7. Projective representations of quivers (with Sergio Estrada), to appear in Comm. Algebra.
8. Relative homological algebra in the category of quasi-coherent sheaves (with Sergio Estrada), Adv. in Math., 194 (2005), 284-295.
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26. Generalized quasi-coherent sheaves (with Sergio Estrada, Juan Ramon Garcia Rozas and Luis Oyonarte), *J. Algebra and Appl.*, 2 (2003), 63-83.
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### **Books**

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### **Ph.D. theses under the direction of Professor Edgar Enochs**

1. **B. Hoyte Maddox**, University of South Carolina, 1964, Absolutely Pure Modules
2. **W. W. Leonard**, University of South Carolina, 1964, Superfluous Submodules

3. **Pelham Thomas**, University of South Carolina, 1966, Maximal Spaces
4. **James Pleasant**, University of South Carolina, 1966, Certain Relations between Objects and Morphisms in a Category
5. **Arthur Van De Water**, University of South Carolina, 1967, A Property of Modules over Rings with a Left Field of Quotients
6. **David R. Stone**, University of South Carolina, 1968, Torsion-Free and Divisible Modules over Matrix Rings
7. **James R. Smith**, University of South Carolina, 1968, Local Domains with Topologically T-nilpotent Radical
8. **Joong Ho Kim**, University of South Carolina, 1968, On Complete Local Rings
9. **Conduff Childress**, University of South Carolina, 1969, Quotients of Hom and Torsionness
10. **C. Bruce Myers**, University of Kentucky, 1970, F-Torsionless and F-Reflexive Modules
11. **Ann F. Bowe**, University of Kentucky, 1970, Some Aspects of Small Modules
12. **James J. Bowe**, University of Kentucky, 1970, Neat Homomorphisms
13. **Thomas J. Cheatham**, University of Kentucky, 1971. Finite Dimensional Rings and Torsion Free Covers
14. **Cary H. Webb**, University of Kentucky, 1972 Tensor and Direct Product
15. **Roger D. Warren**, University of Kentucky, 1972, Free A-Rings
16. **Frank D. Cheatham**, University of Kentucky, 1972, F-Absolutely Pure Modules
17. **David D. Berry**, University of Kentucky, 1975, S-Purity
18. **David D. Adams**, University of Kentucky, 1978, Absolutely Pure Modules
19. **James Patterson, III.**, University of Kentucky, 1979,  $(X, Y)$ -Divisible Modules Over Commutative Rings
20. **Peter McCoart Joyce**, University of Kentucky, 1979, Dual Numbers and Finite Abelian Groups
21. **Walter P. Gerlach**, University of Kentucky, 1980, Connecting Locally Compact Abelian Groups
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23. **Richard G. Belshoff**, University of Kentucky, 1990, On Matlis Reflexive Modules
24. **Mark A. Goddard**, University of Kentucky, 1990, Minimal Projective Resolutions of Complexes
25. **Frank Branner**, University of Kentucky, 1991, On the Projective Functor
26. **Victor K. A. Akatsa**, University of Kentucky, 1991, Flat Envelopes and Negative Torsion Functors
27. **Sangwon Park**, University of Kentucky, 1991, The Macaulay-Northcott Functor



28. **Vivian Cyrus**, University of Kentucky, 1994, The Category of Monoids
29. **Clayton Brooks**, University of Kentucky, 1994, Homotopy Theory of Modules
30. **Albert Bronstein**, University of Kentucky, 1995, On the Representation of Quivers
31. **Okyeon Yi**, University of Kentucky, 1996 Local Nilpotence of Envelopes and Universal Enveloping Algebras
32. **Jinzhong Xu**, University of Kentucky, 1997, Flat Covers of Modules
33. **Christopher Anthony Aubuchon**, University of Kentucky, 1997, A Natural Functor from the Category of Complexes of Left  $R$ -modules to the Category  $\text{left } R(\epsilon)\text{-Modules}$
34. **William Todd Ashby**, University of Kentucky, 1998, The Characterization of Graded Principal Ideal Domains and Graded Torsion Free Covering Modules
35. **David W. Dempsey**, University of Kentucky, 2000, Functors and the Preservation of Covers and Envelopes
36. **Julia Varbalow**, University of Kentucky, 2000, Injective and Projective Representations of Quivers
37. **Makhmud Sagandykov**, University of Kentucky, 2000, On Homological Structures of Transformation Groups
38. **Stephen T. Aldrich**, University of Kentucky, 2000, Exact and Semisimple Differential Graded Algebras and Modules
39. **Naveed Zaman**, University of Kentucky, 2000, Minimal Generators
40. **Chris Bullock**, University of Kentucky, 2001, On Chain Numbers
41. **Molly D. Wesley**, University of Kentucky, 2005, Torsion Free Covers of Graded and Filtered Modules
42. **Katherine R. Pinzon**, University of Kentucky, 2005, Absolutely Pure Modules
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## *About the Editors*

H. PAT GOETERS was born in Houston, Texas, and at age 11 moved with his family to South Bend, Indiana and a year later to New Haven, Connecticut, following his father's academic career. In 1980, Pat finished his undergraduate studies in mathematics and computer science at Southern Connecticut State University in New Haven, and went to University of Connecticut to pursue a Ph.D. which was completed in 1984 under the supervision of William J. Wickless. After spending one year in a post-doctoral position in Middletown under the tutelage of James D. Reid, Goeters was invited for a tenure track position in Auburn by Ulrich F. Albrecht. Soon after, William Ullery and Overtoun Jenda were hired, and so began a lively algebra group.

OVERTOUN M. G. JENDA was born in Malawi and graduated from Chancellor College, University of Malawi with a bachelor's degree in mathematics. Upon graduation, he worked at Chancellor College as an associate lecturer for a year before moving to the U.S. in 1977 for his graduate studies at University of Kentucky. He obtained his Ph.D. in 1981 under the supervision of Professor Edgar Enochs. He then moved back to Chancellor College where he was a lecturer (assistant professor) for three years. In 1984, he moved to University of Botswana for another three year stint as a lecturer before moving back to University of Kentucky as a visiting assistant professor in 1987. In 1988, he joined a lively algebra research group at Auburn University. In addition to traveling within the U.S., he has been to Belgium, Canada, Czech Republic, Iran, Japan, Russia, South Korea, Spain, and several countries in southern Africa visiting mathematics departments and attending conferences. As a result, he has made long-lasting friends from all over the world that have had a great impact on his mathematics career. Overtoun Jenda is married to Claudine and has two children, Emily and Overtoun, Jr.



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## *Preface*

On the occasion of Edgar Earle Enochs' 72nd birthday, many top researchers in algebra gathered at Auburn University on September 9-11, 2004 to honor Ed, exchange ideas, and renew friendships. This book is a collection of refereed papers by the researchers involved in the talks as well as those who were not able to make it to the conference, and represents most of the current research topics in abelian groups, commutative algebra, commutative rings, group theory, homological algebra, lie algebras, and module theory.

We are excited that many of the veteran researchers in algebra took time from their busy schedules to honor Professor Enochs, and present us with their latest research ideas. The book gives the reader access to the current ideas and techniques of leading researchers. We must add that, according to the master of first order, Laszlo Fuchs, the conference was one of the most comfortable he has ever attended; we concur and attribute this to the participants; their devotion to algebra is evident in the articles submitted.

A rarity compared to some proceeding volumes is that due to Edgar Enochs' venerable contributions to a wide range of topics in algebra, we have in this volume a large collection of high-quality papers, as attested by referees' reports, from many high-level algebraists discussing today's hot research topics. Though steeped in veteran techniques, articles in this volume involve topics that are accessible to the beginning mathematician. Also, in many articles, suggestions of problems and programs for future study are made - it is always nice when one can improve on a master's result (or perhaps knock oneself out trying).

This collection of papers is therefore an excellent addition to the literature and will serve as an invaluable handbook for beginning researchers in algebra as well as specialists. This book is indeed a superb way of honoring a legend in algebra, Edgar Enochs.

HPG  
OMGJ