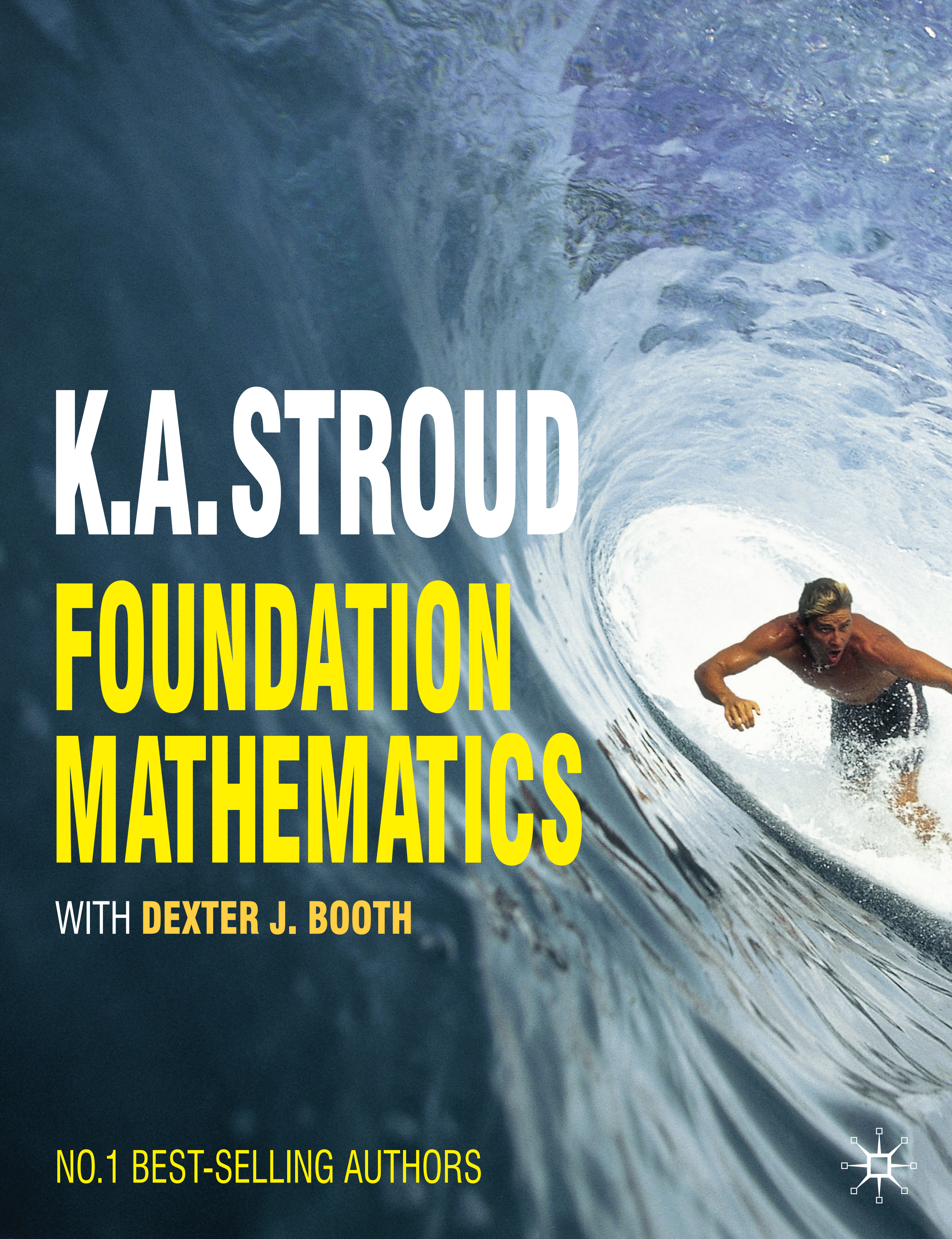

K.A. STROUD

FOUNDATION MATHEMATICS

WITH **DEXTER J. BOOTH**

NO.1 BEST-SELLING AUTHORS

FOUNDATION MATHEMATICS

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Summary of contents

Contents

Preface

It is now nearly 40 years since Ken Stroud first developed his approach to personalized learning with his classic text *Engineering Mathematics*, now in its sixth edition and having sold over half a million copies. That unique and hugely successful programmed learning style is exemplified in this text and I am delighted to have been asked to contribute to it. I have endeavoured to retain the very essence of his style, particularly the time-tested Stroud format with its close attention to technique development throughout. This style has, over the years, contributed significantly to the mathematical abilities of so many students all over the world.

Student readership

Over recent years there have been many developments in a wide range of university disciplines. This has led to an increase in the number of courses that require knowledge of mathematics to enable students to participate in their studies with confidence. Also, by widening access to these courses, many students arrive at university with a need to refresh and amplify the mathematical knowledge that they have previously acquired. This book was written with just those students in mind, starting as it does from the very beginnings of the subject. Indeed, the content of the book ranges from the earliest elements of arithmetic to differential and integral calculus. The material is presented in a manner that will be appreciated by those students requiring to review and extend their current mathematical abilities from a level of low confidence to one of confident proficiency.

Acknowledgements

This is a further opportunity that I have had to work on the Stroud books. It is as ever a challenge and an honour to be able to work with Ken Stroud's material. Ken had an understanding of his students and their learning and thinking processes which was second to none, and this is reflected in every page of this book. As always, my thanks go to the Stroud family for their continuing support for and encouragement of new projects and ideas which are allowing Ken's hugely successful teaching methodology to be offered to a whole new range of students. Finally, I should like to thank the entire production team at Palgrave Macmillan for all their care, and principally my editor Helen Bugler whose dedication and professionalism are an inspiration to all who work with her.

Huddersfield Dexter J Booth
February 2009

How to use this book

This book contains nineteen **Modules**, each module consisting of a number of **Units**. In total there are 62 Units and each one has been written in a way that makes learning more effective and more interesting. It is like having a personal tutor because you proceed at your own rate of learning and any difficulties you may have are cleared before you have the chance to practise incorrect ideas or techniques.

You will find that each Unit is divided into numbered sections called **Frames**. When you start a Unit, begin at Frame 1. Read each frame carefully and carry out any instructions or exercise you are asked to do. In almost every frame, you are required to make a response of some kind, so have your pen and paper at the ready to test your understanding of the information in the frame. You can immediately compare your answer and how you arrived at it with the correct answer and the working given in the frame that follows. To obtain the greatest benefit, you are strongly advised to cover up the following frame until you have made your response. When a series of dots occurs, you are expected to supply the missing word, phrase, number or mathematical expression. At every stage you will be guided along the right path. There is no need to hurry: read the frames carefully and follow the directions exactly. In this way you must learn.

Each Module opens with a list of **Learning outcomes** that specify exactly what you will learn by studying the contents of the Module. The material is then presented in a number of short **Units**, each designed to be studied in a single sitting. At the end of each Unit there is a **Review summary** of the topics in the Unit. Next follows a **Review exercise** of questions that directly test your understanding of the Unit material and which comes complete with worked solutions. Finally, a **Review test** enables you to consolidate your learning of the Unit material. You are strongly recommended to study the material in each Unit in a single sitting so as to ensure that you cover a complete set of ideas without a break.

Each Module ends with a checklist of **Can You?** questions that matches the Learning outcomes at the beginning of the Module, and enables you to rate your success in having achieved them. If you feel sufficiently confident then tackle the short **Test exercise** that follows. Just like the Review tests, the Test exercise is set directly on what you have learned in the Module: the questions are straightforward and contain no tricks. To provide you with the necessary practice, a set of **Further problems** is also included: do as many of the these problems as you can. Remember that in mathematics, as in many other situations, practice makes perfect – or nearly so.

Useful background information

Symbols used in the text

$=$	is equal to	$\to$	tends to
$\approx$	is approximately equal to	$\neq$	is not equal to
$>$	is greater than	$\equiv$	is identical to
$\geq$	is greater than or equal to	$<$	is less than
$n!$	factorial $n = 1 \times 2 \times 3 \times \ldots \times n$	$\leq$	is less than or equal to
$\lvert k \rvert$	modulus of k, i.e. size of k irrespective of sign	∞	infinity
$\sum$	summation	$\underset{n\to\infty}{Lim}$	limiting value as $n \to \infty$

Useful mathematical information

1 Algebraic identities

$(a+b)^2 = a^2 + 2ab + b^2 \qquad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^2 = a^2 - 2ab + b^2 \qquad (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

$a^2 - b^2 = (a-b)(a+b) \qquad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

2 Trigonometrical identities

(a) $\sin^2\theta + \cos^2\theta = 1$; $\sec^2\theta = 1 + \tan^2\theta$; $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

(b) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

(c) Let $A = B = \theta \quad \therefore \quad \sin 2\theta = 2\sin\theta\cos\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

(d) Let $\theta = \frac{\phi}{2}$ $\therefore$ $\sin\phi = 2\sin\frac{\phi}{2}\cos\frac{\phi}{2}$

$$\cos\phi = \cos^2\frac{\phi}{2} - \sin^2\frac{\phi}{2} = 1 - 2\sin^2\frac{\phi}{2} = 2\cos^2\frac{\phi}{2} - 1$$

$$\tan\phi = \frac{2\tan\frac{\phi}{2}}{1 - 2\tan^2\frac{\phi}{2}}$$

(e) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos D - \cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

(f) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

(g) Negative angles: $\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

(h) Angles having the same trigonometrical ratios:

(i) Same sine: θ and $(180° - \theta)$

(ii) Same cosine: θ and $(360° - \theta)$, i.e. $(-\theta)$

(iii) Same tangent: θ and $(180° + \theta)$

(i) $a\sin\theta + b\cos\theta = A\sin(\theta + \alpha)$

$a\sin\theta - b\cos\theta = A\sin(\theta - \alpha)$

$a\cos\theta + b\sin\theta = A\cos(\theta - \alpha)$

$a\cos\theta - b\sin\theta = A\cos(\theta + \alpha)$

where $\begin{cases} A = \sqrt{a^2 + b^2} \\ \alpha = \tan^{-1}\frac{b}{a} \quad (0° < \alpha < 90°) \end{cases}$

3 Standard curves

(a) *Straight line*

Slope, $m = \frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1}$

Angle between two lines, $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

For parallel lines, $m_2 = m_1$

For perpendicular lines, $m_1 m_2 = -1$

Equation of a straight line (slope $= m$)

(i) Intercept c on real y-axis: $y = mx + c$

(ii) Passing through (x_1, y_1): $y - y_1 = m(x - x_1)$

(iii) Joining (x_1, y_1) and (x_2, y_2): $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

(b) *Circle*

Centre at origin, radius r: $x^2 + y^2 = r^2$

Centre (h, k), radius r: $(x - h)^2 + (y - k)^2 = r^2$

General equation: $x^2 + y^2 + 2gx + 2fy + c = 0$

with centre $(-g, -f)$: radius $= \sqrt{g^2 + f^2 - c}$

Parametric equations: $x = r\cos\theta,\ y = r\sin\theta$

(c) *Parabola*

Vertex at origin, focus $(a, 0)$: $y^2 = 4ax$

Parametric equations: $x = at^2,\ y = 2at$

(d) *Ellipse*

Centre at origin, foci $\left(\pm\sqrt{a^2 + b^2}, 0\right)$: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

where $a =$ semi-major axis, $b =$ semi-minor axis

Parametric equations: $x = a\cos\theta,\ y = b\sin\theta$

(e) *Hyperbola*

Centre at origin, foci $\left(\pm\sqrt{a^2 + b^2}, 0\right)$: $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Parametric equations: $x = a\sec\theta,\ y = b\tan\theta$

Rectangular hyperbola:

Centre at origin, vertex $\pm\left(\dfrac{a}{\sqrt{2}}, \dfrac{a}{\sqrt{2}}\right)$: $xy = \dfrac{a^2}{2} = c^2$

i.e. $xy = c^2$ where $c = \dfrac{a}{\sqrt{2}}$

Parametric equations: $x = ct,\ y = c/t$

4 Laws of mathematics

(a) *Associative laws* – for addition and multiplication

$a + (b + c) = (a + b) + c$

$a(bc) = (ab)c$

(b) *Commutative laws* – for addition and multiplication

$a + b = b + a$

$ab = ba$

(c) *Distributive laws* – for multiplication and division

$a(b + c) = ab + ac$

$\dfrac{b + c}{a} = \dfrac{b}{a} + \dfrac{c}{a}$ (provided $a \neq 0$)

Module 1

Arithmetic

Learning outcomes

When you have completed this Module you will be able to:

- Carry out the basic rules of arithmetic with integers
- Check the result of a calculation making use of rounding
- Write a natural number as a product of prime numbers
- Find the highest common factor and lowest common multiple of two natural numbers
- Manipulate fractions, ratios and percentages
- Manipulate decimal numbers
- Manipulate powers
- Use standard or preferred standard form and complete a calculation to the required level of accuracy
- Understand the construction of various number systems and convert from one number system to another.

Units

Types of numbers

Unit 1

1 The natural numbers

The first numbers we ever meet are the *whole numbers*, also called the *natural numbers*, and these are written down using *numerals*.

Numerals and place value

The *whole numbers* or *natural numbers* are written using the ten numerals 0, 1, . . ., 9 where the position of a numeral dictates the value that it represents. For example:

246 stands for 2 hundreds and 4 tens and 6 units. That is $200 + 40 + 6$

Here the numerals 2, 4 and 6 are called the hundreds, tens and unit *coefficients* respectively. This is the place value principle.

Points on a line and order

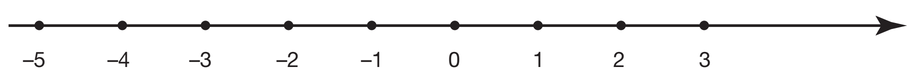
The natural numbers can be represented by equally spaced points on a straight line where the first natural number is zero 0.

The natural numbers are ordered – they progress from small to large. As we move along the line from left to right the numbers increase as indicated by the arrow at the end of the line. On the line, numbers to the left of a given number are *less than* ($<$) the given number and numbers to the right are *greater than* ($>$) the given number. For example, $8 > 5$ because 8 is represented by a point on the line to the right of 5. Similarly, $3 < 6$ because 3 is to the left of 6.

Now move on to the next frame

2 The integers

If the straight line displaying the natural numbers is extended to the left we can plot equally spaced points to the left of zero.



These points represent *negative* numbers which are written as the natural number preceded by a minus sign, for example -4. These positive and negative whole numbers and zero are collectively called the *integers*. The notion of order still applies. For example, $-5 < 3$ and $-2 > -4$ because the point on the line representing -5 is to the *left* of the point representing 3. Similarly, -2 is to the *right* of -4.

The numbers -10, 4, 0, -13 are of a type called . . . .

You can check your answer in the next frame

3

Integers

They are integers. The natural numbers are all positive. Now try this:

Place the appropriate symbol $<$ or $>$ between each of the following pairs of numbers:

(a) $-3 \quad -6$
(b) $2 \quad -4$
(c) $-7 \quad 12$

Complete these and check your results in the next frame

4

(a) $-3 > -6$
(b) $2 > -4$
(c) $-7 < 12$

The reasons being:

(a) $-3 > -6$ because -3 is represented on the line to the *right* of -6
(b) $2 > -4$ because 2 is represented on the line to the *right* of -4
(c) $-7 < 12$ because -7 is represented on the line to the *left* of 12

Now move on to the next frame

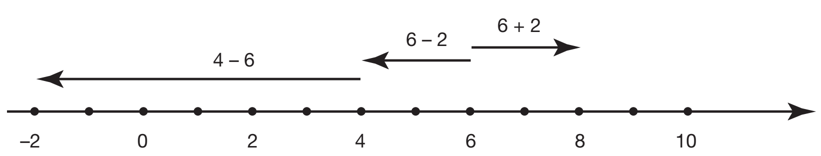
5

Brackets

Brackets should be used around negative numbers to separate the minus sign attached to the number from the arithmetic operation symbol. For example, $5 - -3$ should be written $5 - (-3)$ and 7×-2 should be written $7 \times (-2)$. *Never write two arithmetic operation symbols together without using brackets.*

Addition and subtraction

Adding two numbers gives their *sum* and subtracting two numbers gives their *difference*. For example, $6 + 2 = 8$. Adding moves to the right of the first number and subtracting moves to the left of the first number, so that $6 - 2 = 4$ and $4 - 6 = -2$:



Adding a negative number is the same as subtracting its positive counterpart. For example $7 + (-2) = 7 - 2$. The result is 5. Subtracting a negative number is the same as adding its positive counterpart. For example $7 - (-2) = 7 + 2 = 9$.

So what is the value of:

(a) $8 + (-3)$
(b) $9 - (-6)$
(c) $(-4) + (-8)$
(d) $(-14) - (-7)$?

When you have finished these check your results with the next frame

6

(a)	5
(b)	15
(c)	-12
(d)	-7

Move now to Frame 7

7

Multiplication and division

Multiplying two numbers gives their *product* and dividing two numbers gives their *quotient*. Multiplying and dividing two positive or two negative numbers gives a positive number. For example:

$12 \times 2 = 24$ and $(-12) \times (-2) = 24$
$12 \div 2 = 6$ and $(-12) \div (-2) = 6$

Multiplying or dividing a positive number by a negative number gives a negative number. For example:

$12 \times (-2) = -24$, $(-12) \div 2 = -6$ and $8 \div (-4) = -2$

So what is the value of:

(a) $(-5) \times 3$
(b) $12 \div (-6)$
(c) $(-2) \times (-8)$
(d) $(-14) \div (-7)$?

When you have finished these check your results with the next frame

8

(a)	-15
(b)	-2
(c)	16
(d)	2

Move on to Frame 9

Brackets and precedence rules

9

Brackets and the precedence rules are used to remove ambiguity in a calculation. For example, $14 - 3 \times 4$ could be either:

$11 \times 4 = 44$ or $14 - 12 = 2$

depending on which operation is performed first.

To remove the ambiguity we rely on the precedence rules:

In any calculation involving all four arithmetic operations we proceed as follows:

(a) Working from the left evaluate divisions and multiplications as they are encountered;

this leaves a calculation involving just addition and subtraction.

(b) Working from the left evaluate additions and subtractions as they are encountered.

For example, to evaluate:

$4 + 5 \times 6 \div 2 - 12 \div 4 \times 2 - 1$

a first sweep from left to right produces:

$4 + 30 \div 2 - 3 \times 2 - 1$

a second sweep from left to right produces:

$4 + 15 - 6 - 1$

and a final sweep produces:

$19 - 7 = 12$

If the calculation contains brackets then these are evaluated first, so that:

$$\begin{aligned}(4 + 5 \times 6) \div 2 - 12 \div 4 \times 2 - 1 &= 34 \div 2 - 6 - 1\\ &= 17 - 7\\ &= 10\end{aligned}$$

This means that:

$$\begin{aligned}14 - 3 \times 4 &= 14 - 12\\ &= 2\end{aligned}$$

because, reading from the left we multiply before we subtract. Brackets must be used to produce the alternative result:

$$\begin{aligned}(14 - 3) \times 4 &= 11 \times 4\\ &= 44\end{aligned}$$

because the precedence rules state that brackets are evaluated first.

So that $34 + 10 \div (2 - 3) \times 5 =$ ....

Result in the next frame

10

$\boxed{-16}$

Because

$$\begin{aligned} 34 + 10 \div (2 - 3) \times 5 &= 34 + 10 \div (-1) \times 5 && \text{we evaluate the bracket first} \\ &= 34 + (-10) \times 5 && \text{by dividing} \\ &= 34 + (-50) && \text{by multiplying} \\ &= 34 - 50 && \text{finally we subtract} \\ &= -16 \end{aligned}$$

Notice that when brackets are used we can omit the multiplication signs and replace the division sign by a line, so that:

$5 \times (6 - 4)$ becomes $5(6 - 4)$

and

$(25 - 10) \div 5$ becomes $(25 - 10)/5$ or $\dfrac{25 - 10}{5}$

When evaluating expressions containing *nested* brackets the innermost brackets are evaluated first. For example:

$$\begin{aligned} 3(4 - 2[5 - 1]) &= 3(4 - 2 \times 4) && \text{evaluating the innermost bracket } [\ldots] \text{ first} \\ &= 3(4 - 8) && \text{multiplication before subtraction inside the } (\ldots) \text{ bracket} \\ &= 3(-4) && \text{subtraction completes the evaluation of the } (\ldots) \text{ bracket} \\ &= -12 && \text{multiplication completes the calculation} \end{aligned}$$

so that $5 - \{8 + 7[4 - 1] - 9/3\} = \ldots\ldots\ldots\ldots$

Work this out, the result is in the following frame

11

$\boxed{-21}$

Because

$$\begin{aligned} 5 - \{8 + 7[4 - 1] - 9/3\} &= 5 - \{8 + 7 \times 3 - 9 \div 3\} \\ &= 5 - \{8 + 21 - 3\} \\ &= 5 - \{29 - 3\} \\ &= 5 - 26 \\ &= -21 \end{aligned}$$

Now move to Frame 12

Basic laws of arithmetic

12

All the work that you have done so far has been done under the assumption that you know the rules that govern the use of arithmetic operations as, indeed, you no doubt do. However, there is a difference between knowing the rules innately and being consciously aware of them, so here they are. The four basic arithmetic operations are:

addition and subtraction
multiplication and division

where each pair may be regarded as consisting of 'opposites' – in each pair one operation is the reverse operation of the other.

1 Commutativity

Two integers can be added or multiplied in either order without affecting the result. For example:

$5 + 8 = 8 + 5 = 13$ and $5 \times 8 = 8 \times 5 = 40$

We say that addition and multiplication are commutative operations

The order in which two integers are subtracted or divided *does* affect the result. For example:

$4 - 2 \neq 2 - 4$ because $4 - 2 = 2$ and $2 - 4 = -2$

Notice that $\neq$ means *is not equal to*. Also

$4 \div 2 \neq 2 \div 4$

We say that subtraction and division are not commutative operations

2 Associativity

The way in which three or more integers are associated under addition or multiplication does not affect the result. For example:

$3 + (4 + 5) = (3 + 4) + 5 = 3 + 4 + 5 = 12$

and

$3 \times (4 \times 5) = (3 \times 4) \times 5 = 3 \times 4 \times 5 = 60$

We say that addition and multiplication are associative operations

The way in which three or more integers are associated under subtraction or division does affect the result. For example:

$3 - (4 - 5) \neq (3 - 4) - 5$ because
$3 - (4 - 5) = 3 - (-1) = 3 + 1 = 4$ and $(3 - 4) - 5 = (-1) - 5 = -6$

Also

$24 \div (4 \div 2) \neq (24 \div 4) \div 2$ because
$24 \div (4 \div 2) = 24 \div 2 = 12$ and $(24 \div 4) \div 2 = 6 \div 2 = 3$

We say that subtraction and division are not associative operations

3 Distributivity

Multiplication is distributed over addition and subtraction from both the left and the right. For example:

$$3 \times (4 + 5) = (3 \times 4) + (3 \times 5) = 27 \text{ and } (3 + 4) \times 5 = (3 \times 5) + (4 \times 5) = 35$$

$$3 \times (4 - 5) = (3 \times 4) - (3 \times 5) = -3 \text{ and } (3 - 4) \times 5 = (3 \times 5) - (4 \times 5) = -5$$

Division is distributed over addition and subtraction from the right but not from the left. For example:

$(60 + 15) \div 5 = (60 \div 5) + (15 \div 5)$ because

$(60 + 15) \div 5 = 75 \div 5 = 15$ and $(60 \div 5) + (15 \div 5) = 12 + 3 = 15$

However, $60 \div (15 + 5) \neq (60 \div 15) + (60 \div 5)$ because

$60 \div (15 + 5) = 60 \div 20 = 3$ and $(60 \div 15) + (60 \div 5) = 4 + 12 = 16$

Also:

$(20 - 10) \div 5 = (20 \div 5) - (10 \div 5)$ because

$(20 - 10) \div 5 = 10 \div 5 = 2$ and $(20 \div 5) - (10 \div 5) = 4 - 2 = 2$

but $20 \div (10 - 5) \neq (20 \div 10) - (20 \div 5)$ because

$20 \div (10 - 5) = 20 \div 5 = 4$ and $(20 \div 10) - (20 \div 5) = 2 - 4 = -2$

On now to Frame 13

13 Estimating

Arithmetic calculations are easily performed using a calculator. However, by pressing a wrong key, wrong answers can just as easily be produced. Every calculation made using a calculator should at least be checked for the reasonableness of the final result and this can be done by *estimating* the result using *rounding*. For example, using a calculator the sum $39 + 53$ is incorrectly found to be 62 if $39 + 23$ is entered by mistake. If, now, 39 is rounded up to 40, and 53 is rounded down to 50 the reasonableness of the calculator result can be simply checked by adding 40 to 50 to give 90. This indicates that the answer 62 is wrong and that the calculation should be done again. The correct answer 92 is then seen to be close to the approximation of 90.

Rounding

An integer can be rounded to the nearest 10 as follows:

If the number is less than halfway to the next multiple of 10 then the number is rounded *down* to the previous multiple of 10. For example, 53 is rounded down to 50.

If the number is more than halfway to the next multiple of 10 then the number is rounded *up* to the next multiple of 10. For example, 39 is rounded up to 40.

If the number is exactly halfway to the next multiple of 10 then the number is rounded *up*. For example, 35 is rounded up to 40.

This principle also applies when rounding to the nearest 100, 1000, 10 000 or more. For example, 349 rounds up to 350 to the nearest 10 but rounds down to 300 to the nearest 100, and 2501 rounds up to 3000 to the nearest 1000.

Try rounding each of the following to the nearest 10, 100 and 1000 respectively:

(a) 1846
(b) -638
(c) 445

Finish all three and check your results with the next frame

14

(a) 1850, 1800, 2000
(b) -640, -600, -1000
(c) 450, 400, 0

Because

(a) 1846 is nearer to 1850 than to 1840, nearer to 1800 than to 1900 and nearer to 2000 than to 1000.

(b) -638 is nearer to -640 than to -630, nearer to -600 than to -700 and nearer to -1000 than to 0. The negative sign does not introduce any complications.

(c) 445 rounds to 450 because it is halfway to the next multiple of 10, 445 is nearer to 400 than to 500 and nearer to 0 than 1000.

How about estimating each of the following using rounding to the nearest 10:

(a) $18 \times 21 - 19 \div 11$
(b) $99 \div 101 - 49 \times 8$

Check your results in Frame 15

15

(a) 398
(b) -499

Because

(a) $18 \times 21 - 19 \div 11$ rounds to $20 \times 20 - 20 \div 10 = 398$
(b) $99 \div 101 - 49 \times 8$ rounds to $100 \div 100 - 50 \times 10 = -499$

At this point let us pause and summarize the main facts so far on types of numbers

Review summary

Unit 1

16

1 The integers consist of the positive and negative whole numbers and zero.

2 The integers are ordered so that they range from large negative to small negative through zero to small positive and then large positive. They are written using the ten numerals 0 to 9 according to the principle of place value where the place of a numeral in a number dictates the value it represents.

3 The integers can be represented by equally spaced points on a line.

4 The four arithmetic operations of addition, subtraction, multiplication and division obey specific precedence rules that govern the order in which they are to be executed:

In any calculation involving all four arithmetic operations we proceed as follows:

(a) *working from the left evaluate divisions and multiplications as they are encountered.*

This leaves an expression involving just addition and subtraction:

(b) *working from the left evaluate additions and subtractions as they are encountered.*

5 Multiplying or dividing two positive numbers or two negative numbers produces a positive number. Multiplying or dividing a positive number and a negative number produces a negative number.

6 Brackets are used to group numbers and operations together. In any arithmetic expression, the contents of brackets are evaluated first.

7 Integers can be rounded to the nearest 10, 100 etc. and the rounded values used as estimates for the result of a calculation.

Review exercise

Unit 1

17

1 Place the appropriate symbol $<$ or $>$ between each of the following pairs of numbers:

(a) $-1 \quad -6$ (b) $5 \quad -29$ (c) $-14 \quad 7$

2 Find the value of each of the following:

(a) $16 - 12 \times 4 + 8 \div 2$

(b) $(16 - 12) \times (4 + 8) \div 2$

(c) $9 - 3(17 + 5[5 - 7])$

(d) $8(3[2 + 4] - 2[5 + 7])$

3 Show that:

(a) $6 - (3 - 2) \neq (6 - 3) - 2$

(b) $100 \div (10 \div 5) \neq (100 \div 10) \div 5$

(c) $24 \div (2 + 6) \neq (24 \div 2) + (24 \div 6)$

(d) $24 \div (2 - 6) \neq (24 \div 2) - (24 \div 6)$

4 Round each number to the nearest 10, 100 and 1000:

(a) 2562 (b) 1500 (c) -3451 (d) $-14\,525$

Complete all four questions. Take your time, there is no need to rush.
If necessary, look back at the Unit.
The answers and working are in the next frame.

18

1 (a) $-1 > -6$ because -1 is represented on the line to the right of -6

(b) $5 > -29$ because 5 is represented on the line to the right of -29

(c) $-14 < 7$ because -14 is represented on the line to the left of 7

2 (a) $16 - 12 \times 4 + 8 \div 2 = 16 - 48 + 4 = 16 - 44 = -28$
divide and multiply before adding and subtracting

(b) $(16 - 12) \times (4 + 8) \div 2 = (4) \times (12) \div 2 = 4 \times 12 \div 2 = 4 \times 6 = 24$
brackets are evaluated first

(c)
$$\begin{aligned} 9 - 3(17 + 5[5 - 7]) &= 9 - 3(17 + 5[-2]) \\ &= 9 - 3(17 - 10) \\ &= 9 - 3(7) \\ &= 9 - 21 = -12 \end{aligned}$$

(d)
$$\begin{aligned} 8(3[2 + 4] - 2[5 + 7]) &= 8(3 \times 6 - 2 \times 12) \\ &= 8(18 - 24) \\ &= 8(-6) = -48 \end{aligned}$$

3 (a) Left-hand side (LHS) $= 6 - (3 - 2) = 6 - (1) = 5$
Right-hand side (RHS) $= (6 - 3) - 2 = (3) - 2 = 1 \neq$ LHS

(b) Left-hand side (LHS) $= 100 \div (10 \div 5) = 100 \div 2 = 50$
Right-hand side (RHS) $= (100 \div 10) \div 5 = 10 \div 5 = 2 \neq$ LHS

(c) Left-hand side (LHS) $= 24 \div (2 + 6) = 24 \div 8 = 3$
Right-hand side (RHS) $= (24 \div 2) + (24 \div 6) = 12 + 4 = 16 \neq$ LHS

(d) Left-hand side (LHS) $= 24 \div (2 - 6) = 24 \div (-4) = -6$
Right-hand side (RHS) $= (24 \div 2) - (24 \div 6) = 12 - 4 = 8 \neq$ LHS

4 (a) 2560, 2600, 3000

(b) 1500, 1500, 2000

(c) -3450, -3500, -3000

(d) $-14\,530$, $-14\,500$, $-15\,000$

Now on to the Unit test

Review test

Unit 1

19

1 Place the appropriate symbol < or > between each of the following pairs of numbers:

(a) $-3 \quad -2$ (b) $8 \quad -13$ (c) $-25 \quad 0$

2 Find the value of each of the following:

(a) $13 + 9 \div 3 - 2 \times 5$ (b) $(13 + 9) \div (3 - 2) \times 5$

3 Round each number to the nearest 10, 100 and 1000:

(a) 1354 (b) 2501 (c) -2452 (d) $-23\,625$

Factors and prime numbers

Unit 2

1

Factors

Any pair of natural numbers are called *factors* of their product. For example, the numbers 3 and 6 are factors of 18 because $3 \times 6 = 18$. These are not the only factors of 18. The complete collection of factors of 18 is 1, 2, 3, 6, 9, 18 because

$$\begin{aligned} 18 &= 1 \times 18 \\ &= 2 \times 9 \\ &= 3 \times 6 \end{aligned}$$

So the factors of:

(a) 12
(b) 25
(c) 17 are . . . .

The results are in the next frame

2

(a) 1, 2, 3, 4, 6, 12
(b) 1, 5, 25
(c) 1, 17

Because

(a) $12 = 1 \times 12 = 2 \times 6 = 3 \times 4$
(b) $25 = 1 \times 25 = 5 \times 5$
(c) $17 = 1 \times 17$

Now move to the next frame

3

Prime numbers

If a natural number has only two factors which are itself and the number 1, the number is called a *prime number*. The first six prime numbers are 2, 3, 5, 7, 11 and 13. The number 1 is *not* a prime number because it only has one factor, namely, itself.

Prime factorization

Every natural number can be written as a product involving only prime factors. For example, the number 126 has the factors 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63 and 126, of which 2, 3 and 7 are prime numbers and 126 can be written as:

$$126 = 2 \times 3 \times 3 \times 7$$

To obtain this *prime factorization* the number is divided by successively increasing prime numbers thus:

$$\begin{array}{r|r} 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \quad \text{so that } 126 = 2 \times 3 \times 3 \times 7$$

Notice that a prime factor may occur more than once in a prime factorization.

Now find the prime factorization of:

(a) 84
(b) 512

Work these two out and check the working in Frame 4

4

(a) $84 = 2 \times 2 \times 3 \times 7$
(b) $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Because

(a)
$$\begin{array}{r|r} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \quad \text{so that } 84 = 2 \times 2 \times 3 \times 7$$

(b) The only prime factor of 512 is 2 which occurs 9 times. The prime factorization is:

$$512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

Move to Frame 5

5 Highest common factor (HCF)

The *highest common factor* (HCF) of two natural numbers is the largest factor that they have in common. For example, the prime factorizations of 144 and 66 are:

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$66 = 2 \qquad\qquad \times 3 \quad \times 11$$

Only the 2 and the 3 are common to both factorizations and so the highest factor that these two numbers have in common (HCF) is $2 \times 3 = 6$.

Lowest common multiple (LCM)

The smallest natural number that each one of a pair of natural numbers divides into a whole number of times is called their *lowest common multiple* (LCM). This is also found from the prime factorization of each of the two numbers. For example:

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$66 = 2 \qquad\qquad \times 3 \quad \times 11$$
$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 11 = 1584$$

The HCF and LCM of 84 and 512 are ....

6

$$\boxed{\begin{array}{l}\text{HCF: } 4 \\ \text{LCM: } 10\,752\end{array}}$$

Because

84 and 512 have the prime factorizations:

$$84 = 2 \times 2 \qquad\qquad\qquad\qquad\qquad \times 3 \times 7$$
$$512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \qquad\qquad \text{HCF} = 2 \times 2 = 4$$
$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 10\,752$$

At this point let us pause and summarize the main facts on factors and prime numbers

Review summary

Unit 2

7

1 A pair of natural numbers are called factors of their product.

2 If a natural number only has one and itself as factors it is called a prime number.

3 Every natural number can be written as a product of its prime factors, some of which may be repeated.

4 The highest common factor (HCF) is the highest factor that two natural numbers have in common.

5 The lowest common multiple (LCM) is the lowest natural number that two natural numbers will divide into a whole number of times.

Review exercise

Unit 2

8

1 Write each of the following as a product of prime factors:

(a) 429 (b) 1820 (c) 2992 (d) 3185

2 Find the HCF and the LCM of each pair of numbers:

(a) 63, 42 (b) 92, 34

Complete both questions. Work through Unit 2 again if you need to.
Don't rush. Take your time.
The answers and working are in the next frame.

9

1 (a)

$$\begin{array}{r|r} 3 & 429 \\ \hline 11 & 143 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$429 = 3 \times 11 \times 13$

(b)

$$\begin{array}{r|r} 2 & 1820 \\ \hline 2 & 910 \\ \hline 5 & 455 \\ \hline 7 & 91 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$1820 = 2 \times 2 \times 5 \times 7 \times 13$

(c)

$$\begin{array}{r|r}
2 & 2992 \\ \hline
2 & 1496 \\ \hline
2 & 748 \\ \hline
2 & 374 \\ \hline
11 & 187 \\ \hline
17 & 17 \\ \hline
 & 1 \\ \hline
\end{array} \qquad 2992 = 2 \times 2 \times 2 \times 2 \times 11 \times 17$$

(d)

$$\begin{array}{r|r}
5 & 3185 \\ \hline
7 & 637 \\ \hline
7 & 91 \\ \hline
13 & 13 \\ \hline
 & 1 \\ \hline
\end{array} \qquad 3185 = 5 \times 7 \times 7 \times 13$$

2 (a) The prime factorizations of 63 and 42 are:

$$\begin{aligned}
63 &= \phantom{2 \times{}} 3 \times 3 \times 7 \\
42 &= 2 \times 3 \phantom{{}\times 3} \times 7 \qquad \text{HCF } 3 \times 7 = 21 \\
\text{LCM} &= 2 \times 3 \times 3 \times 7 = 126
\end{aligned}$$

(b) The prime factorizations of 34 and 92 are:

$$\begin{aligned}
34 &= 2 \phantom{{}\times 2} \times 17 \\
92 &= 2 \times 2 \phantom{{}\times 17} \times 23 \qquad \text{HCF } 2 \\
\text{LCM} &= 2 \times 2 \times 17 \times 23 = 1564
\end{aligned}$$

Now for the Review test

Review test

Unit 2

10

1 Write each of the following as a product of prime factors:

(a) 170 (b) 455 (c) 9075 (d) 1140

2 Find the HCF and the LCM of each pair of numbers:

(a) 84, 88 (b) 105, 66

Fractions, ratios and percentages Unit 3

Division of integers

1

A fraction is a number which is represented by one integer – the *numerator* – divided by another integer – the *denominator* (or the *divisor*). For example, $\frac{3}{5}$ is a fraction with numerator 3 and denominator 5. Because fractions are written as one integer divided by another – a *ratio* – they are called *rational* numbers. Fractions are either *proper*, *improper* or *mixed*:

- in a proper fraction the numerator is less than the denominator, for example $\dfrac{4}{7}$
- in an improper fraction the numerator is greater than the denominator, for example $\dfrac{12}{5}$
- a mixed fraction is in the form of an integer and a fraction, for example $6\frac{2}{3}$

So that $-\dfrac{8}{11}$ is a .... fraction?

The answer is in the next frame

2

$\boxed{\text{Proper}}$

Fractions can be either positive or negative.

Now to the next frame

Multiplying fractions

3

Two fractions are multiplied by multiplying their respective numerators and denominators independently. For example:

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$$

Try this one for yourself. $\dfrac{5}{9} \times \dfrac{2}{7} =$ ....

4

$$\boxed{\frac{10}{63}}$$

Because

$$\frac{5}{9} \times \frac{2}{7} = \frac{5 \times 2}{9 \times 7} = \frac{10}{63}$$

Correct? Then on to the next frame

5 Of

The word 'of' when interposed between two fractions means multiply. For example:

Half of half a cake is one-quarter of a cake. That is

$$\frac{1}{2} \text{ of } \frac{1}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$$

So that, for example:

$$\frac{1}{3} \text{ of } \frac{2}{5} = \frac{1}{3} \times \frac{2}{5} = \frac{1 \times 2}{3 \times 5} = \frac{2}{15}$$

So that $\frac{3}{8}$ *of* $\frac{5}{7} = \ldots\ldots\ldots\ldots$

6

$$\boxed{\frac{15}{56}}$$

Because

$$\frac{3}{8} \text{ of } \frac{5}{7} = \frac{3}{8} \times \frac{5}{7} = \frac{3 \times 5}{8 \times 7} = \frac{15}{56}$$

On now to the next frame

7 Equivalent fractions

Multiplying the numerator and denominator by the same number is equivalent to multiplying the fraction by unity, that is by 1:

$$\frac{4 \times 3}{5 \times 3} = \frac{4}{5} \times \frac{3}{3} = \frac{4}{5} \times 1 = \frac{4}{5}$$

Now, $\frac{4 \times 3}{5 \times 3} = \frac{12}{15}$ so that the fraction $\frac{4}{5}$ and the fraction $\frac{12}{15}$ *both represent the same number* and for this reason we call $\frac{4}{5}$ and $\frac{12}{15}$ *equivalent fractions*.

A second fraction, equivalent to a first fraction, can be found by multiplying the numerator and the denominator of the first fraction by the same number.

So that if we multiply the numerator and denominator of the fraction $\frac{7}{5}$ by 4 we obtain the equivalent fraction $\ldots\ldots\ldots\ldots$

Check your result in Frame 8

8

$$\boxed{\frac{28}{20}}$$

Because

$$\frac{7 \times 4}{5 \times 4} = \frac{28}{20}$$

We can reverse this process and find the equivalent fraction that has the smallest numerator by *cancelling out* common factors. This is known as reducing the fraction to its *lowest terms*. For example:

$\frac{16}{96}$ can be reduced to its lowest terms as follows:

$$\frac{16}{96} = \frac{4 \times 4}{24 \times 4} = \frac{4 \times \cancel{4}}{24 \times \cancel{4}} = \frac{4}{24}$$

by cancelling out the 4 in the numerator and the denominator

The fraction $\frac{4}{24}$ can also be reduced:

$$\frac{4}{24} = \frac{4}{6 \times 4} = \frac{\cancel{4}}{6 \times \cancel{4}} = \frac{1}{6}$$

Because $\frac{1}{6}$ cannot be reduced further we see that $\frac{16}{96}$ reduced to its lowest terms is $\frac{1}{6}$.

How about this one? The fraction $\frac{84}{108}$ reduced to its lowest terms is . . . .

Check with the next frame

9

$$\boxed{\frac{7}{9}}$$

Because

$$\frac{84}{108} = \frac{7 \times 3 \times 4}{9 \times 3 \times 4} = \frac{7 \times \cancel{3} \times \cancel{4}}{9 \times \cancel{3} \times \cancel{4}} = \frac{7}{9}$$

Now move on to the next frame

10 Dividing fractions

The expression $6 \div 3$ means the number of 3's in 6, which is 2. Similarly, the expression $1 \div \frac{1}{4}$ means the number of $\frac{1}{4}$'s in 1, which is, of course, 4. That is:

$1 \div \frac{1}{4} = 4 = 1 \times \frac{4}{1}$. Notice how the numerator and the denominator of the divisor are switched and the division replaced by multiplication.

Two fractions are divided by switching the numerator and the denominator of the divisor and multiplying the result. For example:

$$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$$

So that $\frac{7}{13} \div \frac{3}{4} =$ ....

11

$$\boxed{\frac{28}{39}}$$

Because

$$\frac{7}{13} \div \frac{3}{4} = \frac{7}{13} \times \frac{4}{3} = \frac{28}{39}$$

In particular:

$$1 \div \frac{3}{5} = 1 \times \frac{5}{3} = \frac{5}{3}$$

The fraction $\frac{5}{3}$ is called the *reciprocal* of $\frac{3}{5}$

So that the reciprocal of $\frac{17}{4}$ is ....

12

$$\boxed{\frac{4}{17}}$$

Because

$$1 \div \frac{17}{4} = 1 \times \frac{4}{17} = \frac{4}{17}$$

And the reciprocal of -5 is ....

13

$$\boxed{-\frac{1}{5}}$$

Because

$$1 \div (-5) = 1 \div \left(-\frac{5}{1}\right) = 1 \times \left(-\frac{1}{5}\right) = -\frac{1}{5}$$

Move on to the next frame

Adding and subtracting fractions

14

Two fractions can only be added or subtracted immediately if they both possess the same denominator, in which case we add or subtract the numerators and divide by the common denominator. For example:

$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

If they do not have the same denominator they must be rewritten in equivalent form so that they do have the same denominator – called the *common denominator*. For example:

$$\frac{2}{3} + \frac{1}{5} = \frac{10}{15} + \frac{3}{15} = \frac{10+3}{15} = \frac{13}{15}$$

The common denominator of the equivalent fractions is the LCM of the two original denominators. That is:

$$\frac{2}{3} + \frac{1}{5} = \frac{2 \times 5}{3 \times 5} + \frac{1 \times 3}{5 \times 3} = \frac{10}{15} + \frac{3}{15} \text{ where 15 is the LCM of 3 and 5}$$

So that $\frac{5}{9} + \frac{1}{6} = \ldots\ldots\ldots\ldots$

The result is in Frame 15

15

$$\boxed{\frac{13}{18}}$$

Because

The LCM of 9 and 6 is 18 so that
$$\frac{5}{9} + \frac{1}{6} = \frac{5 \times 2}{9 \times 2} + \frac{1 \times 3}{6 \times 3} = \frac{10}{18} + \frac{3}{18}$$
$$= \frac{10+3}{18} = \frac{13}{18}$$

There's another one to try in the next frame

16

Now try $\frac{11}{15} - \frac{2}{3} = \ldots\ldots\ldots\ldots$

17

$$\boxed{\frac{1}{15}}$$

Because

$$\frac{11}{15}-\frac{2}{3}=\frac{11}{15}-\frac{2\times 5}{3\times 5}=\frac{11}{15}-\frac{10}{15}$$
$$=\frac{11-10}{15}=\frac{1}{15}\qquad\text{(15 is the LCM of 3 and 15)}$$

Correct? Then on to Frame 18

18

Fractions on a calculator

The $a^b\!/\!_c$ button on a calculator enables fractions to be entered and manipulated with the results given in fractional form. For example, to evaluate $\frac{2}{3}\times 1\frac{3}{4}$ using your calculator [*note*: your calculator may not produce the identical display in what follows]:

Enter the number 2
Press the $a^b\!/\!_c$ key
Enter the number 3

The display now reads 2 ⌟ 3 to represent $\frac{2}{3}$

Press the $\times$ key
Enter the number 1
Press the $a^b\!/\!_c$ key
Enter the number 3
Press the $a^b\!/\!_c$ key
Enter the number 4

The display now reads 1 ⌟ 3 ⌟ 4 to represent $1\frac{3}{4}$

Press the $=$ key to display the result 1 ⌟ 1 ⌟ 6 $= 1\frac{1}{6}$, that is:

$$\frac{2}{3}\times 1\frac{3}{4}=\frac{2}{3}\times\frac{7}{4}=\frac{14}{12}=1\frac{1}{6}$$

Now use your calculator to evaluate each of the following:

(a) $\frac{5}{7}+3\frac{2}{3}$

(b) $\dfrac{8}{3}-\dfrac{5}{11}$

(c) $\dfrac{13}{5}\times\dfrac{4}{7}-\dfrac{2}{9}$

(d) $4\frac{1}{11}\div\left(-\dfrac{3}{5}\right)+\dfrac{1}{8}$

Check your answers in Frame 19

19

(a) 4 ⌟ 8 ⌟ 21 = $4\frac{8}{21}$
(b) 2 ⌟ 7 ⌟ 33 = $2\frac{7}{33}$
(c) 1 ⌟ 83 ⌟ 315 = $1\frac{83}{315}$
(d) − 6 ⌟ 61 ⌟ 88 = $-6\frac{61}{88}$

In (d) enter the $\frac{3}{5}$ and then press the +/− key.

On now to the next frame

Ratios

20

If a whole number is separated into a number of fractional parts where each fraction has the same denominator, the numerators of the fractions form a *ratio*. For example, if a quantity of brine in a tank contains $\frac{1}{3}$ salt and $\frac{2}{3}$ water, the salt and water are said to be in the ratio 'one-to-two' – written 1 : 2.

What ratio do the components A, B and C form if a compound contains $\frac{3}{4}$ of A, $\frac{1}{6}$ of B and $\frac{1}{12}$ of C?

Take care here and check your results with Frame 21

21

9 : 2 : 1

Because the LCM of the denominators 4, 6 and 12 is 12, then:

$\frac{3}{4}$ of A is $\frac{9}{12}$ of A, $\frac{1}{6}$ of B is $\frac{2}{12}$ of B and the remaining $\frac{1}{12}$ is of C. This ensures that the components are in the ratio of their numerators. That is:

9 : 2 : 1

Notice that the sum of the numbers in the ratio is equal to the common denominator.

On now to the next frame

Percentages

22

A percentage is a fraction whose denominator is equal to 100. For example, if 5 out of 100 people are left-handed then the fraction of left-handers is $\frac{5}{100}$ which is written as 5%, that is 5 *per cent* (%).

So if 13 out of 100 cars on an assembly line are red, the percentage of red cars on the line is . . . .

23

$\boxed{13\%}$

Because

The fraction of cars that are red is $\frac{13}{100}$ which is written as 13%.

Try this. What is the percentage of defective resistors in a batch of 25 if 12 of them are defective?

24

$\boxed{48\%}$

Because

The fraction of defective resistors is $\frac{12}{25} = \frac{12 \times 4}{25 \times 4} = \frac{48}{100}$ which is written as 48%. Notice that this is the same as:

$$\left(\frac{12}{25} \times 100\right)\% = \left(\frac{12}{25} \times 25 \times 4\right)\% = (12 \times 4)\% = 48\%$$

A fraction can be converted to a percentage by multiplying the fraction by 100.

To find the percentage part of a quantity we multiply the quantity by the percentage written as a fraction. For example, 24% of 75 is:

$$\begin{aligned}24\% \textit{ of } 75 &= \frac{24}{100} \textit{ of } 75 = \frac{24}{100} \times 75 = \frac{6 \times 4}{25 \times 4} \times 25 \times 3 = \frac{6 \times \cancel{4}}{\cancel{25} \times \cancel{4}} \times \cancel{25} \times 3 \\ &= 6 \times 3 = 18\end{aligned}$$

So that 8% of 25 is ....

Work it through and check your results with the next frame

25

$\boxed{2}$

Because

$$\frac{8}{100} \times 25 = \frac{2 \times 4}{25 \times 4} \times 25 = \frac{2 \times \cancel{4}}{\cancel{25} \times \cancel{4}} \times \cancel{25} = 2$$

At this point let us pause and summarize the main facts on fractions, ratios and percentages

Review summary

Unit 3

26

1 A fraction is a number represented as one integer (the numerator) divided by another integer (the denominator or divisor).

2 The same number can be represented by different but equivalent fractions.

3 A fraction with no common factors other than unity in its numerator and denominator is said to be in its lowest terms.

4 Two fractions are multiplied by multiplying the numerators and denominators independently.

5 Two fractions can only be added or subtracted immediately when their denominators are equal.

6 A ratio consists of the numerators of fractions with identical denominators.

7 The numerator of a fraction whose denominator is 100 is called a percentage.

Review exercise

Unit 3

27

1 Reduce each of the following fractions to their lowest terms:

(a) $\dfrac{24}{30}$ (b) $\dfrac{72}{15}$ (c) $-\dfrac{52}{65}$ (d) $\dfrac{32}{8}$

2 Evaluate the following:

(a) $\dfrac{5}{9} \times \dfrac{2}{5}$ (b) $\dfrac{13}{25} \div \dfrac{2}{15}$ (c) $\dfrac{5}{9} + \dfrac{3}{14}$ (d) $\dfrac{3}{8} - \dfrac{2}{5}$

(e) $\dfrac{12}{7} \times \left(-\dfrac{3}{5}\right)$ (f) $\left(-\dfrac{3}{4}\right) \div \left(-\dfrac{12}{7}\right)$ (g) $\dfrac{19}{2} + \dfrac{7}{4}$ (h) $\dfrac{1}{4} - \dfrac{3}{8}$

3 Write the following proportions as ratios:

(a) $\dfrac{1}{2}$ of A, $\dfrac{2}{5}$ of B and $\dfrac{1}{10}$ of C

(b) $\dfrac{1}{3}$ of P, $\dfrac{1}{5}$ of Q, $\dfrac{1}{4}$ of R and the remainder S

4 Complete the following:

(a) $\dfrac{2}{5} =$ % (b) 58% of 25 =

(c) $\dfrac{7}{12} =$ % (d) 17% of 50 =

Complete the questions.
Look back at the Unit if necessary but don't rush.
The answers and working are in the next frame.

28

1 (a) $\frac{24}{30} = \frac{2 \times 2 \times 2 \times 3}{2 \times 3 \times 5} = \frac{2 \times 2}{5} = \frac{4}{5}$

(b) $\frac{72}{15} = \frac{2 \times 2 \times 2 \times 3 \times 3}{3 \times 5} = \frac{2 \times 2 \times 2 \times 3}{5} = \frac{24}{5}$

(c) $-\frac{52}{65} = -\frac{2 \times 2 \times 13}{5 \times 13} = -\frac{2 \times 2}{5} = -\frac{4}{5}$

(d) $\frac{32}{8} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = 4$

2 (a) $\frac{5}{9} \times \frac{2}{5} = \frac{5 \times 2}{9 \times 5} = \frac{2}{9}$

(b) $\frac{13}{25} \div \frac{2}{15} = \frac{13}{25} \times \frac{15}{2} = \frac{13 \times 15}{25 \times 2} = \frac{13 \times 3 \times 5}{5 \times 5 \times 2} = \frac{39}{10}$

(c) $\frac{5}{9} + \frac{3}{14} = \frac{5 \times 14}{9 \times 14} + \frac{3 \times 9}{14 \times 9} = \frac{70}{126} + \frac{27}{126} = \frac{70 + 27}{126} = \frac{97}{126}$

(d) $\frac{3}{8} - \frac{2}{5} = \frac{3 \times 5}{8 \times 5} - \frac{2 \times 8}{5 \times 8} = \frac{15}{40} - \frac{16}{40} = \frac{15 - 16}{40} = -\frac{1}{40}$

(e) $\frac{12}{7} \times \left(-\frac{3}{5}\right) = \frac{12 \times (-3)}{7 \times 5} = \frac{-36}{35} = -\frac{36}{35}$

(f) $\left(-\frac{3}{4}\right) \div \left(-\frac{12}{7}\right) = \left(-\frac{3}{4}\right) \times \left(-\frac{7}{12}\right) = \frac{(-3) \times (-7)}{4 \times 12} = \frac{3 \times 7}{4 \times 3 \times 4} = \frac{7}{16}$

(g) $\frac{19}{2} + \frac{7}{4} = \frac{38}{4} + \frac{7}{4} = \frac{45}{4}$

(h) $\frac{1}{4} - \frac{3}{8} = \frac{2}{8} - \frac{3}{8} = -\frac{1}{8}$

3 (a) $\frac{1}{2}, \frac{2}{5}, \frac{1}{10} = \frac{5}{10}, \frac{4}{10}, \frac{1}{10}$ so ratio is 5 : 4 : 1

(b) $\frac{1}{3}, \frac{1}{5}, \frac{1}{4} = \frac{20}{60}, \frac{12}{60}, \frac{15}{60}$ and $\frac{20}{60} + \frac{12}{60} + \frac{15}{60} = \frac{47}{60}$

so the fraction of S is $\frac{13}{60}$

so P, Q , R and S are in the ratio 20 : 12 : 15 : 13

4 (a) $\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100}$ that is 40% or $\frac{2}{5} = \frac{2}{5} \times 100\% = 40\%$

(b) $\frac{58}{100} \times 25 = \frac{58}{4} = \frac{29}{2} = 14\frac{1}{2}$

(c) $\frac{7}{12} = \frac{7}{12} \times 100\% = \frac{700}{12}\% = \frac{58 \times 12 + 4}{12}\% = 58\frac{4}{12}\% = 58\frac{1}{3}\%$

(d) $\frac{17}{100} \times 50 = \frac{17}{2} = 8\frac{1}{2}$

Now for the Review test

Review test

Unit 3

29

1 Reduce each of the following fractions to their lowest terms:

(a) $\frac{12}{18}$ (b) $\frac{144}{21}$ (c) $-\frac{49}{14}$ (d) $\frac{64}{4}$

2 Evaluate the following:

(a) $\frac{3}{7}\times\frac{2}{3}$ (b) $\frac{11}{30}\div\frac{5}{6}$ (c) $\frac{3}{7}+\frac{4}{13}$ (d) $\frac{5}{16}-\frac{4}{3}$

3 Write the following proportions as ratios:

(a) $\frac{1}{2}$ of A, $\frac{1}{5}$ of B and $\frac{3}{10}$ of C

(b) $\frac{1}{4}$ of P, $\frac{1}{3}$ of Q, $\frac{1}{5}$ of R and the remainder S

4 Complete the following:

(a) $\frac{4}{5} =$ % (b) 48% of 50 =

(c) $\frac{9}{14} =$ % (d) 15% of 25 =

Decimal numbers

Unit 4

Division of integers

1

If one integer is divided by a second integer that is not one of the first integer's factors the result will not be another integer. Instead, the result will lie between two integers. For example, using a calculator it is seen that:

$25 \div 8 = 3{\cdot}125$

which is a number greater than 3 but less than 4. As with integers, the position of a numeral within the number indicates its value. Here the number 3·125 represents

3 units + 1 tenth + 2 hundredths + 5 thousandths.

That is $3+\frac{1}{10}+\frac{2}{100}+\frac{5}{1000}$

where the decimal point shows the separation of the units from the tenths. Numbers written in this format are called *decimal numbers*.

On to the next frame

2 Rounding

All the operations of arithmetic that we have used with the integers apply to decimal numbers. However, when performing calculations involving decimal numbers it is common for the end result to be a number with a large quantity of numerals after the decimal point. For example:

$$15{\cdot}11 \div 8{\cdot}92 = 1{\cdot}6939461883\ldots$$

To make such numbers more manageable or more reasonable as the result of a calculation, they can be rounded either to a specified number of *significant figures* or to a specified number of *decimal places*.

Now to the next frame

3 Significant figures

Significant figures are counted from the first non-zero numeral encountered starting from the left of the number. When the required number of significant figures has been counted off, the remaining numerals are deleted with the following proviso:

If the first of a group of numerals to be deleted is a 5 or more, the last significant numeral is increased by 1. For example:

9·4534 to two significant figures is 9·5, to three significant figures is 9·45, and 0·001354 to two significant figures is 0·0014

Try this one for yourself. To four significant figures the number 18·7249 is . . . .

Check your result with the next frame

4

$\boxed{18{\cdot}72}$

Because

The first numeral deleted is a 4 which is less than 5.

There is one further proviso. If the only numeral to be dropped is a 5 then the last numeral retained is rounded up. So that 12·235 to four significant figures (abbreviated to *sig fig*) is 12·24 and 3·465 to three sig fig is 3·47.

So 8·1265 to four sig fig is . . . .

Check with the next frame

5

$\boxed{8{\cdot}127}$

Because

The only numeral deleted is a 5 and the last numeral is rounded up.

Now on to the next frame

Decimal places

6

Decimal places are counted to the right of the decimal point and the same rules as for significant figures apply for rounding to a specified number of decimal places (abbreviated to *dp*). For example:

123·4467 to one decimal place is 123·4 and to two dp is 123·45

So, 47·0235 to three dp is . . . .

7

$\boxed{47{\cdot}024}$

Because

The only numeral dropped is a 5 and the last numeral retained is odd so is increased to the next even numeral.

Now move on to the next frame

Trailing zeros

8

Sometimes zeros must be inserted within a number to satisfy a condition for a specified number of either significant figures or decimal places. For example:

12 645 to two significant figures is 13 000, and 13·1 to three decimal places is 13·100.

These zeros are referred to as *trailing zeros*.

So that 1515 to two sig fig is . . . .

9

$\boxed{1500}$

And:

25·13 to four dp is . . . .

10

$\boxed{25{\cdot}1300}$

On to the next frame

11 Fractions as decimals

Because a fraction is one integer divided by another it can be represented in decimal form simply by executing the division. For example:

$$\frac{7}{4} = 7 \div 4 = 1{\cdot}75$$

So that the decimal form of $\frac{3}{8}$ is ....

12

$$\boxed{0{\cdot}375}$$

Because

$$\frac{3}{8} = 3 \div 8 = 0{\cdot}375$$

Now move on to the next frame

13 Decimals as fractions

A decimal can be represented as a fraction. For example:

$$1{\cdot}224 = \frac{1224}{1000} \text{ which in lowest terms is } \frac{153}{125}$$

So that 0·52 as a fraction in lowest terms is ....

14

$$\boxed{\frac{13}{25}}$$

Because

$$0{\cdot}52 = \frac{52}{100} = \frac{13}{25}$$

Now move on to the next frame

15 Unending decimals

Converting a fraction into its decimal form by performing the division always results in an infinite string of numerals after the decimal point. This string of numerals may contain an infinite sequence of zeros or it may contain an infinitely repeated pattern of numerals. A repeated pattern of numerals can be written in an abbreviated format. For example:

$$\frac{1}{3} = 1 \div 3 = 0{\cdot}3333\ldots$$

Here the pattern after the decimal point is of an infinite number of 3's. We abbreviate this by placing a dot over the first 3 to indicate the repetition, thus:

$0{\cdot}3333\ldots = 0{\cdot}\dot{3}$ (described as zero point 3 recurring)

For other fractions the repetition may consist of a sequence of numerals, in which case a dot is placed over the first and last numeral in the sequence. For example:

$$\frac{1}{7} = 0{\cdot}142857142857142857\ldots = 0{\cdot}\dot{1}4285\dot{7}$$

So that we write $\frac{2}{11} = 0{\cdot}181818\ldots$ as ....

16

$\boxed{0{\cdot}\dot{1}\dot{8}}$

Sometimes the repeating pattern is formed by an infinite sequence of zeros, in which case we simply omit them. For example:

$\frac{1}{5} = 0{\cdot}20000\ldots$ is written as $0{\cdot}2$

Next frame

17

Unending decimals as fractions

Any decimal that displays an unending repeating pattern can be converted to its fractional form. For example:

To convert $0{\cdot}181818\ldots = 0{\cdot}\dot{1}\dot{8}$ to its fractional form we note that because there are two repeating numerals we multiply by 100 to give:

$$100 \times 0{\cdot}\dot{1}\dot{8} = 18{\cdot}\dot{1}\dot{8}$$

Subtracting $0{\cdot}\dot{1}\dot{8}$ from both sides of this equation gives:

$$100 \times 0{\cdot}\dot{1}\dot{8} - 0{\cdot}\dot{1}\dot{8} = 18{\cdot}\dot{1}\dot{8} - 0{\cdot}\dot{1}\dot{8}$$

That is:

$$99 \times 0{\cdot}\dot{1}\dot{8} = 18{\cdot}0$$

This means that:

$$0{\cdot}\dot{1}\dot{8} = \frac{18}{99} = \frac{2}{11}$$

Similarly, the fractional form of $2{\cdot}0\dot{3}1\dot{5}$ is found as follows:

$2{\cdot}0\dot{3}1\dot{5} = 2{\cdot}0 + 0{\cdot}0\dot{3}1\dot{5}$ and, because there are three repeating numerals:

$$1000 \times 0{\cdot}0\dot{3}1\dot{5} = 31{\cdot}5\dot{3}1\dot{5}$$

Subtracting $0{\cdot}0\dot{3}1\dot{5}$ from both sides of this equation gives:

$$1000 \times 0{\cdot}0\dot{3}1\dot{5} - 0{\cdot}0\dot{3}1\dot{5} = 31{\cdot}5\dot{3}1\dot{5} - 0{\cdot}0\dot{3}1\dot{5} = 31{\cdot}5$$

That is:

$$999 \times 0{\cdot}0\dot{3}1\dot{5} = 31{\cdot}5 \text{ so that } 0{\cdot}0\dot{3}1\dot{5} = \frac{31{\cdot}5}{999} = \frac{315}{9990}$$

This means that:

$$2{\cdot}0\dot{3}1\dot{5} = 2{\cdot}0 + 0{\cdot}0\dot{3}1\dot{5} = 2 + \frac{315}{9990} = 2\tfrac{35}{1110} = 2\tfrac{7}{222}$$

What are the fractional forms of $0{\cdot}\dot{2}\dot{1}$ and $3{\cdot}2\dot{1}$?

The answers are in the next frame

18

$$\boxed{\tfrac{7}{33} \text{ and } 3\tfrac{19}{90}}$$

Because

$100 \times 0{\cdot}\dot{2}\dot{1} = 21{\cdot}\dot{2}\dot{1}$ so that $99 \times 0{\cdot}\dot{2}\dot{1} = 21$

giving $0{\cdot}\dot{2}\dot{1} = \dfrac{21}{99} = \dfrac{7}{33}$ and

$3{\cdot}2\dot{1} = 3{\cdot}2 + 0{\cdot}0\dot{1}$ and $10 \times 0{\cdot}0\dot{1} = 0{\cdot}1\dot{1}$ so that $9 \times 0{\cdot}0\dot{1} = 0{\cdot}1$ giving

$0{\cdot}0\dot{1} = \dfrac{0{\cdot}1}{9} = \dfrac{1}{90}$, hence $3{\cdot}2\dot{1} = \dfrac{32}{10} + \dfrac{1}{90} = \dfrac{289}{90} = 3\tfrac{19}{90}$

19 Rational, irrational and real numbers

A number that can be expressed as a fraction is called a *rational* number. An *irrational* number is one that *cannot* be expressed as a fraction and has a decimal form consisting of an infinite string of numerals that does not display a repeating pattern. As a consequence it is not possible either to write down the complete decimal form or to devise an abbreviated decimal format. Instead, we can only round them to a specified number of significant figures or decimal places. Alternatively, we may have a numeral representation for them, such as $\sqrt{2}$, e or π. The complete collection of rational and irrational numbers is called the collection of *real* numbers.

At this point let us pause and summarize the main facts so far on decimal numbers

Review summary

Unit 4

20

1 Every fraction can be written as a decimal number by performing the division.

2 The decimal number obtained will consist of an infinitely repeating pattern of numerals to the right of one of its digits.

3 Other decimals, with an infinite, non-repeating sequence of numerals after the decimal point are the irrational numbers.

4 A decimal number can be rounded to a specified number of significant figures (sig fig) by counting from the first non-zero numeral on the left.

5 A decimal number can be rounded to a specified number of decimal places (dp) by counting from the decimal point.

Review exercise

Unit 4

21

1 Round each of the following decimal numbers, first to 3 significant figures and then to 2 decimal places:

(a) 12·455 (b) 0·01356 (c) 0·1005 (d) 1344·555

2 Write each of the following in abbreviated form:

(a) 12·110110110... (b) 0·123123123...

(c) −3·11111... (d) −9360·936093609360...

3 Convert each of the following to decimal form to 3 decimal places:

(a) $\frac{3}{16}$ (b) $-\frac{5}{9}$

(c) $\frac{7}{6}$ (d) $-\frac{24}{11}$

4 Convert each of the following to fractional form in lowest terms:

(a) 0·6 (b) $1{\cdot}\dot{4}$

(c) $1{\cdot}\dot{2}\dot{4}$ (d) −7·3

Complete all four questions. Take your time, there is no need to rush.
If necessary, look back at the Unit.
The answers and working are in the next frame.

22

1 (a) 12·5, 12·46 (b) 0·0136, 0·01
(c) 0·101, 0·10 (d) 1340, 1344·56

2 (a) $12{\cdot}\dot{1}1\dot{0}$ (b) $0{\cdot}\dot{1}2\dot{3}$
(c) $-3{\cdot}\dot{1}$ (d) $-9360{\cdot}\dot{9}36\dot{0}$

3 (a) $\dfrac{3}{16} = 0{\cdot}1875 = 0{\cdot}188$ to 3 dp

(b) $-\dfrac{5}{9} = -0{\cdot}555\ldots = -0{\cdot}556$ to 3 dp

(c) $\dfrac{7}{6} = 1{\cdot}1666\ldots = 1{\cdot}167$ to 3 dp

(d) $-\dfrac{24}{11} = -2{\cdot}1818\ldots = -2{\cdot}182$ to 3 dp

4 (a) $0{\cdot}6 = \dfrac{6}{10} = \dfrac{3}{5}$

(b) $1{\cdot}\dot{4} = 1 + \dfrac{4}{9} = \dfrac{13}{9}$

(c) $1{\cdot}\dot{2}\dot{4} = 1 + \dfrac{24}{99} = \dfrac{123}{99} = \dfrac{41}{33}$

(d) $-7{\cdot}3 = -\dfrac{73}{10}$

Now for the Review test

Review test

Unit 4

23

1 Round each of the following decimal numbers, first to 3 significant figures and then to 2 decimal places:
(a) 21·355 (b) 0·02456
(c) 0·3105 (d) 5134·555

2 Convert each of the following to decimal form to 3 decimal places:

(a) $\dfrac{4}{15}$ (b) $-\dfrac{7}{13}$ (c) $\dfrac{9}{5}$ (d) $-\dfrac{28}{13}$

3 Convert each of the following to fractional form in lowest terms:
(a) 0·8 (b) 2·8 (c) $3{\cdot}\dot{3}\dot{2}$ (d) – 5·5

4 Write each of the following in abbreviated form:
(a) 1·010101 . . . (b) 9·2456456456 . . .

Powers

Unit 5

Raising a number to a power

1

The arithmetic operation of raising a number to a *power* is devised from repetitive multiplication. For example:

$10 \times 10 \times 10 \times 10 = 10^4$ – the number 10 multiplied by itself 4 times

The power is also called an *index* and the number to be raised to the power is called the *base*. Here the number 4 is the power (index) and 10 is the base.

So $5 \times 5 \times 5 \times 5 \times 5 \times 5 = \ldots\ldots\ldots\ldots$ (in the form of 5 raised to a power)

Compare your answer with the next frame

2

$\boxed{5^6}$

Because the number 5 (the base) is multiplied by itself 6 times (the power or index).

Now to the next frame

The laws of powers

3

The laws of powers are contained within the following set of rules:

- *Power unity*
  Any number raised to the power 1 equals itself.

  $3^1 = 3$

So $99^1 = \ldots\ldots\ldots\ldots$

On to the next frame

4

$\boxed{99}$

Because any number raised to the power 1 equals itself.

- *Multiplication of numbers and the addition of powers*
  If two numbers are each written as a given base raised to some power then the *product of the two numbers* is equal to the same base raised to the *sum of the powers*. For example, $16 = 2^4$ and $8 = 2^3$ so:

$$\begin{aligned} 16 \times 8 &= 2^4 \times 2^3 \\ &= (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2) \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^7 \\ &= 2^{4+3} \\ &= 128 \end{aligned}$$

  Multiplication requires powers to be added.

So $8^3 \times 8^5 =$ .... (in the form of 8 raised to a power)

Next frame

5

$\boxed{8^8}$

Because multiplication requires powers to be added.

Notice that we cannot combine different powers with different bases. For example:

$2^2 \times 4^3$ cannot be written as 8^5

but we can combine different bases to the same power. For example:

$3^4 \times 5^4$ can be written as 15^4 because

$$\begin{aligned} 3^4 \times 5^4 &= (3 \times 3 \times 3 \times 3) \times (5 \times 5 \times 5 \times 5) \\ &= 15 \times 15 \times 15 \times 15 \\ &= 15^4 \\ &= (3 \times 5)^4 \end{aligned}$$

So that $2^3 \times 4^3$ can be written as .... (in the form of a number raised to a power)

6

$\boxed{8^3}$

Next frame

7

- *Division of numbers and the subtraction of powers*
  If two numbers are each written as a given base raised to some power then the *quotient of the two numbers* is equal to the same base raised to the *difference of the powers*. For example:

$$
\begin{aligned}
15\,625 \div 25 &= 5^6 \div 5^2 \\
&= (5 \times 5 \times 5 \times 5 \times 5 \times 5) \div (5 \times 5) \\
&= \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5} \\
&= 5 \times 5 \times 5 \times 5 \\
&= 5^4 \\
&= 5^{6-2} \\
&= 625
\end{aligned}
$$

  Division requires powers to be subtracted.

So $12^7 \div 12^3 =$ .... (in the form of 12 raised to a power)

Check your result in the next frame

8

$\boxed{12^4}$

Because division requires the powers to be subtracted.

- *Power zero*
  Any number raised to the power 0 equals unity. For example:

$$
\begin{aligned}
1 &= 3^1 \div 3^1 \\
&= 3^{1-1} \\
&= 3^0
\end{aligned}
$$

So $193^0 =$ ....

9

$$\boxed{1}$$

Because any number raised to the power 0 equals unity.

- *Negative powers*
  A number raised to a negative power denotes the reciprocal. For example:

$$\begin{aligned} 6^{-2} &= 6^{0-2} \\ &= 6^0 \div 6^2 && \text{subtraction of powers means division} \\ &= 1 \div 6^2 && \text{because } 6^0 = 1 \\ &= \frac{1}{6^2} \end{aligned}$$

Also $6^{-1} = \frac{1}{6}$

A negative power denotes the reciprocal.

So $3^{-5} = \dots\dots\dots\dots$

10

$$\boxed{\frac{1}{3^5}}$$

Because

$$3^{-5} = 3^{0-5} = 3^0 \div 3^5 = \frac{1}{3^5}$$

A negative power denotes the reciprocal.

Now to the next frame

11

- *Multiplication of powers*
  If a number is written as a given base raised to some power then that number *raised to a further power* is equal to the base raised to the *product of the powers*. For example:

$$\begin{aligned} (25)^3 &= \left(5^2\right)^3 \\ &= 5^2 \times 5^2 \times 5^2 \\ &= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \\ &= 5^6 \\ &= 5^{2\times 3} \\ &= 15\,625 && \text{Notice that } \left(5^2\right)^3 \neq 5^{2^3} \text{ because } 5^{2^3} = 5^8 = 390\,625. \end{aligned}$$

Raising to a power requires powers to be multiplied.

So $\left(4^2\right)^4 = \dots\dots\dots\dots$ (in the form of 4 raised to a power)

12

$\boxed{4^8}$

Because raising to a power requires powers to be multiplied.

Now to the next frame.

Powers on a calculator

13

Powers on a calculator can be evaluated by using the x^y key. For example, enter the number 4, press the x^y key, enter the number 3 and press $=$. The result is 64 which is 4^3.

Try this one for yourself. To two decimal places, the value of $1{\cdot}3^{3{\cdot}4}$ is . . . .

The result is in the following frame

14

$\boxed{2{\cdot}44}$

Because

Enter the number 1·3
Press the x^y key
Enter the number 3·4
Press the $=$ key

The number displayed is 2·44 to 2 dp.

Now try this one using the calculator:

$8^{\frac{1}{3}} =$ . . . .   The 1/3 is a problem, use the $a^b\!/_c$ key.

Check your answer in the next frame

15

$\boxed{2}$

Because

Enter the number 8
Press the x^y key
Enter the number 1
Press the $a^b\!/_c$ key
Enter the number 3
Press $=$ the number 2 is displayed.

Now move on to the next frame

16 Fractional powers and roots

We have just seen that $8^{\frac{1}{3}} = 2$. We call $8^{\frac{1}{3}}$ the *third root* or, alternatively, the *cube root* of 8 because

$\left(8^{\frac{1}{3}}\right)^3 = 8$ *the number 8 is the result of raising the 3rd root of 8 to the power 3*

Roots are denoted by such fractional powers. For example, the 5th root of 6 is given as $6^{\frac{1}{5}}$ because

$$\left(6^{\frac{1}{5}}\right)^5 = 6$$

and by using a calculator $6^{\frac{1}{5}}$ can be seen to be equal to 1·431 to 3 dp. Odd roots are unique in the real number system but even roots are not. For example, there are two 2nd roots – *square roots* – of 4, namely:

$$4^{\frac{1}{2}} = 2 \text{ and } 4^{\frac{1}{2}} = -2 \text{ because } 2 \times 2 = 4 \text{ and } (-2) \times (-2) = 4$$

Similarly:

$$81^{\frac{1}{4}} = \pm 3$$

Odd roots of negative numbers are themselves negative. For example:

$$(-32)^{\frac{1}{5}} = -2 \text{ because } \left[(-32)^{\frac{1}{5}}\right]^5 = (-2)^5 = -32$$

Even roots of negative numbers, however, pose a problem. For example, because

$$\left[(-1)^{\frac{1}{2}}\right]^2 = (-1)^1 = -1$$

we conclude that the square root of -1 is $(-1)^{\frac{1}{2}}$. Unfortunately, we cannot write this as a decimal number – we cannot find its decimal value because there is no decimal number which when multiplied by itself gives -1. We have to accept the fact that, at this stage, we cannot find the even roots of a negative number. This would be the subject matter for a book of more advanced mathematics.

Surds

An alternative notation for the square root of 4 is the surd notation $\sqrt{4}$ and, by convention, this is always taken to mean the positive square root. This notation can also be extended to other roots, for example, $\sqrt[7]{9}$ is an alternative notation for $9^{\frac{1}{7}}$.

Use your calculator to find the value of each of the following roots to 3 dp:

(a) $16^{\frac{1}{7}}$ (b) $\sqrt{8}$ (c) $19^{\frac{1}{4}}$ (d) $\sqrt{-4}$

Answers in the next frame

17

(a) 1·486 use the x^y key
(b) 2·828 the positive value only
(c) $\pm$2·088 there are two values for even roots
(d) We cannot find the square root of a negative number

On now to Frame 18

18

Multiplication and division by integer powers of 10

If a decimal number is multiplied by 10 raised to an integer power, the decimal point moves the integer number of places to the right if the integer is positive and to the left if the integer is negative. For example:

$1{\cdot}2345 \times 10^3 = 1234{\cdot}5$ (3 places to the right) and

$1{\cdot}2345 \times 10^{-2} = 0{\cdot}012345$ (2 places to the left).

Notice that, for example:

$1{\cdot}2345 \div 10^3 = 1{\cdot}2345 \times 10^{-3}$ and

$1{\cdot}2345 \div 10^{-2} = 1{\cdot}2345 \times 10^2$

So now try these:

(a) $0{\cdot}012045 \times 10^4$
(b) $13{\cdot}5074 \times 10^{-3}$
(c) $144{\cdot}032 \div 10^5$
(d) $0{\cdot}012045 \div 10^{-2}$

Work all four out and then check your results with the next frame

19

(a) 120·45
(b) 0·0135074
(c) 0·00144032
(d) 1·2045

Because

(a) multiplying by 10^4 moves the decimal point 4 places to the right
(b) multiplying by 10^{-3} moves the decimal point 3 places to the left
(c) $144{\cdot}032 \div 10^5 = 144{\cdot}032 \times 10^{-5}$ move the decimal point 5 places to the left
(d) $0{\cdot}012045 \div 10^{-2} = 0{\cdot}012045 \times 10^2$ move the decimal point 2 places to the right

Now move on to the next frame

20 Precedence rules

With the introduction of the arithmetic operation of raising to a power we need to amend our earlier precedence rules – *evaluating powers is performed before dividing and multiplying*. For example:

$$\begin{aligned} 5\left(3 \times 4^2 \div 6 - 7\right) &= 5(3 \times 16 \div 6 - 7) \\ &= 5(48 \div 6 - 7) \\ &= 5(8 - 7) \\ &= 5 \end{aligned}$$

So that:

$$14 \div \left(125 \div 5^3 \times 4 + 3\right) = \ldots\ldots\ldots\ldots$$

Check your result in the next frame

21

$$\boxed{2}$$

Because

$$\begin{aligned} 14 \div \left(125 \div 5^3 \times 4 + 3\right) &= 14 \div (125 \div 125 \times 4 + 3) \\ &= 14 \div (4 + 3) \\ &= 2 \end{aligned}$$

22 Standard form

Any decimal number can be written as a decimal number greater than or equal to 1 and less than 10 (called the *mantissa*) multiplied by the number 10 raised to an appropriate power (the power being called the *exponent*). For example:

$$\begin{aligned} 57{\cdot}3 &= 5{\cdot}73 \times 10^1 \\ 423{\cdot}8 &= 4{\cdot}238 \times 10^2 \\ 6042{\cdot}3 &= 6{\cdot}0423 \times 10^3 \\ \text{and} \qquad 0{\cdot}267 &= 2{\cdot}67 \div 10 = 2{\cdot}67 \times 10^{-1} \\ 0{\cdot}000485 &= 4{\cdot}85 \div 10^4 = 4{\cdot}85 \times 10^{-4} \text{ etc.} \end{aligned}$$

So, written in standard form:

(a) $52\,674 = \ldots\ldots\ldots\ldots$ (c) $0{\cdot}0582 = \ldots\ldots\ldots\ldots$

(b) $0{\cdot}00723 = \ldots\ldots\ldots\ldots$ (d) $1\,523\,800 = \ldots\ldots\ldots\ldots$

23

(a) $5{\cdot}2674 \times 10^4$	(c) $5{\cdot}82 \times 10^{-2}$
(b) $7{\cdot}23 \times 10^{-3}$	(d) $1{\cdot}5238 \times 10^6$

Working in standard form

Numbers written in standard form can be multiplied or divided by multiplying or dividing the respective mantissas and adding or subtracting the respective exponents. For example:

$$\begin{aligned} 0{\cdot}84 \times 23\,000 &= \left(8{\cdot}4 \times 10^{-1}\right) \times \left(2{\cdot}3 \times 10^4\right) \\ &= (8{\cdot}4 \times 2{\cdot}3) \times 10^{-1} \times 10^4 \\ &= 19{\cdot}32 \times 10^3 \\ &= 1{\cdot}932 \times 10^4 \end{aligned}$$

Another example:

$$\begin{aligned} 175{\cdot}4 \div 6340 &= \left(1{\cdot}754 \times 10^2\right) \div \left(6{\cdot}34 \times 10^3\right) \\ &= (1{\cdot}754 \div 6{\cdot}34) \times 10^2 \div 10^3 \\ &= 0{\cdot}2767 \times 10^{-1} \\ &= 2{\cdot}767 \times 10^{-2} \text{ to 4 sig fig} \end{aligned}$$

Where the result obtained is not in standard form, the mantissa is written in standard number form and the necessary adjustment made to the exponent.

In the same way, then, giving the results in standard form to 4 dp:

(a) $472{\cdot}3 \times 0{\cdot}000564 =$ . . . .
(b) $752\,000 \div 0{\cdot}862 =$ . . . .

24

(a) $2{\cdot}6638 \times 10^{-1}$
(b) $8{\cdot}7239 \times 10^5$

Because

(a) $$\begin{aligned} 472{\cdot}3 \times 0{\cdot}000564 &= \left(4{\cdot}723 \times 10^2\right) \times \left(5{\cdot}64 \times 10^{-4}\right) \\ &= (4{\cdot}723 \times 5{\cdot}64) \times 10^2 \times 10^{-4} \\ &= 26{\cdot}638 \times 10^{-2} = 2{\cdot}6638 \times 10^{-1} \end{aligned}$$

(b) $$\begin{aligned} 752\,000 \div 0{\cdot}862 &= \left(7{\cdot}52 \times 10^5\right) \div \left(8{\cdot}62 \times 10^{-1}\right) \\ &= (7{\cdot}52 \div 8{\cdot}62) \times 10^5 \times 10^1 \\ &= 0{\cdot}87239 \times 10^6 = 8{\cdot}7239 \times 10^5 \end{aligned}$$

For *addition and subtraction in standard form* the approach is slightly different.

Example 1

$4{\cdot}72 \times 10^3 + 3{\cdot}648 \times 10^4$

Before these can be added, the powers of 10 must be made the same:

$$\begin{aligned} 4{\cdot}72 \times 10^3 + 3{\cdot}648 \times 10^4 &= 4{\cdot}72 \times 10^3 + 36{\cdot}48 \times 10^3 \\ &= (4{\cdot}72 + 36{\cdot}48) \times 10^3 \\ &= 41{\cdot}2 \times 10^3 = 4{\cdot}12 \times 10^4 \text{ in standard form} \end{aligned}$$

Similarly in the next example.

Example 2

$13{\cdot}26 \times 10^{-3} - 1{\cdot}13 \times 10^{-2}$

Here again, the powers of 10 must be equalized:

$$\begin{aligned} 13{\cdot}26 \times 10^{-3} - 1{\cdot}13 \times 10^{-2} &= 1{\cdot}326 \times 10^{-2} - 1{\cdot}13 \times 10^{-2} \\ &= (1{\cdot}326 - 1{\cdot}13) \times 10^{-2} \\ &= 0{\cdot}196 \times 10^{-2} = 1{\cdot}96 \times 10^{-3} \text{ in standard form} \end{aligned}$$

Using a calculator

Numbers given in standard form can be manipulated on a calculator by making use of the EXP key. For example, to enter the number $1{\cdot}234 \times 10^3$, enter 1·234 and then press the EXP key. The display then changes to:

1·234 00

Now enter the power 3 and the display becomes:

1·234 03

Manipulating numbers in this way produces a result that is in ordinary decimal format. If the answer is required in standard form then it will have to be converted by hand. For example, using the EXP key on a calculator to evaluate $(1{\cdot}234 \times 10^3) + (2{\cdot}6 \times 10^2)$ results in the display 1494 which is then converted by hand to $1{\cdot}494 \times 10^3$.

Therefore, working in standard form:

(a) $43{\cdot}6 \times 10^2 + 8{\cdot}12 \times 10^3 = \ldots\ldots\ldots\ldots$

(b) $7{\cdot}84 \times 10^5 - 12{\cdot}36 \times 10^3 = \ldots\ldots\ldots\ldots$

(c) $4{\cdot}25 \times 10^{-3} + 1{\cdot}74 \times 10^{-2} = \ldots\ldots\ldots\ldots$

25

(a) $1{\cdot}248 \times 10^4$
(b) $7{\cdot}7164 \times 10^5$
(c) $2{\cdot}165 \times 10^{-2}$

Preferred standard form

In the SI system of units, it is recommended that when a number is written in standard form, the power of 10 should be restricted to powers of 10^3, i.e. 10^3, 10^6, 10^{-3}, 10^{-6}, etc. Therefore in this *preferred standard form* up to three figures may appear in front of the decimal point.

In practice it is best to write the number first in standard form and to adjust the power of 10 to express this in preferred standard form.

Example 1

$5{\cdot}2746 \times 10^4$ in standard form

$= 5{\cdot}2746 \times 10 \times 10^3$

$= 52{\cdot}746 \times 10^3$ in preferred standard form

Example 2

$3{\cdot}472 \times 10^8$ in standard form

$= 3{\cdot}472 \times 10^2 \times 10^6$

$= 347{\cdot}2 \times 10^6$ in preferred standard form

Example 3

$3{\cdot}684 \times 10^{-2}$ in standard form

$= 3{\cdot}684 \times 10 \times 10^{-3}$

$= 36{\cdot}84 \times 10^{-3}$ in preferred standard form

So, rewriting the following in preferred standard form, we have

(a) $8{\cdot}236 \times 10^7 =$ . . . .
(b) $1{\cdot}624 \times 10^{-4} =$ . . . .
(c) $4{\cdot}827 \times 10^4 =$ . . . .
(d) $6{\cdot}243 \times 10^5 =$ . . . .
(e) $3{\cdot}274 \times 10^{-2} =$ . . . .
(f) $5{\cdot}362 \times 10^{-7} =$ . . . .

26

(a) $82{\cdot}36 \times 10^6$	(d) $624{\cdot}3 \times 10^3$
(b) $162{\cdot}4 \times 10^{-6}$	(e) $32{\cdot}74 \times 10^{-3}$
(c) $48{\cdot}27 \times 10^3$	(f) $536{\cdot}2 \times 10^{-9}$

One final exercise on this piece of work:

Example 4

The product of $(4{\cdot}72 \times 10^2)$ and $(8{\cdot}36 \times 10^5)$

(a) in standard form = ...

(b) in preferred standard form = ...

27

(a) $3{\cdot}9459 \times 10^8$

(b) $394{\cdot}59 \times 10^6$

Because

(a) $$(4{\cdot}72 \times 10^2) \times (8{\cdot}36 \times 10^5) = (4{\cdot}72 \times 8{\cdot}36) \times 10^2 \times 10^5$$
$$= 39{\cdot}459 \times 10^7$$
$$= 3{\cdot}9459 \times 10^8 \text{ in standard form}$$

(b) $$(4{\cdot}72 \times 10^2) \times (8{\cdot}36 \times 10^5) = 3{\cdot}9459 \times 10^2 \times 10^6$$
$$= 394{\cdot}59 \times 10^6 \text{ in preferred standard form}$$

Now move on to the next frame

28 Checking calculations

When performing a calculation involving decimal numbers it is always a good idea to check that your result is reasonable and that an arithmetic blunder or an error in using the calculator has not been made. This can be done using standard form. For example:

$$59{\cdot}2347 \times 289{\cdot}053 = 5{\cdot}92347 \times 10^1 \times 2{\cdot}89053 \times 10^2$$
$$= 5{\cdot}92347 \times 2{\cdot}89053 \times 10^3$$

This product can then be estimated for reasonableness as:

$6 \times 3 \times 1000 = 18\,000$ (see Frames 13 – 15 of Unit 1)

The answer using the calculator is 17 121·968 to three decimal places, which is 17 000 when rounded to the nearest 1000. This compares favourably with the estimated 18 000, indicating that the result obtained could be reasonably expected.

So, the estimated value of $800{\cdot}120 \times 0{\cdot}007953$ is ....

Check with the next frame

29

$\boxed{6{\cdot}4}$

Because

$$800{\cdot}120 \times 0{\cdot}007953 = 8{\cdot}00120 \times 10^2 \times 7{\cdot}953 \times 10^{-3}$$
$$= 8{\cdot}00120 \times 7{\cdot}9533 \times 10^{-1}$$

This product can then be estimated for reasonableness as:

$8 \times 8 \div 10 = 6{\cdot}4$

The exact answer is 6·36 to two decimal places.

Now move on to the next frame

Accuracy

30

Many calculations are made using numbers that have been obtained from measurements. Such numbers are only accurate to a given number of significant figures but using a calculator can produce a result that contains as many figures as its display will permit. Because any calculation involving measured values will not be accurate to *more significant figures than the least number of significant figures in any measurement*, we can justifiably round the result down to a more manageable number of significant figures.

For example:

The base length and height of a rectangle are measured as 114·8 mm and 18 mm respectively. The area of the rectangle is given as the product of these lengths. Using a calculator this product is 2066·4 mm^2. Because one of the lengths is only measured to 2 significant figures, the result cannot be accurate to more than 2 significant figures. It should therefore be read as 2100 mm^2.

Assuming the following contains numbers obtained by measurement, use a calculator to find the value to the correct level of accuracy:

$19{\cdot}1 \times 0{\cdot}0053 \div 13{\cdot}345$

31

$\boxed{0{\cdot}0076}$

Because

The calculator gives the result as 0·00758561 but because 0·0053 is only accurate to 2 significant figures the result cannot be accurate to more than 2 significant figures, namely 0·0076.

At this point let us pause and summarize the main facts so far on powers

Review summary

Unit 5

32

1 Powers are devised from repetitive multiplication of a given number.

2 Negative powers denote reciprocals and any number raised to the power 0 is unity.

3 Multiplication of a decimal number by 10 raised to an integer power moves the decimal point to the right if the power is positive and to the left if the power is negative.

4 A decimal number written in standard form is in the form of a mantissa (a number between 1 and 10 but excluding 10) multiplied by 10 raised to an integer power, the power being called the exponent.

5 Writing decimal numbers in standard form permits an estimation of the reasonableness of a calculation.

6 In preferred standard form the powers of 10 in the exponent are restricted to multiples of 3.

7 If numbers used in a calculation are obtained from measurement, the result of the calculation is a number accurate to no more than the least number of significant figures in any measurement.

Review exercise

Unit 5

33

1 Write each of the following as a number raised to a power:

(a) $5^8 \times 5^2$ (b) $6^4 \div 6^6$ (c) $(7^4)^3$ (d) $(19^{-8})^0$

2 Find the value of each of the following to 3 dp:

(a) $16^{\frac{1}{4}}$ (b) $\sqrt[3]{3}$ (c) $(-8)^{\frac{1}{5}}$ (d) $(-7)^{\frac{1}{4}}$

3 Write each of the following as a single decimal number:

(a) $1{\cdot}0521 \times 10^3$ (b) $123{\cdot}456 \times 10^{-2}$

(c) $0{\cdot}0135 \div 10^{-3}$ (d) $165{\cdot}21 \div 10^4$

4 Write each of the following in standard form:

(a) 125·87 (b) 0·0101 (c) 1·345 (d) 10·13

5 Write each of the following in preferred standard form:

(a) $1{\cdot}3204 \times 10^5$ (b) 0·0101 (c) 1·345 (d) $9{\cdot}5032 \times 10^{-8}$

6 In each of the following the numbers have been obtained by measurement. Evaluate each calculation to the appropriate level of accuracy:

(a) $13{\cdot}6 \div 0{\cdot}012 \times 7{\cdot}63 - 9015$ (b) $\dfrac{0{\cdot}003 \times 194}{13{\cdot}6}$

(c) $19{\cdot}3 \times 1{\cdot}04^{2{\cdot}00}$ (d) $\dfrac{18 \times 2{\cdot}1 - 3{\cdot}6 \times 0{\cdot}54}{8{\cdot}6 \times 2{\cdot}9 + 5{\cdot}7 \times 9{\cdot}2}$

Complete all of these questions.
Look back at the Unit if you need to.
You can check your answers and working in the next frame.

34

1 (a) $5^8 \times 5^2 = 5^{8+2} = 5^{10}$ (b) $6^4 \div 6^6 = 6^{4-6} = 6^{-2}$ (c) $\left(7^4\right)^3 = 7^{4\times3} = 7^{12}$

(d) $\left(19^{-8}\right)^0 = 1$ as any number raised to the power 0 equals unity

2 (a) $16^{\frac{1}{4}} = \pm 2{\cdot}000$ (b) $\sqrt[3]{3} = 1{\cdot}442$

(c) $(-8)^{\frac{1}{5}} = -1{\cdot}516$

(d) $(-7)^{\frac{1}{4}}$ You cannot find the even root of a negative number

3 (a) $1{\cdot}0521 \times 10^3 = 1052{\cdot}1$ (b) $123{\cdot}456 \times 10^{-2} = 1{\cdot}23456$

(c) $0{\cdot}0135 \div 10^{-3} = 0{\cdot}0135 \times 10^3 = 13{\cdot}5$

(d) $165{\cdot}21 \div 10^4 = 165{\cdot}21 \times 10^{-4} = 0{\cdot}016521$

4 (a) $125{\cdot}87 = 1{\cdot}2587 \times 10^2$ (b) $0{\cdot}0101 = 1{\cdot}01 \times 10^{-2}$

(c) $1{\cdot}345 = 1{\cdot}345 \times 10^0$ (d) $10{\cdot}13 = 1{\cdot}013 \times 10^1 = 1{\cdot}013 \times 10$

5 (a) $1{\cdot}3204 \times 10^5 = 132{\cdot}04 \times 10^3$ (b) $0{\cdot}0101 = 10{\cdot}1 \times 10^{-3}$

(c) $1{\cdot}345 = 1{\cdot}345 \times 10^0$ (d) $9{\cdot}5032 \times 10^{-8} = 95{\cdot}032 \times 10^{-9}$

6 (a) $13{\cdot}6 \div 0{\cdot}012 \times 7{\cdot}63 - 9015 = -367{\cdot}\dot{6} = -370$ to 2 sig fig

(b) $\dfrac{0{\cdot}003 \times 194}{13{\cdot}6} = 0{\cdot}042794\ldots = 0{\cdot}04$ to 1 sig fig

(c) $19{\cdot}3 \times 1{\cdot}04^{2{\cdot}00} = 19{\cdot}3 \times 1{\cdot}0816 = 20{\cdot}87488 = 20{\cdot}9$ to 2 sig fig

(d) $\dfrac{18 \times 2{\cdot}1 - 3{\cdot}6 \times 0{\cdot}54}{8{\cdot}6 \times 2{\cdot}9 + 5{\cdot}7 \times 9{\cdot}2} = \dfrac{35{\cdot}856}{77{\cdot}38} = 0{\cdot}46337554\ldots = 0{\cdot}463$ to 3 sig fig

Now for the Review test

Review test

Unit 5

35

1 Write each of the following as a number raised to a power:

(a) $3^6 \times 3^3$ (b) $4^3 \div 2^5$ (c) $(9^2)^3$ (d) $(7^0)^{-8}$

2 Find the value of each of the following to 3 dp:

(a) $15^{\frac{1}{3}}$ (b) $\sqrt[5]{5}$ (c) $(-27)^{\frac{1}{3}}$ (d) $(-9)^{\frac{1}{2}}$

3 Write each of the following as a single decimal number:

(a) $3{\cdot}2044 \times 10^3$ (b) $16{\cdot}1105 \div 10^{-2}$

4 Write each of the following in standard form:

(a) 134·65 (b) 0·002401

5 Write each of the following in preferred standard form:

(a) $16{\cdot}1105 \div 10^{-2}$ (b) 9·3304

6 In each of the following the numbers have been obtained by measurement. Evaluate each calculation to the appropriate level of accuracy:

(a) $11{\cdot}4 \times 0{\cdot}0013 \div 5{\cdot}44 \times 8{\cdot}810$

(b) $\dfrac{1{\cdot}01 \div 0{\cdot}00335}{9{\cdot}12 \times 6{\cdot}342}$

Number systems

Unit 6

1

Denary (or decimal) system

This is our basic system in which quantities large or small can be represented by use of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 together with appropriate place values according to their positions.

For example	2	7	6	5	·	3	2	1_{10}
has place values	10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}
	1000	100	10	1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

In this case, the place values are powers of 10, which gives the name *denary* (or *decimal*) to the system. The denary system is said to have a *base* of 10. You are, of course, perfectly familiar with this system of numbers, but it is included here as it leads on to other systems which have the same type of structure but which use different place values.

So let us move on to the next system

Binary system

2

This is widely used in all forms of switching applications. The only symbols used are 0 and 1 and the place values are powers of 2, i.e. the system has a base of 2.

For example	1	0	1	1	·	1	0	1_2
has place values	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}
i.e.	8	4	2	1		$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

So: 1 0 1 1 · 1 0 1 in the binary system

$= \quad 1\times 8 \quad 0\times 4 \quad 1\times 2 \quad 1\times 1 \quad 1\times\frac{1}{2} \quad 0\times\frac{1}{4} \quad 1\times\frac{1}{8}$

$= \quad 8 + 0 + 2 + 1 + \frac{1}{2} + 0 + \frac{1}{8}$ in denary

$= \quad 11\frac{5}{8} = 11{\cdot}625$ in the denary system. Therefore $1011{\cdot}101_2 = 11{\cdot}625_{10}$

The small subscripts 2 and 10 indicate the bases of the two systems. In the same way, the denary equivalent of $1\ 1\ 0\ 1 \cdot 0\ 1\ 1_2$ is ………… to 3 dp.

3

$\boxed{13{\cdot}375_{10}}$

Because

$$\begin{aligned} &1 \quad 1 \quad 0 \quad 1 \cdot 0 \quad 1 \quad 1_2 \\ &= 8 + 4 + 0 + 1 + 0 + \tfrac{1}{4} + \tfrac{1}{8} \\ &= 13\tfrac{3}{8} = 13{\cdot}375_{10} \end{aligned}$$

Octal system (base 8)

This system uses the symbols

0, 1, 2, 3, 4, 5, 6, 7

with place values that are powers of 8.

For example	3	5	7	·	3	2	1	in the octal system
has place values	8^2	8^1	8^0		8^{-1}	8^{-2}	8^{-3}	
i.e.	64	8	1		$\frac{1}{8}$	$\frac{1}{64}$	$\frac{1}{512}$	

So 3 5 7 · 3 2 1_8

$= \quad 3\times 64 \quad 5\times 8 \quad 7\times 1 \quad 3\times\frac{1}{8} \quad 2\times\frac{1}{64} \quad 1\times\frac{1}{512}$

$= \quad 192 + 40 + 7 + \frac{3}{8} + \frac{1}{32} + \frac{1}{512}$

$= \quad 239\frac{209}{512} = 239{\cdot}4082_{10}$

That is

$357{\cdot}321_8 = 239{\cdot}408_{10}$ to 3 dp