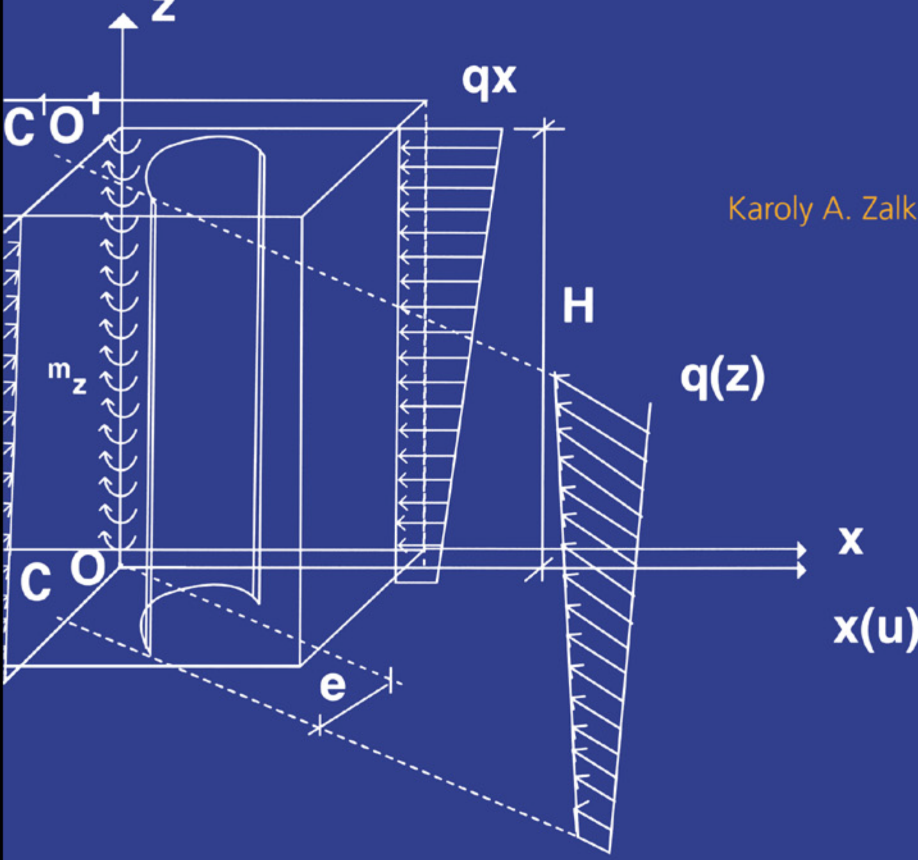
Global Structural Analysis of Buildings



Karoly A. Zalka

Global Structural Analysis of Buildings

Global Structural Analysis of Buildings

Karoly A.Zalka

E & FN SPON
ALERE FLAMMAM
Taylor & Francis Group

London and New York

First published 2000 by E & FN Spon
11 New Fetter Lane, London EC4P 4EE

Simultaneously published in the USA and Canada
by E & FN Spon
29 West 35th Street, New York, NY 10001

E & FN Spon is an imprint of the Taylor & Francis Group

This edition published in the Taylor & Francis e-Library, 2002.

Publisher´s Note

This book has been prepared from camera-ready copy provided by the author.

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloging in Publication Data

Zalka, K.A.

Global structural analysis of buildings/Karoly A.Zalka.

p. cm.
Includes bibliographical references and index.
1. Structural analysis (Engineering) 2. Global analysis (Mathematics)
I. Title.
TA645 .Z35 2000
690´.21–dc21 00–026451

ISBN 0-203-18429-7 Master e-book ISBN

ISBN 0-203-18456-4 (Adobe eReader Format)
ISBN 0-415-23483-2 (Print Edition)

Contents

Preface

A shift in emphasis can be seen in the approach to structural design. More often structures are looked at 'globally', as whole structural units, rather than a group of individual elements. The investigation of this global behaviour, also described as 'holistic' or 'whole building' behaviour has been made possible by new theoretical achievements and the spectacular advance in computer technology during the last decades.

The global structural analysis of buildings can be carried out following two routes. First, sophisticated and complex computer packages based on the finite element method offer endless facilities and can handle even huge structures with a great number of elements. Second, analytical methods can also deal with whole structures leading to simple closed-form solutions, with the additional benefit of providing fast checking facilities for the computer-based methods.

This book follows the latter route and, after describing and solving the complex theoretical problems of bracing systems covering many practical cases, intends to achieve the following three objectives:

- To present simple procedures and closed-form formulae which make it possible for the practising structural engineer to carry out a general structural analysis of the bracing system of building structures in minutes.
- To show that the main areas of structural design (stability, stress and frequency analyses) are not independent; indeed they can be linked by the global critical load ratio which can be used to achieve optimum structural solutions with high performance and adequate safety.
- To help to understand global behaviour better and to develop structural engineering common sense through the introduction of the most representative stiffness characteristics for the stability, stress and frequency analyses.

Notations

CAPITAL LETTERS

A	cross-sectional area; area of the plan of the building; floor area
A_a	area of lower flange
A_b	cross-section of beams
A_c	cross-section of columns
A_d	cross-section of diagonal bars in cross-bracing
A_h	cross-section of horizontal bars in cross-bracing
A_f	area of upper flange; contact area between foundation and soil
A_g	area of web
A_i	cross-sectional area of the ith bracing element
A_j	incremental area
A_o	area of closed cross-section defined by the middle line of the walls
A_{ref}	reference area for the force coefficient
B	plan breadth of the building (in direction y)
C	centre of vertical load; centroid
C_1, C_2, C_3, C_4	constants of integration
E	modulus of elasticity
E_b	modulus of elasticity of beams
E_c	modulus of elasticity of columns
E_d	modulus of elasticity of diagonal bars in cross-bracing
E_h	modulus of elasticity of horizontal bars in cross-bracing
F	concentrated load (on top floor level); horizontal force
F_{cr}	critical concentrated load
$F_{cr,X}$, $F_{cr,Y}$	critical concentrated load in directions X and Y
$F_{cr,\varphi}$	critical concentrated load for pure torsional buckling
F_g	full-height (global) bending critical concentrated load
F_i	vertical load on the ith bracing element/framework; vertical force at x_i, y_i
F_l	local bending critical concentrated load
F_m	total horizontal load due to misalignment
F_x, F_y	components of the resultant of the horizontal load in directions x and y
F_w	global wind force
$F_{w,x}$,$F_{w,y}$	wind force in directions x and y
F_{Wj}	global wind force for height/width>2
G	modulus of elasticity in shear
H	height of building/frame/coupled shear walls; horizontal force
I	second moment of area
I_b	second moment of area of beams

I_c second moment of area of columns

I_{cg} fictitious 'global' second moment of area of a column of storey height

I_e effective second moment of area

I_f second moment of area of the foundation

I_g global second moment of area of the columns of the framework; gross (uncracked) second moment of area

I_o polar moment of inertia of the ground plan area with respect to the shear centre of the bracing system

I_x, I_y second moments of area with respect to centroidal axes x and y

I_X, I_Y second moments of area with respect to principal axes X and Y

I_{xp}, I_{yp} second moments of area of the plan of the building with respect to centroidal axes x and y

I_{xy} product of inertia with respect to axes x and y

I_{XY} product of inertia with respect to principal axes X and Y

I_ω warping (bending torsional) constant

J Saint-Venant torsional constant

K shear stiffness of frameworks; shear critical load; seismic constant

K^* shear stiffness/shear critical load of coupled shear walls

K_d shear stiffness representing the effect of the diagonal bars in cross-bracing

K_g^* full height (global) shear stiffness; global shear critical load

full height (global) shear stiffness of coupled shear walls; full height (global) shear critical load of coupled shear walls

K_h shear stiffness representing the effect of the horizontal bars in cross-bracing

K_l local shear stiffness; local shear critical load

L width of framework; plan length of building (in direction x); width of equivalent shear wall

M_p bending moments due to load of intensity p

M_Q bending moments due to virtual force Q=1

M_t Saint-Venant torsional moment

$M_{x,i}$, $M_{y,i}$ bending moment in the ith bracing element in planes xz and yz

M_Z total torque

M_ω warping torsional moment

M^* bending moment on the equivalent column

$\bar{M}$ concentrated moments representing the supporting effect of the beams

MSC Mercalli-Sieberg-Cancani seismic scale

N total applied uniformly distributed load (measured at ground floor level)

$N_{combined}$ combined sway-torsional critical load for the monosymmetric case

N_{cr} critical load for the uniformly distributed load

N_{cr}^D combined (F+N) critical load

N_{cr}^{flex} critical load of rigid column on flexible support

N_{cr}^{int} critical load which takes into consideration soil-structure interaction

$N_{cr,X}$, $N_{cr,Y}$ critical UDL in directions X and Y

$N_{cr,\varphi}$ critical UDL for pure torsional buckling

N_g full-height (global) bending critical UDL for frameworks

N_l local bending critical UDL for frameworks
$N(z)$ total vertical load at z
O shear centre
Q intensity of uniformly distributed floor load; weight per unit area of top floor; shear force on floor level
R radius
S, S_x, S_y seismic force
S'_x, S'_y first (statical) moments of area about the neutral axis
S_ω sectorial static moment
T natural period; shear force at contraflexure point
$T_{x,i}, T_{y,i}$ shear force in the *i*th bracing element in planes *xz* and *yz*
T_X, T_Y first natural period for lateral vibration
UDL uniformly distributed load
V vertical load
V_{CR} critical load for model structure
W bimoment; width of structure
X, Y principal axes
Z_1, Z_2 auxiliary functions defined by formulae (5.18)

SMALL LETTERS

a length of wall section
a_0, a_1, a_2 coefficients
b length of wall section; width of perforated wall section
b_w width of diagonal strip for infill
c translation; length of wall section; depth of lintel with coupled shear walls; critical load parameter for different end conditions
c_F translation due to F passing through the shear centre
c_{yF} translation in direction y due to F passing through the shear centre
c_M translation due to M acting around the shear centre
c_{yM} translation in direction y due to M acting around the shear centre
c_{ALT} altitude factor
c_d dynamic coefficient
c_{DIR} wind direction factor
$c_e(z_e)$ exposure coefficient
c_f force coefficient
c_{fj} force coefficient for incremental area A_j
c_i coefficients in a series
c_{TEM} temporary (seasonal) factor
d length of wall section; length of diagonal with cross-bracing
dz length of elementary section
e perpendicular distance between the line of action of the horizontal load and the shear centre; distance of upper flange from centroid
e^* distance of lower flange from centroid (with bracing cores)

f	frequency
f_c	frequency when the effect of the compressive force is taken into account
$f_{combined}$	combined lateral-torsional frequency for the monosymmetric case
f_{int}	natural frequency which takes into account soil-structure interaction
f_{flex}	natural frequency of rigid column on flexible support
f_d	frequency when the effect of damping is taken into account
f_X, f_Y	natural frequency of lateral vibration in principal directions X and Y
f_φ^t	pure torsional frequency associated with the Saint-Venant stiffness
f_φ^ω	pure torsional frequency associated with the warping stiffness
f_φ	fundamental frequency of torsional vibration
g	global axis; gravity acceleration
h	height of storey; length of wall section
h^*	equivalent height of storey
$\bar{h}$	height of first storey columns
$h_{i,k}$	length of the kth section of the ith bracing element
i	parameter relating to the number of bracing elements (from 0 to n)
i_p	radius of gyration
k	torsion parameter; translational stiffness; parameter relating to the number of elements in a series; spring constant; stiffness of a framework
k_x, k_y	translational stiffness with respect to axes x and y, respectively
k_s	modified torsion parameter; modulus of subgrade reaction
l	width of bay; distance between shear walls; local axis
l_1, l_2, l_3, l_4	'torsion arm' of shear walls; distance between the wall sections of coupled shear walls; width of bay
l_i	frequency factor
m	distributed moments for the stability analysis; length of section of beam for cross-bracing
m_z	distributed torque
m_{z0}	uniformly distributed part of torque
$\bar{m}$	distributed moments for the stress analysis, representing the supporting effect of the beams
n	number of columns/walls/bracing elements; number of storeys
n_h	number of structural elements on one floor level
p	intensity of the uniformly distributed load on the beams
p_s	intensity of seismic load
q	intensity of the uniformly distributed load on the floors; intensity of the uniformly distributed vertical load; horizontal load
q_0	intensity of the uniformly distributed part of the horizontal load
q_{0x}, q_{0y}	components of the uniformly distributed part of the horizontal load in directions x and y
q_1	intensity of the variable part of the horizontal load at top
q_{1x}, q_{1y}	components of the variable part of the horizontal load in directions x and y
q_{ref}	reference mean wind velocity pressure
q_x, q_y	components of the horizontal load in directions x and y

$q_{x,i}$, $q_{y,i}$	components of the horizontal load in directions x and y on the ith bracing element
$\bar{q}_x$, $\bar{q}_y$	auxiliary parameters defined by formula (5.19)
r	combination factor; modifier
r_1, r_2	critical load ratio; frequency ratio (for mode coupling); radius
r_f	reduction factor for the frequency analysis
r_{flex}	reduction factor for flexible support
r_s	reduction factor for the stability analysis
s	width of wall; distance between bays; length of arc (with closed cross-sections)
s_{flex}	reduction factor for flexible support
t	distance between the shear centre and the centre of the vertical load; wall thickness; global centroidal axis of the cross-sections of the columns; time
t^*	thickness of the equivalent wall
t_i	distance between the axis of the *i*th column and the global centroidal axis; wall thickness of the *i*th bracing element
$t_{i,k}$	wall thickness of the *k*th section of the *i*th bracing element
u	horizontal displacement of the shear centre in direction x
u_B	horizontal displacement of corner point B of the building in direction x
u_{flex}	top translation of rigid column on flexible support in direction x
u_g	horizontal displacement of the bracing element at top floor level
u_i	horizontal displacement of the shear centre of the *i*th bracing element in direction x
u_l	accumulative top level horizontal displacement due to the storey level displacements of the columns
$u_{\max}$	maximum horizontal displacement in direction x
v	horizontal displacement of the shear centre in direction y
v_A	horizontal displacement of corner point A of the building in direction y
v_{flex}	top translation of rigid column on flexible support in direction y
v_i	horizontal displacement of the shear centre of the *i*th bracing element in direction y
$v_{\max}$	maximum horizontal displacement in direction y
$v_{ref,0}$	basic value of the wind velocity given by means of wind maps
x	horizontal coordinate axis; horizontal coordinate
$\bar{x}$	horizontal coordinate axis; coordinate in coordinate system $\bar{x}$—$\bar{y}$
x_A	coordinate of corner point A of the building in direction x
x_c	coordinate of the centroid in direction x in the x—y coordinate system of the shear centre
x_i	coordinate of the shear centre of the ith bracing element in direction x
$x_{\max}$	location of maximum translation
$\bar{x}_i$, $\bar{y}_i$	coordinates of the shear centre of the ith bracing element in the coordinate system $\bar{x}$—$\bar{y}$
$\bar{x}_o$	coordinate of the shear centre in coordinate system $\bar{x}$—$\bar{y}$
y	horizontal coordinate axis; horizontal coordinate
$\bar{y}$	horizontal coordinate axis; coordinate in coordinate system $\bar{x}$—$\bar{y}$
y_B	coordinate of corner point B of the building in direction y
y_c	coordinate of the centroid in direction y in the x—y coordinate system of the shear centre

y_i	coordinate of the shear centre of the ith bracing element in direction y
$\bar{y}_0$	coordinate of the shear centre in coordinate system $\bar{x}$—$\bar{y}$
z	vertical coordinate axis; vertical coordinate
z_j	height of the centre of gravity of incremental area A_j
z_{max}	location of maximum bending moment in the beams of the framework

GREEK LETTERS

α	eigenvalue; critical load parameter for frames on fixed supports; critical load parameter for pure torsional buckling; angle between axes x and X; auxiliary parameter
α_p	eigenvalue; critical load parameter for frames on pinned supports
α_s	Southwell estimate for eigenvalue α; eigenvalue and critical load parameter for the sandwich model
β	stiffness ratio; damping ratio; dynamic constant for seismic load; stiffness ratio for soil-structure interaction (stability; frequencies)
βs	stiffness ratio for the sandwich model
δ	lateral displacement (storey drift)
ε	mode coupling parameter for the monosymmetrical case
γ	weight per unit volume; angular displacement
η	first natural frequency parameter for pure torsional vibration; height/width ratio
η_i, η_2, η_3	ith, 2nd and 3rd natural frequency parameters for pure torsional vibration
η_q	load factor (due to rotation)
η_T	shear force factor (due to rotation)
$\eta_{T'}$	shear force factor (due to rotation) for the concentrated force load case
η_M	bending moment factor (due to rotation)
$\eta_{M'}$	bending moment factor (due to rotation) for the concentrated force load case
κ	mode coupling parameter for the 3-dimensional case
λ	stiffness ratio for beams/columns of frameworks
φ	rotation; angle between the diagonal and horizontal bars in cross-bracing
φ_{max}	maximum rotation
μ	slope of the function of the horizontal load; construction misalignment
ν	global critical load ratio
ρ	mass density per unit volume; shape factor; air density
ρ^*	mass density per unit area
σ_z	compressive stress
τ_i	shear stress in the ith bracing element
τ_i^b	shear stress due to unsymmetrical bending
τ_i^t	shear stress due to Saint-Venant torsion
τ_i^ω	shear stress due to warping torsion
τ_X, τ_Y	eccentricity parameters for the 3-dimensional torsional-flexural buckling
ω	sectorial coordinate
ω_X, ω_Y	circular frequencies for vibration in principal directions X and Y
ω_φ	circular frequency for pure torsional vibration

1

Introduction

In applying physical and mathematical models which are based on the global behaviour of building structures, a unified treatment of the stress, stability and frequency analyses of bracing systems is presented for carrying out the structural analysis of buildings. In complementing the conventional ‘element-based’ design process, closed-form formulae and simple procedures are given for the global analysis of individual bracing elements and 3-dimensional bracing systems.

1.1 BACKGROUND

The conventional design process is normally based on the ‘local’ structural analysis of individual elements (columns, beams, floor slabs, walls, etc). This attitude is natural, since the structural system consists of individual elements. However, theoretical research, small-scale and large-scale tests and failures (and in some cases the lack of failures) in structural systems have indicated that complex structures cannot be considered simply as a collection of individual elements. The response of the structure is often more than the ‘sum’ of the responses of the individual elements since structural integrity ensures that the elements work together in a properly designed system and the structure develops some ‘global’ response through the complex interaction of its elements.

The ‘local’ or ‘global’ approach to structural design may affect the level of safety of the structure and can lead to considerable advantages or disadvantages as far as structural economy is concerned. A structure based on the optimum solution of the individual elements may not be economic when the system is considered as a whole. It is an interesting fact from the point of view of structural safety and economy, that when some elements in the system are supposed to fail when checked individually, they do not do so because other elements can help out. It is equally important to point out that interaction among the structural elements may also result in unfavourable phenomena.

Well-established and well-publicized methods have been made available for the analysis of individual structural elements. These methods are relatively simple and usually do not require much theoretical knowledge. Being suitable for automation, most of them have been built into computer procedures and are widely used in design offices.

The area of global structural analysis, however, is not so well developed. Many reasons have contributed to the relatively slow progress in developing methods for global design. Because of the complexity of the problem and the great number of structural elements involved in the analysis, deeper theoretical background, more sophisticated techniques and computers of great capacity are needed.

Global analysis, as structural design itself, can be carried out on two levels. An 'exact' analysis—or an analysis called exact—relies on a mathematical model as exact as possible and uses a static model which takes into account as many structural elements, material properties, geometrical and stiffness characteristics as possible. Taking everything into consideration, however, can result in problems. Even using a powerful computer, the job can be too big to handle. Because of the complexity of the results, they can be difficult to interpret. The lengthy and time-consuming procedure of handling all the data can always be a source for errors. Another disadvantage of this approach may be that the importance of the key structural elements is sometimes hidden behind the great number of input and output data.

Simplified procedures relying on carefully chosen approximations represent the other possibility for the analysis. A good approximate method relies on the most important structural characteristics and ignores those which have no real influence on the response of the structure. It is therefore simple, fast and offers a clear picture of the structural behaviour.

As both the local and the global approaches are important since they complement each other, both the exact and approximate procedures have their own significance. In the design process, the main structural characteristics are often established using an approximate method. Relying on the results of the approximate method, an exact procedure can follow, which eventually gives the final structural solution. The approximate method can fulfil another important task. As the theories behind the exact and approximate methods are often different—the exact methods are normally based on the finite element approach while the approximate methods are often based on analytical procedures—the results of the approximate analysis can be used as independent checks on the results of the exact method. When the two sets of result show the same structural behaviour, it is a strong indication that the results are correct. This endorsement is important to the structural designer, since it is sometimes not easy to detect an error with the exact analysis where thousands of data may be involved [MacLeod, 1995]. The significance of the independent verification of the results has widely been recognized and the importance of avoiding a 'Computer Aided Disaster' has been discussed at different conferences [Brohn, 1996; Knowles, 1996].

With the widespread use of more slender structural elements and lighter materials, and with the increasing demand for more economic structures, design for stability has become more and more important. In recognizing this tendency, the methods and procedures presented in this book for the stress, stability and dynamic analyses are linked together through the global critical load ratio, which is also shown to be a generic performance indicator.

With the increasing availability of more and more sophisticated and user-friendly computer procedures, the engineering society seems to be falling into two groups. Those developing the software packages manage to build on, and even develop further, their theoretical knowledge acquired at the university. On the other hand, office pressure to produce more and quickly forces some of the practising structural designers to concentrate only on pressing the right key on the keyboard. Some might say that it is not really important for the designer to understand the theory behind the analysis since the computer knows everything anyway. The tendency in the last twenty years indicates that the general knowledge of the young generation regarding basic structural behaviour is less than satisfactory [Brohn and Cowan, 1977; Brohn, 1992]. The discussion and debate on the advantages and disadvantages of the use of computers in the design office are still going on [Smart, 1997; Gardner, 1999].

1.2 GENERAL ASSUMPTIONS

The majority of the formulae and procedures are based on the application of the equivalent column concept and the summation theorems of civil engineering. The equivalent column concept is applicable to *regular* structures when the geometrical and stiffness characteristics of the bracing system do not vary over the height of the building. In addition, the following conditions also have to be fulfilled.

a) The material of the structures is homogeneous, isotropic and obeys Hooke's law.
b) The floor slabs are stiff in their plane and flexible perpendicular to their plane.
c) The structures have no geometrical imperfections, they develop small deformations and the third-order effect of the axial forces is negligible.
d) The loads are applied statically and maintain their direction (conservative forces).
e) The location of the shear centre only depends on geometrical characteristics.

When applied, adsditional assumptions concerning the different structures and types of analysis may be given in the introduction of the corresponding chapter.

The formulae presented for the stability analysis are applicable to structures subjected to concentrated top load, distributed load (or concentrated forces) on floor levels or the two loads together. It is assumed that the frameworks and coupled shear walls are sway structures and the critical load defines the bifurcation load.

The formulae given for the dynamic analysis were derived assuming distributed mass over the floors and concentrated mass on top floor level. The horizontal load for the stress analysis is assumed to be of trapezoidal distribution.

1.3 THE STRUCTURE OF THE BOOK

The objective of the book and general assumptions are given in Chapter 1. Chapter 2 briefly describes the 3-dimensional behaviour of the loadbearing elements and introduces the equivalent column for the bracing system, which is mostly used later for the analyses. Chapter 3 covers the stability and frequency analyses of buildings. Closed-form formulae, tables and diagrams are given for the quick calculation of the basic critical loads and natural frequencies for the stability and dynamic analyses. The combination of the basic modes in both the buckling and the frequency analyses is taken into account in two ways: by a simple interaction formula and by a cubic equation. Both multistorey and single-storey buildings are covered. The effect of soil-structure interaction is taken into account approximately by simple summation formulae. An elementary approach is presented in Chapter 4 for the stress analysis of buildings braced by parallel shear walls, a system of perpendicular shear walls, or frameworks, subjected to a uniformly distributed horizontal load. Worked examples facilitate practical application. The scope of the stress analysis is extended in Chapter 5 where a comprehensive method is given for bracing systems having (Saint-Venant and warping) torsional stiffnesses as well. Closed-form solutions are derived and diagrams and tables are given for the load distribution and the stresses and deformations when the building is under horizontal load of trapezoidal distribution. Simple expressive formulae are also presented for the maximum values of the stresses and deformations. Formulae for single-storey buildings are also presented. In addition to sizing the elements of the bracing system, the technique is potentially useful both at the concept design stage and for final analysis for checking of structural adequacy, assessing the suitability of structural layouts, verifying the results of other methods, evaluating computer packages, facilitating theoretical research and developing new techniques and procedures.

Based on a series of worked examples, a comprehensive qualitative and quantitative evaluation is given in Chapter 6, which also shows how the procedures are used in practice for actual design and for finding optimum structural arrangement. The global critical load ratio, also identified as a performance indicator of the bracing system, is introduced in Chapter 7. It is demonstrated that the global critical load ratio can be used to increase the efficiency of the bracing system while ensuring adequate level of safety, leading to more economic construction. Monitoring the value of the global critical load ratio for different structural layouts offers a simple tool for increasing the global critical load and the fundamental frequencies and reducing the maximum stresses and deformations in the bracing system.

To improve the accuracy of the procedures, which are based on mathematical and physical models, alternative formulae are also derived in Chapter 8 which, instead of the theoretical values of the stiffnesses, the key elements in the structural analysis, use stiffness values based on frequencies measured on the building. These formulae are applicable to the stress and stability analyses of doubly symmetrical bracing systems.

Chapter 9 deals with the stability analysis of planar structures and presents a closed form formula for an equivalent wall. The equivalent wall can then be used in the 3-dimensional analysis. It is shown that all 'frame-like' planar structures can be characterized by four distinct deformations and the corresponding four stiffnesses. Closed-form formulae are then given for frameworks on pinned and fixed supports with and without cross bracing, for coupled shear walls and for infilled frameworks. The efficiency of planar bracing elements is investigated through two sets of representative one-bay and two-bay, four to ninety-nine storey high structures.

Chapter 10 presents the results of a series of small-scale tests and a short summary of a comprehensive accuracy analysis which show that the accuracy of the procedures is acceptable for practical structural engineering applications. A brief evaluation of the procedures is given in Chapter 11 with practical guidelines for the structural designer.

Cross-sectional characteristics for commonly used bracing elements are collected in Appendix A. Appendix B introduces the power series method and shows how a complex eigenvalue problem in stability analysis can be transformed into a simple problem of finding the smallest root of a polynomial. In providing a complete set of tables, Appendix C reduces the task of producing the critical load of combined sway-torsional buckling and the fundamental frequency of combined lateral-torsional vibration to a single-step calculation.

References cited in the book are given following the Appendices. Further reading is helped by a list of additional bibliographical items.

Name and subject indexes conclude the book.

With the exception of Chapter 4, the book does not give the detailed derivations and proofs of the formulae; these are available in separate publications cited in the text and listed in the References. Chapter 4, however, demonstrates how carefully chosen elementary static considerations can lead to the simple solution of complex problems.

2

Spatial behaviour

2.1 BASIC PRINCIPLES

The primary structural elements of buildings are the vertical and horizontal load bearing structures. These structures carry the horizontal and vertical loads of the building. The vertical (dead and live) loads are transmitted to the vertical load bearing elements (shear walls, frames and columns) by the horizontal load bearing elements (floor slabs). The horizontal loads (wind, construction misalignment, and seismic forces) are transmitted by the floor slabs to those vertical load-bearing elements that are capable of passing them on the foundation. These dedicated structural elements (shear walls, frames and cores) are called the *bracing elements* of the building, whose main task is to provide the building with adequate lateral stiffness. They represent a system, which will be referred to as the *bracing system* of the building.

Of the vertical load-bearing elements, the frameworks are basically responsible for carrying the vertical loads and the main task of the cores is to provide the necessary lateral and torsional stiffnesses. The shear walls and coupled shear walls contribute to both tasks. The floor slabs act as horizontal load bearing elements and are also responsible for transmitting the applied vertical loads to the vertical load bearing elements and for distributing the horizontal loads among the vertical load bearing elements. Compared to the shear walls and cores, the frameworks are more flexible and they are often neglected when the lateral stiffness of a building is assessed. If, however, the effect of the frameworks also has to be taken into account for some reason, then, as an approximation, they can be replaced by fictitious walls. These fictitious walls can then be included in the analysis which can be carried out in a relatively simple manner when the bracing system only comprises walls and cores. Several methods are available for the calculation of the size of the cross-section of the fictitious walls; one of them is to stipulate that the critical load of the fictitious wall and that of the framework be identical. A simple procedure is given in section 9.9 where the limitations of the use of fictitious walls are also discussed.

Subjected to external loads, a system of shear walls and cores can develop three kinds of deformation: sway in the two principal planes of inertia and rotation around the shear centre. Apart from some special symmetrical arrangements, the three modes couple resulting in a combined sway-torsional behaviour. The exact spatial analysis of these structures is rather complicated, partly because of the interaction between the horizontal and vertical load bearing systems and among the elements of the horizontal and vertical systems themselves and partly because of the great number of elements to be involved in the analysis. By applying the equivalent column approach, however, the analysis can be simplified and closed-form solutions can be produced for the stresses and deformations, the load distribution among the structural elements, the elastic critical loads and the natural frequencies.

2.2 THE EQUIVALENT COLUMN AND ITS CHARACTERISTICS

The 3-dimensional analysis is based on the analysis of the equivalent column. The equivalent column is obtained by combining the bracing shear walls and cores of the building to form a single cantilever. Its bending and torsional stiffnesses represent the whole building. As the equivalent column is situated concurrent with the shear centre (centre of stiffness) of the bracing elements, the first step is to locate this global shear centre (*O* in Fig. 2.1). The position of the shear centre is found by making use of the basic geometrical and stiffness characteristics of the bracing elements [Beck and Schäfer, 1969].

The calculation is carried out in the coordinate system $\bar{x}$—$\bar{y}$, whose origin lies in the upper left corner of the plan of the building (Fig. 2.1) and whose axes are aligned with the sides of the building:

$$\bar{x}_o = \frac{I_{xy}\left(\sum_1^n I_{y,i}\bar{y}_i - \sum_1^n I_{xy,i}\bar{x}_i\right) - I_y\left(\sum_1^n I_{xy,i}\bar{y}_i - \sum_1^n I_{x,i}\bar{x}_i\right)}{I_x I_y - I_{xy}^2}, \tag{2.1}$$

$$\bar{y}_o = \frac{I_x\left(\sum_1^n I_{y,i}\bar{y}_i - \sum_1^n I_{xy,i}\bar{x}_i\right) - I_{xy}\left(\sum_1^n I_{xy,i}\bar{y}_i - \sum_1^n I_{x,i}\bar{x}_i\right)}{I_x I_y - I_{xy}^2}. \tag{2.2}$$

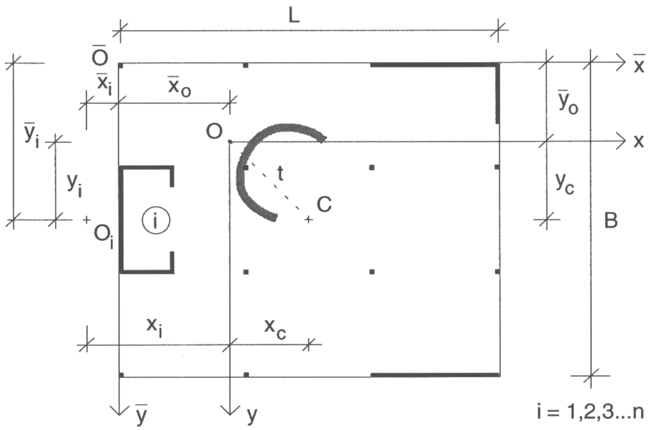


Fig. 2.1 Building layout with bracing elements and the equivalent column.

In formulae (2.1) and (2.2) $I_{x,i}$, $I_{y,i}$ and $I_{xy,i}$ represent the moments of inertia and the product of inertia of the ith element of the bracing system with respect to its local centroidal coordinate axes which are parallel to axes $\bar{x}$ and $\bar{y}$. Coordinates $\bar{x}_i$ and $\bar{y}_i$ stand for the location of the shear centre of the individual bracing elements in the coordinate system $\bar{x}$—$\bar{y}$ The sums of the moments of inertia and the product of inertia of the bracing elements are also needed in formulae (2.1) and (2.2):

$$I_x = \sum_1^n I_{x,i}, \qquad I_y = \sum_1^n I_{y,i}, \qquad I_{xy} = \sum_1^n I_{xy,i}, \tag{2.3}$$

where i=1…n, and n is the number of bracing elements. The sums of the moments of inertia of the bracing elements are important characteristics of the building in relation to its global bending. The product of inertia plays an important role in determining the orientation of the principal axes of the bracing system.

In addition to the above bending characteristics, there are two more characteristics of the equivalent column, which are associated with torsion: the Saint-Venant torsional constant and the warping (bending torsional) constant.

The Saint-Venant torsional constant is the sum of the Saint-Venant torsional constants of the bracing elements and is obtained in a similar manner as with the moments of inertia:

$$J = \sum_{1}^{n} J_i \,, \tag{2.4}$$

where J_i is the Saint-Venant torsional constant of the ith bracing element.

The warping constant is a weighted sum which is calculated in a coordinate system whose origin is the global shear centre [Beck, König and Reeh, 1968]. For this purpose, after making use of formulae (2.1) and (2.2), coordinate system $\bar{x}$—$\bar{y}$ is transferred to coordinate system x—y, whose origin coincides with the global shear centre and whose axes are parallel with axes $\bar{x}$ and $\bar{y}$ (Fig. 2.1). The warping constant of the equivalent column in this coordinate system assumes the form:

$$I_\omega = \sum_{1}^{n} (I_{\omega,i} + I_{x,i} x_i^2 + I_{y,i} y_i^2 - 2I_{xy,i} x_i y_i) \,, \tag{2.5}$$

where $I_{\omega,i}$ is the warping constant and x_i and y_i are the coordinates of the shear centre of the ith bracing element in the x—y coordinate system.

The warping (bending torsional) constant of the equivalent column represents two types of contribution: the first term in formula (2.5) represents the warping torsion of the bracing elements with regard to their own shear centre and the second, third and fourth terms stand for the bending torsion of the elements with regard to the global shear centre of the whole system. This part of contribution is realized through the bending of the bracing elements ($I_{x,i}$, $I_{y,i}$, $I_{xy,i}$) utilizing their 'torsion arm' (x_i, y_i) around the global shear centre.

Formulae (2.3), (2.4) and (2.5) reflect the assumption that the floors of the building are stiff in their own plane and flexible perpendicular to their plane. It is the out-of-plane flexibility of the floor slabs that leads to the simple sums in formulae (2.3) and (2.4) and it is their in-plane stiffness that results in the second, third and fourth terms in formula (2.5).

When the bending and torsional stiffnesses of reinforced concrete bracing elements are calculated for the establishment of the equivalent column, the effect of cracking on stiffness may have to be taken into account. The phenomenon is not discussed here as detailed information is available elsewhere; it is only mentioned that l_e=0.8I_g is normally considered an adequate reduction in the value of the second moment of area, where I_e and I_g are the effective and the gross (uncracked) second moments of area. Detailed information and guidelines are given in [Council, 1978d].

Apart from the above constants of the equivalent column, the radius of gyration is also needed for the establishment of the equivalent column for the stability and dynamic analyses. The radius of gyration is determined by the load of the building and the area over which it is distributed. The load is represented either by the mass (for the dynamic analysis) or by the vertical load (for the stability analysis).

In the general case when the building is subjected to a load of arbitrary distribution on a layout of arbitrary shape, the radius of gyration is calculated from

$$i_p = \sqrt{\frac{\int\limits_{(A)} q(x,y)(x^2+y^2)dA}{\int\limits_{(A)} q(x,y)dA}}, \tag{2.6}$$

where $q(x,y)$ is the intensity of the load.

Concentrated forces can also be taken into consideration: when the load consists of concentrated forces formula (2.6) assumes the form

$$i_p = \sqrt{\frac{\sum_i F_i(x_i^2+y_i^2)}{\sum_i F_i}}, \tag{2.7}$$

where F_i is the ith concentrated force and x_i and y_i are its coordinates in the x—y coordinate system whose origin is in the shear centre.

Formula (2.6) is a general formula. In many practical cases, simpler formulae can be used for the analysis. Assuming uniformly distributed load over the plan of the building, for example, the radius of gyration is obtained from

$$i_p = \sqrt{\frac{I_o}{A}} = \sqrt{\frac{I_{xp}+I_{yp}+A(x_c^2+y_c^2)}{A}} = \sqrt{\frac{I_{xp}+I_{yp}}{A}+t^2}, \tag{2.8}$$

where

I_o is the polar moment of inertia of the ground plan with respect to the shear centre of the bracing system,

I_{xp}, I_{yp} are the second moments of area of the plan of the building with respect to the centroidal axes,

A is the area of the plan,

t is the distance between the shear centre (O) and the centre of vertical load (C) (Fig. 2.1), according to the formula

$$t = \sqrt{x_c^2 + y_c^2} \,. \tag{2.9}$$

Coordinates x_c and y_c are the coordinates of the geometrical centre of the plan of the building in coordinate system x—y whose origin is in the shear centre:

$$x_c = \frac{L}{2} - \bar{x}_o, \qquad y_c = \frac{B}{2} - \bar{y}_o \,. \tag{2.10}$$

When the building has a rectangular plan, the formula for the radius of gyration simplifies to

$$i_p = \sqrt{\frac{L^2 + B^2}{12} + t^2} \,, \tag{2.11}$$

where L and B are the plan length and breadth of the building (Fig. 2.1).

Coordinate systems $\bar{x}$—$\bar{y}$ and x—y have been used for convenience as they make the calculation of the basic characteristics straightforward. However, the stability and dynamic analyses can be carried out in a much simpler way in the coordinate system whose origin is placed at the global shear centre and whose horizontal axes coincide with the principal axes.

In many practical cases the product of inertia of the elements of the bracing system is zero. In such cases, axes x and y are already the principal axes and the equivalent column is established by the stiffness and geometrical characteristics defined by formulae (2.1) to (2.5) which, due to I_{xy}=0, simplify considerably. [The simplified versions of formulae (2.1), (2.2) and (2.5) are given in section 5.7.] However, I_{xy} is not zero, for example, for Z and L shaped bracing cores and axes x and y are not the principal axes. A transformation of the coordinate system is needed: coordinate system x—y should be rotated around the origin (the global shear centre) in such a way that the new axes X and Y are the principal axes (Fig. 2.2). The angle of principal axis X with axis x is obtained from the formula

$$\alpha = \frac{1}{2}\arctan\frac{2I_{xy}}{I_y - I_x}. \tag{2.12}$$

Principal axis *Y* is perpendicular to axis *X*. The change in the coordinate system only affects the moments of inertia. The moments of inertia in the new coordinate system are obtained from

$$I_X = I_x \cos^2\alpha + I_y \sin^2\alpha - I_{xy}\sin 2\alpha \tag{2.13}$$

and

$$I_Y = I_x \sin^2\alpha + I_y \cos^2\alpha + I_{xy}\sin 2\alpha. \tag{2.14}$$

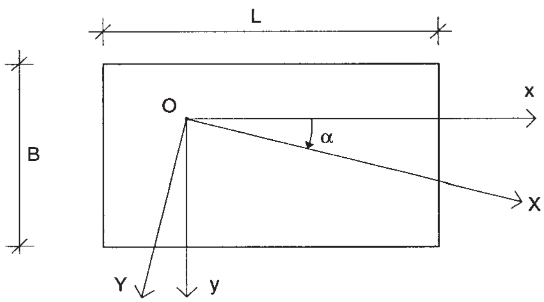


Fig. 2.2 Principal axes *X* and *Y*.

It should be noted that the product of inertia vanishes in the coordinate system whose axes are the principal axes, i.e. $I_{XY}=0$.

Low-rise and medium-rise buildings are sometimes braced by frameworks or by a mixture of frameworks and shear walls. The equivalent column approach can still be used but the frameworks have to be replaced first by fictitious walls. When such a wall is incorporated into the equivalent column, only its in-plane bending stiffness has to be calculated and all the other stiffness characteristics are set to be zero. This procedure is considered more accurate for frameworks with cross-bracing and for infilled frameworks as they develop predominantly bending type deformations. However, when moment resistant frameworks on fixed or pinned supports are replaced by fictitious shear walls, the procedure is only approximate. The level of approximation depends on to what extent the deformation of the framework differs from the bending type deformation of the fictitious wall. Each case should be treated very carefully as an individual case. Detailed information is given in section 9.9 regarding the calculation of the size of the fictitious wall and in section 9.10 where the behaviour and efficiency of different planar structures are considered.

When the lateral stiffness of the bracing system is evaluated for practical structural engineering calculations, the contribution of the stiffnesses of the individual columns is normally neglected, being small compared to that of the shear walls and cores. This approximation simplifies the calculation and leads to conservative estimates. However, experimental evidence shows that this contribution can be significant in certain cases (e.g. when there are many columns and only few bracing walls, or when the elements of the bracing system are in a special arrangement) [Zalka and White, 1992 and 1993]. The approximate method presented below can be used in such cases. The method is based on a simple formula which converts the 'local' stiffness of a column of storey height into a 'global' stiffness of a bracing element of building height. This element can then be simply added to the other 'ordinary' bracing elements and can be incorporated into the equivalent column in the usual manner.

The accumulation of the local sway of a column of storey height h with the same cross-sectional size (Fig. 2.3/a) over the height of the building results in a total top translation of

$$u_l = 2n\frac{F(h/2)^3}{3EI_c}, \tag{2.15}$$

where n is the number of storeys and I_c is the second moment of area of the cross-section of the column. Assuming a horizontal load of trapezoidal distribution, the top translation of a single bracing element of building height H and of a fictitious second moment of area I_{cg} (Fig. 2.3/b) is

$$u_g = \frac{q_0 H^4}{EI_{cg}}\left(\frac{1}{8}+\frac{11}{120}\mu\right), \tag{2.16}$$

where μ is a parameter defining the slope of the function of the horizontal load, according to formula (5.2) in section 5.1 and q_0 is the intensity of the uniformly distributed part of the load (Fig. 5.1).

By combining the right-hand sides of equations (2.15) and (2.16) and making use of the relationship $H=hn$, the 'local' moment of inertia of a column can be transformed into a 'global' moment of inertia for the global analysis of the building:

$$I_{cg} = 12n^2 I_c \left(\frac{1}{8} + \frac{11}{120}\mu \right), \tag{2.17}$$

where I_{cg} is the 'global' moment of inertia of the column. The 'global' moment of inertia defined by formula (2.17) can now be used directly for the global analysis. When the stiffnesses of the equivalent column are assembled, the columns can be considered as 'ordinary' bracing elements and their 'global' moments of inertia can simply be included in the summations in formulae (2.1) to (2.5).

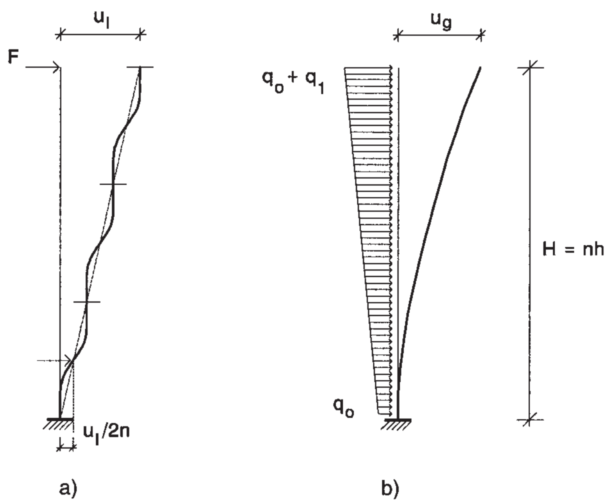


Fig. 2.3 Top translation. a) Accumulation of storey-sway over the height, b) translation of a bracing element of height *H*.

Formula (2.17) is considered an approximation and can only be used for the global analysis of the building. The approximation is due to the two assumptions made for the derivation of the formula, namely, it is assumed that

- the columns of storey height have fixed ends,

- the global deflection of the building resulting from the double-curvature bending of the columns between the floors assumes a straight line (dashed line in Fig. 2.3/a).

These assumptions seem to represent strict restrictions but it has to be kept in mind that the effect of the columns on the global stiffnesses is of secondary nature anyway. The results of tests on small-scale, 10-storey building models given in Chapter 10 indicate that the displacements and rotations obtained by using formula (2.17) are of acceptable accuracy.

When the geometrical characteristics of the equivalent column are needed, several sources can be used for the calculation of the Saint-Venant torsional constant and the moments of inertia of the individual bracing elements [Griffel, 1966; Hrobst and Comrie, 1951; Roark and Young, 1975]. However, the situation is different when the warping constant and the location of the shear centre of the cores are needed. Only a limited number of publications deal with these and they either give a detailed theoretical background for the calculation with only one or two worked examples or present ready-to-use formulae for only some special cases [Gjelsvik, 1981; Kollbrunner and Basler, 1969; Murray, 1984; Vlasov, 1940]. Unfortunately, printing errors, which are difficult to detect in some cases, make their application somewhat risky. It is possible to develop computer procedures of general validity [Roberts, 1985; Waldron, 1986] but they are usually not widely available and their accuracy and reliability are difficult to check. To make the task of producing the bending and torsional characteristics of the individual bracing cores as simple as possible, a collection of closed-form formulae for cross-sections widely used for bracing cores is given in Appendix A. For bracing elements of special cross-sections where no closed-form solution is available, the excellent computer procedure PROSEC [1994] can be used, whose accuracy has been established and proved to be within the range required for structural engineering calculations [Zalka, 1994a].

2.3 THE SPATIAL BEHAVIOUR OF THE EQUIVALENT COLUMN

Apart from some special cases, building structures develop a combination of the three basic modes (sway in the principal planes and torsion). The nature of the behaviour (and the extent of the combination) depend on the relative position of the shear centre of the bracing system and the centre of the external load—and for lateral loads, the direction of the load.

Assuming vertical load and stability analysis, possibilities for the coupling of modes are shown in Fig. 2.4, where the whole bracing system of the building is represented by the equivalent column of open, thinwalled cross-section. The distance between the shear centre of the bracing system *(O)* and the centre of the vertical load *(C)* is marked with *t*.

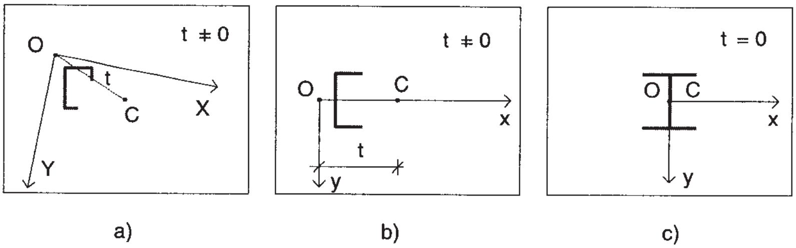


Fig. 2.4 Coupling of basic modes. a) Triple coupling, b) double coupling, c) no coupling.

When the centre of the vertical load does not lie on either principal axis (Fig. 2.4/a), sway in the two principal directions *X* and *Y* is coupled by pure torsion. The critical load which belongs to this combined sway-torsional buckling is the global critical load of the building.
When the centre of vertical load lies on one of the principal axes (*x* on Fig. 2.4/b), sway in that principal plane develops independently of the other two modes. Sway in the perpendicular direction combines with torsion. Both critical loads have to be calculated, i.e. the independent sway critical load in the principal plane and the combined sway-torsional critical load in the perpendicular direction, and the smaller one is the global critical load of the building. The simplest case arises when the shear centre and the centre of the vertical load coincide (Fig. 2.4/c). Sway in the principal directions and torsion about the shear centre develop independently. The global critical load is the smallest one of the independent basic critical loads.

The above principles outlined for the stability analysis can also be applied to the dynamic analysis and—assuming a horizontal load system of arbitrary direction—to the stress analysis. Sections 3.1 and 3.2 also deal with the spatial behaviour of the equivalent column when the 3-dimensional stability and vibrations of the building are investigated using the governing differential equations. In sections 5.1 to 5.3 the 3-dimensional behaviour of the equivalent column (and the building) under horizontal load is analysed in detail.