HOW TO READ

Historical Mathematics

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PRINCETON UNIVERSITY PRESS
Princeton and Oxford

Published by Princeton University Press,
41 William Street, Princeton, New Jersey 08540

In the United Kingdom:
Princeton University Press,
6 Oxford Street,
Woodstock, Oxfordshire OX20 1TW

Library of Congress Cataloging-in-Publication Data

Wardhaugh, Benjamin, 1979–
How to read historical mathematics / Benjamin Wardhaugh.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-691-14014-8 (hardcover : alk. paper)
1. Mathematics—History. 2. Guided reading. I. Title.
QA21.W324 2010
510.9—dc22 2009041704

British Library Cataloging-in-Publication Data is available

This book has been composed in Adobe Garamond Pro
with IM Fell Double Pica Pro display

Printed on acid-free paper. ∞

press.princeton.edu

Printed in the United States of America

7 9 10 8 6 4 2

CONTENTS

Preface *vii*

CHAPTER 1
What Does It Say?
1

CHAPTER 2
How Was It Written?
21

CHAPTER 3
Paper and Ink
49

CHAPTER 4
Readers
73

CHAPTER 5
What to Read, and Why
92

Bibliography *111*

Index *115*

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PREFACE

Read Euler, read Euler, he is the master of us all.

—Pierre-Simon Laplace, quoted by Gugliemo Libri
in the *Journal des Savants*, January 1846, p. 51

If you've ever read a historical mathematical text and then thought, *what on earth was that?* this book is for you.

Let me explain.

Reading historical mathematics is fascinating, challenging, enriching, and endlessly rewarding. A huge wealth of mathematics and mathematical experience are contained in the past; the enthralling task of finding out about them can transform your study and enjoyment of modern mathematics, and it can turn into the study of a lifetime in its own right.

Nations and societies commemorate people and events from their past, and by doing so they create and strengthen their own sense of identity. Mathematicians do the same thing when they commemorate the past in the names they use: the Isaac Newton Institute, Lebesgue integrals, Noether's theorem. . . . We all know who Isaac Newton was, but most of us are a bit more shaky about Henri Lebesgue or Emmy Noether. That's where the history of mathematics comes in. Who were the people who made our mathematics the way it is? What did they do? Why do we remember them? (There are more awkward questions, too: What about the people we don't remember? Why do some mathematicians and their writings become part of the mathematical "hall of fame" and others not? Why, in the end, is our mathematics the way it is?)

Answering those questions is fun, and it need not be hard. More and more people are now experiencing the fascination of the history of mathematics, through television programs, popular books, and public lectures. Courses on the history of mathematics are often offered as part of a degree in mathematics.

With those courses come sourcebooks, like the ever-popular *Reader* edited by John Fauvel and Jeremy Gray and the recent *Mathematics Emerging* edited by Jacqueline Stedall. On the web there are vast treasure troves of resources for the history of mathematics. Today you can read not just Euler but a whole range of mathematical "masters" if you want to.

Reading the mathematical "classics" is a way to enrich ourselves, to engage with our predecessors and learn from them not just what they did but also how they did it, and why. Just as reading the classics of literature can help us to see our own world with new eyes, and reading the classics of science writing can both inspire modern science and help us to remember why we are doing it, so reading historical mathematics can help us to see modern mathematics afresh, to reconnect with the distinctively mathematical way of thinking that runs through human history and to recall the reasons for doing mathematics in the first place. By reading their works, we are privileged to have access to the great minds of the past; and we learn something about who we are, too.

At a more mundane level, reading mathematics from the past can help us to learn mathematics itself: it often provides a different perspective on familiar ideas, showing what really motivated their development. And it can make mathematics more engaging by bringing to life some of the human stories which make up what is fundamentally a very human enterprise.

So, as I said above, reading historical mathematics is fascinating, enriching, and rewarding. Learning to savor the mathematical language of the past, to understand it more deeply and to enter into its authors' ways of thought, and to be inspired by it in our own intellectual endeavors are valuable and worthwhile activities.

Reading historical mathematics can also be hard. The sourcebooks I mentioned above give a lot of guidance to their readers, but it can still be daunting to approach mathematical writings from the past for the first time—particularly if you are taking a course which asks you to engage with them, think about them, or write about them.

Many things might be unfamiliar: the language, the notation, the fact that you are reading a translation rather than the author's actual words. The sources for historical mathematics were written in times and places that are now mostly unfamiliar to us, by people whose ideas and values were very different from our own and whose mathematical culture, methods, and assumptions may have been very different from anything we are familiar with. When you think about a piece of historical writing more deeply, there are still more questions: What was the actual book like that this text appeared in? Who read it? Who was meant to read it? Why are we, now, studying it? Who says that it's important, and what does that mean?

It can be hard even to know what questions to ask, let alone to find any answers. What can we really know about a piece of mathematical writing from the past, given all the distance there is between us and it, between us and its author, between us and its readers, and between us and its time and context?

Well, plenty—and this book will show you how to do it. It will show you how to delve deeper into the historical texts that are often read in the history of mathematics—what things to look for and what questions to ask. And it will show you how to discover from historical texts—and about them—the things you want to know about the history of mathematics. How to read, in other words.

Chapter 1 will show you how to think about *what a historical piece of mathematics really says*, and how to find out about the processes of translation and interpretation it may have gone through before it reached us.

Chapter 2 will introduce you to the *author* of a text, and show you how to find and choose sources of information about the time and place in which it was written.

In *Chapter 3* you will learn about the *physical objects*—books, but not only books—in which historical mathematical texts are found, and what you can learn from them.

Chapter 4 will introduce you to the *people who read* historical mathematics when it was new, and will help you to ask and answer questions about who was meant to read a mathematical work, who really read it, and why it matters.

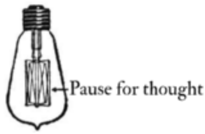
Finally *Chapter 5* will show you how to approach the elusive idea of *"significant" historical mathematics*, and help you to think about why we choose to read some historical writings and not others.

This book will, I hope, give students the skills and the confidence to tackle a course on the history of mathematics in which reading historical writings is required. It will also give general readers—or students taking a course that relies on a modern textbook rather than the historical writings themselves—the nerve to pick up a historical mathematics book or an anthology and have a go at making sense of it.

The history of mathematics is, as much as anything, a set of skills—ways of reading a particular kind of text. Historians of mathematics deal with a unique kind of writings, and they use a unique combination of skills to make sense of them. This book can begin to make you into a historian of mathematics.

The book uses specific examples to guide you through the experience of looking at a historical mathematical text, from thinking *what on earth was that?* to asking and answering detailed questions about what it is and what it means. The examples come from the mathematics of Europe between about 1550 and 1900—the historical mathematics that I happen to be most familiar with—though I hope the historical lessons will carry over to other cultures and periods. I've tried to provide some examples that are well-known classics and some that are a bit more unusual. The actual mathematics contains some undergraduate-level material (a little analysis and group theory, for instance), but much of the discussion doesn't require you to follow the mathematics in detail.

From time to time I'll invite you to stop reading and think through a particular point yourself before going on. These places are marked with the icon:



Summaries set out in boxes provide handy reminders of the key techniques you have learned in each section and chapter, and in some cases give you quick help with particular situations.

Finally, at the end of each chapter there is a set of points for you to think about, taking the material further in various ways. These can be approached in many different ways, and there's no "right" answer to any of them: in a study setting they might be used for class dicussion or essay writing, but they could just as well be springboards to further thought and reading for the fun of it.

Acknowledgments

This book has grown out of my own experience in teaching the history of mathematics at the University of Oxford, and I am grateful to all of my students and colleagues there who have taught me—knowingly or otherwise—how to read historical mathematics. I would also like to thank the editor and director of *Contemporary Review* for his kind permission to reproduce a section from Oliver Lodge's article, "Einstein's Real Achievement," which appeared in the *Fortnightly Review* on 1 September 1921. This excerpt is reprinted in question 7 at the end of chapter 4 of this volume.

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