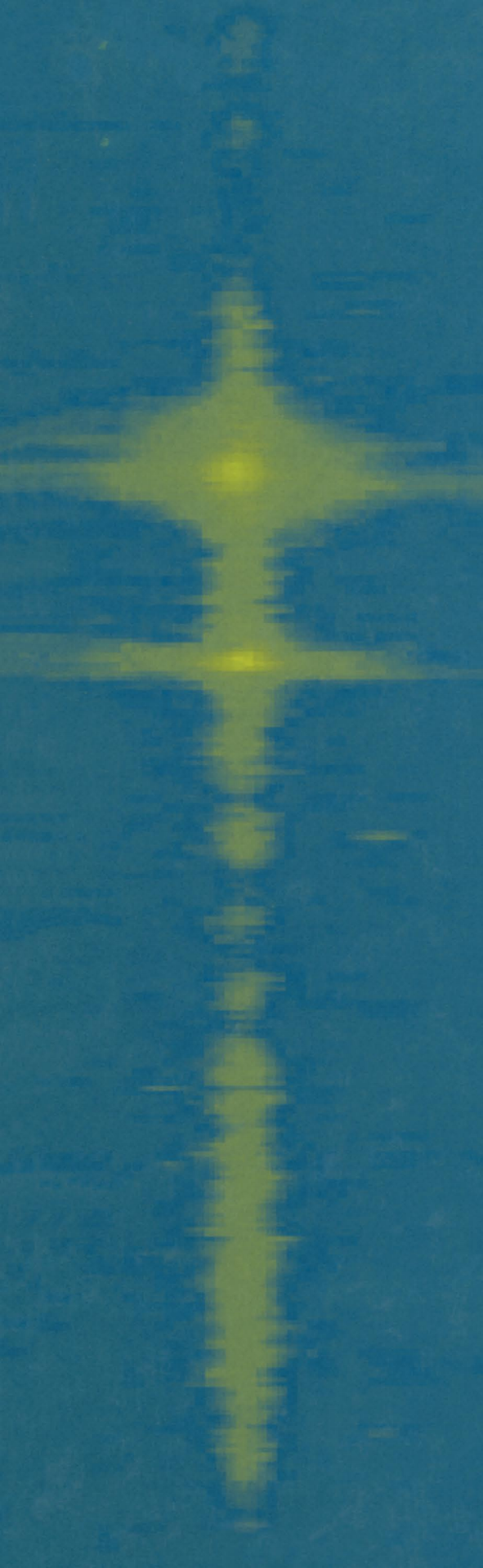




High Resolution X-ray Diffraction and Topography

D. Keith Bowen
and Brian K. Tanner



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Preface

The purpose of this book is to provide the theoretical and practical background necessary to the study of single-crystal materials by means of high resolution X-ray diffractometry and topography. Whilst some of these techniques have been available for over fifty years, and the basic theory for even longer, it is in the last decade that they have grown enormously in importance, essentially for two technological reasons. On the one hand there has been the development of the powerful sources for scientific and industrial research: dedicated electron storage rings for synchrotron radiation, which have enabled the ideas and techniques developed in the earlier years to be applied and extended. On the other hand has been the industrial need for characterisation and control of the high-quality crystals that now form the heart of so many of the devices used for the electronics, communications and information engineering industries: integrated circuits, sensors, optoelectronic and electro-acoustic devices and microprocessors. Techniques such as high resolution X-ray diffractometry, that for half a century were rather obscure research tools, are now in daily use for industrial quality control.

Educational establishments have not yet fully responded to this growth, and the availability of trained ‘characterisation’ scientists and engineers is a limiting factor in many industries that manufacture devices based upon high-quality crystals. In consequence, many research or quality control laboratories are staffed by people trained in different fields, who may be unaware of both the potentialities and the pitfalls of the X-ray techniques. In this book we attempt to redress these problems. We assume only a knowledge of the materials structures and defects whose characterisation is required. We provide the elementary and advanced theory of X-ray diffraction that is required to utilise the techniques; besides covering the mathematical formalism we have taken pains to present the conclusions of the theory qualitatively and visually, to make its consequences more accessible. We discuss in depth the techniques of X-ray diffractometry and topography, including both the practical details and the application of the theory in the interpretation of the data. The limits of the techniques are explored throughout, an attitude that takes particular technological importance in the light of the present concern with very thin films and surface layers.

Preface

We are pleased to acknowledge the contributions and the stimulation of our graduate students and research associates over many years of research into materials characterisation. The content and approach of this book emerged over almost as many years of teaching advanced courses, beginning with a NATO Advanced Study Institute project in 1979 and including workshops and courses for the Denver X-ray Conference and the European Union COMETT and TEMPUS schemes. We owe special gratitude to our colleagues at Bede Scientific Instruments, where we are both directors, for funding the development of these courses and, especially, for posing interesting and difficult problems for us arising from the real needs of industrial scientists working with advanced materials. Finally, we thank the European Synchrotron Radiation Laboratory and the University of Denver for being our hosts during periods of intensive writing on sabbatical leave.

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Introduction: Diffraction Studies of Crystal Perfection

In this chapter we introduce high resolution diffraction studies of materials, beginning from the response of a perfect crystal to a plane wave, namely the Bragg law and rocking curves. We compare X-rays with electrons and neutrons for materials characterisation, and we compare X-rays with other surface analytic techniques. We discuss the definition and purpose of high resolution X-ray diffraction and topographic methods. We also give the basic theory required for initial use of the techniques.

The experimental beginning of the study of materials by diffraction methods took place in 1911 when von Laue, Friedrich and Knipping¹ obtained the first X-ray diffraction patterns, on rock salt. The theoretical foundation was laid between then and 1917 by Bragg,² von Laue,³ Darwin⁴ and Ewald.⁵ Many workers then contributed to the steady development of the subject, and we highlight in particular the work of Lang, Authier, Takagi, Taupin and Kato in developing the experimental and theoretical tools for the study of distorted and defective crystals by X-ray methods which form the main topic of this book.

In its first sixty or seventy years, two great methods dominated X-ray diffraction research. Single-crystal diffraction for structure analysis revealed the crystal and molecular structure of inorganic, organic and more recently biological compounds, and is now an essential tool in the understanding of complex processes such as enzyme action and in drug design. The powder diffraction method has become indispensable for the practical analysis of materials ranging from corrosion products to contraband. However, two developments in the early 1980s stimulated the explosive growth of high resolution X-ray diffraction, which began in the 1920s but had remained a small, specialised method. The push was the ample intensity available from synchrotron radiation sources, which allowed the rapid exploration of many new X-ray scattering techniques. The pull was the industrial requirement for rapid, non-destructive analysis of defects in highly perfect materials for the electronics industry, which was beyond the capability of the lower-resolution techniques. Interestingly, developments in X-ray optics which were enabled by the availability of large, highly perfect crystals of silicon from the electronics industry have permitted the development of laboratory-based instruments for rapid industrial materials characterisation.

The development of electron diffraction and microscopy was parallel but separate. Although there are many obvious similarities between the theories, they are usually expressed in different notations and it is common that workers trained in the one have little knowledge of the other. This is particularly unfortunate in that the

techniques are almost always complementary, and many investigations will benefit by the use of both electron and X-ray (or neutron) methods. The emphasis in this book is on the X-ray methods. Electron techniques have been amply covered in a number of excellent publications;^{6,7} we have not attempted to include any detailed description or assessment of them, but we shall take the opportunity to draw parallels and complementarities between the electron, X-ray and neutron beam methods.

1.1 The response of a crystal to a plane wave

Any radiation striking a material is both scattered and absorbed. Scattering is most easily approached by thinking of a plane wave. This is formally defined as one whose phase is constant over any plane normal to its direction of travel, its wavefront is a plane. It is more easily thought of as a point source of waves at an infinite distance; a perfectly collimated wave. When such a wave strikes a three-dimensional atomic lattice, each scattering point (electron or nuclear particle) acts as a source of spherical waves, whose wavefronts lie on spheres centred on the scattering points. The addition of the amplitudes of all these waves in given directions results in almost zero intensity in most directions but strong beams in some directions if the wavelengths of the wave are comparable with the spacing of the scattering centres. This is the phenomenon of diffraction.

The simplest and most useful description of crystal diffraction is still that obtained by Bragg.² Strong diffraction occurs when all the wavelets add up in phase. By considering an entire crystal plane as the scattering entity, rather than each individual electron, it is easy to see from Figure 1.1 that strong diffraction results when

$$n\lambda = 2d \sin \theta \quad (1.1)$$

where n is an integer representing the order of diffraction, λ is the wavelength, d the interplanar spacing of the reflecting (diffracting) plane and θ the angle of incidence and of diffraction of the radiation relative to the reflecting plane. The requirement for the angle of incidence to equal that of diffraction is not seen directly from Figure 1.1, but arises from the incorporation of scattering from many planes normal to the

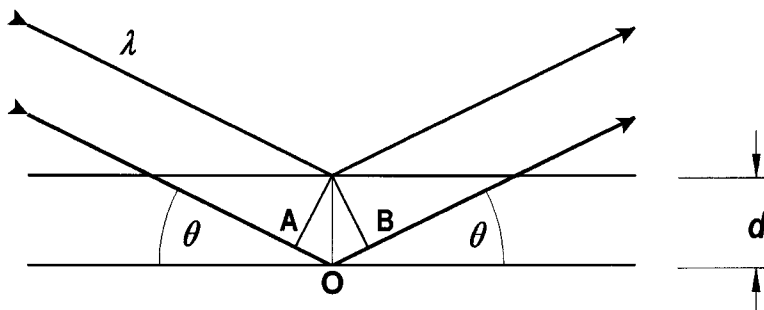
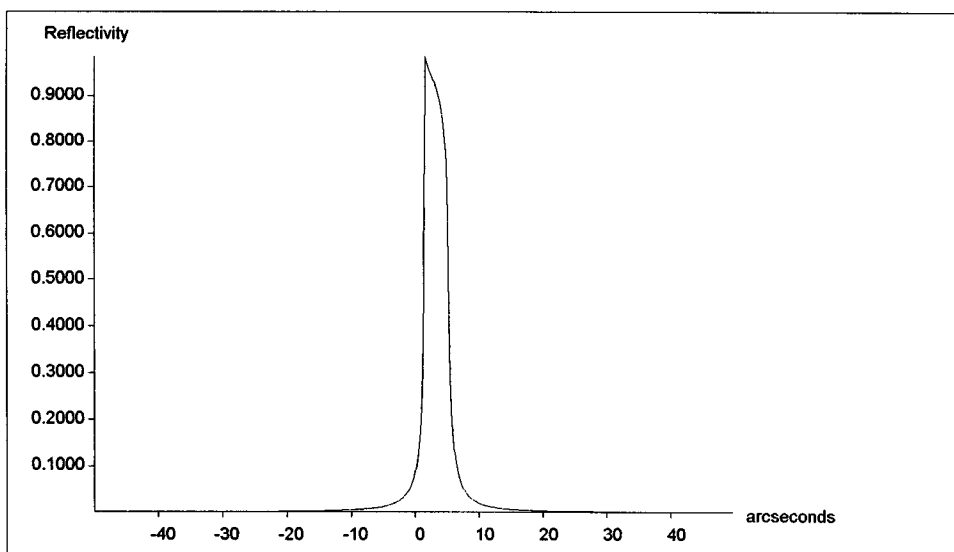
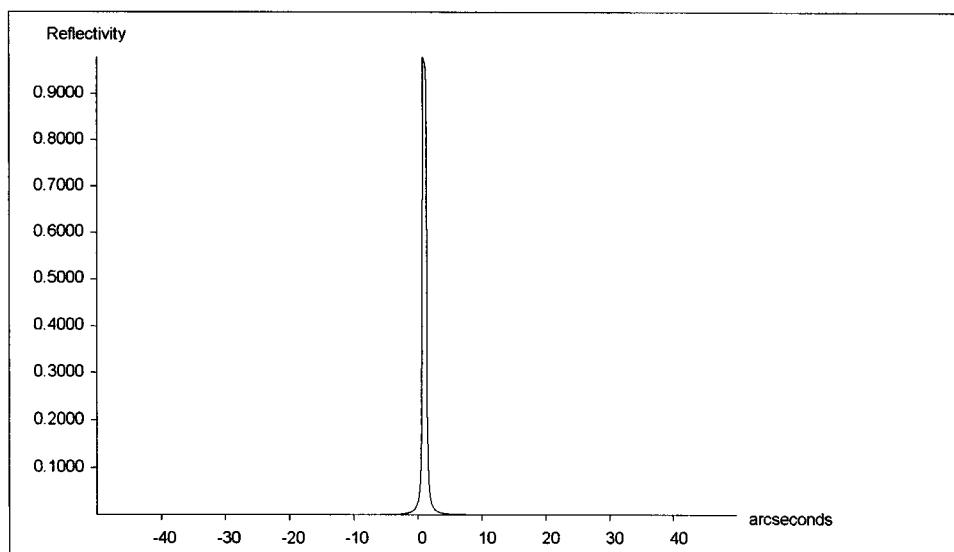


Figure 1.1 Diffraction of a plane wave off successive crystal planes. Strong diffraction results when the angles of incidence and diffraction, θ , are equal and the path difference AOB between the two beams is equal to $n\lambda$, an integral number of wavelengths. Hence the Bragg law, $n\lambda = 2d \sin \theta$

surface. A small number of planes give a very broad peak, and large numbers of planes a narrow peak, converging to a value characteristic of a thick crystal. Thus, diffraction for a given plane and wavelength does not take place over the zero angular range defined by the Bragg law, but over a small finite range. This range,

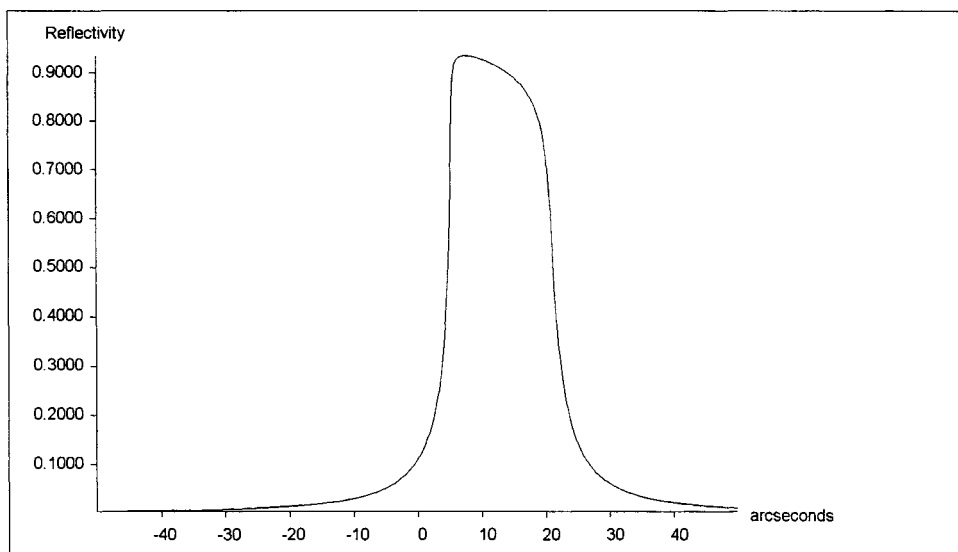


(a)

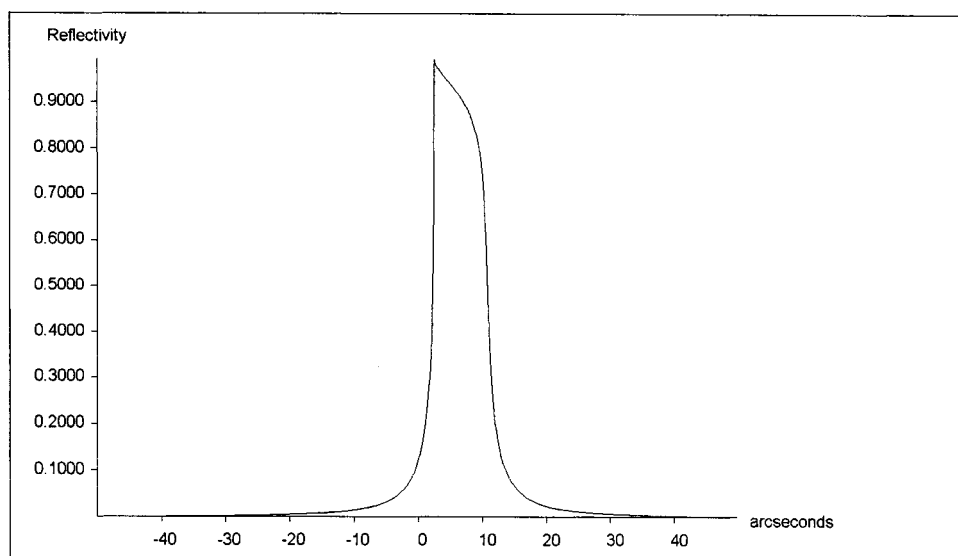


(b)

Figure 1.2 Calculated plane wave X-ray rocking curves. (a) Si 004 with CuK $_{\alpha 1}$ (0.154 nm), FWHM = 3.83 arcsec, (b) Si 333 with MoK $_{\alpha 1}$ (0.071 nm), FWHM = 0.73 arcsec, (c) Ge 111 with CuK $_{\alpha 1}$, FWHM = 16.69 arcsec, (d) GaAs 004 with CuK $_{\alpha 1}$, FWHM = 8.55 arcsec



(c)



(d)

Figure 1.2 (cont.)

called the *rocking curve* width, varies tremendously and it governs the strain sensitivity of the technique. Examples of X-ray rocking curves for perfect crystals are shown in Figure 1.2.

Any radiation is also absorbed to a greater or lesser extent by a material. This is well described by the usual absorption equation:

$$I/I_0 = \exp\{-\mu t\} \quad (1.2)$$

where I is the transmitted and I_0 the incident intensity, μ is the linear absorption coefficient and t the specimen thickness in the direction of the X-ray beam.

The intensities of diffracted beams, or reflections as they are commonly called, depend upon the strength of the scattering that the material inflicts upon the radiation. Electrons are scattered strongly, neutrons weakly and X-rays moderately. The basic scattering unit of a crystal is its *unit cell*, and we may calculate the scattering at any angle by multiplying

- the scattering strength of an electron or nucleus
- the scattering strength of an atom
- the scattering strength of a unit cell
- the scattering strength of the total number of unit cells

all with regard to the direction of scattering and the relative phase of the scattered waves.

Electron, nuclear and atomic scattering factors are either calculated or measured and tabulated for X-rays.⁸ The angular dependence of the intensity is most usefully represented by calculating the scattering strength of the unit cell for a particular lattice plane reflection, hkl , wavelength and, of course, crystal structure. This most important parameter is the *structure factor* F_{hkl} . This is a fundamental quantity, which appears in all the expressions for diffracted intensity, penetration depth and rocking curve width, and is calculated as follows. Waves of vector $\mathbf{k}$ scattered from two points in the unit cell which are separated by a vector $\mathbf{r}$ (see Figure 1.3), will have a phase difference

$$\exp\{-2\pi i \mathbf{k} \cdot \mathbf{r}\} = \exp\{-2\pi i(hu + kv + lw)\} \quad (1.3)$$

for the hkl reflection, where (uvw) are the fractional coordinates of the vector $\mathbf{r}$. We now need to add up the waves scattered from each atom with regard to the atomic scattering factor of each atom and the phase of the wave from each atom and sum this over the unit cell:

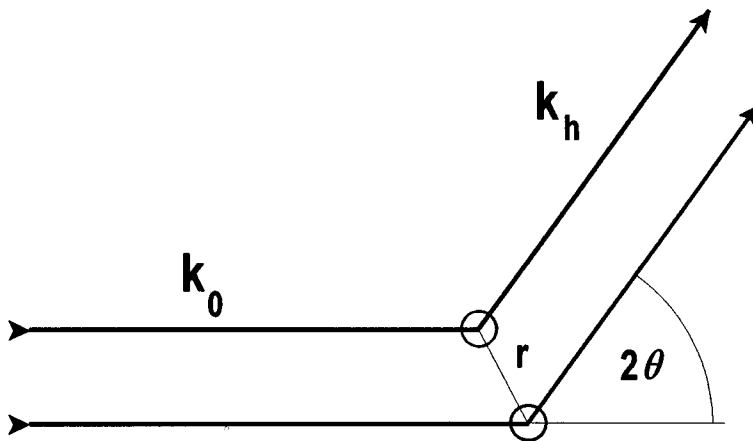


Figure 1.3 The addition of waves scattered by an angle 2θ from an atom at the origin and one at a vector $\mathbf{r}$ from the origin. The wavevectors $\mathbf{k}_0$ and $\mathbf{k}_h$ are in the directions of the incident and diffracted beams, respectively, and $|\mathbf{k}_0| = |\mathbf{k}_h| = 1/\lambda$

$$F_{hkl} = \sum_i f_i \exp\{-2\pi i(hu + kv + lw)\} \quad (1.4)$$

The f_i are the atomic scattering factors of the atoms of type i . These depend upon both θ and λ . The phases may cause the waves to add, as in for example Si 004, or cancel, as in for example Si 002. The larger the structure factor, the broader is the rocking curve.

There is a relationship between scattering strength and absorption strength, in that strong scatterers are also in principle strong absorbers. The parameters are often combined, as will be seen in Chapter 4, into a complex structure factor in which the real part represents the scattering factor (equation (1.4)) and the imaginary part the absorption coefficient (equation (1.2)).

1.2 Comparison of radiations for diffraction

Later chapters will deal with a more complete description of the diffraction process, but we now have enough to discuss the selection of radiations and techniques. If the structure factor and scattering strength of the radiation are high, the penetration is low and the rocking curve is broad. This is the case with electron radiation. For X-rays and even more for neutrons, the structure and absorption factors are small, penetration is high and rocking curves are narrow. These factors have three main consequences for X-rays and also for neutrons:

- 1 The techniques are normally non-destructive, since adequate penetration and coverage of industrial-scale samples is possible.
- 2 The strain sensitivity is high, since narrow rocking curves are strongly influenced by small rotations caused by strains in the crystal.
- 3 The spatial resolution of diffraction imaging is poor, compared with electrons, since small areas are not scattering enough radiation for good imaging, and the high strain sensitivity means that defect images are very broad.

Table 1.1 summarises the characteristics. Electron microscopy and diffraction and X-ray topography and diffraction are complementary techniques in almost every respect. The neutron techniques have applications similar to X-rays but decisive advantages in some cases, such as the study of magnetic materials and of very thick samples. The theory is well understood for all three. Two great advantages of X-ray techniques are their convenience and non-destructive nature; they

Table 1.1 Characteristics of electron, neutron and X-radiations

Characteristic	Electrons	Neutrons	X-rays
Absorption	high	very low	low
Penetration	<1 μm	~cm	~mm
Rocking curve width	degree	<arc second	arc seconds
Strain sensitivity	10^{-3}	10^{-7}	10^{-7}
Spatial resolution	1 nm	30 μm	1 μm
Destructive?	yes	no	no
Cost	high	very high	medium
Convenience	good	poor	good

may thus be performed before, say, sectioning for electron microscopy and the results for both techniques combined, or they may be used for on-line quality control.

1.3 Comparison of surface analytic techniques

The penetrating power of X-rays is a major factor in their applicability to problems such as crystal growth. However, there is strong and increasing demand for methods of characterisation of very thin layers of material. Devices are now being marketed that depend for their operation on the existence of a controlled layer of material a few nanometres thick. If such layers are near the surface, as they often are for reasons of manufacture, they may also be characterised by X-ray methods using the new *grazing incidence* methods. This term is used for angles of incidence close to the critical angle for total external reflection, in practice, up to a few degrees. As seen in the previous section, the X-ray methods are particularly sensitive to strain (as are many modern devices) and it is relevant to compare the applicability of strain-sensitive surface techniques. Figure 1.4 shows a comparison of the application range

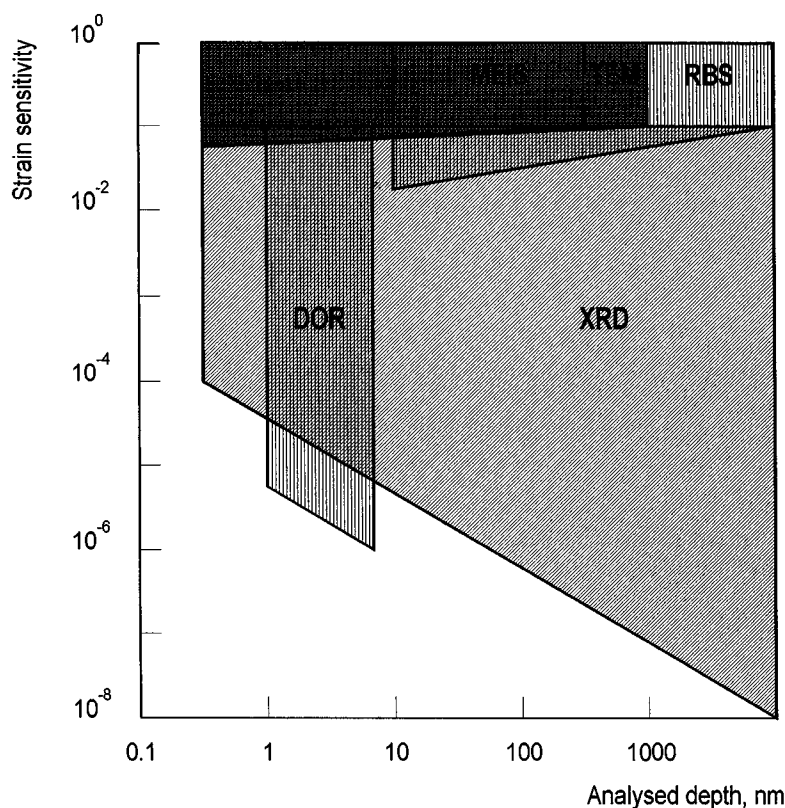


Figure 1.4 Comparison of the application ranges of techniques that are sensitive to near-surface strains. Minimum detection limits are plotted against depth resolution of the measurement. XRD: X-ray diffraction; DOR: differential optical reflectometry. RBS: Rutherford back scattering; MEIS: medium energy ion scattering; TEM: transmission electron microscopy

of the major strain-measuring techniques that are sensitive to strains in the top micrometre of crystalline material. The important role of X-ray methods is clear, even though the methods lack the spatial resolution that can be achieved with charged particle probes.

1.4 High resolution X-ray diffractometry

The term ‘diffractometry’ here means the measurement of the rocking curve of a sample. It is always necessary to define the incident radiation, both in wavelength and divergence, and a clear understanding of the latter is particularly important. The reference is always the plane wave rocking curve, such as would be measured with a perfectly parallel, monochromatic incident beam, and theoretical calculations are based upon this imagined radiation. Of course, any real radiation has both a wavelength spread and a divergence, and a good approximation to a plane wave curve is only found if the broadening effects of these are small compared with the width of the theoretical plane wave rocking curve. The examples in Figure 1.4 make this clear. The rocking curve widths range from $0.6''$ to $12''$. The divergence of a good synchrotron radiation beam could be about $1''$, and that of a sealed tube X-ray source at 1 m is about $80''$. Synchrotron radiation is a continuous spectrum and single-crystal rocking curves map the spectrum. The rocking curve obtained in all other cases will be dominated by the source profile, little influenced by the specimen. In other words, the instrument function for conventional powder and single-crystal diffractometers is far too great to measure subtle changes in rocking curves of nearly perfect crystals.

Better conventional collimation will not do, except for the largest synchrotron radiation installations; to obtain sub-arc-second collimation in the laboratory would require a collimator some 100 m long with a sealed-tube source, and at this distance the intensity would be impracticably low. The problem is solved by the use of a beam conditioner, which is a further diffracting system before the specimen. The measured rocking curve is then the correlation of the plane wave rocking curves of

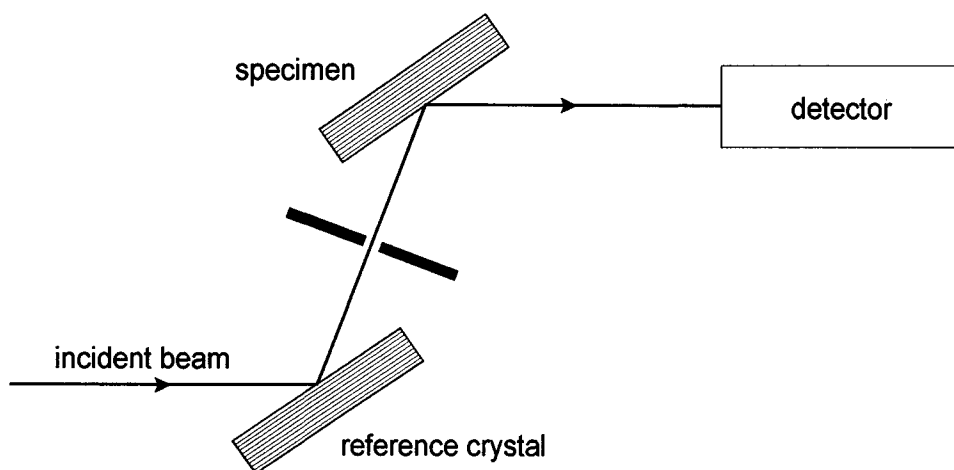


Figure 1.5 The symmetrical double crystal (+, -) setting for measuring rocking curves

the beam conditioner and the specimen crystals, from which most of the diffracting characteristics of the specimen crystal may be deduced.

The simplest conditioner is a perfect crystal of the same type as the specimen, using the same reflecting planes, with the deviation of the diffracted beam in the opposite sense to that at the specimen. This is the classic ‘+,- symmetrical double crystal method’, as shown in Figure 1.5, which gives excellent and easily interpreted results. Many variations are, however, possible, for example to maximise the sensitivity to strain, or to emphasise the contribution of near-surface layers to the diffraction, and we shall treat these in detail in this book.

The great utility of rocking curve measurements is based upon two fundamental properties:

- 1 The details of the rocking curve are extremely sensitive to the strains and strain gradients in the specimen.
- 2 For a given structural model, the rocking curve may be computed to high accuracy using fundamental X-ray scattering theory.

In addition, the measurements are rapid and simple, and are now even used in 100% inspection for quality control of multiple-layer semiconductors. An example is shown in Figure 1.6. This is a GaAs substrate with a ternary layer and a thin cap. The mismatch between the layer and the substrate is obtained immediately from the separation between the peaks, and more subtle details may be interpreted with the aid of computer simulation of the rocking curve. This curve can be obtained in a matter of minutes. Routine analysis of such curves gives the composition of ternary epilayers, periods of superlattices and thicknesses of layers, whilst more advanced analysis can give a complete strain and composition profile as a function of depth.

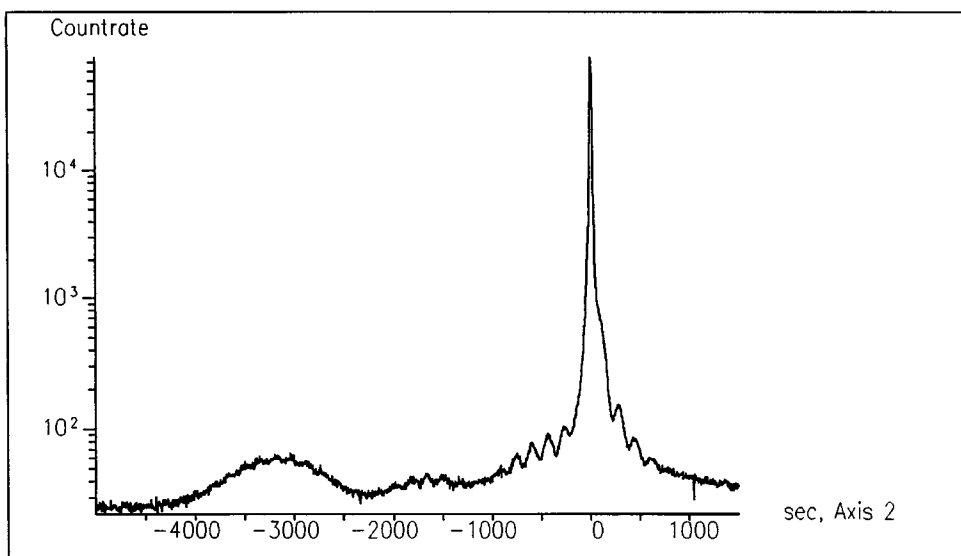
1.5 Triple-axis diffractometry

Triple-axis diffractometry is an extension of high resolution diffractometry in which an analyser crystal (or crystals) is placed before the detector in order to restrict its angular acceptance. This has the effect of separating the effects of strains and tilts on the measurement, and permitting the measurement of diffuse scatter. In reciprocal space notation, which will be explained in Chapter 5, the intersection of the angular collimation of the beam conditioner crystal(s) and the analyser crystal(s) in reciprocal space defines a small volume of reciprocal space that is sampled by the detector at a given setting. Thus, complete two-dimensional reciprocal-space maps may be obtained, giving very detailed information on thin layers and surface conditions.

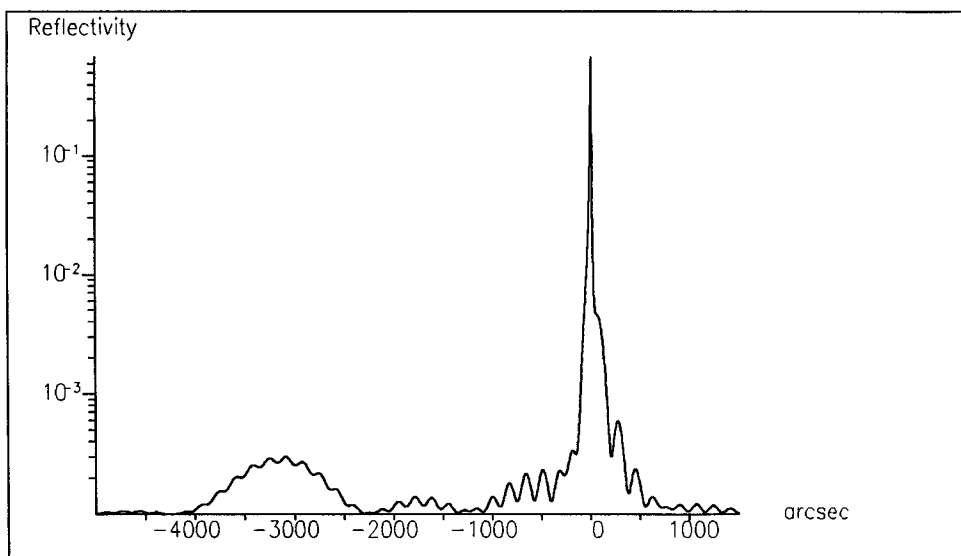
In later chapters we shall give both the fundamental theoretical treatment necessary to understand the complex rocking curves that arise from complex structures, and the practical experimental details required to measure them reliably and unambiguously.

1.6 X-ray topography

The essence of the topographic methods is that they map the intensity of the diffracted beam over the surface of the crystal. Defects affect the diffracted intensity,



(a)



(b)

Figure 1.6 (a) An example of a measured rocking curve, for an epitaxial layer 17 nm thick of $\text{In}_{0.18}\text{Ga}_{0.82}\text{As}$, grown on GaAs, with a $0.1\text{ }\mu\text{m}$ layer of GaAs on the top. $\text{CuK}\alpha_1$ radiation, GaAs single-crystal beam conditioner. (b) A theoretical simulation of this structure, by means of which the composition of the InGaAs layer was determined to $<1\%$ and its thickness to 0.1 nm

so give contrast in the image. The methods are quite sensitive enough to reveal individual dislocations, precipitates, magnetic domains and other long-range strain fields but cannot reveal point defects except in dense clusters.

1.6.1 X-ray topography with conventional sources

There are many methods of X-ray topography, though the most popular are the Lang method with slit-collimated radiation and the double-crystal methods, which may be thought of as high resolution diffractometry with an imaging detector. The principles of image formation and diffracted intensity are common, and may be applied to any method. The two examples show opposite extremes. Figure 1.7 shows a high resolution image of a silicon wafer, taken on an ultra-high resolution photographic plate, in which details of the strain fields of individual dislocations may be observed. Figure 1.8 shows a high-speed image taken on a television detector in which the broad features of the defect density and distribution are quickly visualised.

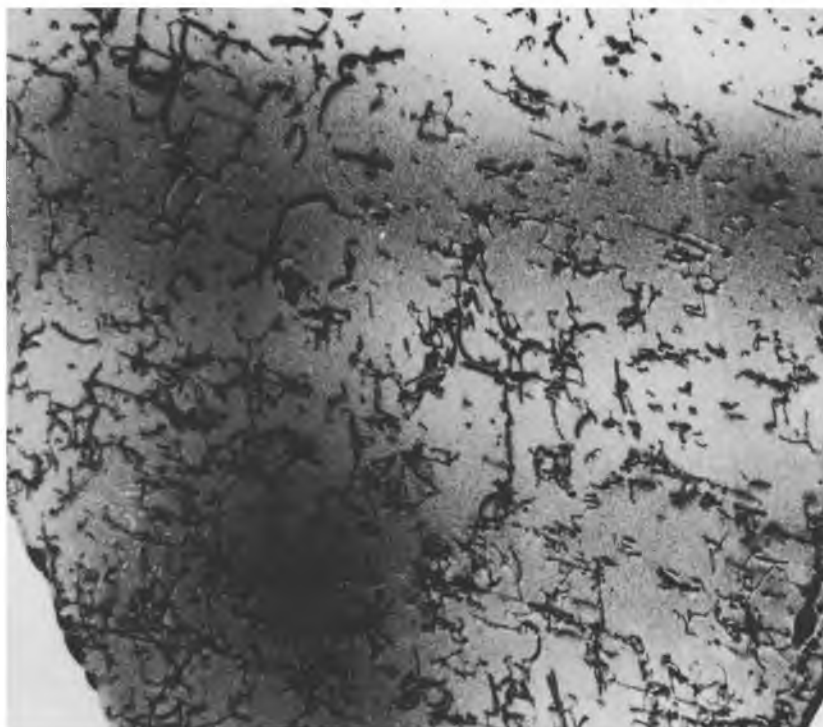


Figure 1.7 A Lang topograph (in transmission) of a silicon wafer. $\text{MoK}_{\alpha 1}$ radiation (0.07 nm), conventional X-ray tube, Ilford L4 ultra-high resolution plate. Field size 3 mm by 3.3 mm 022 reflection. Diffraction vector in direction up the page. The black lines are images of individual dislocations

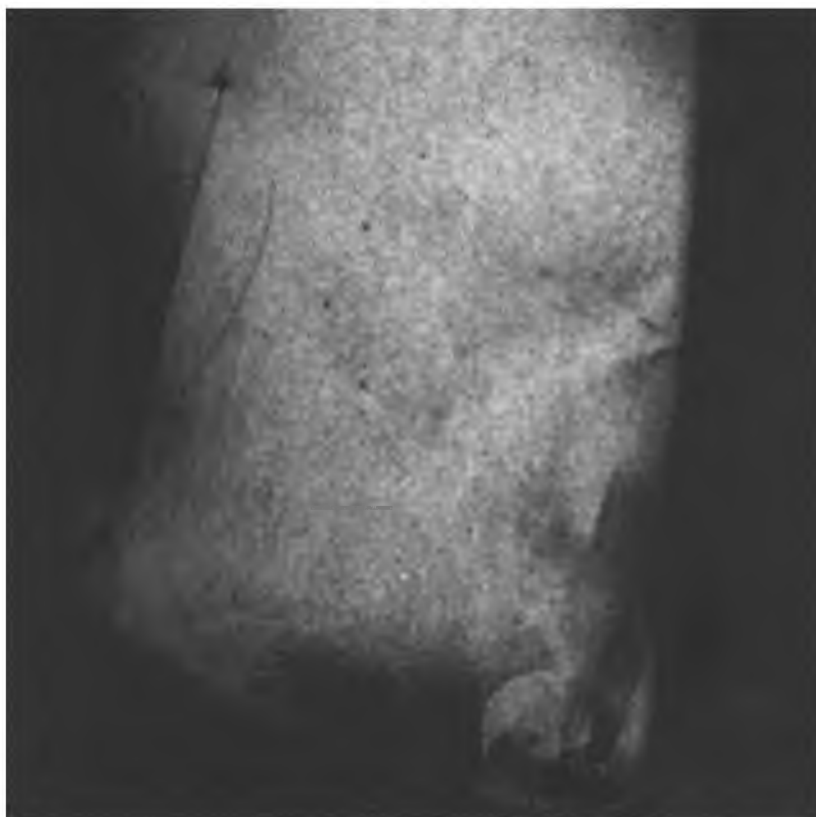


Figure 1.8 A double-crystal topograph of a GaP crystal, taken with $\text{CuK}_{\alpha 1}$ radiation from a conventional 1 kW X-ray tube. Direct imaging X-ray detector (Bede Scientific), 1 minute exposure. Cracks, sub-grain boundaries and variations in dislocation density are seen

1.6.2 Synchrotron radiation topography

The very high intensities and good collimations available make synchrotron radiation very suitable for topography. Against this must be set the inconvenience and slow turn-round necessitated by transport to a central synchrotron radiation laboratory. Hence, the technique is most appropriately used in the following cases:

- 1 For dynamic experiments, such as modelling of crystal growth or circuit processing.
- 2 For extensive survey topography, where the time taken on conventional machines would be prohibitive.
- 3 For obtaining information inaccessible to conventional methods, e.g. distorted crystals, stroboscopic topography, very thin layers, anomalous dispersion, simultaneous topography and fluorescence.
- 4 For rapid research into new methodologies and techniques, which may in some cases be transferred later to laboratory systems.

1.7 Summary

Crystals diffract radiation of comparable wavelength to the atomic spacing, as described by Bragg's law. The range of diffraction, or reflection, is of the order of seconds of arc for X-rays, and depends on the structure factor; this in turn depends upon the atomic species and their arrangement and indicates the strength of reflection from a given crystal plane. X-rays may probe millimetres, sometimes centimetres, into a material, or, using grazing incidence methods, penetration can be restricted to nanometres to make the techniques surface-sensitive. High (angular) resolution methods measure the reflection curve of a material to a pseudo-plane wave. This curve may be simulated using diffraction theory and a model of the material structure to determine this structure. Topographic methods give a direct image of many types of crystal defect and may be used to assess crystal perfection with high (strain) resolution.

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High Resolution X-ray Diffraction Techniques

We first define the geometry and instrumental parameters common to high resolution diffractometry. As a reference, we then develop the duMond diagram for visualisation of X-ray optics and use it to discuss practical beam conditioners. Next we treat the principal aberrations of high resolution diffractometry: tilt, curvature and dispersion. We discuss the requirements on X-ray detectors, and finally show how to set up a high resolution measurement in practice.

2.1 Limitations of single-axis diffraction

As we have seen in Chapter 1, we need something near a plane wave in order to see the finest details of the specimen structure. A single-axis diffractometer utilises a beam that is very far from a plane wave. Thus, single-crystal rocking curves are broadened due to the beam divergence, and the spectral width of the characteristic X-ray lines.

The divergence is a function of the source size h and slit size s and the source-specimen distance a . As seen in Figure 2.1,

$$\delta\theta = \frac{h+s}{a} \tag{2.1}$$

Thus, for a typical case of $h = 0.4\text{ mm}$, $s = 1\text{ mm}$, $a = 500\text{ mm}$, $\delta\theta \sim 500\text{ arc seconds}$, far above the width of the rocking curve for highly perfect crystals, which is typically a few arc seconds.

The spectral width of the X-ray characteristic lines is approximately $\delta\lambda/\lambda \sim 10^{-4}$. This rises to 10^{-3} if both the $K_{\alpha 1}$ and $K_{\alpha 2}$ lines are diffracted by the specimen. The effect this has upon the rocking curve depends on the dispersion of the whole system of beam conditioner and specimen, and ranges from zero to very large. This will be discussed below, in section 2.6.

We therefore need to limit the divergence and wavelength spread of the beam incident upon the specimen. Beam conditioners are used to collimate and to monochromate the beam. We see that the requirement is for a system with high *angular* resolution and sufficient *monochromatisation*. We shall first discuss the characteristics of sources and the general features of high resolution diffraction systems, then analyse the requirements for beam conditioning.

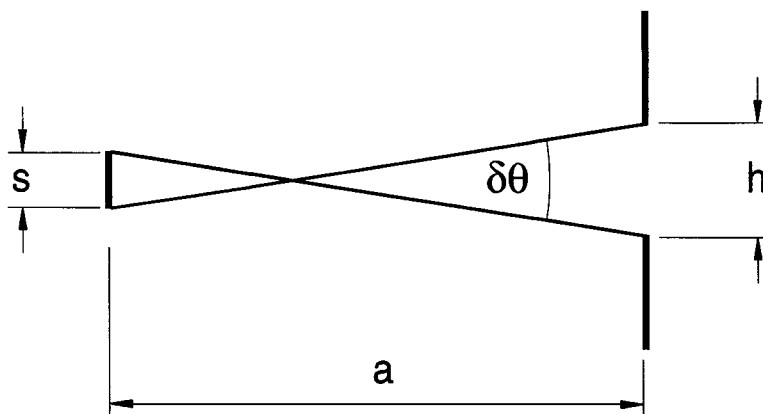


Figure 2.1 The divergence of a single-axis system

2.2 X-ray sources

Electromagnetic radiation is generated by the acceleration or deceleration of charged particles. If the acceleration is sufficiently high and the particles sufficiently energetic, the radiation is in the X-ray regime, forming a continuous spectrum. In addition, transitions of electrons between energy levels in atoms cause emission of radiation of wavelength $\lambda = hc/\Delta E$, where h is Planck's constant, c the velocity of light, and ΔE the energy difference between the levels. This gives rise to a line spectrum, whose most energetic components are in the hard X-ray regime for most elements. Both laboratory and synchrotron radiation sources are in common use for high resolution diffractometry.

2.2.1 Laboratory generators

In a laboratory generator, electrons are accelerated by a potential around 30kV towards a solid target, where they are stopped by impact. The output contains the line spectrum superimposed upon a continuous spectrum. The line, or *characteristic* spectrum is characteristic of the element and is used in X-ray fluorescent analysis to identify the type and amount of an element present in a sample. The continuous radiation is also called the *Bremsstrahlung*, from the German for 'braking radiation', as it is emitted when the electrons are 'braked' by the solid. The complete spectrum from a copper target as a function of accelerating voltage is shown in Figure 2.2. It is clear that the characteristic radiation is far more intense than the continuous, and it is used almost exclusively in high resolution diffractometry.

The characteristic lines are labelled K, L, M, etc., according to the label of the electron shell to which the transition occurs, with the subscript α , β , etc. indicating where the transition started. Thus the K_α lines result from $L \Rightarrow K$ transitions, the K_β line from $M \Rightarrow K$ transitions, and so on. By far the most popular choice for high resolution diffractometry is the $\text{CuK}_{\alpha 1}$ line, because of its convenient wavelength (0.154nm), which is useful for most interplanar spacings, and because of its high intensity. The copper target is the most easily cooled of all metallic targets, thanks

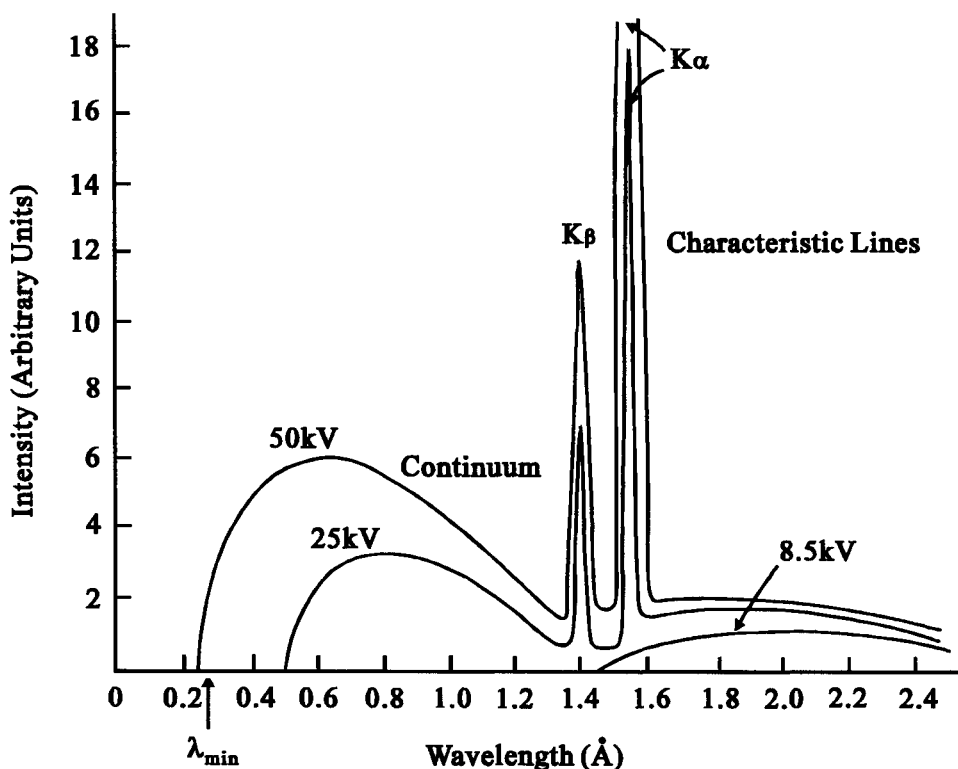


Figure 2.2 Spectrum of X-ray emission from a copper target at various excitation voltages

to its high thermal conductivity, and the output of an X-ray target is limited only by heat dissipation. The cooling may be by air, water or a combination of these with a rotating target to spread the heat more effectively. X-ray tubes run from fractions of a milliamp for a microfocus source up to about an amp for a very high power rotating target tube.

The K_{α} line is a doublet with separation about 10^{-3} . It is often important to remove the $K_{\alpha 2}$ line, which has approximately half the intensity of the $K_{\alpha 1}$, by the use of a beam conditioner as shown later.

2.2.2 Synchrotron radiation sources

Synchrotron radiation sources remove the limitation of the thermal properties of solids by confining the electron beam in an ultra-high vacuum. Magnetic fields are used to bend the beam into a closed 'ring' (in fact, a polygon with rounded corners), and the installations are called electron storage rings. Since the phenomenon was first created on earth in synchrotrons (although it had been first postulated for explaining astronomical X-ray emission), the name *synchrotron radiation* has stuck. A storage ring is simply a synchrotron run at stable rather than cyclic energy and magnetic field conditions. The installations are large, with diameters up to ~ 100 m, and are built on a national or international scale. The radiation spectrum is shown