Dynamics of Railway Vehicle Systems

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Foreword

"The continual rattling during the motion . . . is principally produced by the fact that it is scarcely possible to retain the four points of the rails, on which the wheels of the locomotive rest, continually in one plane. . . ." This excerpt from a 1829 study of railroad operation, written when the very first railway in the world was barely 5 years old, represents an astute observation. Although the "rattling during the motion" may have been reduced, the problems of rail – vehicle dynamics resulting from the moving contact points between wheels and rails are of as much concern today as they were over 150 years ago when the first railway system was designed.

The importance of railways to world commerce and world development is self-evident. Canada owes its existence as a nation extending from the Atlantic to the Pacific to the decision to build a railway. Railways are still the most energy-efficient overland movers of heavy freight. Railway networks penetrate all corners of the world, and the construction of a railway is one of the first requirements to be met after the decision has been made to develop a new area of a country.

Yet, there are problems in the maintenance and operation of railways that go back to the early days. The use of flanged steel wheels running on steel tracks to provide simultaneously support, guidance, and traction was a brilliant concept. But the simplicity of the concept masked the complexity of the dynamics of the resulting motions. The motions of railway wheels and vehicles are the result of complex interactions among contact forces, component geometry, suspension, and vehicle masses — stiffness and damping coefficients that challenge analysis. Yet, an understanding of railway dynamics is fundamental to the control of rail – wheel wear and vehicle stability and reliability.

The railroad industry is essentially conservative. Only during recent years have modern scientific methods of analysis been applied to the problem of rail-wheel-vehicle dynamics. The complexity is extreme, but the demands for increased speeds and greater load capacity, which bring new problems of wear and stability, have forced railway operators and equipment suppliers to address these problems in a more systematic and fundamental way.

This book is the first devoted to a thorough, modern, analytical treatment of the rail-wheel interaction problem and its effect on vehicle dynamics. As well as providing a comprehensive source for information on the latest theories and results of studies on rail dynamics, it has been organized so that the relevant equations are clearly derived and the limitations to applications well defined. Thus, this work will provide a guide for future analysts and research workers seeking to improve our understanding of rail-vehicle dynamics, an understanding that is fundamental to the continued successful development of railway transportation.

Although the first century and a half of railway technological development was noted for the ingenuity and determination of its protagonists, the second will be founded firmly on a more complete understanding of the phenomena involved. This volume is a valuable contribution to scientific analysis and research in this area.

National Research Council Canada E. H. DUDGEON
Ottawa, Ontario, Canada

Preface

Although the subject of railway vehicle dynamics is constantly gaining importance in all aspects of modern railway engineering, currently there is no book available that deals with this rapidly expanding discipline. This book covers the development of mathematical models and their applications to dynamic analyses and the design of railway vehicles. It should help to put in proper perspective the role of analytical models in various railway vehicle design activities.

The book contains all the information that is usually needed to formulate general procedures and conduct detailed design analyses of common railway vehicle systems. Special attention is given to a clear presentation of the equations and methods of their solution. Related references to guide the reader further in this field are given at the end of each chapter.

The authors have developed the material in a way that allows the book to be used in courses in railway vehicle dynamics. Design and research engineers will be able to draw upon the book in selecting and developing mathematical models for analytical and design purposes.

The distinctive features of this book are as follows: Chapters 1–4 cover the necessary background material required to study the dynamics of railway vehicles. In Chapter 1, a review of the analytical techniques used in determining the dynamic response of single- and multiple-degree-of-freedom systems is given, covering deterministic and nondeterministic approaches. Chapter 2 deals with a condensed presentation of numerical solutions of linear and nonlinear dynamic systems; explicit and implicit numerical integration schemes are presented. Chapter 3 outlines various problems associated with the dynamic behavior of railway vehicles and train consists. Several mathematical models are proposed to study these problems. Both deterministic and nondeterministic approaches are used to represent various track irregularities. Chapter 4 deals with the wheel–rail rolling contact theories being applied in railway vehicle dynamics problems. Brief descriptions, applications, and limitations of these theories are also given.

Chapters 5–8 are devoted to the modeling of the vehicle and its components on both tangent and curved track. In Chapter 5, a complete derivation of the equations of motion for a single wheel–axle set traveling on tangent and curved track is presented. Chapter 6 is devoted to developing analytical models for the dynamic response of railway vehicles on tangent track. The equations of motion for a freight car, locomotive, and passenger car are developed by using deterministic and random track inputs. In Chapter 7, the lateral stability of these vehicles on tangent track is discussed. Chapter 8 presents the formulation of mathematical models for vehicle response on curved track.

Chapter 9 deals with the dynamic behavior of a train consist. The problems associated with the longitudinal, lateral, and vertical dynamic behavior of a train consist are outlined; quasi-static and quasi-dynamic approaches are presented. Analytical models for longitudinal, lateral, and vertical train dynamics are formulated.

Chapter 10 presents vehicle–bridge interaction models, whereas in Chapter 11 an introduction to validating railway vehicle dynamics models is given.

Vector and matrix notation is used throughout the book. This usage presupposes an elementary knowledge of calculus, ordinary differential equations, vector and matrix algebra, and dynamics.

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The format of this book has been influenced by many people working in the field of railway vehicle dynamics. The authors are indebted to their colleagues for their suggestions and to numerous writers who have made contributions to the literature in this field. We gratefully acknowledge the encouragement given by the Research and Test Department of the Association of American Railroads* and the National Research Council of Canada. Encouragement and guidance during all stages of the work that were freely rendered by Dr. W. J. Harris, Mr. E. H. Dudgeon, Mr. G. H. Way, Mr. J. G. Britton, Mr. K. L. Hawthorne, and Mr. C. A. M. Smith are greatly appreciated. Thanks are also due to Dr. R. Breese for editing the manuscript and to Mr. M. Farahmandpour for preparing the illustrations. We wish to thank Mrs. H. S. Cuccaro and Mrs. Sandra Lobo for the excellent typing of the manuscript. We gratefully acknowledge the assistance given by the staff of Academic Press. Finally, we acknowledge the encouragement, patience, and support of our families, especially our wives, Pushpa and Sudha.

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Chapter 1 Analysis of Dynamic Systems

1.1 Introduction

In this chapter we briefly discuss the analytical techniques that are used with dynamic systems. The initial portion of the chapter is devoted to the analysis of a linear single-degree-of-freedom system. Both free and forced responses of the system are discussed. In the following sections of the chapter, the analytical methods for a multiple-degree-of-freedom system are presented. Dynamic systems with and without damping are considered. Eigenvalue problems for these systems are formulated. The modal superposition technique for calculating the response of a multiple-degree-of-freedom system is also presented. Finally, a brief discussion of the theory of random vibration is given, and a method for calculating the response of a linear system subjected to stationary random excitations is presented.

1.2 Constraints, Generalized Coordinates, and Degrees of Freedom

The position of a system of particles is called its *configuration.* Usually, because of constraints on the system, actual coordinates need not be assigned to each particle. In a dynamic system, constraints may be at its boundary or at points internal to the system. Constraints may be either static or kinematic in nature. The static constraints result from relationships among forces, whereas the kinematic constraints are due to the relationships among displacements. In selecting the coordinates to describe a dynamic system, the static and kinematic constraints must be considered. Relationships among coordinates, which exist because of constraints on the system, are termed *constraints equations*. Based on this discussion, it can be said that systems of *unconstrained* or *independent* coordinates exist. In general, this is true in dynamic systems, and such a system can be described by a system of constrained coordinates. As an example, we can consider a dynamic system that is defined in terms of M coordinates. If there are R constrained displacements, then R coordinates can be expressed in terms of the remaining $M - R$ coordinates, which are independent. Thus, if

$$N = M - R, \tag{1.1}$$

N is the number of independent coordinates, and the forces and displacements are fully defined by these N coordinates. The independent coordinates required to specify completely the configuration of a dynamic system are called *generalized coordinates*. It is assumed that the generalized coordinates may be varied arbitrarily and independently without violating the constraints. Such a dynamic system is called a *holonomic system*. The number of generalized coordinates is called the *number of degrees of freedom* of a dynamic system.

To illustrate a dynamic system with constraint, we consider a rigid body attached to a point that is constrained to translate in the y direction, as shown in Fig. 1.1. In three-dimensional space, the motion of the rigid body would be described by five coordinates, i.e., two translations, one each along the x and z axes, and three rotations about the x, y, and z axes, respectively. In this case, the number of degrees of freedom for the system is five. Let us suppose that the rigid body is further constrained and that it undergoes motion in the x–y plane only, as shown in Fig. 1.2. The rigid body in a planar motion configuration would require two degrees of freedom to describe its motion. These degrees of freedom would correspond to the translation along the x axis and the rotation about the z axis.

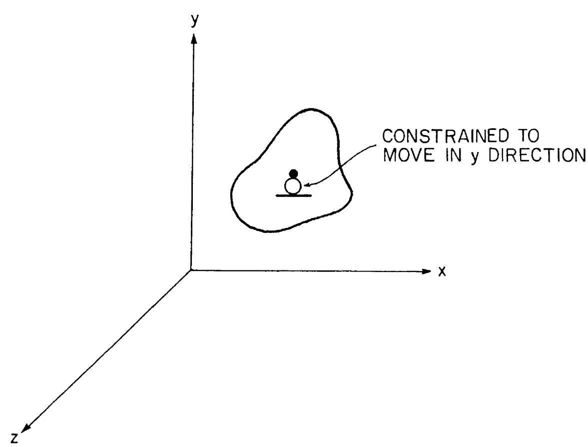

Fig. 1.1. Rigid body in general motion (five degrees of freedom).

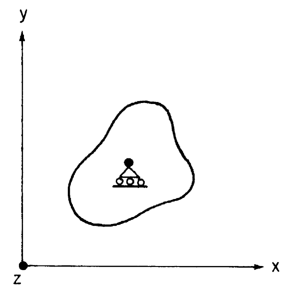

Fig. 1.2. Rigid body in planar motion (two degrees of freedom).

1.3 Linear Dynamic Systems

We have already seen that the number of degrees of freedom of a dynamic system is the number of independent coordinates required to describe its motion completely. A discrete model of a dynamic system possesses a finite number of degrees of freedom, whereas a continuous model has an infinite number of degrees of freedom. Of the discrete mathematical models, the simplest one is the single-degree-of-freedom linear model. The advantages of linear models are these:

(1) their response is proportional to input,
(2) superposition is applicable,
(3) they closely approximate the behavior of many dynamic systems,
(4) their response characteristics can be obtained from the form of system equations without a detailed solution,
(5) a closed-form solution is often possible,
(6) numerical analysis techniques are well developed, and
(7) they serve as a basis for understanding more complex nonlinear system behaviors.

It should, however, be noted that in most nonlinear problems it is not possible to obtain closed-form analytic solutions for the equations of motion. Therefore, a computer simulation is often used for the response analysis. Numerical analysis techniques used in computer simulations are discussed in the next chapter.

1.4 Classification of Vibrations

Vibrations can be classified into three categories: free, forced, and self-excited. *Free vibration* of a system is vibration that occurs in the absence of forced vibration, where damping may or may not be present. In the absence of damping, the total mechanical energy due to the initial conditions is conserved, and the system can vibrate forever because of the continuous exchange between the kinetic and potential energies. Because almost all mechanical systems exhibit some form of damping, the application of such free-vibration theories lies in the areas of celestial mechanics, space dynamics, and structural dynamics problems, in which the amount of damping is so small that the system can be treated as an undamped system.

Forced vibrations are caused by an external force that acts on the system. In this case, the exciting force continuously supplies energy to the system to compensate for that dissipated by damping. Forced vibrations may be either deterministic or random. The differential equations of motion of the dynamic systems considered in this book are all deterministic; i.e., the parameters are not randomly varying with time. However, the exciting force may be either a deterministic or a random function of time. In deterministic vibrations, the amplitude and frequency at any designated future time can be completely predicted from the past history; whereas random forced vibrations are defined in statistical terms, and only the probability of occurrence of designated magitudes and frequencies can be predicted.

Self-excited vibrations are periodic and deterministic oscillations. Under certain conditions, the equilibrium state in such a vibration system becomes unstable, and any disturbance causes the perturbations to grow until some effect limits any further growth. The energy required to sustain these vibrations is obtained from a nonalternating power source. In self-excited vibrations, the periodic force that excites the vibrations is created by the vibrations themselves. If the system is prevented from vibrating, then the exciting force disappears. In contrast, in the case of forced vibrations, the exciting force is independent of the vibrations and can still persist even when the system is prevented from vibrating.

1.5 Linear Single-Degree-of-Freedom (SDOF) System

We now consider a single-degree-of-freedom model of a linear dynamic system, as shown in Fig. 1.3. From Newton's third law we write

$$F(t) - F_s(t) - F_d(t) = m\,\ddot{x}(t), \tag{1.2}$$

where $F(t)$, $F_s(t)$, and $F_d(t)$ are the exciting, spring, and damping forces, respectively; m denotes the mass of the body and $\ddot{x}(t)$ its acceleration. Because $F_s(t) = kx(t)$ and $F_d(t) = c\,\dot{x}(t)$, Eq. (1.2) becomes

$$m\,\ddot{x}(t) + c\,\dot{x}(t) + kx(t) = F(t), \tag{1.3}$$

where c and k are the damping and stiffness coefficients, respectively.

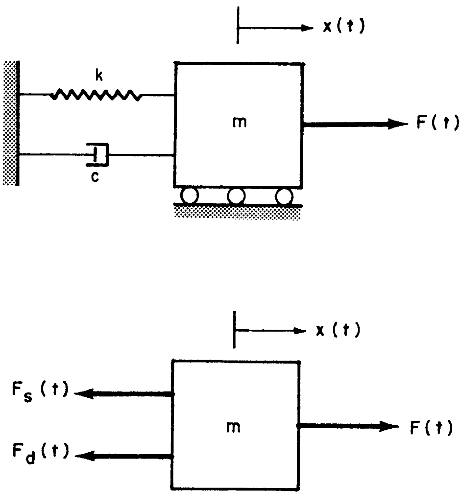

Fig. 1.3. Linear single-degree-of-freedom system.

Equation (1.3) is the equation of motion of the linear single-degree-of-freedom system and is a second-order linear differential equation with constant coefficients.

1.5.1 *Free Vibration of an SDOF System*

In the case of the free vibration of an SDOF system, the exciting force $F(t) = 0$ and the equation of motion is

$$m\,\ddot{x}(t) + c\,\dot{x}(t) + kx(t) = 0. \tag{1.4}$$

If we define $\omega_n^2 = k/m$ and $\xi = c/2m\omega_n$, Eq. (1.4) can be written as

$$\ddot{x}(t) + 2\xi\omega_n\,\dot{x}(t) + \omega_n^2 x(t) = 0. \tag{1.5}$$

To solve Eq. (1.5), we assume that

$$x(t) = Ae^{st}, \tag{1.6}$$

where A is a constant and s a parameter that remains to be determined. By substituting (1.6) into (1.5), one obtains

$$(s^2 + 2\xi\omega_n s + \omega_n^2)Ae^{st} = 0. \tag{1.7}$$

Since $Ae^{st} \neq 0$, then

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0. \tag{1.8}$$

Equation (1.8) is known as the *characteristic equation* of the system. This equation has the following two roots

$$s_1, s_2 = (-\xi \pm \sqrt{\xi^2 - 1})\omega_n. \tag{1.9}$$

Solution a. $\xi < 1$ (underdamped condition):

$$s_1, s_2 = (-\xi \pm i\sqrt{1 - \xi^2})\omega_n,$$

$$x(t) = A \exp(-i\omega_n t) \cos(\omega_n\sqrt{1 - \xi^2}t - \phi), \tag{1.10}$$

$$x(t) = A \exp(-i\omega_n t) \cos(\omega_d t - \phi), \tag{1.11}$$

where ω_n is the natural circular frequency, ξ the damping factor, and $\omega_d = \omega_n\sqrt{1 - \xi^2}$ the damped frequency of the system. Constants A and ϕ are determined from the initial conditions.

Solution b. $\xi > 1$ (overdamped condition):

$$s_1, s_2 = (-\xi \pm \sqrt{\xi^2 - 1})\omega_n,$$

$$x(t) = A_1 \exp(-\xi + \sqrt{\xi^2 - 1})\omega_n t + A_2 \exp(-\xi - \sqrt{\xi^2 - 1})\omega_n t. \tag{1.12}$$

The motion is aperiodic and decays exponentially with time. Constants A_1 and A_2 are determined from the initial conditions.

Solution c. $\xi = 1$ (critically damped condition):

$$s_1 = s_2 = -\omega_n,$$

$$x(t) = (A_1 + A_2 t) \exp(-\omega_n t). \tag{1.13}$$

Equation (1.13) represents an exponentially decaying response. The constants A_1 and A_2 depend on the initial conditions.

For this case, the coefficient of viscous damping has the value

$$c_c = 2m\omega_n = 2\sqrt{km}.$$

Hence,

$$\xi = c/c_c. \tag{1.14}$$

The locus of the roots s_1 and s_2 can be represented on a complex plane, as shown in Fig. 1.4. This permits an instantaneous view of the effect of the parameter ξ on system response. For an undamped system with $\xi = 0$, the imaginary roots are $\pm i\omega_n$. For a system $0 < \xi < 1$, the roots s_1 and s_2 are complex conjugates that are located symmetrically with respect to the real axis on a circle of radius ω_n. For $\xi = 1$, $s_1 = s_2 = -\omega_n$, and as $\xi \to \infty$, $s_1 \to 0$, and $s_2 \to -\infty$.

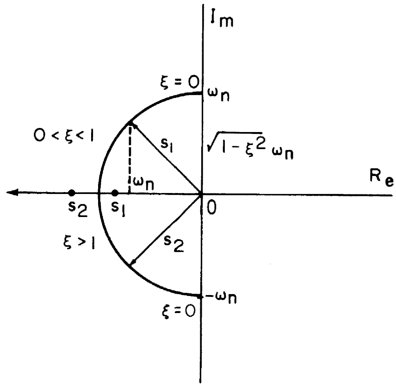

Fig. 1.4. Complex planar representation of roots s_1, s_2.

We further consider the underdamped condition in which t_1 and t_2 denote the times corresponding to the consecutive displacements x_1 and x_2, measured one cycle apart, as shown in Fig. 1.5. By using Eq. (1.11), we can write

$$\frac{x_1}{x_2} = \frac{A \exp(-i\omega_n t_1) \cos(\omega_d t_1 - \phi)}{A \exp(-i\omega_n t_2) \cos(\omega_d t_2 - \phi)}. \tag{1.15}$$

Since $t_2 = t_1 + T = t_1 + 2\pi/\omega_d$, then $\cos(\omega_d t_1 - \phi) = \cos(\omega_d t_2 - \phi)$. Equation (1.15) then reduces to

$$x_1/x_2 = \exp(\xi\omega_n T). \tag{1.16}$$

If we define

$$\delta = \ln(x_1/x_2) = \xi\omega_n T = 2\pi\xi/\sqrt{1 - \xi^2}, \tag{1.17}$$

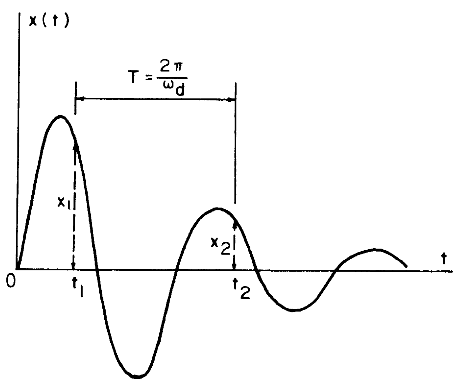

Fig. 1.5. Response of an underdamped system.