

H.C. van de Hulst

# MULTIPLE LIGHT SCATTERING

Tables, Formulas,  
and Applications

VOLUME 1

# **MULTIPLE LIGHT SCATTERING**

## **Tables, Formulas, and Applications**

Volume 1

This page intentionally left blank

# MULTIPLE LIGHT SCATTERING

## Tables, Formulas, and Applications

Volume 1

*H. C. VAN DE HULST*

*Astronomical Observatory  
University of Leiden  
Leiden, The Netherlands*



1980

ACADEMIC PRESS

A Subsidiary of Harcourt Brace Jovanovich, Publishers

New York London Toronto Sydney San Francisco

COPYRIGHT © 1980, BY ACADEMIC PRESS, INC.  
ALL RIGHTS RESERVED.  
NO PART OF THIS PUBLICATION MAY BE REPRODUCED OR  
TRANSMITTED IN ANY FORM OR BY ANY MEANS, ELECTRONIC  
OR MECHANICAL, INCLUDING PHOTOCOPY, RECORDING, OR ANY  
INFORMATION STORAGE AND RETRIEVAL SYSTEM, WITHOUT  
PERMISSION IN WRITING FROM THE PUBLISHER.

ACADEMIC PRESS, INC.  
111 Fifth Avenue, New York, New York 10003

*United Kingdom Edition published by*  
ACADEMIC PRESS, INC. (LONDON) LTD.  
24/28 Oval Road, London NW1 7DX

**Library of Congress Cataloging in Publication Data**

Hulst, Hendrik Christoffel van de.  
Multiple light scattering

Includes bibliographies and index.

1. Light--Scattering--Handbooks, manuals, etc.
2. Radiative transfer--Handbooks, manuals, etc.

I. Title.

QC427.6.H84      535'.4      79-51687

ISBN 0-12-710701-0 (v. 1)

PRINTED IN THE UNITED STATES OF AMERICA

80 81 82 83      9 8 7 6 5 4 3 2 1

# □ Contents

PREFACE	ix
CONTENTS OF VOLUME 2	xi

## Part I GENERAL THEORY

### 1 Concepts, Terms, Notation

1.1 Directions for Use	3
1.2 Some Hard Choices	3
References	6

### 2 Exponential Integrals and Related Functions

2.1 Quick Survey	8
2.2 Exponential Integrals ( $E$ Functions)	9
2.3 Generalized Exponential Integral	10
2.4 $F$ Functions	11
2.5 $G$ Functions	13
References	15

### 3 Reciprocity

3.1 Reciprocity and Detailed Balance	16
3.2 Far-Field Scattering by a Single Particle	18
3.3 Arbitrary Configurations	24
3.4 Plane Surfaces and Plane-Parallel Slabs	30
References	33

**4 Methods**

4.1	Posing the Problem	34
4.2	Criteria for a Choice	39
4.3	Method of Successive Orders	46
4.4	Ambartsumian's Method	50
4.5	The Adding or Doubling Method	54
	References	65

**5 Very Thick Layers with Arbitrary Anisotropic Scattering**

5.1	Method and Terminology	67
5.2	Basic Concepts and Relations	70
5.3	Very Thick Layers	76
5.4	Transition to Conservative Scattering	78
5.5	Internal Source Layer	83
5.6	Asymptotic Fitting	85
	References	87

**6 Results Obtained by Expanding the Phase Function in Legendre Polynomials**

6.1	Introduction and Conclusions	89
6.2	Unbounded Medium	92
6.3	The Ambartsumian Functions	99
6.4	Reduction to $H$ Functions	104
6.5	The Radiation Field at Arbitrary Depth	117
	References	122

**Part II ISOTROPIC SCATTERING****7 Isotropic Scattering; Solutions by Use of the Milne Operator**

7.1	Matrices in $\tau$ and $\mu$	127
7.2	Solving the Milne Equation	131
7.3	Resulting Quantities	133
7.4	Eigenvalues of the Milne Operator	140
7.5	The Adding or Doubling Method Derived from the Milne Equation	146
	References	148

**8 Isotropic Scattering, Semi-Infinite Atmospheres**

8.1	Specifications	149
8.2	The Unbounded Medium	149
8.3	The $H$ Functions and Their Moments	161
8.4	Moments and Bimoments of the Reflection Function	172
8.5	Point-Direction Gain in a Semi-Infinite Atmosphere	174
8.6	Radiation Emerging from a Semi-Infinite Atmosphere	183
	References	191

**9 Isotropic Scattering, Finite Slabs**

9.1	Reflection and Transmission	192
9.2	Fate of Incident Energy	203
9.3	Point-Direction Gain and Its Moments	206
9.4	Integrals of Gain over Optical Depth: Homogeneously Embedded Sources	218
9.5	The Intensity inside the Atmosphere	221
9.6	Some Special Functions	224
	References	235



This page intentionally left blank

## □ Preface

The play of radiation by repeated scattering in a cloud layer or any other slab of particles poses a problem that is common to atmospheric physics, astronomy, ocean optics, and branches of industrial research. Methods for solving this problem in diverse situations have been known for decades but their complexity has given the subject the reputation of being accessible only to specialists.

This book is aimed at the nonspecialist, e.g., an expert in an applied field, who needs a result from multiple scattering theory but does not wish to spend excessive time in solving it himself or searching the very extensive literature.

Numerical results form the core of these two volumes. Since users from diverse fields should be served, the tabulated quantities are named by their physical meaning, e.g., reflection function, gain, diffusion pattern, net flux, but are presented in the form of functions of a few dimensionless parameters. Most tables have five-figure accuracy in order to enable readers to use them for checking their own computer programs. The graphical illustrations have been chosen to serve as a quick orientation and also to highlight key phenomena such as asymptotic behavior.

Special cases such as the limits adopted for each quantity for conservative scattering ( $a = 1$ ), or in a semi-infinite atmosphere ( $b = \infty$ ), or at large depth ( $\tau \gg 1$ ) have been included in each tabulation. The same is true for moments and bimoments of the functions of the angles of incidence and emergence.

The formulas expressing these results show a similar ramification of special cases and asymptotic forms. For clarity and ease of access, they have been arranged, where possible, in a “Display,” which is a collection of formulas in tabular form. Derivations have been kept to a minimum. They are presented in

a form emphasizing the physical content and the use of certain intermediate results. Only rarely does an intricate derivation require the use of numbered equations.

Although the author's prime intention is to present known results, new discoveries or new light shed on the meaning and use of known forms was unavoidable. The major findings have been published in scientific journals and several have come into general use. Subjects like doubling, similarity relations, reduction to  $H$  functions, and, generally, the interpretation of mathematical results in physical terms, are presented here in their proper context.

The volumes have a strict organization: Part I on general relations and Part II on isotropic scattering (Volume 1), Part III on anisotropic scattering and Part IV on applications to selected fields (Volume 2). The division of parts into chapters again follows a strictly logical scheme as the table of contents for each volume shows.

What I started as a sideline has become a major project. This would not have been possible without the help and encouragement of a great many people. Among this long list I wish to record my special gratitude to K. G. Grossman and J. W. Hovenier for their support throughout the work and to W. M. Irvine and V. V. Ivanov, whose enthusiasm helped the project gain momentum in the early years.

# □ Contents of Volume 2

## Part III ANISOTROPIC SCATTERING

- 10 Phase Functions
- 11 Results for the Henyey–Greenstein Phase Function,  
Unbounded and Semi-Infinite Medium
- 12 Other Phase Functions, Semi-Infinite Atmospheres
- 13 Henyey–Greenstein Functions, Results for Finite Layers
- 14 Results for Other Phase Functions, Finite Layers
- 15 Polarization and Azimuth-Dependent Terms
- 16 Rayleigh Scattering

## Part IV SAMPLE APPLICATIONS

- 17 Photon Optical Paths and Absorption Lines
- 18 Planets
- 19 Scattered Light in the Earth's Atmosphere
- 20 Miscellaneous Applications

INDEX

This page intentionally left blank

# **MULTIPLE LIGHT SCATTERING**

## **Tables, Formulas, and Applications**

Volume 1

This page intentionally left blank

**Part I ○ GENERAL THEORY**



This page intentionally left blank