

the
Calculus **of**
Happiness

**How a Mathematical
Approach to Life
Adds Up to
Health,
Wealth,
and
Love**

Oscar E. Fernandez

The Calculus of Happiness



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To Emilia

One day when you can read this

Sabrás que estaba pensando en ti

And when that day comes

Búscame y dame un beso y un abrazo

Because there's no better gift

Que una hija le puede dar a su papá.

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Preface



NUMBERS—THEY'RE EVERYWHERE. They describe the prices you pay for things, the calories you consume each day, and even your dating life (How many people have you dated? How long did those relationships last?). What you may not have realized is that many of the numbers we care about—like the answers to the aforementioned questions—are the *outputs* of various *inputs*. (Example: the calories you consume in a day—the output number—depends on the calories of the individual foods eaten—the inputs.) When thought of this way, an intriguing question emerges: can we use mathematics to improve our lives?

This book is dedicated to answering that question for three aspects of life we think about constantly: health, wealth, and love. Fortunately, many of the input-output relationships in that trio are described by *functions*, special types of equations well-understood by mathematicians. I'll guide you through the formulas behind health, wealth, and love in the chapters that follow. We'll see that they arise quite naturally from careful observations about our everyday experiences, and we'll learn how to extract valuable and powerful insights from them. Here's a sampling of the results to give you a sense of what I mean.

- Chapter 1: A research-backed equation that determines how many calories you should eat each day given certain inputs, including age and weight.
- Chapter 2: A research-based diet that can lower your bad cholesterol, raise your good cholesterol, decrease your risk for developing heart disease and diabetes, and help you lose weight.
- Chapter 2: A research-backed equation that estimates how many years of future life you're losing (and then tells you how to get them back).

- Chapter 3: Several math-backed strategies for increasing your monthly take-home pay.
- Chapter 3: An equation that estimates how soon you'll be able to retire.
- Chapter 4: An investment portfolio whose average annual return since 1926 is 8% during recessions and 10% during expansions.
- Chapter 5: An algorithm to build couples from a group of men and women that *guarantees no one will cheat* with another member in the group.
- Chapter 6: A math-backed way to make joint decisions in a relationship that are perceived as fair and transparent by both parties.

To make the book accessible I've built in several features:

- *Calculations relegated to each chapter's appendix.* Math requires calculations. But rather than force you to read through them, I put them in each chapter's appendix. (Calculations are signaled via starred superscripts, like this one.^{*1}) There are some exceptions, but in these cases the details of the calculation contain valuable insights. The main body of the book describes the applications and main concepts of the math and each chapter's appendix details the actual calculations. Moreover, you can skip all the appendixes and still enjoy the book!
- *Online customizable equations linked to book content.* Certain equations have a computer icon next to them (like the one you see in the margin), indicating that I've created online customizable calculators based on the equation. You can access these interactive calculators by visiting the book's website:



<http://press.princeton.edu/titles/10952.html>

- *Various ways of presenting information.* This book is jam-packed with information. To help you absorb it all, I present information in a variety of ways, including graphically, in tables, with equations, and organized into lists. Anything in *italics* represents information I consider especially important, or is a word or phrase that is being defined.
- *Chapter summaries and tips.* I summarize math and nonmath results at the end of each chapter. I also discuss practical tips related to the content discussed.

- *Very short refresher on background math and notation.* Appendix A contains a quick review of several background (math) concepts you'll encounter throughout the book as well as a glossary of common math symbols.
- *Focus on conciseness.* I'd like you to read this book cover to cover. That's why I've tried my hardest to make it as succinct as possible.
- *Annotated bibliography.* Many of the references I cite—citations are in bracketed numbers (e.g., [4])—are free to read (they're marked as such in the bibliography). For some references I include brief comments on the study's limitations.
- *List of math topics covered.* Following this preface you'll find a list of math topics covered in this book along with the chapter (or appendix) they appear in. As you'll see there, I've organized the book so that the mathematics used gets gradually more advanced as you proceed. I did this to help ease you back into math. You'll also notice that *mainly precalculus-level mathematics is used in this book*. The only exception is in Chapter 6, where I use basic calculus concepts to help describe love dynamics (I provide a brief introduction to the mathematics behind those concepts in the chapter's appendix). This means you almost certainly studied nearly *all* the math discussed in this book at some point in your K–12 education.

Despite my best attempts, there may be times when the material (the math) gets challenging. But don't give up. At those times I suggest rereading the paragraph(s) a few times, pausing in between to think carefully about what I presented. I also recommend that you feel free to contact me. "What?! Write the *author*?" Yes. Here's my email: math@surroundedbymath.com. (If enough people take me up on this offer I may not get back to you as quickly as I'd like to.)

One last thing I'll ask of you: *run any major changes to your life inspired by this book by the relevant expert closest to you first* (e.g., your doctor). These people know the details of your particular situation (e.g., your medical history) and can help you assess how the findings presented here might affect you.

You're about to embark on a journey through mathematics you may not have thought about in a while. As you'll soon discover, math

is hidden in things you think about *every day*. Taking the time to understand the mathematics will yield huge rewards, including insights that may dramatically improve your quality of life. I hope this book inspires you to adopt a mathematical approach to life, and helps convince you that math is worth learning. Let's get to work.

Oscar E. Fernandez
Wellesley, MA

Math Topics Covered by Chapter



Here's a list of the math topics covered in each chapter (or appendix).

Math Topic	Chapter (or Appendix)
Linear functions	1, 3–4, 6
Piecewise linear functions	3
Multilinear functions	1–2
Quadratic functions	1, 4, 6
The quadratic formula	A1
Cubic polynomials	2
Polynomials	A1
Rational functions	1
Exponential functions	3
Comparison of exponential and linear growth	A3
Logarithmic functions	3
Standard deviation	4
Probability	5
Dynamical systems	6
Game theory (specifically, the bargaining problem)	6
3D graphing	A1
Proof of why $1/0$ is not defined	A2
Proof of Gale-Shapley algorithm stability	A5
Brief introduction to derivatives (calculus)	A6

PART I



A Healthier You Is Just a Few Equations Away

CHAPTER 1



How Many Calories Should You Eat Each Day?

A FEW YEARS AGO, I used to love going to a chain restaurant—let’s call it “Tuscan Fields”—and ordering my favorite dish: fettuccine alfredo. The creamy, cheesy sauce smothering warm strips of fettuccine noodles did it for me, a self-confessed carb lover. I loved the dish so much that at one point I was eating at Tuscan Fields twice a month and making the dish at home. One day I spotted the restaurant’s nutritional information pamphlet while waiting for a table. I knew there was nothing in there but bad news. I wanted to continue living my delusion. But I couldn’t; I had to know. On page 3 I got the news—the fettuccine alfredo had 1,100 calories and 41 grams of saturated fat!

I was shocked. Every doctor I’ve had has urged me to keep my daily saturated fat intake under 10 grams. They didn’t give me a daily calorie limit to go along with that recommendation, but I was pretty sure that 1,100 calories was too much for one meal. That got me thinking: How many calories should I be eating, anyway? What foods should I avoid, and which ones should I eat more of? There are probably thousands of books on diet, nutrition, and exercise out there that address these questions. But read through the research studies mentioned in their bibliographies and you’ll quickly notice something: *mathematics is at the heart of it all*. From calculating “best fits” to the data to determining the error bars, math helps health researchers draw conclusions from their data. If you can understand the math, you have a better chance of understanding the research findings (and their limitations).

That’s the premise of this part of the book. I want to show you how to translate the research on nutrition and exercise into mathematics and