LYMAN S. SPITZER

Dynamical Evolution of Globular Clusters

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To my colleagues

whose support and stimulation
have been invaluable

Contents

Preface

The dynamical evolution of an isolated spherical system composed of very many mass points has an appealing simplicity. The Newtonian laws of motion are exact, and all average quantities are functions only of radial distance r and time t. Nevertheless, it is only recently, with the availability of fast computers, that a systematic understanding of how such systems develop through time has emerged. Since these idealized systems should provide a very good approximation for globular clusters in this and other galaxies, the theory of their development is an important part of astronomy as well as an interesting branch of theoretical particle dynamics.

This book analyzes the various processes that occur as a globular cluster evolves dynamically. Written for the use of astronomers and physicists interested in this active research field, the book presents a deductive approach, with most of the essential results derived from first principles. The treatment is somewhat compressed, with some of the derivations sketched in outline rather than spelled out in detail, an approach which may have some educational value in developing a clearer understanding of the principles involved.

Chief emphasis is placed on those aspects which are believed to play a major role in the evolution of most clusters and which have been studied in sufficient detail to permit a coherent and somewhat realistic theory. Thus the book is focussed on the changes of stellar velocities resulting from random encounters between pairs of stars (chapter 2) and on the various evolutionary changes which result (chapters 3 and 4). A major complicating factor considered is the galactic gravitational field, which perturbs stellar orbits in the clusters moving about in this field and modifies their dynamical evolution (chapter 5). An important phase of most model clusters is the collapse of the central core; when the distances between stars become relatively short, binary stars are formed, and the role of these objects in terminating core collapse and affecting post-collapse evolution is analyzed (chapters 6 and 7).

In view of the emphasis on pure theory, no detailed comparison is given here between theoretical models and the observations. However, the introductory chapter 1 summarizes the observed properties of globular clusters, providing observational guidance for the subsequent theoretical developments, also summarized in this chapter.

The origin and early evolution of globular clusters is ignored because so little is known about this important early phase. Evolution after core collapse is discussed only briefly since our understanding of this phase is still very incomplete. Several topics are ignored because their relevance to real clusters is uncertain; these include the effect of a massive black hole at the cluster center, the dynamical instability of a cluster in which the distribution of velocities is anisotropic, and the effect of special and general relativity on clusters with random velocities comparable with the light velocity. Elegant and extensive research has been done on such topics, which are possibly relevant to galactic nuclei, but the observations do not indicate any direct connection of these subjects with globular clusters.

The references are restricted to publications which give further technical details on some of the specific subjects discussed in the book. Thus the bibliography is not in any sense complete and omits a number of papers of substantial importance in the history of this field. More complete lists of relevant papers may be found in the Proceedings of two IAU Symposia, Nos. 69 and 113, each dealing specifically with the dynamical evolution of clusters. Reference numbers here are given in square brackets, with references listed at the end of each chapter. The subject index and the directory of symbols may be helpful in the use of this volume.

This book was started in late 1984, and the early chapters include very little that was published after that time. For chapters 6 and 7 the effective cut-off date was the late spring of 1986. Ignoring important preprints that reached me during the summer of 1986 was a painful necessity.

It is a great pleasure to acknowledge the information and helpful suggestions I have received from many astronomical colleagues, including J. Binney, J. Goodman, M. Hénon, P. Hut, S. Inagaki, I. King, D. Lynden-Bell, S. McMillan and S. Shapiro. H. Cohn kindly provided me with much of the unpublished analysis summarized in §2.3. Very detailed comments on the manuscript were made by D. Heggie, A. Lightman, J. Ostriker, T. Statler and S. Tremaine. I am especially indebted to Venita Nixon, my secretary during 30 years, who has converted my jumbled drafts of several books into elegant manuscripts. Much of this book was written during a four-month stay, from December 1984 to March 1985, at the Institut d'Astrophysique, Paris, France. As in the past, I greatly appreciate the warm hospitality of the Institut and of its Director, Professor Jean Audouze.

Princeton University Observatory
August 1986

Dynamical Evolution of Globular Clusters

1

Overview

The structure of a globular cluster—the spatial distribution and random velocities of its stars—changes slowly with time as a result of simple dynamical principles. In much of this dynamical evolution, the changes in the cluster are a consequence of Newton's laws of motion and of gravitation as applied to an assembly of mass points. The theory of such changes is the general topic of this book.

The following sections in this first chapter present the observational and theoretical background for the later chapters. Relevant observations on globular clusters, especially data on the structure, age and mass of these systems, are summarized in §1.1. Little is said about most of the physical properties of the individual stars, including their composition and internal structures; these properties affect dynamical evolution of clusters only under unusual circumstances, not yet studied in detail. The subsequent sections give the theoretical point of view adopted in the rest of the book. Thus §1.2 describes the basic zero-order approximation in which the cluster is spherically symmetric, the gravitational potential, ϕ, varies smoothly with distance, r, from the cluster center, and many types of steady solutions, with no evolution, are possible. As detailed in the subsequent section §1.3, such zero-order solutions are subject to a variety of perturbations, mostly resulting from time dependent gravitational fields, including small-scale fields within the cluster and larger-scale fields produced by external masses, especially the Galaxy. The specific effects which these various perturbations can gradually produce on the structure of the cluster are also summarized in this section.

1.1 OBSERVATIONS

In common with other galaxies, our own system includes [1] some 150–200 globular clusters, each containing typically some 10^5 stars. More than half of the observed clusters are within 10 kpc of the galactic center, but the distribution extends to much greater distances, with several beyond 50 kpc. Their z distances (measured perpendicular to the galactic plane) do not differ much on the average from those in x and y.

A photograph of one such system, NGC 2808, is shown in Fig. 1.1. The image appears nearly circular, which is typical for globular clusters accompanying our Galaxy. In more than half these systems the observed

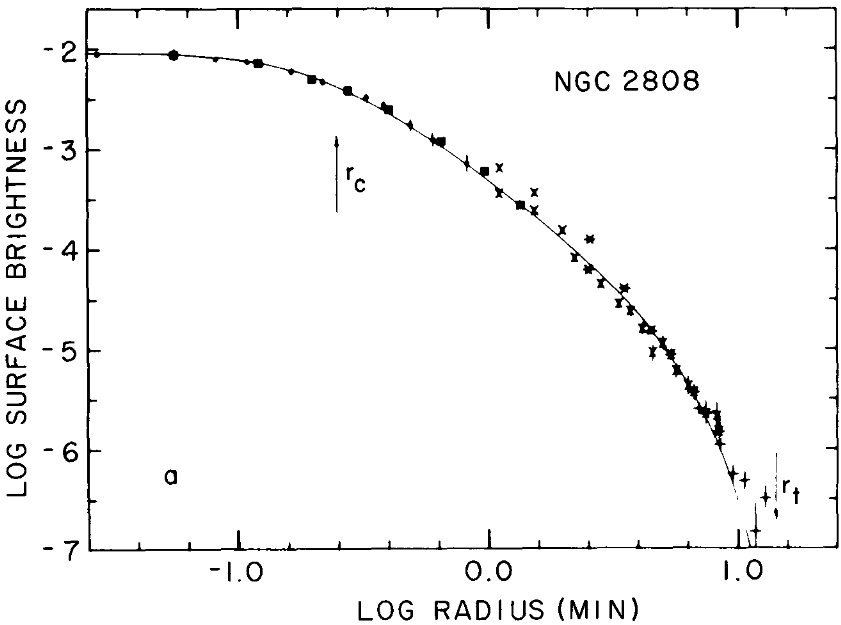
Fig. 1.1. Photograph of NGC 2808. This picture was obtained with the Anglo-Australian Telescope by D. Malin, using an "unsharp masking" technique to bring out both the core and the halo.

[2] ratio of minor axis $2b$ to major axis $2a$ exceeds 0.9; this ratio is generally greater than 0.8. To a first approximation most globular clusters are nearly spherical.

The ages of these systems are determined from their HR diagrams, which show virtually no main-sequence stars with spectra earlier than the "turn-off" point, at which the stars leave the main sequence and evolve for the first time up the giant branch. As time goes on, the position of the turn-off, at which the hydrogen in the stellar core has been converted to helium, moves to stars of lower mass and later spectral type. Thus in principle the position of this point on the observed HR diagram of a cluster can be used to determine the time since the stars, and presumably the entire cluster, were formed. In practice the age is determined by fitting the points in each HR diagram with one of a series of curves, computed with detailed stellar models and with different assumed ages. Some determinations have shown a difference of age between clusters with different abundances of heavy elements with respect to hydrogen (i.e., different metallicities). However, more recent work [3] gives ages between 15 and 18 times 10^9 years for systems with widely different metallicities. Evidently these clusters, in common with other objects of Baade's Population Type II, are generally very old and were likely born in the early phases of galaxy formation. Individual clusters share the high random space velocities generally found for old (Population II) objects.

As indicated in Fig. 1.1, a globular cluster generally shows a large range in surface brightness with increasing radius r. A plot of the observed brightness distribution [4] for this same cluster NGC 2808 is given in Fig. 1.2a, showing a surface brightness, $\sigma(r)$, decreasing by more than four orders of magnitude from the central region, or core, to the outer envelope, or halo. Two quantities used to parametrize such a distribution are the core radius, r_c, defined as the value of r at which the surface brightness is half its central value, and the tidal radius r_t, at which $\sigma(r)$ reaches zero. Determination of r_t requires an extrapolation of the observed points, and depends on the model used for fitting the data. For five-sixths of the clusters measured [5], r_c lies between 0.3 and 10 pc, with r_t/r_c between 10 and 100.

While most systems show a central region or core within which $\sigma(r)$ changes slowly, as indicated in Fig. 1.1, in some systems $\sigma(r)$ increases down to the smallest radius resolved by the seeing. Such systems are said to possess central "cusps," as shown in Fig. 1.2b by the plot [6] of $\sigma(r)$ for the cluster NGC 6624. This cluster is one of the few which contain a strong X-ray burster source, presumably formed in the compact core of the cluster. More detailed knowledge of such cusps would be very relevant to the topic of core collapse, discussed in chapter 4.



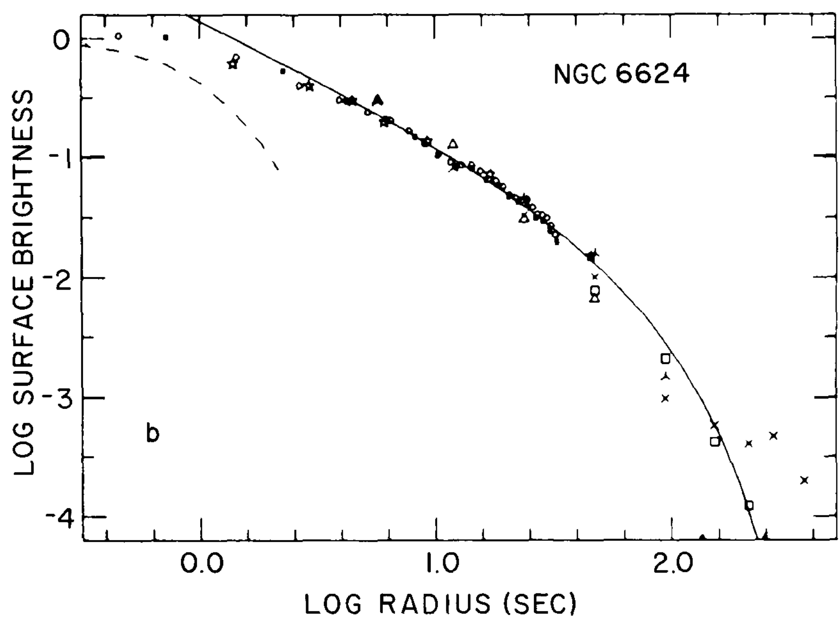


Fig. 1.2. Surface Densities in Globular Clusters. The surface brightness observed in two clusters is plotted against radial distance. Plot a [4] is for NGC 2808; the solid curve shows a theoretical King model with $r_t/r_c = 56$—see §1.2b. Plot b [6] is for NGC 6624, which contains a bright X-ray source within the central core; the solid curve shows a singular King model ($r_c = 0$), while the instrumental profile is indicated by the dashed line. In each curve, different symbols denote results by different observers, with photometry used in the central regions and star counts further out.

Different clusters differ among themselves not only in the distribution of surface density but in other ways as well. Values of the surface brightness at the cluster center [5], denoted by $\sigma(0)$, lie mostly within a range of about 300 to 1. The total visual luminosity L_V varies [1] over a somewhat smaller range, with a dispersion of 0.5 about a mean value of 4.8 for $\log (L_V/L_\odot)$.

To determine accurately the total mass of the cluster requires rather detailed knowledge of the velocity dispersion, which is not easily measured. For M3, the velocity data obtained on about a hundred stars [7] indicate an rms radial velocity of about 5 km/s in the inner regions, decreasing to 2.9 km/s at r equal to 20 pc, some 16 times r_c. These M3 results may be combined with the observed light and stellar luminosity distributions and the best theoretical model, based on equation (1-35) below, to yield a mass of 6.0×10^5 $M_\odot$ and a mass-to-light ratio, M/L_V in solar units, of 2.1. A more approximate method makes use of the central velocity dispersion, obtained from line widths in the integrated spectrum of the cluster center, interpreted with use of the virial theorem; for 10 clusters the values of M/L_V found [8] in this way ranged from about 1 to 4, again in solar units, with masses between 1.7×10^5 and 1.5×10^6 times $M_\odot$. If we assume that the same values of M/L_V apply to the more numerous clusters of lower L_V, for which no velocity data are available, we find that the cluster masses are between 10^4 and 10^6 $M_\odot$ with a peak at about 10^5 $M_\odot$.

One important result of the M3 data is a definite indication of an anisotropic velocity distribution in the cluster halo, with velocities towards or away from the cluster center significantly exceeding the transverse velocities. Indication of a similar conclusion for M3 has been obtained [9] more directly from proper motion data.

With this determination of M/L_V from the velocity observations, together with the measured profiles of $\sigma(r)$, it is possible to compute the particle density of stars, their rms velocity, and finally the value at each point of t_r, the time of relaxation. A precise definition of t_r is postponed until chapter 2, but this quantity may be regarded as the time required for the velocity distribution to approach its Maxwell-Boltzmann form as a result of mutual deflections of stars by each other, accompanied by exchanges of kinetic energy. The time required for dynamical evolution generally exceeds t_r, sometimes by several orders of magnitude. The computed values [5] of $t_r(0) \equiv t_{rc}$, the relaxation time at the cluster center, range mostly from 10^7 up to 10^{10} years.

As we shall see in later chapters, the time interval required for evolution of a cluster as a whole is usually comparable with the value of t_{rh}, the half-mass relaxation time. This reference time is defined—in equation (2-63)—as the value of t_r for average conditions within the radius r_h containing half of the cluster mass. The value of t_{rh} significantly exceeds the central relaxation time. The distribution [10] of estimated t_{rh} values for 32 observed clusters in our Galaxy is shown in Fig. 1.3. Since these values are mostly much less than the cluster ages, there are only a few of these clusters for which relatively little evolution has occurred; many may be highly evolved systems.

One can also compute for each cluster the core dynamical time, t_{dc}, defined as $r_c/v_m(0)$, where $v_m(0)$ is the central rms scalar velocity, equal to $3^{1/2}$ times the dispersion of radial velocities in the core. A typical value of t_{dc} is about 10^5 years, usually less than t_{rc} by a factor 10^{-2} or smaller. In the outer regions of the cluster, for $r \gtrsim r_h$, t_d/t_r is appreciably smaller still.

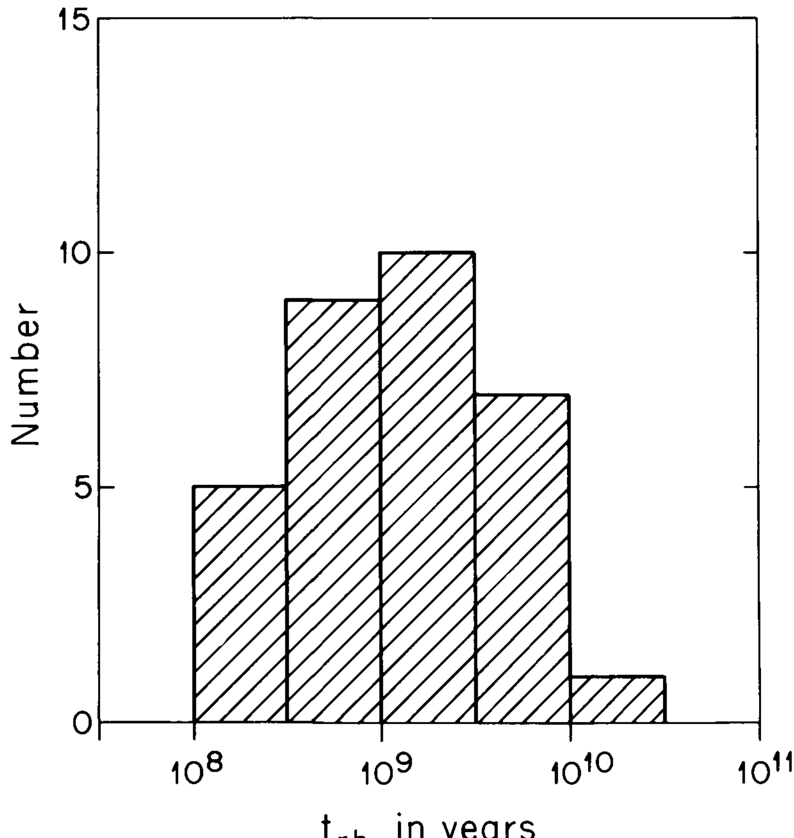


Fig. 1.3. Distribution of Half-Mass Relaxation Times. The histogram shows [10] for a sample of 32 clusters the distribution of values of t_{rh}, the dynamical relaxation time for mean conditions within the inner half of the cluster's mass.

Information on the presence of binary stars in globular clusters is provided [11] by the observed X-ray sources in these systems. Such sources have a bimodal distribution of luminosity, L_x, with bright ones having L_x between 3×10^{35} and 3×10^{37} erg/s, while for a fainter, more numerous group, L_x is less than 10^{34} erg/s; no sources are observed within this gap. The brighter sources are all located within about 1 core radius, r_c, from the cluster centers. The theory of mass stratification, discussed in chapter 3, indicates that the average mass of these systems exceeds the average mass of giant stars in the cluster core (probably about 0.7 $M_\odot$) by a factor between 1.5 and 4.9. If these sources are binaries, the compact object in each of the stronger sources is presumably a neutron star, with a main-sequence dwarf as a companion. The weaker sources are more widely distributed in the clusters where they are observed, consistent with a degenerate dwarf (with a mass somewhat less than that of the giants) as a compact emitting object, and again a dwarf companion which provides the gas for accretion by the X-ray emitting star. The statistics of these two types of objects are consistent with an abundance ratio of roughly 1 to 100 for neutron stars to white dwarfs, though more detailed models would be required for definite results.

Other data do not show many binaries in globular clusters [12], though problems of observational selection complicate the interpretation. For example, of 33 giants observed several times in M3, only one was observed to be a spectroscopic binary [13], although for giant stars in the galactic disc about a third show variations of radial velocity amounting to a few kilometers per second over a few years [7], readily detectable in the M3 data. There appears to be a similar scarcity of spectroscopic binaries among high-velocity (Population II) stars in the galactic disc [14]; in marked contrast, for the more widely separated visual binaries the ratio of single to double stars is independent of stellar velocity. According to the evolutionary theories presented in chapter 7, the X-ray binary stars have been formed in late stages of cluster evolution. The significant correlation between the scarce central cusps and the scarce strong X-ray sources provides strong support for this point of view. The truly primordial binaries present initially in globular clusters may have constituted a relatively small fraction of the total stellar population, though this conclusion is not yet firmly established.

One final observational area where much work has been done on globular clusters is the chemical composition, as determined mostly from spectroscopic studies. The increased average ratio of heavy elements (for example, of Fe) relative to hydrogen with decreasing distance from the galactic center seems firmly established [1]. This result is of great importance for theories of the birth and early evolution of globular clusters in this and other galaxies. However, such theories are still in an early state and are not considered here. More relevant to the evolution of present clusters is the study of chemical inhomogeneities within a single such system [15]. The one cluster in which a marked increase of metallicity all the way into the cluster center seems well established is ω Cen (NGC 5139), for which even in the center the relaxation time has the relatively high value of 0.4×10^{10} years, and scarcely any dynamical evolution may be expected. Hence these results also are more relevant to cluster formation than to evolution. Such effects may exist in some other clusters, especially those with relaxation times longer than average [16], but the evidence is controversial.

1.2 STATIONARY EQUILIBRIA

To facilitate a quantitative discussion of cluster dynamics we introduce idealized models in which the effect of encounters is ignored and steady-state solutions are possible. These models provide a "zero-order" solution for the dynamical equations. Stellar encounters and other effects ignored can then be regarded as perturbations which gradually produce cluster evolution along a sequence of these zero-order models. We first consider the basic assumptions made in the zero-order approximation, together with the basic principles governing their stationary equilibria. Next we review some of the particular zero-order models that have been considered in this connection.

In all these discussions relativistic effects are ignored. The theory of self-gravitating systems with random velocities comparable to the velocity of light may be relevant to galactic nuclei [17], but, as far as we know, not to the globular clusters.

a. Assumptions and principles

The basic simplifying assumption made in the zero-order approximation is:

(A) The granularity of the self-gravitating matter in the cluster can be ignored, and the gravitational potential, taken to be a slowly varying function of position. It is this assumption that makes possible the existence of stationary equilibrium states.

Once this assumption is made, it is useful to define a velocity distribution function $f(\mathbf{r},\mathbf{v},t)$, dependent on the vector position, $\mathbf{r}$, the velocity $\mathbf{v}$, and the time t. This quantity is defined so that $f(\mathbf{r},\mathbf{v},t)\,\mathbf{dr}\,\mathbf{dv}$ is the number of stars at time t within the volume element $\mathbf{dr} \equiv dx\,dy\,dz$ centered at $\mathbf{r}$ and within the velocity space element $\mathbf{dv} \equiv dv_x\,dv_y\,dv_z$ centered at $\mathbf{v}$. This quantity is meaningful if one can construct volume elements which are large enough to