Guidance and Control

Edited by ROBERT E. ROBERSON
JAMES S. FARRIOR

Progress in Astronautics and Rocketry — Volume 8

An American Rocket Society Series

Guidance and Control

Progress in ASTRONAUTICS and ROCKETRY

A series of volumes sponsored by

American Rocket Society

500 Fifth Avenue, New York 36, New York

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ACADEMIC PRESS • NEW YORK AND LONDON

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A Selection of Technical Papers
based mainly on
A Symposium of the American Rocket Society
held at
Stanford University, Stanford, California
August 7-9, 1961

ACADEMIC PRESS · NEW YORK · LONDON · 1962

ACADEMIC PRESS INC.
111 FIFTH AVENUE
NEW YORK 3, N. Y.

United Kingdom Edition
Published by
ACADEMIC PRESS INC. (London) LTD.
BERKELEY SQUARE HOUSE, LONDON W. 1

Library of Congress Catalog Card Number 62-13119

PRINTED IN THE UNITED STATES OF AMERICA

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PREFACE

The fields of space guidance and attitude control, as well as advanced systems and components for terrestrial guidance and control, are growing at a phenomenal rate. The literature in some of these areas is doubling itself every one to three years. To help cope with this kind of explosive growth in limited technical areas, the profession needs both personal meeting grounds and a publication forum in which to present current work. Normal publication channels in technical journals serve the latter purpose, but, except for occasional survey or tutorial treatment, it is not the purpose of the ordinary technical paper to give an integrated picture of an entire field. Thus the usual literature in the field, in this case guidance and control, does not adequately mirror the growth of the field or sharply delineate the areas of keenest current interest. In effect, isolated technical papers give one the same viewpoint as looking at single frames of a strip of motion picture film--important pictures, perhaps, but not a complete substitute for running the entire film occasionally.

By combining into one volume a number of papers carefully chosen to illuminate important areas of technical development, it is possible to get this broader view of the whole field, its state of the art, and its pattern of development. This is the purpose of the present volume. The papers it contains have been selected from the Guidance, Control, and Navigation Conference sponsored by the American Rocket Society at Stanford University, Stanford, California, August 7-9, 1961. However, the book is in no sense a proceedings of the meeting, since the editors chose only certain papers that would best illustrate current problem areas and trends.

In arranging the papers, the editors have grouped them broadly into space guidance and path control, terrestrial guidance concepts and components, and other control topics. The first of these comprises three major mission phases: ascent from Earth to an orbit or space trajectory operations in space requiring navigation or guidance, and descent to the surface of Earth or the moon. The second group includes the system aspects of inertial navigation, gyroscopes as basic components of inertial navigation, and topics in optical navigation. The third group is less homogeneous. Its major divisions are adaptive control, a subject currently in the forefront of modern control theory developments, and attitude control, the major control phase in a space environment.

The boost or ascent phase represents the beginning of any space or missile operation. The ingredient problem areas are the launch itself, control of the boost trajectory, and control of the velocity increment at injection so that the resulting free flight trajectory is close to the one desired. The first paper, by C. E. Kohlhase, in Section A, Ascent, is an analysis of the geometric effects resulting from variations in launch time, particularly the resulting variations in asymptotic direction and path velocity. These error effects can be compensated by suitable changes in the ascent guidance constants, provided that the guidance rationale is sufficiently flexible to accommodate these changes. The editors remark that the launch time problem involved in near-simultaneous launch of more than one vehicle has arisen as an especially important topic in relation to the technique of in-transit rendezvous proposed by C. E. Kaempen (at the 11th International Astronautical Congress, held in Stockholm, August 1960). The second paper, by D. Lukes, illustrates the way in which the Pontryagin Maximum Principle, of so much current interest in control theory, can be applied to the optimum boost control problem. The formulation is carried to the explicit display of the differential equations for a two-point boundary value problem, and Lukes remarks on the solutions of these by digital computation. The fact that solutions are not displayed is merely a manifestation of the fact well recognized in control circles that it is very difficult, in general, to find numerical solutions to the optimum control problem using Pontryagin's method. Nevertheless, the Principle is considered to be of basic importance in control theory, which makes this early application of it to boost control of considerable interest. The third paper also is based on the Pontryagin Maximum Principle. W. Schmaedeke and G. Swanlund here apply it to the derivation of optimum injection guidance. Again the entire boost history is followed, a key assumption in the development being that the deviations from the nominal (presumably "optimum") trajectory during ascent are sufficiently small so that the error behavior can be represented by a linear description. Because of the relative unfamiliarity of the method, the authors include a brief survey of the Maximum Principle, also helping to provide background for the preceding paper.

In relation to Space Operations, Section B, there are three major categories of guidance and control problems. One type of control which may be exercised over an extended time period is attitude control, a topic sufficiently extensive so that it is treated separately in Section H. The remaining guidance and control problems are related to relatively short powered maneuvers, the two major instances of which are embraced by "rendezvous" and "orbit and trajectory correction." The first paper in this section, by R. S. Swanson, P. W. Soule, and N. V. Petersen, motivated by a rendezvous situation but actually treating station keeping, can be considered an example of the orbit correction problem whose goal is to maintain a vehicle in a "rendezvous-compatible" situation (a term originated by Petersen and his colleagues). Whether corrective maneuvers must be initiated at all, of course, is a function of the growth of initial errors. The paper by H. J. Gordon considers the expected initial errors for a lunar or interplanetary trajectory and develops their interpretation in terms of errors at the target. An interesting inverse situation is presented by A. Peske and M. Ward. They show how deviations in flight can be related to terminal rather than initial errors, thus providing a very direct basis for determining the size of needed en route corrections at any instant. W. C. Marshall, in the fourth paper, also uses a linear perturbation technique to examine the propagation of initial errors along an arbitrary trajectory, as well as the growth of error from disturbing forces of several types. Although the formalism is quite general, the lunar mission is especially in mind. An important question in space guidance or control operations is the accuracy with which vehicle position can be determined. This determination can be done aboard, when it is known as the space navigation problem, or on Earth, when it is known as the tracking problem. Since the former seems to have received the lion's share of past treatments, the final paper in this section, by C. R. Woods and E. B. Mullen, is an attempt to restore balance to the subject.

As manned space missions come more to the fore, the re-entry phase becomes increasingly important. Even for unmanned systems, soft landing on a surface can be of great interest when it is desired to deliver an instrument package intact. These cases can be subsumed by the term "descent," the title of Section C. The papers in this section by no means cover all of the problems of descent, but they do hit several important high spots. R. K. Cheng and I. Pfeffer's article concerns the guidance for a soft lunar landing, about which little has heretofore been published. Controlled re-entry, specifically longitudinal range control, is treated by R. Rosenbaum, whose way of achieving such control uses a lifting vehicle, with the result that the importance of accurate lift to drag prediction is clearly seen. The remaining paper, by P. C. Dow, D. P. Fields, and F. H. Scammell, considers the guidance and control problems that arise during two methods of re-entry at escape velocity. The first of these uses an apparent target and proportional steering, the second a method of explicit guidance in which the impact point is predicted.

Section D contains four papers on the subject of inertial navigation. The Transit satellite navigation system, which is now in operation, represents a major breakthrough in navigation technology. J. W. Crooks, R. C. Weaver, and M. M. Cox in their paper describe how maximum accuracy can be obtained from such a system through the use of sideband folding techniques. In any inertial navigation system, damping must be introduced in an optimum manner if maximum performance is to be obtained. The way in which servo techniques may be used to describe system performance and permit the design of specific damping equalizers is discussed in the paper by C. Broxmeyer. Redundancy has often been proposed as a technique for improving reliability. R. R. Palmer and D. F. McAllister's article considers how, for long term navigation, redundancy in the form of multiple system operation also can be used to improve navigational accuracy. In the final paper of this section, M. Kayton treats the fundamental limitations on inertial measurements.

Section E, Inertial Components, is directed toward the design of gyroscopes, which are the basis of any inertial guidance or navigation system. Design features are described which permit the designer to obtain the maximum possible performance from these precision instruments. Papers by C. O. Swanson, S. Osband, and R. P. Durkee discuss the more conventional designs, and a paper by A. Nordsieck provides a timely look at the electric vacuum gyroscope.

Optical techniques and devices for navigation are considered in Section F. The subject is introduced in a paper by E. M. Wormser and M. H. Arck which treats the application of infrared navigation sensors to a variety of space projects. R. G. Franklin and D. L. Birx discuss how velocity indications may be derived from the measurement of optical Doppler shift and describe how lasers might be employed for optical heterodyning to shift the optical Doppler frequencies to the radio-frequency range where they may be measured by existing methods. Optical heterodyning is further discussed by W. C. Reisener, who describes an interesting technique involving a traveling wave tube mixer. Microwave currents are generated due to the interference of the two optical signals at the photosensitive cathode of the mixer tube.

One of the most frequently discussed topics in modern control theory is adaptive control. Adaptive systems have been applied in practice to terrestrial flight control, but only recently have astronautical applications been developed. Section G contains two papers on adaptive control. The first, by H. P. Whitaker and A. Kezer, actually is rather general, that is, not specifically astronautical in character, but concerns a subject of special importance in both terrestrial and astronautical applications: the way in which reliability can be improved by means of adaptive systems. On the other hand, the paper by W. E. Miner, D. H. Schmieder, and N. J. Braud is directed toward the booster guidance and control problem with special application to Saturn.

The satellite attitude control problem has a number of interesting facets, of which four are represented by the papers in Section H. One of the important questions in this field is the nature of the torque on the vehicle. Gravitational, magnetic, and other torques have been treated in the literature; in this section, R. J. McElvain adds an analysis of solar radiation pressure. One of the methods of closed loop active control employs combined reaction wheel and jet actuators, about which relatively little detailed analysis has been published. The paper by D. B. DeBra and R. H. Cannon is a good discussion of many aspects of this problem. The other method of closed loop control which is of special current interest is pure jet actuation, in relation to which the major problem is the choice of a control logic and the resulting limit cycle response that is typical of such on-off devices. The paper by P. R. Dahl, G. T. Aldrich, and L. K. Herman concerns limit cycles in the presence of external torques, often neglected in other analyses, while that of R. S. Gaylord and W. N. Keller presents a control logic that is effective in reducing the limit cycle without the use of direct or derived rate information. Another major class of attitude control systems is spin stabilization. Even there, however, spin vector control often is necessary. C. Grubin, in the final paper, presents a generalized two-impulse scheme for reorienting the spin vector.

The editors feel that, because the forementioned papers have been selected from those given at the first Guidance, Control, and Navigation Conference sponsored by the ARS Guidance and Control Committee, special acknowledgment and recognition are due those whose efforts made that conference possible. Appreciation is expressed particularly to Stanford University for its unstinting support of the conference. Donald P. LeGalley, Program Chairman, and Robert H. Cannon, Jr., Vice Chairman, deserve special mention. Together with the session chairmen, they were largely responsible for the high technical quality of the papers at the meeting and thus, indirectly, of the papers in this volume. An equally important role was played by Daniel B. DeBra, who, as Arrangements Chairman, did much to insure the success of the meeting.

Robert E. Roberson
James S. Farrior

April 1962

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A.

ASCENT

LAUNCH-ON-TIME ANALYSIS FOR SPACE MISSIONS

C. E. Kohlhase[1]

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ABSTRACT

Lunar and interplanetary trajectories are dependent on time of launch as a result of the relative motion between the launch site and destination. It is therefore essential to understand the geometric aspects of this dependency in order to establish the guidance criteria necessary to correctly direct the vehicle in the presence of firing-time delays that may occur at the launching complex.

The launch-on-time problem is analyzed by realizing that the primary defining quantities for deep-space missions are the pseudo-asymptote and energy of the departure conic (coast trajectory). The goal of the injection guidance system is therefore to steer the vehicle so that at injection (when final burning is terminated) the coast trajectory will exhibit the desired energy and pseudo-asymptotic direction. Practical trajectories for deep-space missions will generally use parking orbits in order to relieve geometric constraints. The launch-on-time problem can be handled by changing the firing azimuth and parking-orbit coasting arc. This eliminates the necessity for any dramatic vehicle maneuvers that would result in performance degradation. It is this consideration that severely limits the firing window (allowable launch delay) for direct-ascent missions, as the direct-ascent vehicle must fly a steeper flight path in order to compensate for launch-time delay.

INTRODUCTION

Trajectory dependency on time of launch is present whenever there is relative motion between the launch site and destination. This situation does not arise for ballistic-

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Presented at ARS Guidance, Control, and Navigation Conference, Stanford University, Stanford, Calif., Aug. 7-9, 1961; this paper gives results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract NASw-6, sponsored by NASA.

[1]Research Engineer, Systems Analysis Section.

missile or Earth-satellite trajectories, but for lunar and planetary missions, the geometry between the launch site and destination continuously changes with time, due, primarily, to Earth's rotation about its axis and, secondarily, to the motion of the target body relative to Earth. It is therefore important to understand trajectory behavior with launch time in order to establish the guidance criteria necessary to properly direct the vehicle in the presence of unforeseen firing-time delays that may occur at the launching complex during an attempt to launch at some preselected standard firing time.

GUIDANCE ELEMENTS

It has been shown by other authors (1,2)[2] that the outward radial direction $\vec{S}$ (also termed the pseudo-asymptote) and the departure conic energy[3] C_3 are the primary defining quantities for lunar and planetary trajectories. In view of this important dependency, the goal of the injection guidance system will be to steer the vehicle so that the prescribed asymptote and energy will be achieved at injection. As can be seen from Fig. 1, for planetary missions, the unit vector $\vec{S}$ lies along the asymptote to the standard departure hyperbola and, for lunar missions, $\vec{S}$ lies along the position vector of the "massless" Moon at the predicted time of lunar encounter. It can usually be assumed that over a period of a few hours $\vec{S}$ and C_3 remain essentially constant for planetary missions, and $\vec{S}$ moves with the Moon for lunar missions.

In order to satisfy the asymptote-energy requirements, the guidance system could employ:

1) Yaw steering to force the vehicle plane of motion to contain $\vec{S}$;
2) pitch steering to properly orient the departure conic trajectory within the vehicle plane of motion;
3) termination of final-stage burning upon reaching the nominal[4] value of C_3.

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[2]Numbers in parentheses indicate References at end of paper.

[3]The departure conic energy is twice the total energy per unit mass.

[4]For lunar trajectories, this is essentially equivalent to maintaining the nominal flight time, so that the vehicle will encounter the Moon late by the amount of the launch-time delay.

Figs. 2 and 3 illustrate the in-plane points of interest for parking-orbit and direct-ascent powered-flight trajectory profiles for a two-stage vehicle with second-stage restart capability. Point B represents the position of the launch site after Earth has rotated during the launch-time-delay interval. Yaw steering may be accomplished by nulling a signal proportional to $\vec{R} \times \vec{V} \cdot \vec{S}$. Because of aerodynamic (and other) constraints, the first portion of the booster stage is usually flown "open loop"; that is, in a preprogrammed manner without guidance steering. When truly guided flight begins, the vehicle might well be off course, requiring a significant maneuver to return to the proper plane of motion. If the integral of thrust acceleration in the plane of motion is to be maintained at some fixed value, then it is very important that the vehicle not be required to execute any large yaw maneuvers.

Launch-time delays can be compensated more conveniently and efficiently with parking-orbit trajectories than with direct-ascent trajectories. In the former case, it is a simple matter to vary the parking-orbit interval in order to maintain the proper in-plane orientation of the departure conic, but in the latter case, injection must occur at larger values of true anomaly and can only be accomplished by flying a steeper flight path with its associated reduction in vehicle performance (3).

FIRING AZIMUTH FROM NONROTATING SPHERICAL EARTH

Although the guidance system could, in theory, achieve the desired injection conic regardless of the initial direction in which the vehicle is launched, from a practical standpoint it is mandatory that the vehicle not be required to execute any dramatic maneuvers. For this reason, it makes sense to determine an initial firing azimuth that corresponds to the desired vehicle plane of motion. An approximate value of the desired launch azimuth σ_L can be obtained analytically by considering the simple model of a nonrotating spherical Earth (fixed at the instant of lift-off) shown in Fig. 4. Normally, the vehicle roll axis is erected along the plumb line or geodetic vertical at the launching complex, and the firing azimuth σ_L is then the angle measured clockwise from north to the projection of the vehicle thrust vector (as soon as the vehicle is pitched over from the vertical) onto the local geodetic horizontal plane. For the simple spherical-Earth model of Fig. 4 the geodetic and geocentric verticals are coincident.

The unit vector $\vec{r}_L$, which points from the center of Earth through the launching site, is given by

$$\vec{r}_L = \vec{i} \cos\psi_L \cos \textcircled{H}_L + \vec{j} \cos\psi_L \sin \textcircled{H}_L + \vec{k} \sin\psi_L \quad [1]$$

where ψ_L and $\textcircled{H}_L$ are the geocentric latitude and right ascension of the launch site, and $\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors defined by a space fixed, equatorial, rectangular coordinate system with the x-axis towards the vernal equinox (Υ). The unit vector $\vec{a}$, pointing down the firing azimuth, is given by

$$\vec{a} = \left\{ \left[\vec{k} \times \vec{r}_L\right] \sin\sigma_L - \left[(\vec{k} \times \vec{r}_L) \times \vec{r}_L\right] \cos\sigma_L \right\} \sec\psi_L \quad [2]$$

The unit normal vector $\vec{N}$ to the plane of motion is given by $\vec{r}_L \times \vec{a}$ and, for a nonrotating, spherical Earth, the correct firing azimuth may be obtained by solving the equation $\vec{N} \cdot \vec{S} = 0$ for σ_L

$$\sigma_L = \tan^{-1} \left[\frac{S_x \sin \textcircled{H}_L - S_y \cos \textcircled{H}_L}{(S_x \cos \textcircled{H}_L + S_y \sin \textcircled{H}_L) \sin\psi_L - S_z \cos\psi_L} \right] \quad [3]$$

Because of Earth rotation, the right ascension of the launch site $\textcircled{H}_L$ is related to the launch-time delay Δt_L by

$$\textcircled{H}_L = \textcircled{H}_{L_s} + \omega_e \Delta t_L$$

where $\textcircled{H}_{L_s}$ is the launch-site right ascension at the standard firing time, and ω_e is the average angular velocity of the Earth.

A situation is imagined that is defined by assuming $\textcircled{H}_{L_s} = 0$, $\psi_L = 28.3^\circ$ (Atlantic Missile Range), and $\vec{S} = -\vec{i} \cos\psi_S + \vec{k} \sin\psi_S$, where ψ_S is the declination of $\vec{S}$. This situation is described by Fig. 5, which illustrates firing azimuth behavior with launch time for several values of ψ_S. Although Eq. 3 admits two possible firing azimuths for any given firing time, only the easterly values ($0 \le \sigma_L \le 180^\circ$) have been shown in Fig. 5. The curves exhibit two characteristic patterns, with the critical boundary occurring at $|\psi_S| = |\psi_L|$. It is noted that for $|\psi_S| \le |\psi_L|$, it is possible to fire at all azimuths (within range-safety limits), but for $|\psi_S| > |\psi_L|$, a symmetric band of firing azimuths about due east is eliminated (3, 4). Launch-on-time considerations generally favor launching when the rate of change of firing azimuth with launch time is a minimum, if possible, as the associated firing windows are usually longer and the tracking geometry varies at the slowest possible rate. Accordingly, it is least desirable to select a nominal firing time for which the $\partial\sigma_L/\partial t_L$ is very large. Although Fig. 5 illustrates firing-azimuth behavior with launch time for what appears (from the symmetry involved) to be a very special situation, it is actually representative of any real situation (for $\psi_L = 28.3^o$) by a simple translation of the launch-time axis. This is apparent when it is realized that any $\vec{S}$ vector may be expressed in the form assumed for Fig. 5 by performing a rotation of the equatorial coordinate axes in order to null S_y.

Fig. 5 is very useful in determining the expected firing-window[5] width, given the limiting azimuths for adequate tracking coverage. For example, good tracking facilities exist for trajectories launched to the southeast from the Atlantic Missile Range from about 95 to 110°. For most lunar and planetary missions, this would correspond to a firing window of between one and three hours during each day.

ROTATING SPHERICAL EARTH

The curves of Fig. 5 have assumed that, at the instant of launch, Earth is nonrotating. The actual firing azimuth from a rotating Earth will, in general, lie slightly away from east of the azimuth given by Eq. 3. This deviation is essentially due to the initial crossrange-rate component $\dot{Z}_{L_o}$ present at launch. Fig. 6 displays an inertial, rectangular, launch-site coordinate system, defined at the instant of launch. $\vec{Y}_L$ is perpendicular to the spherical Earth model, $\vec{X}_L$ points along the downrange or azimuthal heading, and $\vec{Z}_L = \vec{X}_L \times \vec{Y}_L$.

If expressions are developed for Z_L and $\dot{Z}_L$, then the amount by which $\vec{N}$ has been rotated may be determined. If drag is neglected and the assumption is made that vehicle thrust is maintained parallel to the $\vec{X}_L \vec{Y}_L$ plane, then

$$\ddot{Z}_L = - \frac{\mu}{R^2} \left(\frac{Z_L}{R}\right) \simeq -K \; Z_L \qquad [4]$$

where K may be thought of as a time-averaged value of μ/R^3 over the powered flight from launch to the point where the rotation of $\vec{N}$ is to be determined. Integration of Eq. 4 leads to

$$\dot{Z}_L = \dot{Z}_{L_o} \cos (K^{\frac{1}{2}} t) \qquad [5]$$

[5]The <u>firing window</u> refers to that period of time during which the vehicle may be launched without violating any of several constraints.

$$Z_L = K^{-\frac{1}{2}} \dot{Z}_{L_0} \sin (K^{\frac{1}{2}} t) \qquad [6]$$

where $\dot{Z}_{L_0}$ is equal to the product of Earth's eastward surface velocity at the launch site and the cosine of the firing azimuth σ_L.

It is imagined that the vehicle is launched from a nonrotating Earth, flown to some point, and then an instantaneous $\dot{Z}_L$ is applied. This would have the effect of rotating the plane of motion negatively about $\vec{R}$ by an amount $\dot{Z}_L (V \cos \Gamma)^{-1}$. The application of an instantaneous Z_L would be equivalent to a rotation of $Z_L(R \cos \Gamma)^{-1}$ about a line through the center of Earth and parallel to $\vec{V}$. It would therefore seem appropriate to define the rotation vector $\vec{\rho}$ [6]

$$\vec{\rho} = C_1^{-1} (Z_L \vec{V} - \dot{Z}_L \vec{R}) \qquad [7]$$

where the angular momentum $C_1 = R V \cos \Gamma$. Eq. 7 is a valid approximation as $\vec{\rho}$ is a small rotation. An inertial observer located far above the launch site and looking in the $-\vec{Y}_L$ direction would observe the trajectory curving to the right (for a southeast firing) as the vehicle accelerates downrange, after being launched with an eastward inertial velocity imparted by Earth rotation. In order that the actual plane of motion defined at injection contain the desired $\vec{S}$, it is necessary that

$$(\vec{N} + \vec{\rho} \times \vec{N}) \cdot \vec{S} = 0 \qquad [8]$$

Solution of Eq. 8 for σ_L yields the same expression as Eq. 3 with S_x, S_y, S_z replaced by S_x', S_y', S_z', where

$$S_x' = S_x + S_y \rho_z - S_z \rho_y$$

$$S_y' = S_y + S_z \rho_x - S_x \rho_z$$

$$S_z' = S_z + S_x \rho_y - S_y \rho_x$$

Fig. 7 illustrates the effect of Earth rotation (initial cross-range rate) upon firing azimuth for a typical firing situation.

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[6] An approximate method for determining $\vec{\rho}$ is given in the Appendix.

As might have been expected, there is no rotation of the plane of motion for trajectories fired due east (or west) from a spherical Earth and maximum rotation for those launched due south (or north). The quantity $\Delta\sigma_L$ represents the additional amount by which the firing aximuth must be rotated away from east in order to compensate for initial crossrange rate and attain the desired plane of motion at injection. Experience has shown that the value of σ_L computed for a rotating, spherical Earth is very close (within 0.5°) to the actual value for a rotating, oblate Earth. There is no need to conduct a detailed analysis of the effects of oblateness in order to obtain an exact value for σ_L, as guidance system yaw steering will achieve the desired plane of motion with negligible loss in vehicle performance.

It should be noted that changing launch azimuth away from due east causes a performance loss because of the diminished component of "Earth's rate" in the plane of motion. The launch site is moving with speed $V_L = \omega_e R_L \cos\psi_L$ relative to the center of Earth, and this contributes to meeting the inertial energy requirements at first-stage burnout. The component of V_L in the plane of motion is

$$V_L \sin\sigma_L = \omega_e R_L \cos\psi_L \sin\sigma_L$$

which implies that additional thrusting is required if σ_L is changed away from 90°.

COAST-TIME CORRECTION

The second part of the launch-on-time problem, from the guidance point of view, is controlling the orientation of the departure conic in the plane of motion. This can be handled rather simply for trajectories with parking orbits by assuming that the last burn profile will stay essentially fixed (the departure conic will be standard) and by igniting the last stage at the proper place in inertial space by varying the parking-orbit interval (see Fig. 2). This can be done by initiating last burn when the signal $\vec{R}_s \cdot \vec{S}_s - \vec{R} \cdot \vec{S}$ goes to zero in the guidance computer, thereby causing the in-plane angle between the beginning of the last burn and $\vec{S}$ to have the standard value. The coast-time variation Δt_c is given by

$$\Delta t_c = R_c V_c^{-1} \left[\cos^{-1}(\vec{r}_{L_S} \cdot \vec{S}_s) - \cos^{-1}(\vec{r}_L \cdot \vec{S}) \right] \qquad [9]$$

where R_c and V_c are the parking-orbit radius and velocity. If $\vec{a} \cdot \vec{S} > 0$ for $\vec{r}_{L_S}$ and $\vec{r}_L$, the coast-time correction is given by

$-\Delta t_c$ as defined by Eq. 9. For lunar trajectories and launch-time delays not in excess of a few hours, $\vec{S}$ varies approximately as

$$\vec{S} \simeq \frac{\vec{R}_{m_s} + \Delta t_L \vec{V}_{m_s}}{\left|\vec{R}_{m_s} + \Delta t_L \vec{V}_{m_s}\right|} \tag{10}$$

where $\vec{R}_{m_s}$ and $\vec{V}_{m_s}$ are the geocentric position and velocity of the "massless" Moon at the standard time of expected encounter. Eq. 10 has assumed that since the standard injection energy is maintained, the flight time from injection to lunar encounter does not change appreciably for nominal launch-time delays. Eq. 10 also neglects Δt_c, which is small ($\Delta t_c \simeq -0.05\ \Delta t_L$) in comparison with Δt_L. For planetary missions, $\vec{S} \simeq \vec{S}_s$, the asymptote to the standard departing geofocal hyperbola. Strictly speaking, the heliocentric geometry has undergone a small change after the passage of a launch-time delay, but during the short time associated with a typical firing window, negligible[7] error is made by assuming $\vec{S} = \vec{S}_s$. In actual practice, however, several trajectories from launch to planet encounter would be run (on an accurate digital computer trajectory program) at launch-time intervals every fifteen minutes or so after the nominal firing time, over the firing window. Then a simple $\vec{S}(\Delta t_L)$ fit would be obtained from the trajectory data and used in the asymptote-guidance equations.

Fig. 8 illustrates coast-time variation (based upon 100-n mi circular parking orbit) with launch time for the symmetric situation described by Fig. 5. The discontinuity at $\Delta t_L = 12$ results from considering only the easterly firing azimuths ($0 \leq \sigma_L \leq 180^o$). Fig. 8 further assumes that whenever the downrange angle from $\vec{r}_L$ to $\vec{S}$ is less than 180^o, then the vehicle must coast around Earth before departure. The value of this minimum downrange angle is dependent upon the type of vehicle and the particular mission (1). For many of the current vehicles and anticipated missions, this angle may vary from about 150 to 200^o, and 180^o was merely chosen as a typical value. For all possible parking-orbit trajectories, $0 \leq |\partial t_c/\partial t_L| \leq \omega_e R_c V_c^{-1}$. Typically, $\partial t_c/\partial t_L \simeq -0.05$ about the nominal firing time for many of the envisaged space missions that employ the parking-orbit technique.

[7]That is, negligible in comparison with target dispersions that result from component error sources in the injection guidance system.

Attaining the proper in-plane orientation of the departure conic for direct-ascent trajectories cannot be achieved in the same manner as for parking-orbit missions. Since there is no coast interval to vary, the equivalent compensation for a direct-ascent trajectory must be accomplished by varying the true anomaly at injection; that is, by injecting at a different point on the coast trajectory (see Fig. 3). Since the optimum injection point (near perigee) usually corresponds to a launching time close to the nominal, it follows that the last stage must be pitched up for a late launching. This causes a loss in vehicle performance because of the less efficient flight path. It is for this reason that the firing window is shorter for direct-ascent than for parking-orbit trajectories.

CONCLUSIONS

It has been shown that launch-time variations may be compensated very simply by changing the firing azimuth and coasting arc for parking-orbit trajectories. The allowable firing-time delay for direct-ascent missions is severely limited, however, due to the necessity of flying a steeper and less efficient flight path.

In order to verify the efficacy of energy-asymptote guidance, several standard parking-orbit trajectories were rerun (with launch-time variations) on the IBM 704 digital computer. The results of three typical missions have been summarized in Table 1.

APPENDIX

The amount of rotation of the powered-flight plane of motion depends upon the firing azimuth. Neglecting oblateness, there would be no rotation for trajectories fired due east or west and maximum rotation for those launched due north or south. Therefore, in order to compute σ_L, $\vec{\rho}$ must be known, but in order to determine $\vec{\rho}$, σ_L must be known. This situation may be handled without difficulty by first computing the firing azimuth from Eq. 3. Use of this equation is consistent with the assumption that the vehicle is flown to some point (Earth-fixed at instant of lift-off) and that instantaneous Z_L and $\dot{Z}_L$ are then applied to determine the rotation of the powered-flight plane of motion. If it is desired to determine $\vec{\rho}$ at injection

$$\vec{\rho}_I = C_{1_I}^{-1} \left(Z_{L_I} \vec{V}_I - \dot{Z}_{L_I} \vec{R}_I \right)$$

where C_{1_I} is simply the standard injection angular momentum, and $\vec{R}_I$ and $\vec{V}_I$ may be determined approximately by utilizing the nonrotating spherical Earth model. For parking-orbit trajectories, it can be assumed that injection occurs at the standard in-plane angle from $\vec{S}$. Fig. 9 illustrates the in-plane quantities.

It can be seen that

$$\vec{R}_I = R_{I_s} (\vec{S} \cos \alpha + \vec{T} \sin \alpha) \qquad [A1]$$

$$\vec{V}_I = V_{I_s} (\vec{S} \sin \beta - \vec{T} \cos \beta) \qquad [A2]$$

where

$$\alpha = \cos^{-1}(\vec{R}_{I_s} \cdot \frac{\vec{S}_s}{R_{I_s}}),\ \beta = \alpha + \Gamma_{I_s},$$

and

$$\vec{T} = \frac{(\vec{r}_L \cdot \vec{S})\vec{S} - \vec{r}_L}{\left[1 - (\vec{r}_L \cdot \vec{S})^2\right]^{\frac{1}{2}}} \qquad [A3]$$

If $\vec{a} \cdot \vec{S} > 0$, it will be necessary to use $-\vec{T}$ as defined by Eq. A3. Finally, it is necessary to determine Z_{L_I} and $\dot{Z}_{L_I}$. In Eqs. 5 and 6

$$\dot{Z}_{L_0} = \omega_e R_L \cos \psi_L \cos \sigma_L \qquad [A4]$$

where R_L is the radius of Earth at the launch site and σ_L may be computed from Eq. 3. The time from launch to injection t_I is given by

$$t_I = t_{I_s} + \Delta t_c \qquad [A5]$$

where Δt_c is given by Eq. 9.

NOMENCLATURE

- $\vec{a}$ = unit vector along azimuthal heading at launch
- C_1 = angular momentum defined by $RV \cos \Gamma$
- C_3 = twice total energy per unit mass and equal to $V^2 - 2\mu R^{-1}$
- K = time-averaged value of μR^{-3} over powered flight from launch to point where $\vec{\rho}$ is desired
- $\vec{N}$ = unit vector normal to vehicle (launched from nonrotating Earth) plane of motion

$\vec{R}$ = position vector of vehicle
R_c = circular parking-orbit radius
$\vec{R}_{m_s}$ = position vector of "massless" Moon at time of expected lunar encounter
$\vec{r}_L$ = unit vector pointing from center of Earth through launching site
$\vec{S}$ = unit vector along asymptote to departure hyperbola, for interplanetary missions; lies along lunar position vector at time of predicted encounter with "massless" Moon, for lunar missions
t = time measured from lift-off
$\vec{T}$ = unit vector normal to $\vec{S}$ in plane of motion
$\vec{V}$ = inertial velocity of vehicle
$\vec{V}_{m_s}$ = inertial velocity (relative to Earth's center) of Moon at standard time of predicted lunar encounter
$\vec{X},\vec{Y},\vec{Z}$ = space fixed, equatorial rectangular coordinate system with X-axis toward vernal equinox; prescribes unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$
$\vec{X}_L,\vec{Y}_L,\vec{Z}_L$ = inertial launch site coordinate system, defined at instant of launch
$Z_L, \dot{Z}_L$ = vehicle crossrange and crossrange rate for simplified mathematical model of Fig. 6
α = nominal downrange angle from injection to pseudo-asymptote, for parking-orbit missions
$\beta = \alpha + \Gamma_{I_s}$
Γ = angle from local horizontal plane to inertial velocity vector
Δt_c = parking-orbit coast-time correction
Δt_L = launch-time variation (positive for late launch)
Ⓗ = right ascension
μ = gravitational constant for Earth (GM_e)
$\vec{\rho}$ = rotation vector of powered-flight plane of motion
σ_L = firing azimuth measured clockwise from north
ψ = geocentric latitude or declination
ω_e = average angular velocity of Earth

SUBSCRIPTS

c = circular parking-orbit conditions
I = injection values
L = launch site
m = lunar quantities
s = values associated with the standard, no launch time variation trajectory
S = pseudo-asymptote
x,y,z = components in X, Y, Z coordinate system
o = initial value of given parameter

REFERENCES

1 Clarke, V. C., "Design of lunar and interplanetary ascent trajectories," Jet Propulsion Lab. Tech. Rept. 32-30, Calif. Instit. Technology, Pasadena, Calif., July 26, 1960.

2 Sauer, C. G., Jr., "Interplanetary injection guidance," Jet Propulsion Lab. Tech. Rept. 34-88, Calif. Instit. Technology, Pasadena, Calif., Oct. 25, 1960.

3 Seifert, Howard S., Space Technology (John Wiley and Sons, Inc., New York, 1959), Chap. 8, pp. 88-90.

4 "Lunar probe flight; introduction to flight geometry and accuracy problems," ABMA DA Tech. Note No. 58-58, Redstone Arsenal, Ala., Aug. 22, 1958, pp. 17-22.

Table 1 Launch-on-time results

Key parameters	66-hour lunar	66-hour lunar	176-day Mars
Launch-time delay, min	60	60	30
Firing-azimuth change, deg	108.0 116.8	96.0 105.0	112.0 112.9
Coast-time correction, sec	-175.9	-185.3	-90.1
Miss distance from target center with no correction for launch-time delay, mi	4.90×10^4	4.60×10^4	4.87×10^6
Miss distance from target center with energy asymptote injection guidance[a] but no midcourse maneuver, mi	190 (impact)	350 (impact)	1.60×10^4

[a]Assuming no performance or component errors

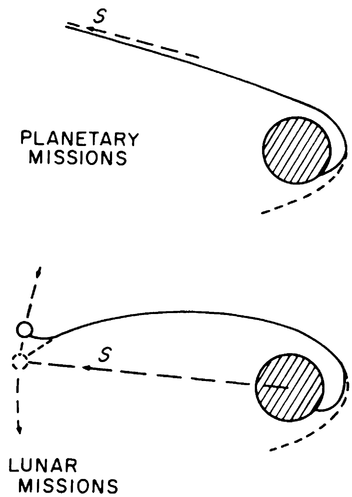
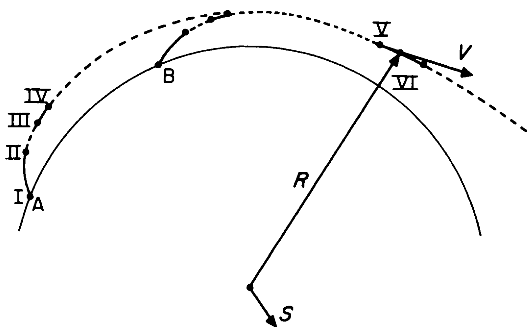


Fig. 1 Psuedo-asymptotic direction



I LAUNCH
II FIRST-STAGE SEPARATION
III SECOND-STAGE IGNITION
IV PARKING-ORBIT ENTRY
V SECOND-STAGE RESTART
VI INJECTION (DEPARTURE CONIC DEFINED)

Fig. 2 Parking-orbit trajectory profile

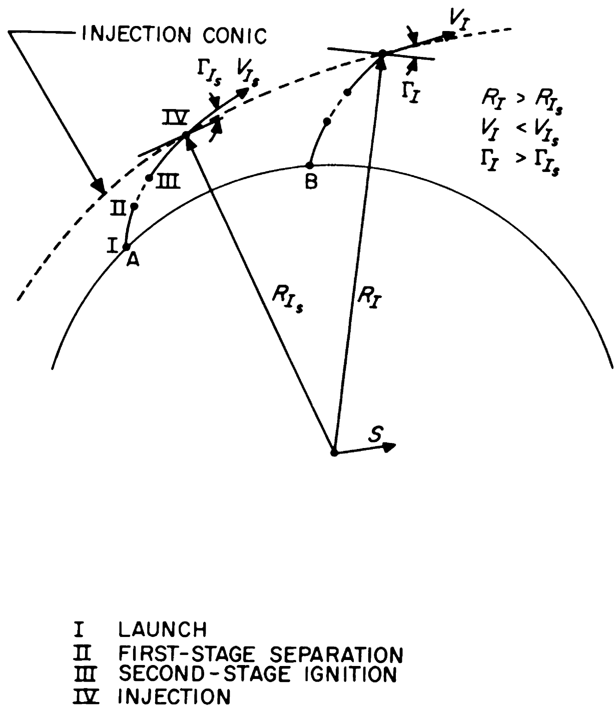


Fig. 3 Direct-ascent trajectory profile

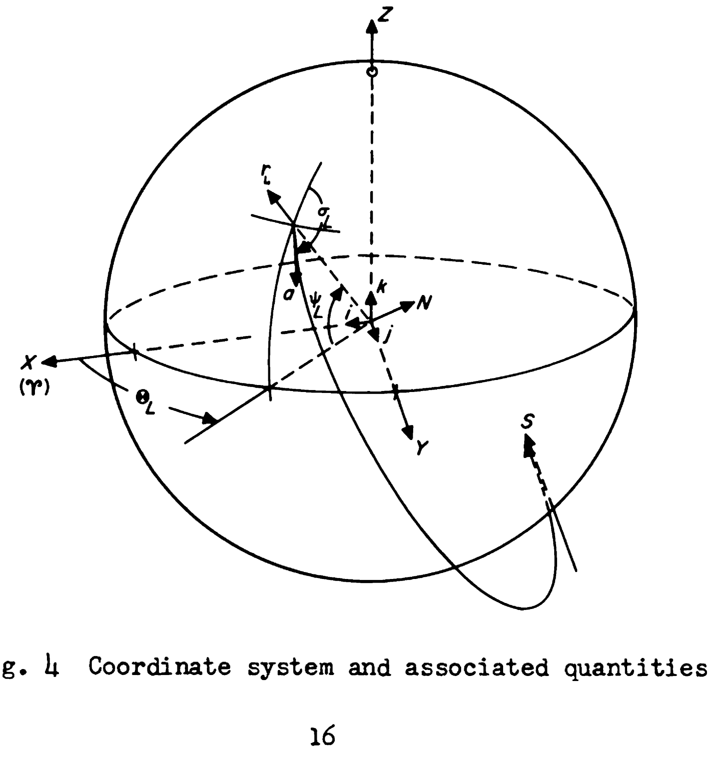


Fig. 4 Coordinate system and associated quantities

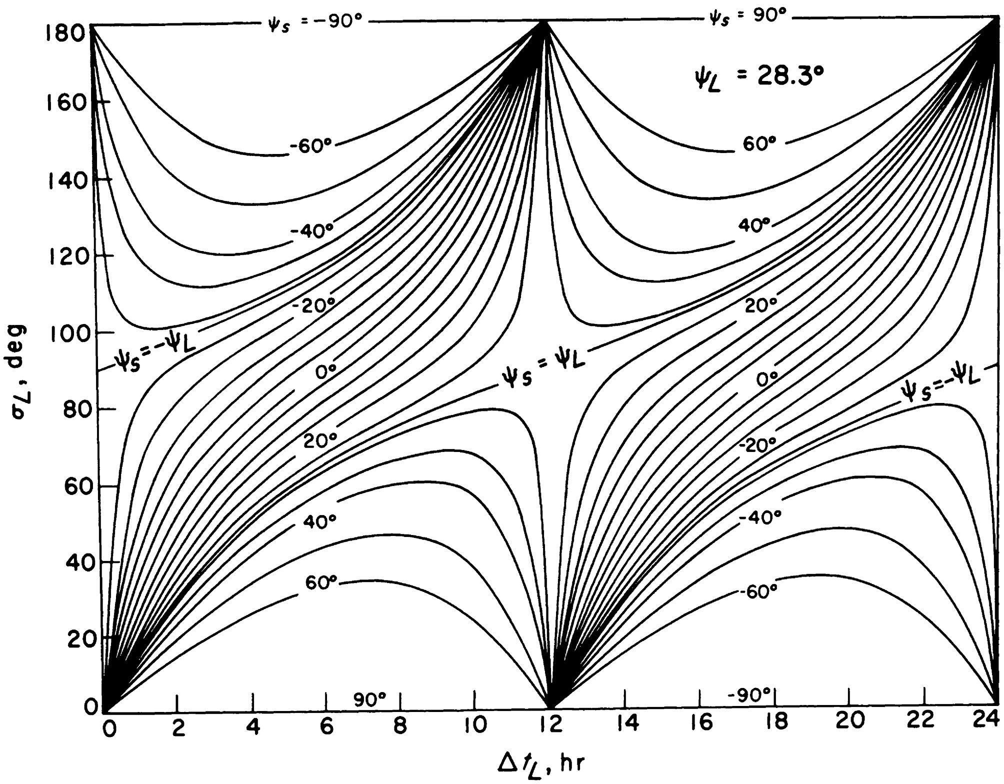


Fig. 5 Firing azimuth vs. launch time for symmetric situation

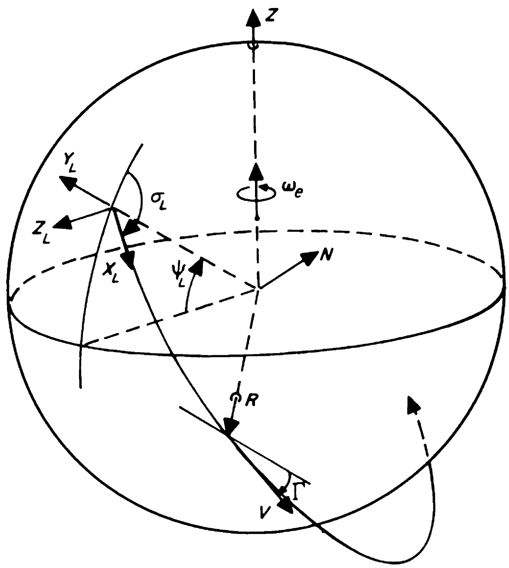


Fig. 6 Launch site and vehicle coordinates

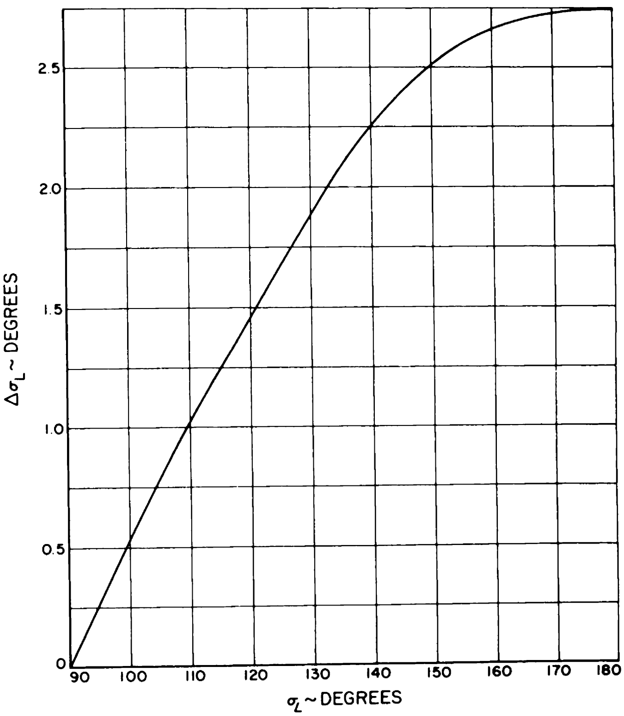


Fig. 7 Effect of Earth rotation upon firing azimuth