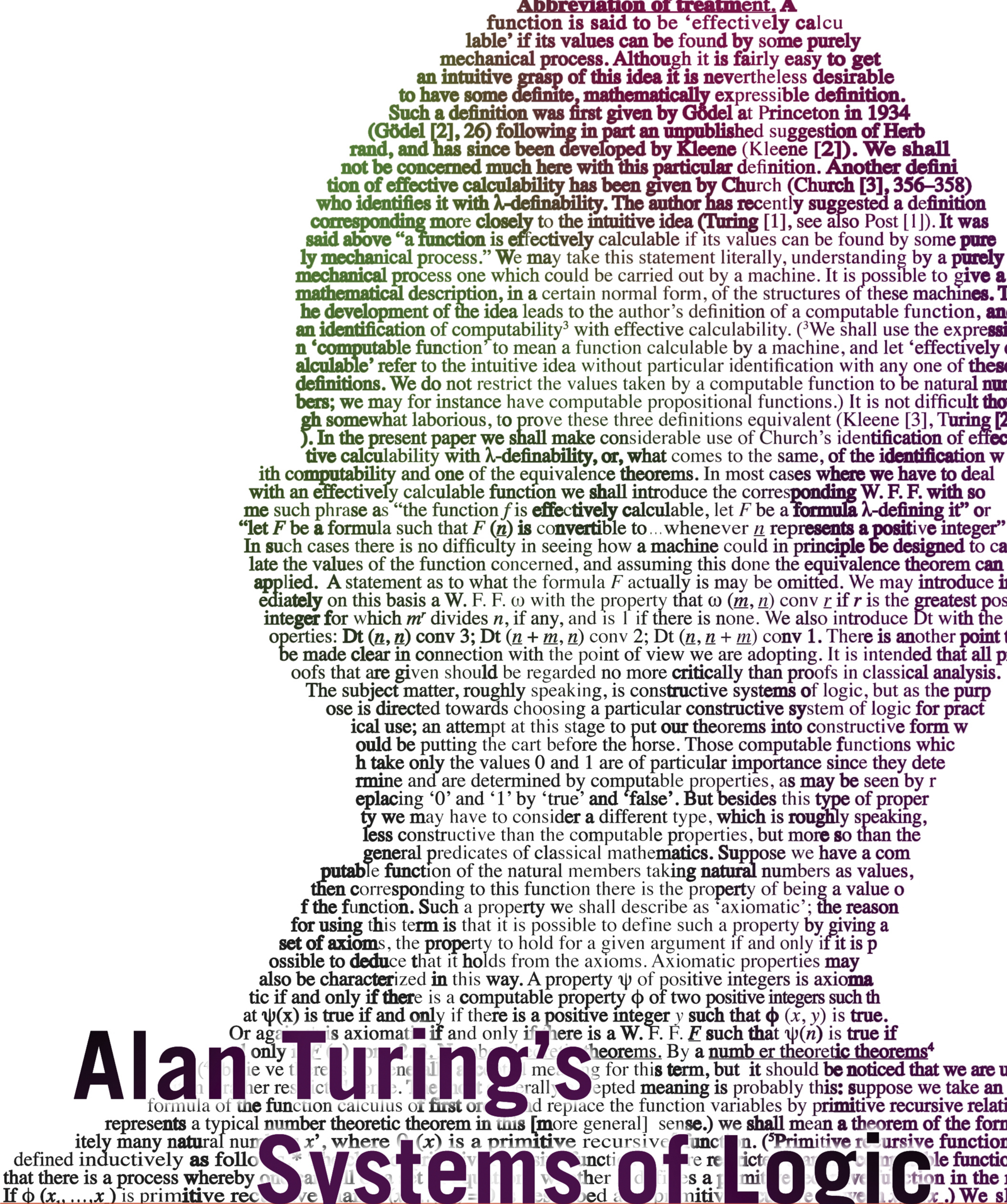

Alan Turing's Systems of Logic

THE PRINCETON THESIS

Edited and introduced by **Andrew W. Appel**

Alan Turing's Systems of Logic

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September 10, 1936

TURING,
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FOR COMING YEAR. LETTER FOLLOWS.

LUTHER EISENHART, DEAN

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Send as cable night letter

24 July

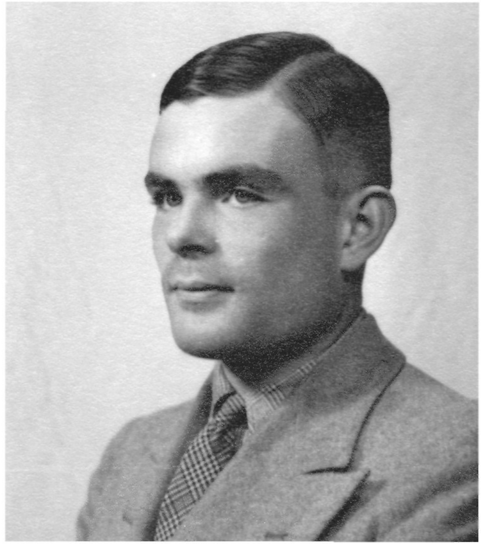
King's College,
Cambridge.

Dear Mr Eisenhart,

The American Consulate in London tells me that I shall again need a letter in duplicate giving evidence that I am accepted as a student at Princeton, before I can be readmitted to the U.S. I should be grateful if you would send me such letters: it would be convenient if you would mention at the same time that I have been awarded a Fellowship which will cover all expenses; this would save my obtaining any other evidence about finances,

Yours sincerely

A. M. Turing



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Preface

Alan M. Turing, after his great result in 1936 discovering a universal model of computation and proving his incompleteness theorem, came to Princeton in 1936–38 and earned a PhD in mathematics. Before 1936 there were no universal computers. By 1955 there was not only a theory of computation, but there were real universal ("von Neumann") computers in Philadelphia, Cambridge (Massachusetts), Princeton, Cambridge (England), and Manchester. The new field of computer science had a remarkably short gestation.

The great engineers who built the first computers are well known: Konrad Zuse (Z3, Berlin, 1941); Tommy Flowers (Colossus, Bletchley Park, 1943); Howard Aiken (Mark I, Harvard, 1944); Prosper Eckert and John Mauchley (ENIAC, University of Pennsylvania, 1946).

But computer science is not just the construction of hardware. Who were the creators of the intellectual revolution underlying the theory of computers and computation?

Turing is very well known as a founder and pioneer of this discipline. In 1936 at the age of twenty-four he discovered the universal model of computation now known as the Turing machine; in 1938 he developed the notion of "oracle relativization"; in 1939–45 he was a principal figure in breaking the German Enigma ciphers using computational devices (though not "Turing machines"); in 1948 he invented the LU-decomposition method in numerical computation; in 1950 he foresaw the field of artificial intelligence and made remarkably accurate predictions about the future of computing and computers. And, of course, he famously committed suicide in 1954 after prosecution and persecution for practicing homosexuality in England.

But as significant as Turing is for the foundation of computer science, he was not the only scholar whose work in the 1930s led to the birth of this field.

In Fine Hall,[1] home in the 1930s of the Princeton Mathematics Department and the newly established Institute for Advanced Study, were mathematicians whose students would form a significant part of the new fields of computer science and operations research.

This volume presents the manuscript of Alan Turing's PhD thesis. It is accompanied by two introductory essays that explore both the work and the context of Turing's stay in Princeton. My essay elucidates the significance of Turing's work (and that of his adviser, Alonzo Church) for the field of computer science; Solomon Feferman's essay describes its significance for mathematics. Feferman also explains how to relate some of Turing's 1938 terminology to more current usage in the field. But on the whole, the notation and terminology in this field have been fairly stable: "Systems of Logic Based on Ordinals" is still readable as a mathematical and philosophical work.

Andrew W. Appel
Princeton, New Jersey

1 Fine Hall was built in 1930, named for the mathematician Henry Burchard Fine. During the 1930s it housed the Mathematics Department of Princeton University and the mathematicians (e.g., Gödel and von Neumann) and physicists (e.g., Einstein) of the Institute for Advanced Study. In 1939, the Institute moved to its own campus about a mile away from Princeton University's central campus. In 1969, the University's Mathematics Department moved to the new Fine Hall on the other side of Washington Road. The old building was renamed Jones Hall, in honor of its original donors, and now houses the departments of East Asian Studies and Near Eastern Studies.

OSWALD VEBLEN, chairman of the Princeton University Mathematics Department and first professor at the Institute for Advanced Study. His students include Alonzo Church (PhD 1927), and his PhD descendants through Philip Franklin (Princeton PhD 1921) via Alan Perlis (Turing Award 1966) include David Parnas, Zohar Manna, Kai Li, Jeannette Wing, and 500 other computer scientists. Veblen has more than 8000 PhD descendants overall. He helped oversee the development of the pioneering ENIAC digital computer in the 1940s.

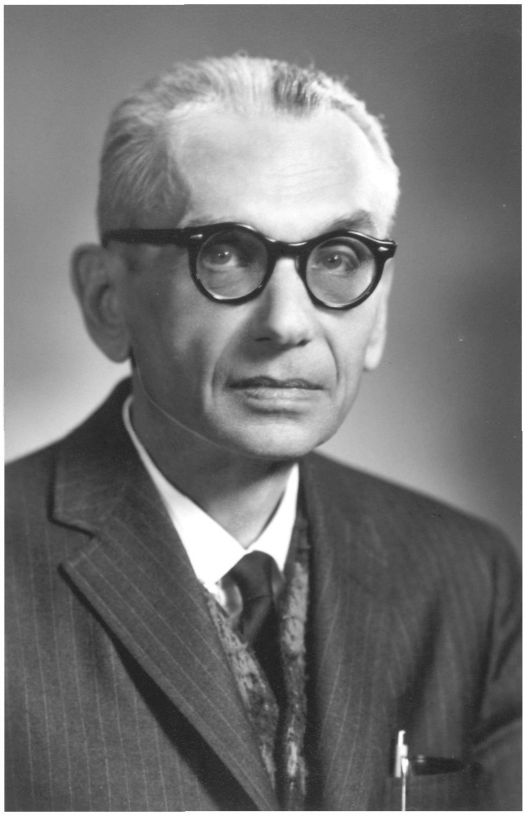
(Photographer unknown, from the Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA.)

ALONZO CHURCH, professor of mathematics, whose students include Alan Turing, Leon Henkin, Stephen Kleene, Martin Davis, Michael Rabin (Turing Award 1976), Dana Scott (Turing Award 1976), and Barkley Rosser, and whose PhD descendants include 3000 other mathematicians and computer scientists, among them Robert Constable, Edmund Clarke (Turing Award 2007), Allen Emerson (Turing Award 2007), and Les Valiant (Turing Award 2010).

(Photo from the Alonzo Church Papers. Department of Rare Books and Special Collection. Princeton University Library.)

SOLOMON LEFSHETZ, professor of mathematics, whose students include John McCarthy (Turing Award 1971), John Tukey, Ralph Gomory, and Richard Bellman (inventor of dynamic programming), and whose 6181 PhD descendants include John Nash (Nobel Prize 1994), Marvin Minsky (Turing Award 1969), Manuel Blum (Turing Award 1995), Barbara Liskov (Turing Award 2008), Gerald Sussman, Shafi Goldwasser, Umesh and Vijay Vazirani, Persi Diaconis, and Peter Buneman.

(Photo courtesy of the Princeton University Archives. Department of Rare Books and Special Collection. Princeton University Library.)



KURT GÖDEL, visitor to the Institute in 1933, 1934, and 1935, and professor at the Institute from 1940, had no students but had an enormous influence on the fields of mathematics and computer science. His 1931 incompleteness result—that it will never be possible to enumerate in logic the true statements of mathematics—stunned mathematicians and philosophers with its unexpectedness. His methods—the numerical encoding of syntax and the numerical processing of logic—set the stage for many techniques of computer science. Major results of Church, Kleene, Turing, and von Neumann clearly and explicitly owe much to Gödel.

(Photo from the Kurt Gödel Papers, the Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA, on deposit at Princeton University.)

JOHN VON NEUMANN, at Princeton University from 1930 and professor at the Institute for Advanced Study from 1933, had only a few students (including the pioneer in parallel computer architecture Donald Gillies), but also had an enormous influence on the development of physics, mathematics, logic, economics, and computer science. In 1931 he was the first to recognize the significance of Gödel's work, and toward 1950 he brought Turing's ideas of program-as-data to the engineering of the first stored-program computers. Stored-program computers are called "von Neumann machines," and essentially all computers today are von Neumann machines.

(Photographer unknown, from the Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA.)