TRANSLATED BY E. S. KENNEDY

Planetary Equatorium of Jamshīd Ghiyath al-Dīn al-Kāshī

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The Planetary Equatorium
of
Jamshīd Ghiyāth al-Dīn al-Kāshī

Princeton Oriental Studies

Volume 18

THE PLANETARY EQUATORIUM

OF JAMSHĪD GHIYĀTH AL-DĪN AL-KĀSHĪ
(d. 1429)

An Edition of the Anonymous Persian
Manuscript 75[44b] in the Garrett Collection
at Princeton University
Being a Description of Two Computing Instruments

The Plate of Heavens
and the Plate of Conjunctions

WITH TRANSLATION AND COMMENTARY BY

E. S. KENNEDY

1960

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To

my foster mother

Annie E. Kennedy

PREFACE

This book presents, with translation and commentary, the text of a Persian manuscript in the Garrett Collection at Princeton University. The manuscript describes the construction and use of two astronomical computing instruments invented by the fifteenth century Iranian scientist Jamshīd ibn Masʿūd ibn Maḥmūd, Ghiyāth al-Dīn al-Kāshī (or al-Kāshānī). Our fund of knowledge concerning the work of this individual has been materially increased in recent years, particularly by the studies of the late Paul Luckey in Germany, and by the fruitful collaboration of B.A. Rosenfeld and A.P. Yushkevich in Russia. The reader will find their publications listed in the bibliography at the end of this volume.

Little enough has been known about the life of Kāshī (as we shall call him hereafter), and we have been able to supplement that little, principally by examining the introductions and colophons of Kāshī manuscripts. The results are assembled in the biographical sketch immediately following the table of contents.

The succeeding section recites the relations between our anonymous Persian text and the two versions of a book by Kāshī himself on which it is based.

There follows the facsimile text and translation, arranged for immediate reference with corresponding pages and lines opposite each other. After this comes the commentary to the text. In general, topics are treated in the order in which they appear in the manuscript. Where it has been found convenient to digress from this order, a statement has been inserted indicating the proper section. When sections are related to a particular passage in the text, this passage has been specified in parentheses following the section title. All such references to the text give folio and line, separated by a colon.

It is assumed that the reader is familiar with the leading concepts of Ptolemaic astronomy, of which [41] and [42] contain

PREFACE

readily available expositions. Here and in the sequel, numbers enclosed in square brackets refer to items in the bibliography at the end of the book.

The bulk of the manuscript is taken up with an instrument called by Kāshī <u>Ṭabaq al-Manāṭiq</u>, which we translate as "Plate (<u>or</u> Tray) of Heavens." It is an example of the class of devices known as equatoria, analogue computers on which the various Ptolemaic planetary configurations were laid out to scale. They thus yield solutions to such problems as that of finding the true longitude of a planet at a given time. Kāshī's equatorium is only one element in a tradition stretching through sixteen centuries in time, and in space from Western Europe to Central Asia via North Africa and the Near East. The story of the other instruments, and the Plate of Heavens' place therein, has been delineated admirably by D.J. Price in his publication [42] of the Chaucerian equatorium - there is no need to repeat it here.

The second instrument, the "Plate of Conjunctions" (<u>Lawḥ al-Ittiṣālāt</u>), is a simple device for performing a linear interpolation.

A number of the problems raised by the manuscript have been dealt with in a preliminary way in papers published over a period of years. This is an attempt simultaneously to dispose of the remaining problems, to revise, complete, and correct previous solutions, and to put before the public the primary source on which they are based. In addition to presenting the philologians with an opportunity to pick apart the work of a translator whose formal training has been in mathematics, this will serve to present intact a medieval scientific work, a very small link in the chain which leads from the tallystick to the electronic computer. For the manifold shortcomings of the result the editor, like the anonymous author of the manuscript, craves correction "with the musk-dripping pen" of forebearance.

Acknowledgments

In thus bringing to a close a task, the working out of which has been a source of much satisfaction over a long time, it is pleasant indeed to set down the names of some of the many friends who have been helpful. Professor Yahya Armajani called to my attention the equatorium manuscript in the first place. In the office of the late Professor George Sarton I was privileged to associate with Dr. Alexander Pogo, and from him gained my first real insight into planetary motions. Professors N. Seifpur Fatemi and Jalal Homa'i, and Dr. Iraj Dehqan answered many questions connected with the reading of the Persian text. The commentary has been improved as a result of many discussions with Professor Derek Price, whose book on the Chaucer equatorium will continue to maintain itself as the model of publications of this genre.

The contributions acknowledged above, important as they are, can be regarded as peripheral and as not involving the individuals in errors which the book may contain. It remains to name another associate, one who cannot escape his portion of responsibility for whatever merit this work has. To the counsel and example of O. Neugebauer, best of friends and keenest of critics, I owe such competence as I have acquired in the history of the exact sciences.

The work was made possible by the Rockefeller Foundation, Brown University, the Institute for Advanced Study and a Fulbright grant. The Princeton University Department of Oriental Languages and Literatures and the Princeton University Press have been uniformly cooperative in making arrangements for publication.

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E.S.K.

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INTRODUCTION

Biographical Material on the Inventor

Our earliest fixed point in Kāshī's life is 2 June, 1406 (12 Dhū al-Ḥijjah, 808). On this date he observed in Kāshān, his home town in central Iran, the first of a series of three lunar eclipses (see [23]*, f.4r.)

He is the author of an Arabic treatise on the sizes and distances of the heavenly bodies, called Sullam al-samā' [22]. Most of the extant copies of this work are undated, but in [34], p.50, Krause reports that one of the Istanbul copies claims the book was finished on 1 March, 1407 (21 Ramaḍān, 809). This is confirmed by Ṭabāṭabā'ī ([52], p.23) on the basis of a copy at the Shrine Library in Meshed, Iran. Since this date falls during the run of eclipse observations, it follows that the treatise must have been written in Kāshān. It is dedicated to a certain wazīr designated in the manuscript only as Kamāl al-Dīn Maḥmūd. Search through general histories of the period has failed thus far to produce any information whatsoever as to the identity, jurisdiction, or political affiliation of this individual. A certain Maulā Kamāl al-Dīn Maḥmūd al-Sāghirjī mentioned by Khwāndamīr ([33], vol.3, p.513) fulfills the requirements as to name and rank, but cannot have been contemporary with Kāshī. It is probably to this individual that Ṭabāṭabā'ī ([52], p.23) refers.

Sometime during the year 816 A.H. (1413/14) Kāshī completed the Khāqānī Zīj [23], written in Persian and the first of his two major works. In the introduction to it he complains that he had been working on astronomical problems for a long period, living in penury in the cities of ʿIrāq (doubtless ʿIrāq-i ʿAjamī, Persian ʿIrāq), and most of the time in Kāshān. Having undertaken the composition of a zīj, he would have been unable

* Numbers in square brackets refer to items in the bibliography.

to complete it except for the timely beneficence of the prince Ulugh Beg, he says, and to him he dedicated the finished work. The fact that the longitude of Shīrāz was taken as base for the mean motion tables (see Section 18 in the commentary below) does not prove that he worked there part of the time, although he may have done so. The place had been a center of astronomical activity on and off for many centuries, and Kāshī may simply have chosen as base a location better known than Kāshān. From a remark in his Miftāḥ ([47], p.176) it is clear that he was in the neighboring city of Isfahān at one time or another.

Next comes a very short (seven page) Persian treatise [20] on astronomical instruments written in January 1416 (Dhū al-Qaʿda, 818) and dedicated to a Sultan Iskandar.

Soon after this, on 10 February, 1416 (10 Dhū al-Ḥijjah, 818), and in Kāshān, the first version of the Nuzhat al-Ḥadāʾiq was completed. The book is Kāshī's own description of the equatorium he invented. It names no patron. When next heard from, Kāshī has joined the group of scientists at the Samarqand court of Ulugh Beg. There he stayed for the rest of his life.

His career commenced during the long reign of Tamerlane. When the latter died in 1405 he was eventually succeeded by his son Shāhrukh, whose rule outlasted Kāshī's life span, and who throughout evidently retained some sort of hegemony over all of Iran. During this time Shāhrukh's son Ulugh Beg was the ruler of Samarqand, and eventually as head of the Timurid dynasty survived his father for a short time.

The Iskandar referred to above can hardly have been any other than the son of Qara Yūsuf ([12], p.127) second ruler of the Black Sheep Turcoman dynasty which established itself mainly in Azarbaijan and Mesopotamia, encroaching eventually into Fārs. Iskandar was twice defeated by Shāhrukh. Like Ulugh Beg, he achieved primacy in his dynasty only long after the death of Kāshī.

INTRODUCTION

The curious spectacle of a scientist dedicating successive writings to minor potentates in rival dynasties invites speculation. It is tempting to liken his actions to those of a modern scholar alternately wooing one and another of the affluent learned foundations in hopes of an ever more princely grant. Kāshī's short treatise on astronomical instruments [20] contains little beyond what must have been common knowledge to any competent astronomer of the day, and its composition can have cost him only the time required to write it down. He may have turned it out and dedicated it to Iskandar in order to counteract the effect of the earlier dedication to a Timurid. But ignorance of the details of political vicissitudes makes further conjectures unprofitable.

In 1417 Ulugh Beg commenced building in Samarqand a madrasah ([16], p.54), a school to house students of theology and the sciences. This impressive tiled structure is still admired by tourists from all over the world. Shortly after its completion the construction of the observatory was begun.

It was during these operations that Kāshī arrived; we do not know exactly when. Several sources (e.g. [33], vol.4, p.21) have him accompanied by a fellow-townsman and astronomer, Muʿīn al-Dīn al-Kāshī, according to Ṭabāṭabā'ī ([52], p.6), a nephew. From this period also is a document of the greatest interest, a letter from Kāshī to his father in Kāshān. It has been published (as [53]) in the original Persian, collated and annotated by M. Ṭabāṭabā'ī from two manuscripts, one of which is in the library of the Madrasah Sepahsālār in Tehrān. The letter merits publication in facsimile with a translation into some European language. Pending this we outline its contents below.

The epistle begins with a quotation from the Qur'ān indicative of filial piety, and a statement that the writer has been too busy with the observatory to do anything else. He writes that the sultan is an extremely well educated man, in the Qur'ān, in Arabic grammar, in logic, and the mathematical sciences. As an illustration of the latter he tells how the king, while on horseback, once computed in his head a solar position correct to minutes of arc. Kāshī then goes on to describe how, upon his arrival at Samarqand, he was put through his paces by the sixty or seventy other mathematicians and astronomers in attendance there. He gives as examples four of the problems propounded to him. The first involved a method of determining the projections of 1022 fixed stars on the rete of an astrolabe one cubit in diameter. The second required the laying out of the hour lines on an oblique wall for the shadow cast by a certain gnomon. The third problem demanded the construction of a hole in a wall, of such a nature that it would admit the sun's light at, and only at, the time of evening prayer, the time to be that determined by the rule of Abū Ḥanīfah. Lastly, he was asked to find the radius, in degrees of arc on the earth's surface, of the true horizon of a man whose height is three and a half cubits. All these and others, says Kāshī, which had baffled the best minds of the entourage, he solved with ease, thus quickly gaining intellectual paramountcy among them.

He held a low opinion of the rest of the sultan's scientific staff in general, in spite of the fact that Ulugh Beg's astronomical bent had stimulated the study of mathematics in Samarqand for the past ten years. The only one who gave him any competition at all was Qāḍīzādah al-Rūmī ([51], p.174), and Kāshī recounts in detail two occasions on which he worsted this individual. One of these was brought on by Qāḍīzādah, who had been expounding the famous zīj of al-Bīrūnī, the Masudic Canon [5], when he ran into difficulties with the proof of a theorem. Using the immemorial gambit of any mathematics teacher when faced with a like situation, he told the class, which included Ulugh Beg himself, that there must be a fault in the text, he had best compare it with a better copy elsewhere. After two days he was still stuck at the same place, when Kāshī just happened to turn up, explained the proof off-hand, and showed that the manuscript was correct.

In spite of this, and other exhibitions of tact and erudition on the part of Kāshī, he assures his father that relations between the two of them are of the most amicable. He agrees that Qāḍīzādah is the only one of the lot who knows much about the Almagest, although he is deficient in observational technique. The views of Qāḍīzādah on the matter have not come down to us, but it may or may not be of significance that in the prolegomena to Ulugh Beg's zīj ([54], p.5) Qāḍīzādah is the first of the two to receive honorable mention and extravagant praise.

Kāshī uses up a good deal of space telling his father about the ignorance of a certain Badr al-Dīn. This man, he writes, having been through a few propositions of Euclid which he is unable to apply, is like one who knows several rules of (Arabic) grammar but can write no Arabic. He states further that Badr al-Dīn is a liar. Kāshī's motive in reporting on this person is not clear.

The letter describes the status of the work in progress on the construction of instruments for the observatory. It closes with a detailed explanation intended to make clear to the layman why the taking of observations for a complete set of planetary parameters is a long process, and cannot be completed in as short a time as a year, or anything like it.

In the middle of Shaʿbān, 827 (July, 1424), Kāshī finished his unprecedentedly precise π-determination, al-Risālat al-muḥīṭīyah. Written in Arabic, this masterpiece of computational technique has been published in German [36] and Russian [47] editions, both excellent. It has no dedicatory passage.

From the same month two lunar years later (June, 1426) dates the Samarqand rescension of the Nuzhah, and on 3 Jumādī I, 830 (2 March, 1427), Kāshī completed his second major work, the Miftāḥ. It is dedicated to Ulugh Beg, and has recently been published [47] with Arabic text, Russian translation, and commentary, by Rosenfeld and Yushkevich. Studies based on it are [35], [37], [44], and [7]. In the introduction to the Miftāḥ Kāshī gives a partial list of his own works. Other than titles already mentioned above, the following appear:

Risālat al-watar w'al-jaib, also known as Risālah fī istikhrāj jaib darajat wāḥidah, is apparently extant both in a lithographed edition printed in Tehran in 1889, and in manuscript ([51], p.174). Marginal notes in [23], f.32r, and the lithograph edition of [17], p.3, state that the treatise was incomplete when Kāshī died and that it was finished by Qāḍīzādah. A copy of the lithographed collection of which it is a part is in the Parliament (Majlis) Library in Tehran. It describes an elegant iterative method of computing the sine of one degree to any required accuracy. No translation of this risālah has been published, although a commentary on it is available in French [54] and in Russian [47] translations, and a considerable literature in European languages has accumulated about it (see, e.g. [1]).

The Zīj al-tashīlāt is not extant as such, but is probably the set of tables and accompanying explanation for a simplified method of computing planetary positions as worked out by Kāshī. In his Khāqānī Zīj ([23], ff.142r-155r) these tables occupy a section distinct from the planetary tables of standard type.

In addition to those listed in the Miftāḥ al-ḥisāb, the following treatises were written by or attributed to him:

Miftāḥ al-asbāb fī ʿilm al-zīj, listed in the Mosul catalogue [40].

Risālah dar sākht-i asṭurlāb listed in the Meshed catalogue

([38], Ms. math. 84).

Risālah fī maʿrifat samt al-qiblah min dā'irat hindiyah, also listed in the Meshed catalogue.

Risālat ʿamal al-ḍarb bi'l-takht w'al-turāb is also included in the Tehran lithographed edition of 1889 of which a copy is in the Parliament Library in Tehran.

Al-risālat al-iqlīlāminah is mentioned by Kāshī himself in the Samarqand version of the Nuzhah ([19], p.311).

Kāshī's statements about the length of time required to complete the observations proved all too true. In the prolegomena to the zīj ([54], p.5) based on them, his royal patron Ulugh Beg, laments his death early in the course of the work. His collaborator and rival, Qādīzādah, also died before the zīj was finished. On the title page, the India Office copy of the Khāqānī Zīj [23] bears a note saying that on the morning of Wednesday, 19 Ramaḍān, 832 (22 June, 1429), at the observatory outside Samarqand there died the "mighty master, Ghiyāth al-Millah w'al-Dīn, Jamshīd." According to Ṭabāṭabā'ī ([52], p.19) the incomplete copy of the same zīj in the Shrine Library at Meshed has the same annotation. That Kāshī left behind him more than his scientific works is witnessed by the British Museum manuscript of his Miftāḥ ([6], p.199) which has the following curious colophon:

> Verily I finished copying this honorable manuscript on the second day of the month of Shawwāl, year 997, (14 August, 1589). It was copied (or checked ?) by the one (who is) indigent (for the sake) of God, al-Razzāq, son of ʿAbdallāh, son of ʿAbd al-Razzāq, son of Jamshīd, son of Masʿūd, son of Jamshīd, the author of this noble book.

We lack evidence on which to pass judgment, or even to assess, Kāshī's personal character. Concerning him the author of the

Haft iqlīm (see [45], vol.ii, p.45) has this to say:

> The former (Kāshī) was ignorant of the etiquette of courts, but Ulugh Beg was obliged to put up with his boorish manners because he could not dispense with his assistance.

In the letter to his father he does not depict himself as the proverbially shrinking violet. At the same time this was a personal communication addressed to a parent, and presumably not intended for publication. And on the basis of the evidence at hand we can only agree with his own estimate - he was the best of the lot at Samarqand.

In the closing appendix to the revised Nuzhah, "On the Naming of the Instrument," ([19], p.312) Kāshī whimsically relates a suggestion from some of his friends, that his equatorium be called Jām-i Jamshīd,

> ... Jamshyd's Sev'n-ring'd Cup, where no one knows,

a likening of the instrument with its seven planetary deferents to the magic divining goblet of the mythical Iranian king, discoverer of the uses of wine, and whose name Kāshī bore.

> They say the Lion and the Lizard keep
> The courts where Jamshyd gloried and drank deep.

As for his scientific attainments and his place in the history of science, here we can operate from much firmer ground. He was first and foremost a master computer of extraordinary ability, witness his facile use of pure sexagesimals, his invention of decimal fractions (cf. [44] and [37], p.102), his wide application of iterative algorisms, and his sure touch in so laying out a computation that he controlled the maximum error and maintained a running check at all stages.

His equatorium marked the most extensive development ever given to this class of instrument. In particular his was the only mechanical device with which a determination of the planetary latitudes was possible. If we retain mental reservations as to the practicality of the results, there can be no doubt of the ingenuity of the descriptive geometry involved.

He seems to have been a completely competent observer and astronomical technician, neither ahead of nor behind his times. The same statement can be made of his work in planetary theory. He accepted unreservedly the notion, not contained in the Almagest, that the moon, inner planets, sun, and other planets move in contiguous bands about the fixed earth, and that hence it was possible to compute in terrestrial units the mean distance of, say, Saturn from the earth. His contemporaries were therefore overgenerous in calling him "the second Ptolemy" ([33], vol.iv, p.21), but the next generation was equally sanguine in calling a mathematician of their own time "the second Ghiyāth al-Dīn Jamshīd" ([53], p.60).

The History of the Text and Its Versions

The Persian manuscript published in this volume is an apparently unique copy of a tract composed by some individual now unknown, between the years 1481 and 1512, and probably in Constantinople. This time spans the reign of the Ottoman Sultan Bayazid II, to whom the book is dedicated. Constantinople is mentioned in the text as this ruler's capital, and its longitude is taken as base for the planetary mean motion tables.

If the author is unknown, his prime source is not. He specifically states that the astronomical instruments he describes were invented by Jamshīd (al-Kāshī), and his work can be characterized as largely a translation into Persian of selected parts from Kāshī's own Arabic description of the same two instruments.

The latter book is called Nuzhat al-Ḥadā'iq (A Fruit-

Garden Stroll) and is extant in two versions. The first was written in Kāshān and completed on 10 February, 1416. The only copy known to have survived is Number 210 in [48], a microfilm of which has been made available by the officials of the India Office. The copy is modern, having been completed on 2 December, 1863. It is written in an easily legible nastaʿlīq hand, but very carelessly, and all the figures are missing. This Kāshān version we refer to as NK in the sequel.

In June, 1426, just three years before he died, Kāshī completed a rescension of the Nuzhah. This was after he had moved to Samarqand to work at Ulugh Beg's observatory there. Following the colophon of the original material, which was changed only in minor details, he added a set of ten appendices. These describe additional techniques for utilizing the instruments, and improvements or changes in the construction of their parts. It will be shown in the commentary (Section 42) to our text that the author made use of the Samarqand rescension, which we will abbreviate as NS. No manuscript copy is known to exist, but in 1889 it was printed in the Tehran lithograph edition of several of Kāshī's works. The example in the translator's possession is bound and paginated (pp.250-313) with the Miftāḥ al-Ḥisāb. It is written in a fair naskh hand, very carefully, and with text corrections in the margin. The space on page 261 for the main figure, however, has been left blank.

A table of contents of NS and NK follows, combined with a concordance of corresponding sections in our text.

I N T R O D U C T I O N

Table of Contents of the *Nuzhat al-Ḥadā'iq* With a Concordance of the Kāshān (NK), Samarqand (NS, Tehrān lithograph), and Persian Versions

THE PLANETARY EQUATORIUM

	NK page	NS page	Persian Anonymous treatise and chapter	Persian Anonymous folio
Numerical Example	43	268	-	-
Section 2. On the Arrangement of the Instrument	46	269	II,1	18r
Section 3. On the Determination of the Sun's True Longitude, Equation and Distance	47	270	II,3	19v
Section 4. On the Determination of the True Longitudes of the Other Planets	48	270	II,4 II,5	20r 21r
Section 5. On the Determination of Equations	57	274	II,6	21v
Section 6. On the Determination of the Distances of Planets from the Earth	58	274	II,8	26v
Section 7. On the Determination of Retrogradations and Forward Motions	60	275	II,9	28r
Section 8. On the Knowledge of (the Planetary) Sectors	64	276	II,10	30r
Section 9. On the Determination of Planetary Latitudes	66	277	-	-
Section 10. On the Determination of the Lunar Third Equation (i.e., the correction due to the moon's orbit being inclined with respect to the ecliptic) and its (the moon's) True Longitude in the Parecliptic	72	280	-	-
Section 11. On the Determination of Lunar Eclipses	73	280	II,11	31r
Section 12. On the Determination of Solar Eclipses	76	281	II,12	33r
Section 13. On the Determination of the Mean of Transfer by Means of the Sun's True Longitude	77	282	II,13	34v