H.C. van de Hulst

# MULTIPLE LIGHT SCATTERING

## Tables, Formulas, and Applications

## VOLUME 1

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Volume 1

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# Tables, Formulas, and Applications

## Volume 1

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## 9 Isotropic Scattering, Finite Slabs

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# □ Preface

The play of radiation by repeated scattering in a cloud layer or any other slab of particles poses a problem that is common to atmospheric physics, astronomy, ocean optics, and branches of industrial research. Methods for solving this problem in diverse situations have been known for decades but their complexity has given the subject the reputation of being accessible only to specialists.

This book is aimed at the nonspecialist, e.g., an expert in an applied field, who needs a result from multiple scattering theory but does not wish to spend excessive time in solving it himself or searching the very extensive literature.

Numerical results form the core of these two volumes. Since users from diverse fields should be served, the tabulated quantities are named by their physical meaning, e.g., reflection function, gain, diffusion pattern, net flux, but are presented in the form of functions of a few dimensionless parameters. Most tables have five-figure accuracy in order to enable readers to use them for checking their own computer programs. The graphical illustrations have been chosen to serve as a quick orientation and also to highlight key phenomena such as asymptotic behavior.

Special cases such as the limits adopted for each quantity for conservative scattering ($a = 1$), or in a semi-infinite atmosphere ($b = \infty$), or at large depth ($\tau \gg 1$) have been included in each tabulation. The same is true for moments and bimoments of the functions of the angles of incidence and emergence.

The formulas expressing these results show a similar ramification of special cases and asymptotic forms. For clarity and ease of access, they have been arranged, where possible, in a "Display," which is a collection of formulas in tabular form. Derivations have been kept to a minimum. They are presented in

a form emphasizing the physical content and the use of certain intermediate results. Only rarely does an intricate derivation require the use of numbered equations.

Although the author's prime intention is to present known results, new discoveries or new light shed on the meaning and use of known forms was unavoidable. The major findings have been published in scientific journals and several have come into general use. Subjects like doubling, similarity relations, reduction to $H$ functions, and, generally, the interpretation of mathematical results in physical terms, are presented here in their proper context.

The volumes have a strict organization: Part I on general relations and Part II on isotropic scattering (Volume 1), Part III on anisotropic scattering and Part IV on applications to selected fields (Volume 2). The division of parts into chapters again follows a strictly logical scheme as the table of contents for each volume shows.

What I started as a sideline has become a major project. This would not have been possible without the help and encouragement of a great many people. Among this long list I wish to record my special gratitude to K. G. Grossman and J. W. Hovenier for their support throughout the work and to W. M. Irvine and V. V. Ivanov, whose enthusiasm helped the project gain momentum in the early years.

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# Part I ○ GENERAL THEORY

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