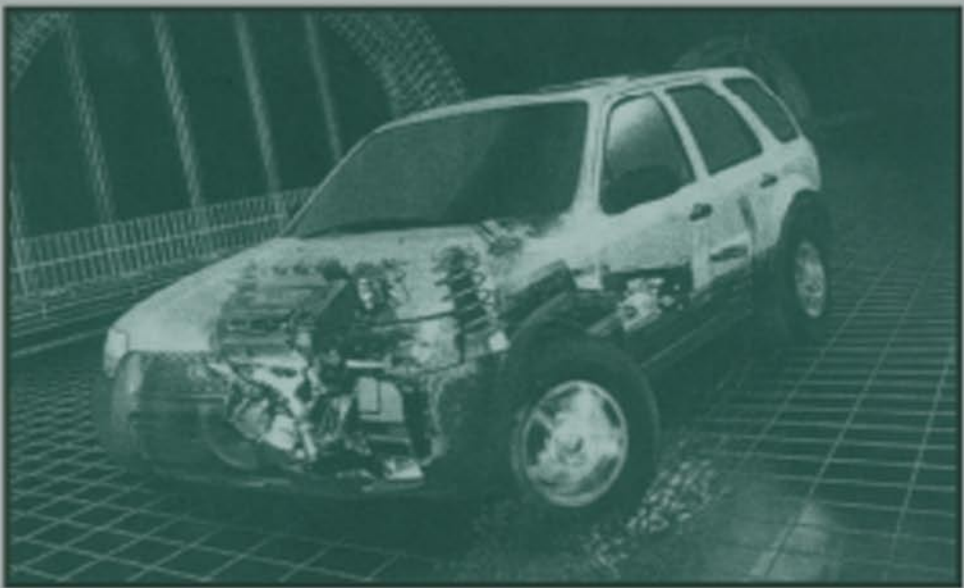
Vehicular Electric Power Systems

Land, Sea, Air, and Space Vehicles



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Vehicular Electric Power Systems

POWER ENGINEERING

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Publisher's Note
The publisher has gone to great lengths to ensure the quality of this reprint
but points out that some imperfections in the original may be apparent.

To the memory of my sister, Annahita
Ali Emadi

to my wife, Zohreh
Mehrdad Ehsani

for Doreen and for Mike
John M. Miller

Series Introduction

Power systems are the most fundamental aspect of electrical engineering, because such systems create and control the energy that enables—literally powers—all electric and electronic capabilities. Power engineering is by far the oldest and most traditional of the various areas within electrical engineering. While initially restricted to only stationary electric system applications (in the early 20th century), power systems—engineered combinations of generation, distribution, and control—gradually worked their way into all manner of marine, automotive, and aerospace applications. Today, no significant vehicle—whether robotic or manned, whether military or civilian, whether designed to move on, under, or above land or sea, or in outer space—is designed without its power system being a core part of its overall design.

There are significant differences between vehicular and stationary (utility, industrial, building) power systems engineering. But there are also great similarities. Ultimately, all share the same overall mission, are subject to the same physical principals and limitations, and, perhaps in different measure, use the same basic technical approaches and rules. Most important, however, there is tremendous transfer of technology between the two main branches of power systems engineering. Early marine and airborne power systems borrowed much from existing electric utility power system technology. Today, vehicular technologies such as fuel cells and advanced power electronics controls are making their way into electric utility and industrial power applications.

Vehicular Electric Power Systems: Land, Sea, Air, and Space Vehicles provides a very thorough and complete discussion of theory and application of power systems engineering to anything that moves. The book's greatest strength is its combination of a thorough exploration of the needs and limitations of each type of vehicular power application (e.g., automotive), with very detailed discussions of the specific technologies available for power systems, such as fuel cells and multiconverter systems, and their control.

Like all the books in Marcel Dekker's Power Engineering series, *Vehicular Electric Power Systems: Land, Sea, Air, and Space Vehicles* puts modern technology in the context of practical application; it is useful as a reference book as well as for self-study and advanced classroom use. The series includes books covering the entire field of power systems engineering, in all of its specialties and subgenres, all aimed at providing practicing electrical and design engineers with the knowledge and techniques they need to meet our society's energy and engineering challenges in the 21st century.

H. Lee Willis

Preface

Mechanical, electrical, hydraulic, and pneumatic systems are conventional power transfer systems in different land, sea, air, and space vehicles. In order to improve vehicle fuel economy, emissions, performance, and reliability, the more electric vehicle (MEV) concept emphasizes the utilization of electrical power systems instead of non-electrical power transfer systems. In addition, the need for improvement in comfort, convenience, entertainment, safety, communications, maintainability, supportability, survivability, and operating costs necessitates more electric vehicular systems. Therefore, electric power distribution systems with larger capacities and more complex configurations are required to facilitate increasing electrical demands in advanced vehicles.

In MEVs, solid-state switching power converters are extensively used for generating, distributing, and utilizing electrical energy throughout the system. Different converters such as DC/DC choppers, DC/AC inverters, AC/AC converters, and AC/DC rectifiers are used in source, load, and distribution subsystems to provide power at different voltage levels and in both DC and AC forms. Most of the loads are also in the form of power electronic converters and electric motor drives. Therefore, in these vehicles, different converters are integrated together to form complex and extensively interconnected multi-converter systems. The number of power electronic converters in these systems varies from a few converters in a conventional car to tens of converters in the advanced aircraft and spacecraft power systems, to hundreds of converters in the international space station. Recent advancements in the areas of power electronics, electric motor drives, fault tolerant electrical power distribution systems, control electronics, digital signal processors (DSPs), and microprocessors are already providing the impetus towards MEVs.

These unconventional power systems have unique system architectures, characteristics, dynamics, and stability problems that are not similar to those of conventional electrical power systems. The purpose of this book is to present a conceptual definition and a comprehensive description of these systems. In addition, an inclusive explanation of the conventional and advanced architectures, role of power electronics, and present trends is given. Furthermore, this book addresses the fundamental issues faced in these systems, both before and after their implementation.

This book consists of thirteen chapters. It starts with an introduction to electrical power systems, basics of electric circuits, and principles of control systems in Chapter 1. Chapters 2 and 3 are also introductory chapters about fundamentals of power electronics and electric machines, respectively. Conventional and advanced power electronic AC/DC, DC/DC, DC/AC, and AC/AC converters are presented in Chapter 2. Chapter 3 deals with the conventional DC, AC induction, and AC synchronous machines and their associated power electronic drivers. Advancements in the areas of power electronics and motor drives facilitate electrification of vehicular systems and enable the introduction of more electric vehicles with improved performance, efficiency, volume, and weight.

Chapter 4 presents a comprehensive description of automotive power systems including conventional automobiles and more electric cars. At present, most automobiles use a 14V DC electrical system. However, demands for higher fuel economy, performance, and reliability as well as reduced emissions push the automotive industry to seek electrification of ancillaries and engine augmentations. In advanced cars, throttle actuation, power steering, anti-lock braking, rear-wheel steering, air conditioning, ride-height adjustment, active suspension, and the electrically heated catalyst will all benefit from the electrical power system. Therefore, a higher system voltage, such as the proposed 42V PowerNet, is necessary to handle these newly introduced loads.

Chapter 5 deals with electric and hybrid electric vehicles. Principles of hybrid electric drivetrains, system configurations, electrical distribution system architectures, control strategies, hybridization effects, low-voltage traction systems, and design methodologies are presented. In addition, electrical systems of heavy duty vehicles and electric dragsters are explained. Modeling and simulation of automotive power systems are also described in Chapter 5.

Chapter 6 concentrates on air vehicles. Conventional aircraft power systems, electrical loads, power generation systems, AC and DC distribution systems, and the concept of more electric aircraft are presented in Chapter 6. In addition, space power systems including spacecraft and the international space station are described in Chapter 7. Chapter 7 also explains modeling, real-time state estimation, and stability assessment of aerospace power systems.

In Chapter 8, sea and undersea vehicles are comprehensively studied. Propulsion and non-propulsion electric loads, more electric ships, integrated power systems, and pulsed power technology, as well as advanced sea and undersea vehicles are introduced. Chapter 9 concentrates on the applications of fuel cells in different land, sea, air, and space vehicles. It explains structures and operations of fuel cells as well as their utilization. In Chapter 10, modeling techniques for energy storage devices including batteries, fuel cells, photovoltaic cells, and ultracapacitors are presented in detail.

Advanced motor drives for vehicular applications are reviewed in Chapter 11. Brushless DC (BLDC) and switched reluctance motor (SRM) drives are comprehensively presented as advanced motor drive technologies for different vehicles. Furthermore, motoring and generating modes of operation as well as sensorless techniques are explained.

Chapter 12 introduces multi-converter vehicular dynamics. In Chapter 12, electrical loads of advanced vehicles are categorized to two groups. One group is constant voltage loads, which require constant voltage for their operation. The other group is constant power loads that sink constant power from the source bus, which is a destabilizing effect for the system and known as negative impedance instability. Effects of such loads on the dynamic behavior of different vehicles are comprehensively studied.

The purpose of Chapter 13 is to present an assessment of the effects of constant power loads in AC vehicular distribution systems. Furthermore, recommendations for the design of AC vehicular systems to avoid negative impedance instability are provided. Guidelines for designing proper distribution architectures are also established.

The material in this book is recommended for a graduate or senior-level undergraduate course. Depending on the background of the students in different disciplines such as electrical and mechanical engineering, course instructors have the flexibility to choose the material or skip the introductory sections/chapters from the book for their lectures. This text has been taught at Illinois Institute of Technology as a graduate level course titled Vehicular Power Systems. An earlier version of this text has been revised based on the comments and feedback received from the students in this course. We are grateful to the students for their help.

This book is also an in-depth source for engineers, researchers, and managers who are working in vehicular and related electrical, electronic, electromechanical, and electrochemical industries.

We would like to acknowledge gratefully the contributions of many graduate and undergraduate students at Illinois Institute of Technology in different sections/chapters of this book. They are Mr. Ranjit Jayabalan contributing in Chapter 1 and Section 4.5, Mr. Ritesh Oza contributing in Sections 2.1 and 2.2, Mr. Sheldon S. Williamson contributing in Sections 2.3 and 7.1-7.3, Mr. Basem Fahmy contributing in Chapter 3, Mr. Erwin Uy contributing in Section 4.6, Mr. Fernando Rodriguez contributing in Section 4.6, Mr. Arjun Shrinath contributing in Section 4.8, Mr. Srdjan M. Lukic contributing in Sections 5.5 and 5.10, Ms. Valliy Dawood contributing in Section 5.8, Mr. Rajat Bijur contributing in Section 5.9, Mr. Sachin A. Borse contributing in Chapter 8, and Mr. Yogesh P. Patel contributing in Section 11.2. In addition, Chapters 9 and 10 draw heavily from the graduate research work of Mr. Sheldon S. Williamson, which is gratefully acknowledged.

We would also like to acknowledge the efforts and assistance of the staff of Marcel Dekker, Inc.

Ali Emadi
Mehrdad Ehsani
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Contents

Vehicular Electric Power Systems

1

Introduction to Electrical Power Systems

1.1 Fundamentals of Electric Circuits

1.1.1 Ohm's Law

Ohm's law states that the potential difference across any element is directly proportional to the current carried by it, if the physical conditions (temperature, dimensions, etc.) remain unchanged.

$$V \alpha R \tag{1.1}$$

$$V = RI \tag{1.2}$$

R is a constant of proportionality and is called the resistance of an element. R may be represented in terms of element length (l), area of the cross-section (A), and resistivity of the element (ρ).

$$R = (\rho l) / A \tag{1.3}$$

Resistance of an element is its opposition to the flow of current. Not all materials follow Ohm's law like superconductors, which have zero resistance. Such materials are called non-ohmic materials, while all others that follow the Ohm's law are called ohmic materials.

1.1.2 Kirchhoff's Law

Any electric circuit is composed of elements such as resistors, capacitors, and inductors, which carry current and have a voltage that exists across them. As a result, these elements give rise to a relationship with current and voltage referred to as Kirchhoff's current and voltage laws.

Kirchhoff's current law states that the current at any node of an electric circuit is zero. Here, a node in an electric circuit is defined as the point of intersection of two or more electrical components. Kirchhoff's current law is shown in Figure 1.1.

$$\sum i = i_1+i_2+i_3-i_4 = 0 \tag{1.4}$$

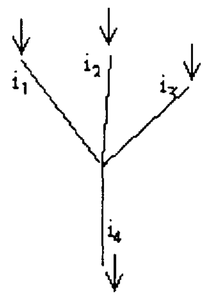

Figure 1.1 Representation of Kirchhoff's current law.

Kirchhoff's voltage law states that the sum of the products of the voltage and current across each element is equal to the sum of the potential sources in the closed circuit. Figure 1.2 shows the representation of Kirchhoff's voltage law.

$$\sum V = 0 \tag{1.5}$$

$$V = iR_1 + iR_2 \tag{1.6}$$

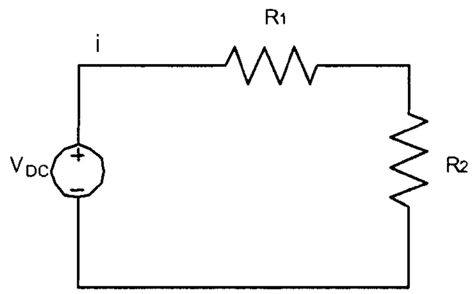

Figure 1.2 Representation of Kirchhoff's voltage law.

1.1.3 Network Elements Voltage Current Relations

The common network elements in an electric circuit are resistors, inductors, and capacitors. The relation of these elements with the current they carry and the voltage that exists across their terminals are given below with equations.

Voltage current relation for resistor:

$$V = RI \tag{1.7}$$

V = Voltage across the element

I = Current flowing in the element

Voltage current relation for inductor:

$$V = L\ (di/dt) \tag{1.8}$$

L = Inductance of the element

Voltage current relation of capacitor:

$$I = C\ (dv/dt) \tag{1.9}$$

C = Capacitance of the element

1.1.4 Transient Circuit Analysis for RC, RL and RLC Circuits

In general cases, the voltage and current measured in a circuit are during the steady state condition, that is, when the source provides a constant DC or AC signal. However, when a circuit is switched on or off, it tends to change from one steady state to another. This transition period between the two steady states is called transient period and its analysis is referred to as transient analysis. The transient behavior of a system primarily exists due to the presence of energy storage devices like inductances and capacitances that have a high inertia to a sudden change in current and voltage.

In the transient analysis of RL circuits, consider the switch initially open. At this point, the current and voltage in the inductor is zero. When the switch is closed, the source is across the resistor and inductor. There is an instantaneous change in the voltage across the inductor, but the current in it cannot change instantaneously. The current before and after the switching is almost the same. Once the switched is closed and the circuit is in steady state, the inductor behaves as a short circuit. The RL circuit and the change in voltage and current with time on switching are shown in the Figure 1.3.

In the transient analysis of RC circuits, when the switch is closed, the capacitor acts as an open circuit. The capacitor current becomes zero; but, the voltage is that of the source. The change in capacitor voltage is not instantaneous and the voltage before and after the switching is almost the same.

A first order differential equation of the circuit gives the solution. Figure 1.4 shows the RC circuit and its transient behavior.

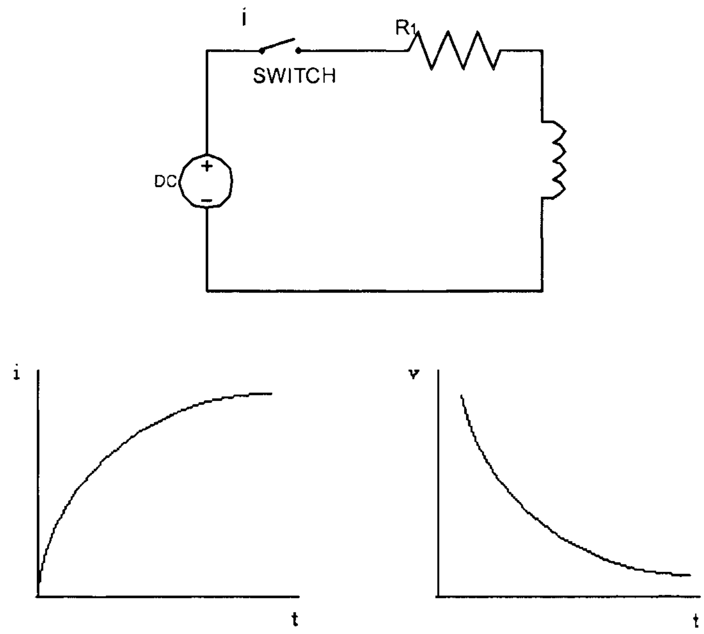

Figure 1.3 The RL circuit and the change in current and voltage during transient period.

The transient analysis of RLC circuits is similar to that for RL and RC circuits. In an RLC circuit, the components may be arranged in parallel or series combination. The only difference in this analysis is that a second order differential equation will be used to give the solution of the RLC circuit. An RLC circuit is shown in Figure 1.5.

1.1.5 Introduction to Laplace Transforms

The Laplace transform is a powerful tool in solving a wide range of initial value problems. It transforms ordinary and partial differential equations to simple algebraic problems where solution can be easily obtained. Applying inverse Laplace transform on the algebraic problem gives the solution to the ordinary and partial differential equations.

A function $f(t)$ has a Laplace transform $F(s)$ when defined over the interval of $0 < t < \infty$,

$$L[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)dt \tag{1.10}$$

where s is real and L is called the Laplace transform.

Some of the conditions for the existence of the Laplace transform are that it should be piecewise continuous on $0 < t < \infty$ and $f(t)$ should be of exponential order as t reaches infinity. Although these two conditions are sufficient, they do not necessitate the existence of $F(s)$.

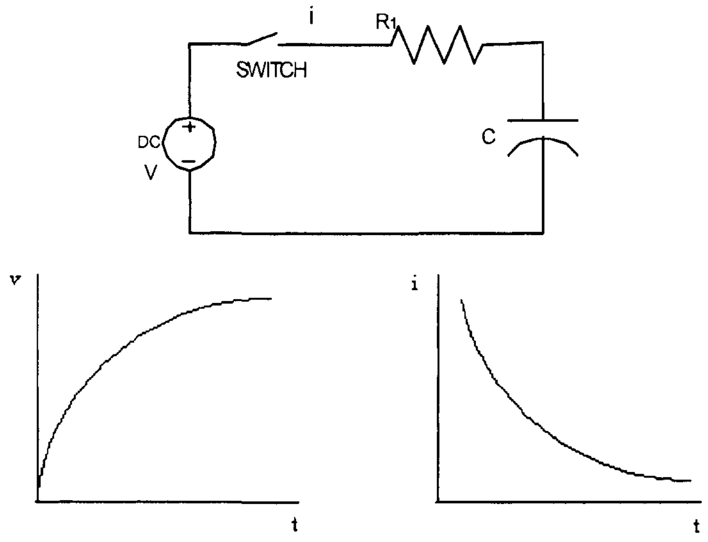

Figure 1.4 RC circuit and its transient behavior.

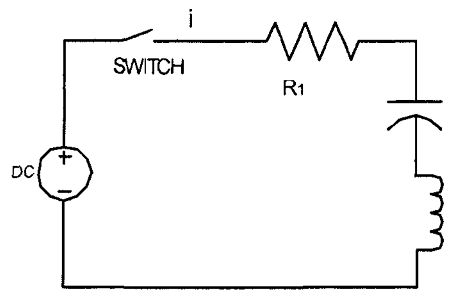

Figure 1.5 RLC circuit.

1.1.6 Sinusoidal Excitation and Phasors

A sinusoidal waveform is an alternating current (AC) [as opposed to direct current (DC)] that flows first in one direction and then in the opposite direction, as shown in Figure 1.6. The ampere current is a function of time and is not constant like in DC. Such a waveform may be represented as

$$V = V_m \, Sin\ (\omega t) \text{ and } I = I_m \, Sin\ (\omega t) \tag{1.11}$$

where V_m and I_m are peak values. A system excited by such a source will give a linear or non-linear output depending on whether the system is a linear or non-linear.

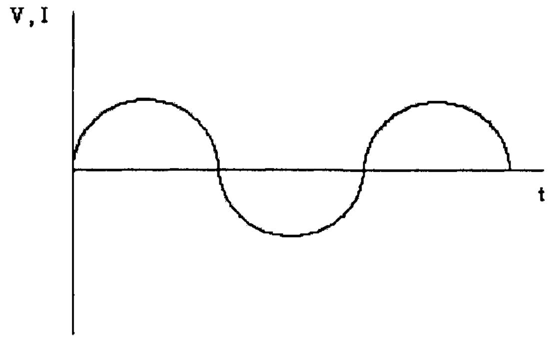

Figure 1.6 A sinusoidal signal.

Phasors are the complex numbers that we multiply with e^{jwt} in the expression for $i(t)$ and $v(t)$.

V is a voltage phasor,

$$V(t) = V e^{jwt} = |V| \, e^{j} e^{jwt} = |V| \, e^{(jwt + j)} \tag{1.12}$$

Phasor laws:

Ohms law, $V=RI$

Inductance, $V = jwLI$

Capacitor, $V = [1/(jwC)]I$

Impedance, $(V/I) = Z$

Impedance in a generalized circuit plays the role of resistance in a resistive circuit. It is a complex number (not a phasor) and has a real (resistive) component and an imaginary (reactive) component. An important note that needs to be made when dealing with phasors is that they are useful for finding the forced response of a system.

1.1.7 Fourier Series

Fourier series are expansions of periodic functions *f(x)* in terms of an infinite sum of sine and cosine functions such as

$$F(x) = a_n Cos(nx) + b_n Sin(nx) \tag{1.13}$$

The values of the coefficients a_n and b_n are determined from the orthogonality of the sine and cosine functions. Fourier series are computed using the following integrals.

$$\int_{-\pi}^{\pi} Sin(mx)Sin(nx)dx = \pi\delta_{mn}, \quad m,n \neq 0 \tag{1.14}$$

$$\int_{-\pi}^{\pi} Cos(mx)Cos(nx)dx = \pi\delta_{mn}, \quad m,n \neq 0 \tag{1.15}$$

$$\int_{-\pi}^{\pi} Sin(mx)dx = 0 \tag{1.16}$$

$$\int_{-\pi}^{\pi} Cos(mx)dx = 0 \tag{1.17}$$

δmn is Kronekar delta, which is a discrete version of delta function defined by $\delta ij = 0$ for $i \neq j$ and *1* for $i = j$.

1.1.8 Digital Systems

Primarily, a system may be classified as an analog or digital system based on the nature of the signal used by them. An analog system uses signals (voltages and currents) which are continuous and may be sinusoidal or even constant DC. Digital systems, on the other hand, have a discontinuous signal, whose waveform is composed of pulse with a non-zero voltage level and a zero voltage level.

A digital system may use a unipolar signal having two voltage levels – zero and a positive or negative voltage level – or may be a bipolar signal, consisting of a zero, positive, and negative voltage level. Digital systems are generally related to low signal low power applications such as calculators; however, they can be used in high power applications as well, such as in power converter circuits.

Digital systems are primarily composed of discrete elements such as transistors, resistors, capacitors, and logic gates or may be integrated together to form integrated circuits (IC). Microprocessors and memory chips are examples of digital circuits packaged together to form a single digital component that can be used in larger digital circuits. Digital components such as analog to digital and digital to analog converters help in interfacing analog and digital circuits to provide a more comprehensive system design.

1.2 Control Systems

1.2.1 Signal Flow Graphs and Block Diagrams

Signal flow graphs and block diagrams serve as an invaluable means of system representation in control system analysis. They help in defining the basic control concept and simplifying the representation of complex system. Both can represent almost all possible systems. The signal flow graphs and block diagrams for a simple control system are shown in Figure 1.7.

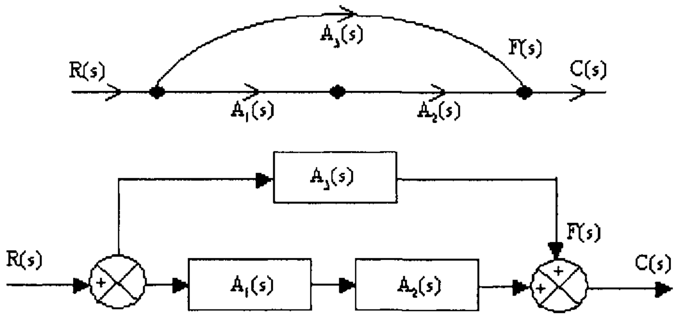

Figure 1.7 Signal flow graph and block diagram representation of a simple control system.

The input/output relationship is given as

$$A(s) = C(s)/R(s) \tag{1.18}$$

where

$A(s)$ = Transfer function of the system

$R(s)$ = Input or reference variable

$C(s)$ = Output or controlled variable

$F(s)$ = Error variable

$F(s)$ may be a feedback or feedforward variable depending on whether the connector is negative or positive, respectively. These variables may represent any physical parameter such as velocity and temperature. The upper parallel path in Figure 1.7 is the forward loop. In the case of the signal flow graph, the sign of the variable in the parallel branch indicates whether it is a feedback loop or not.

As stated above, the signal flow graph is a simplified graphical representation of linear systems only that makes use of linear algebraic equations to establish an input/output relationship. Additionally, the representation of signal flow graph is more mathematically constrained compared to a block diagram.

1.2.2 Feedback Control Techniques

Feedback control techniques refer to a specific concept or an idea that is involved with the control of a physical system. The control here is taken in the context of the control of an entire system as an entity and not the control of a segment of a system. The concept of feedback control system and techniques have largely evolved from linear system theory, which forms the foundation for much of the advanced technologies that exists today. A generalized representation of feedback control system is given in Figure 1.8.

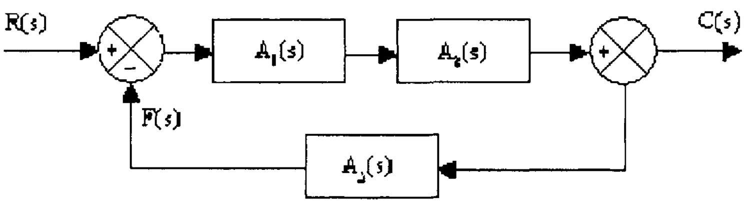

Figure 1.8 A generalized form of feedback control system.

There are numerous feedback control techniques and they are classified based on the approach of the design. However, the common ones are those like linear and non-linear control. Almost all systems are non-linear in nature; but, linear feedback control techniques are idealized models for test purposes. They function as linear systems as long as the applied signals are within the linear operating range. Non-linear control techniques are used sometimes to make the control more effective and precise and to achieve minimum time to carry out a desired control.

In time invariant systems, the control parameters are stationary with time; but, in time variant systems, the control parameters tend to vary. A linear time variant system design and analysis is however more complex to solve compared to a linear time invariant system. In the case of continuous data control system, the control signal is a function of time at various parts of the system. If the signal has a carrier, it is an AC system; otherwise, it is a DC system. However, a DC system may have a few signals that are of AC nature.

A sampled data control system involves the use of pulse signals at various parts of the system, while a digital control system makes use of a binary array of numbers for the control. Both control techniques are collectively referred to as discrete control systems. The inherent benefit of such a control is component sharing for multiple functions, downsizing control layout, and an increased degree of flexibility.

1.2.3 Stability and Routh-Hurwitz Criterion

Stability of a system refers to its ability to come back to its stable operating condition, which is also often referred to as a steady state in which the input and output variables or functions are related through numerical constants. Mathematically, the system may be stated as stable if it does not have any positive real poles, which, in other words, sharply defines the system as stable and unstable otherwise. In practical systems, the possibility to reach instability exists, but many times due to system component saturation, the system remains in stable state. Systems with negative feedbacks ensure that the system stability is maintained and are used when system components natural limitation are insufficient in maintaining system stability.

The Routh-Hurwitz criterion is one of the methods of determining system stability without solving for the roots of the characteristics equation of a system. Instead, it involves computation of a triangular array that is a function of the coefficients of the characteristic equation. The necessary and sufficient condition for stability is that the element in the first column be positive. If these are all positive, then all the zeros are on the left hand half of the s-plane. However, if all the roots are not positive, then the number of zeros lying on the right hand half plane equals the number of sign changes in the first column.

1.2.4 Time Domain and Frequency Domain Analysis

In most control systems, time is taken as an independent variable with respect to which the output or the system state is referred to. In time domain analysis, a sample or reference input signal is applied to the system and the output obtained is used for evaluating the system. The system behavior with time or time response $C(t)$ has two segments – transient response $C_t(t)$ and steady state response $C_s(t)$.

$$C(t) = C_t(t) + C_s(t) \tag{1.19}$$

In control systems, the transient response is that part of the response that goes to zero at infinity.

$$Limit_{t\to\infty} C_t(t) = 0 \tag{1.20}$$

The steady state response, on the other hand, is that portion of the response at which the system response becomes constant at infinity or changes uniformly with time. For designing a new control system, transient and steady state responses are given as the system specifications.

Although time domain is more realistic for control system analysis, the frequency domain analysis is preferred for certain systems like those used in communications. This is primarily due to the fact that high order systems are difficult to handle. There is no specific approach to design systems with specifications like rise time and delay time in the time domain analysis approach. However, the frequency domain analysis has graphical tools as a simplified approach for solving linear control systems. Also, the relationship that exists between time domain and frequency domain makes it possible to interpret the time domain system parameters from the frequency domain parameters. The frequency domain analysis takes the approach of using the system transfer function.

1.2.5 State Space Description

The state space representation provides a holistic and complete representation of a system. This representation can be used to describe a large family of systems like single input single output, multi-input single output, multi-input multi-output, time variant and time invariant systems. The state space formulation is given as

$$v(n+1) = Av(n) + Bu(n) \tag{1.21}$$

$$y(n) = Cv(n) + Du(n) \tag{1.22}$$

This formulation describes a discrete time system, but can be easily extended to describe continuous time systems. Here, $v(n+1)$ is a vector of states at strategically placed nodes in the system at time $n+1$. The matrix A here is termed a state transition matrix and models the dynamic behavior of the system. The state space vector is especially powerful for multi-input multi-output linear systems and also for time varying systems.

1.3 Electrical Systems

1.3.1 Power Network Models

A power network model is a simplified representation of a power network (single or three phase) by a single line diagram. It involves the use of standard symbols to denote different components of the system. A simple power system network comprises an alternator, transmission line, and load unit. The power network diagram is shown in Figure 1.9.

1.3.2 Per-Unit Quantity

In a power network, all the components like alternators, transformers, and loads may work at different voltages, current, and power levels. It will be convenient for the analysis of such power systems if the voltage, current, power, and impedances are expressed with reference to a common base value. This base value is any arbitrary value chosen for the simplification of analysis only and does not have any physical meaning. All the voltages, currents, powers, and impedances of the components are then expressed as a percent or per unit of the base value.

The base values may thus be defined as the ratio of the actual value to the base value expressed in decimals or in percent.

Per unit value = Actual Value/Base Value (1.23)

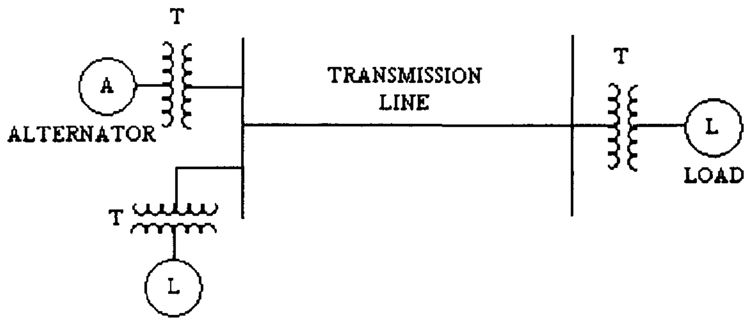

Figure 1.9 Power network model.

1.3.3 Gauss-Seidel Load Flow

Load flow studies involve the solution of an electric power system under steady state condition. On the basis of certain inequality constraints imposed on the node voltages and reactive power of the generators, the solution is obtained. The load flow studies relieve information about magnitude and phase angle of the voltages at each bus and the real and reactive power flowing in each segment of the system. It also gives the initial system conditions when transient behaviors are to be studied. The load flow analysis is essential from the point of operating the power network in the most optimal condition, to expand an existing network system, and also while designing new power network.

The Gauss-Seidel method is an iterative procedure for solving a set of non-linear load flow equations. The Gauss-Seidel load flow equation is

$Vp = [1/Y_{pp}] * [(P_p - jQ_p)/V_p - \sum Y_{pq} V_p]$ (1.24)

where

$P = 1, 2, 3, \ldots.$

V_1, V_2, V_3 – Bus voltages

V^0_1, V^0_2, V^0_3 – Initial bus voltages

P and Q are real and reactive power, respectively. The iterative process of computing the bus voltages is repeated until the bus voltage converges to the desired accuracy. The choice of initial values significantly affects rate of convergence and the knowledge of selecting these initial values is obtained by experience.

1.3.4 DC Power Systems

The most commonly used is the AC power system, where power is generated, transmitted, and used in the AC form. In the case of DC power systems, power could be generated as DC and then transmitted in the same DC form. Another option would be to generate power conventionally as AC and rectify it to convert to DC and then have it transmitted in the same DC form. The latter is preferred as rectification is a simple process involving conversion of AC to DC (rectifier) compared to conversion of DC to DC (chopper) when power is generated in the DC form. Also, a rectifier is more economical and has higher efficiency compared to choppers. In vehicular electrical power systems, both in land and space vehicles, power is essentially generated in the AC form and then rectified to DC to be used for storage in the battery or for use by the loads. At the transmitted end of the DC power system if required, DC is converted back to AC using inverters for using AC loads. Figure 1.10 depicts a typical block diagram layout of a DC power system.

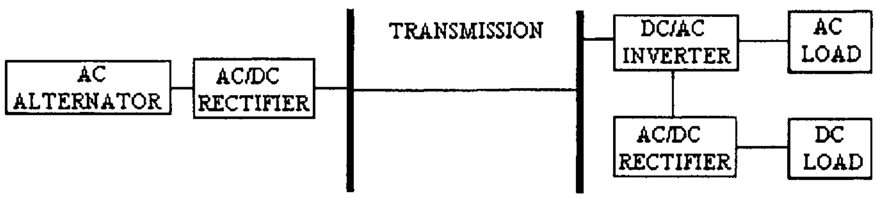

Figure 1.10 Block diagram of a DC power system.

1.4 References

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2

Fundamentals of Power Electronics

Power electronics is the technology for conversion of one type or level of an electric waveform to another. Power electronic converters are increasingly utilized in different vehicular applications. These converters include AC/DC rectifiers, DC/DC choppers, DC/AC inverters, and AC/AC voltage controllers. The impetus towards this expansion of power electronics has been provided by recent advancements in the areas of semiconductor switching devices, control electronics, and advanced microcontrollers and digital signal processors (DSPs). In fact, these advancements enable the introduction of power electronic converters with reduced cost, highest performance, maximum efficiency, and minimum volume and weight.

Power diodes, thyristors, transistors, MOSFETS, and isolated gate bipolar transistors (IGBTs) are the main power electronic switches. Power diodes are the simplest, uncontrollable power electronic switches. Power diodes are forward biased (ON) when their current is positive and reverse biased (OFF) when their voltage is negative. Thyristors are controllable three-terminal devices. If a current pulse applies to its gate, a thyristor can be turned on and conduct current from its anode to its cathode provided there is a positive anode-to-cathode voltage. However, in order to turn a thyristor on, gate current must be above a minimum value called I_{GT}.

Power transistors have the characteristics of conventional transistors. However, they have the capability of conducting higher collector current. They also have higher breakdown voltage (V_{CEO}). Power transistors are designed for high current, high voltage, and high power applications. They are usually operated either in the fully on or fully off state.

Power MOSFETs are voltage-controlled devices. They are usually N-channel and of the enhancement type. Most power MOSFETs are off when

V_{GS} <2v and are on when V_{GS} >4v. When a power MOSFET is on, there is a small resistance, i.e., less than 1 Ω, between drain and source and when it is off, there is a large resistance (almost open circuit) between drain and source.

IGBTs are equivalent to power transistors whose bases are driven by MOSFETs. Similar to a MOSFET, an IGBT has a high impedance gate, which requires only a small amount of energy to switch the device. Like a power transistor, an IGBT has a small on-state voltage.

2.1 AC/DC Rectifiers

An AC/DC rectifier is a power electronic circuit, which converts its input AC voltage into a DC output voltage. In this section, single-phase types of AC/DC converters and their operating modes in different loading conditions are presented.

2.1.1 Single-Phase, Half-Wave, Uncontrolled Rectifiers

The diagram shown in Figure 2.1 is the power stage of a single-phase, half-wave, uncontrolled rectifier. The input to the converter is single-phase supply and the switch used in this converter is diode, which is uncontrollable. If we look at the output voltage, the waveform is half of the input voltage. Thus, this converter is known as a single-phase, half-wave, uncontrolled rectifier.

The operation of this converter is very simple. During the positive half cycle of the input supply voltage, positive Vs will appear across the diode, which is forward biased (Figure 2.2 (a)). Thus, the output voltage in a positive half cycle will be the input or supply voltage Vs. The current will be Vs/R. When supply voltage enters into the negative half cycle, the diode is reverse biased. Thus, no voltage appears at the output and there is no current.

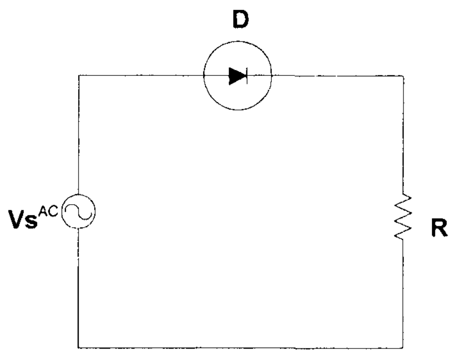
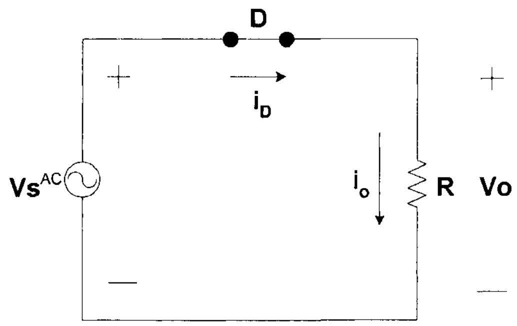
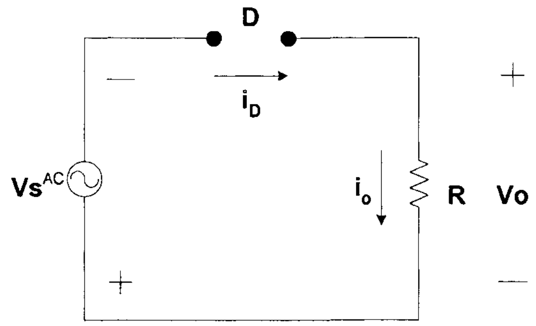


Figure 2.1 Single-phase, half-wave, uncontrolled rectifier with resistive load.



(a)



(b)

Figure 2.2 Circuit configuration: (a) diode is on (b) diode is off.

Power stage is the same for an inductive load. This converter includes only one diode. Again, during the positive half cycle of the input voltage, diode is forward biased and Vs appears across the resistor and inductor. When supply voltage goes in negative half cycle, due to the stored energy into the inductor, the output current will not be zero as soon as the input voltage becomes zero and then negative. Therefore, the diode will still conduct. The duration of this period is decided by the size of the inductor. Once the energy stored in the inductor is dissipated completely into the resistor, the output current will go to zero and diode will be reverse biased. Output voltage will be zero at this instant.

For loads with an internal battery or back-emf in case of DC motors, there is one source on each side of the diode. If amplitude of the battery is greater than the input or source voltage, the diode will be reverse biased and it will not conduct. Thus, output voltage of the rectifier will be equal to that of battery. As soon as input (AC source) voltage gets higher than the battery voltage, the diode will be forward biased and it will start conducting. Output voltage will be source voltage following input voltage until the current through the inductor dies to zero. Once energy stored in the inductor is completely depleted, the current through it will be zero. Output voltage will be equal to the battery voltage.

2.1.2 Single-Phase, Half-Wave, Controlled Rectifiers

As is shown in Figure 2.3, these rectifiers have a controlled switch (fully controlled or semi-controlled) instead of a simple diode (which is an uncontrolled switch). Input to the rectifier is single-phase supply and the output waveform is half of the input voltage; thus, these are single-phase, half-wave, controlled rectifiers. Similar to the uncontrolled rectifiers, operation of controlled rectifiers is explained below in different loading conditions and different operating modes.

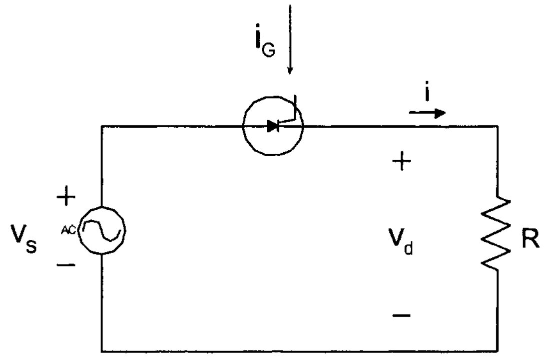


Figure 2.3 Single-phase, half-wave, controlled rectifier with resistive load.

Here the load is purely resistive and the switch is a silicon controlled rectifier (SCR) or thyristor. A thyristor conducts only when its gate is given signal and it is forward biased. Once the gate signal is received, it starts conducting. It stops conducting only if either current through it dies to zero or it gets reverse biased. Figure 2.4 shows the power stage and operating mode circuit diagrams of this rectifier with a purely resistive load. In the first half cycle of the input voltage, even though the thyristor is forward biased, it will not conduct unless gate signal is applied. Thus, at this time output current is zero. Output voltage is also zero. If the gate signal is applied at angle α, thyristor starts conducting. Now, the output voltage is equal to the supply voltage. The output current is given by output voltage divided by the load resistance. This will continue until the half period. Then, supply voltage will be negative and, thus, the thyristor will be reverse biased. As mentioned earlier, as soon as thyristor gets reverse biased, it will turn off and there will not be any output current and, thus, no output voltage. Thyristor will conduct again only when next gate signal is applied and the sequence will continue. Figure 2.5 shows the input and output waveforms.

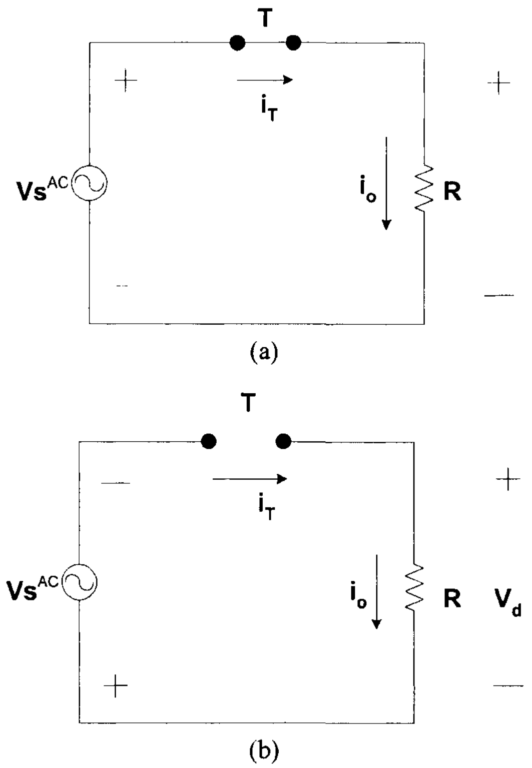


Figure 2.4 Circuit configuration: (a) thyristor is on (b) thyristor is off.

For an inductive load, during the first half cycle of the input voltage, the thyristor will be forward biased and ready to conduct. It will start conducting as soon as the gate signal is applied. Once the thyristor is triggered, input voltage will appear across the load. Now, current starts flowing through switch and load. The inductor will store energy during this time. At the end of the half period, current through the inductor is not zero and, thus, it has to pass through the switch in the circuit. Until current through the inductor becomes zero, the source voltage will continue appearing at the output of the rectifier. When the current reaches zero, the thyristor will be reverse biased. Output voltage will remain zero until SCR is fired again.

For a load with an internal battery, when thyristor is not triggered, there is no output current and the voltage is equal to the battery voltage. Here one important point is that if the thyristor is triggered before input voltage becomes greater than the battery voltage, it will not turn on because it is reverse biased. Once input or source voltage is greater than the battery voltage, the switch is ready to turn on. When the thyristor is triggered, current starts flowing through the load. Energy will be stored in the inductor. Once load current becomes zero, the switch will turn off and output voltage will no more be the source voltage. But, as soon as current becomes zero, voltage will have a jump from source voltage to the battery voltage. Output voltage will stay at this level until the switch is triggered again. Once the switch is triggered, the whole sequence will be repeated.

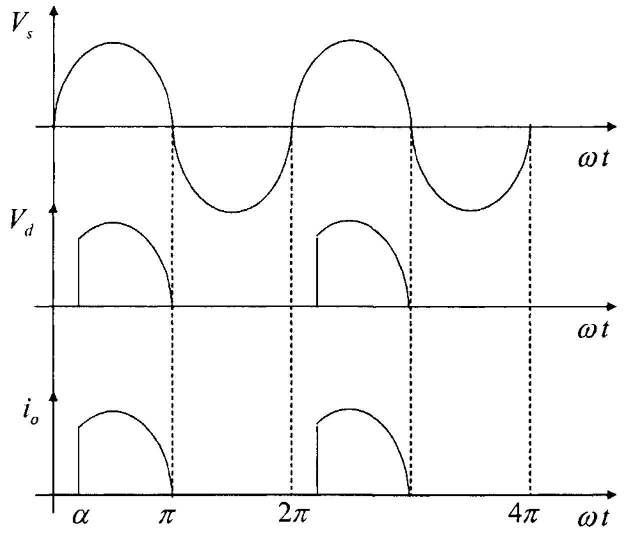


Figure 2.5 Input voltage and output voltage and current waveforms.

2.1.3 Single-Phase, Full-Wave, Uncontrolled Rectifiers

These rectifiers are supplied from single-phase AC source. Output voltage waveform has both halves of the source voltage. Diodes, which are uncontrollable power switches, are used. Figure 2.6 depicts a single-phase, full-wave, uncontrolled rectifier. Looking at the power stage diagram of these rectifiers, they resemble bridge topology and, thus, they are also called diode bridge rectifiers.

Circuit state diagrams, in different switching conditions, are shown in Figure 2.7. In the first half cycle of source voltage, diodes D1 and D2 are forward biased. Thus, source voltage will appear at the load and current starts flowing through the resistive load. After the half period, voltage goes in negative region; therefore, diodes D3 and D4 are forward biased. Due to this, negative voltage will appear across diodes D1 and D2 and they will be reverse biased. Current through output will be still there in the same direction and negative of the source voltage will appear at the output. Output voltage and output current will always be positive and in the same direction.

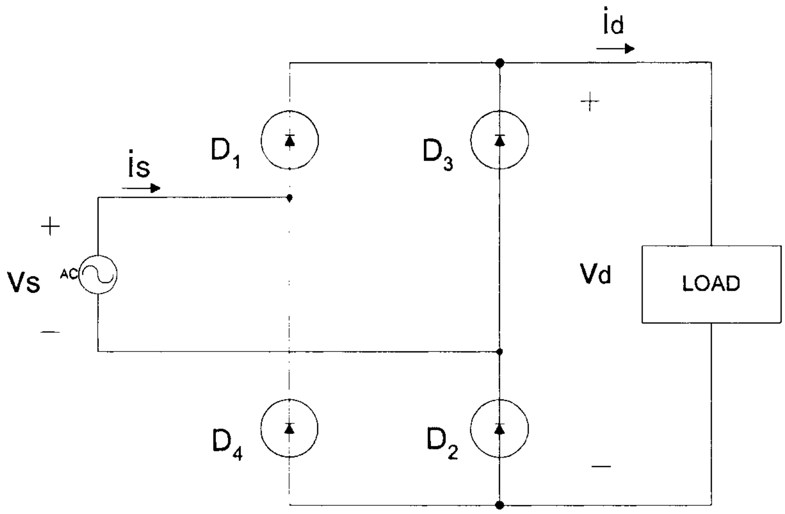
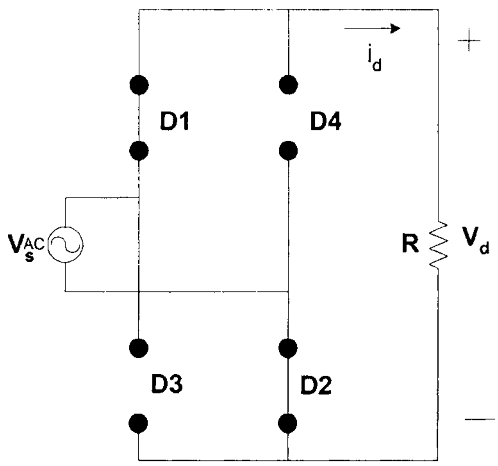
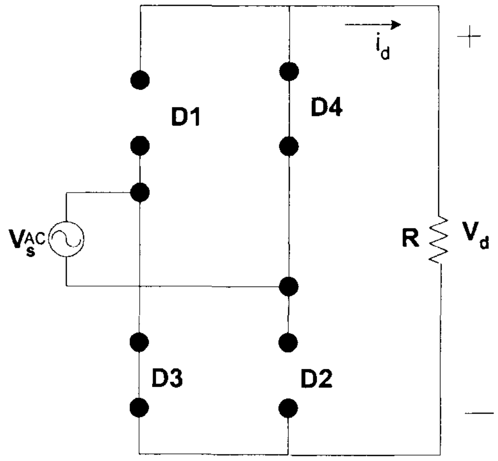


Figure 2.6 Single-phase, full-wave, uncontrolled rectifier.



(a)

Figure 2.7 Circuit configuration: (a) D1, D2 are conducting (b) D3, D4 are conducting.



(b)

Figure 2.7 (Continued)

2.1.4 Single-Phase, Full-Wave, Controlled Rectifiers

Figure 2.8 depicts a single-phase, full-wave, controlled rectifier with resistive load. All the switches are thyristors, which are controlled devices.

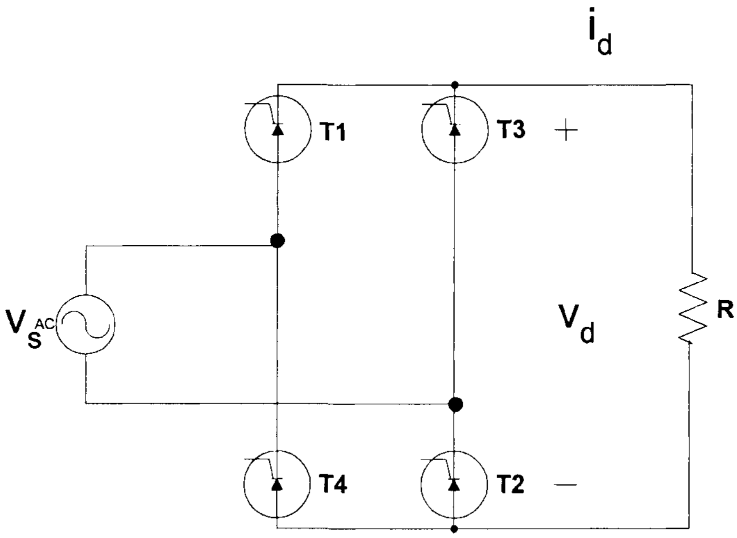
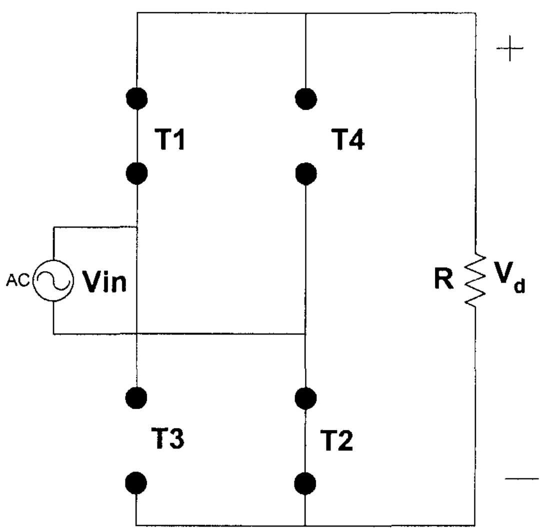


Figure 2.8 Single-phase, full-wave, controlled rectifier with resistive load.

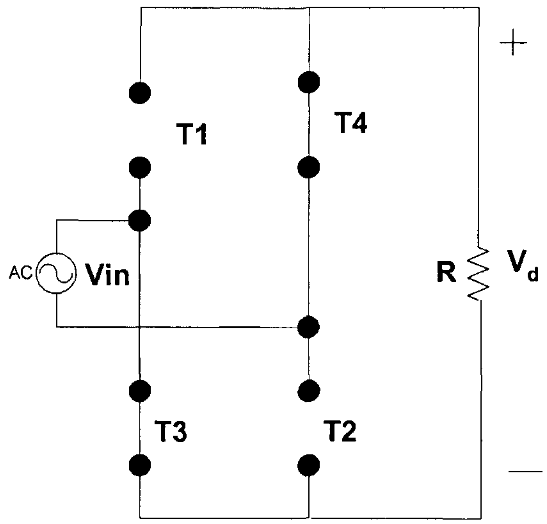
Below is the operating stage diagram when the load is resistive. We can see that, at any time, two switches are closed. But, a point to be kept in mind is that at no instant are two switches from one leg closed. This is simply because if it happens the source gets short-circuited. This phenomenon is also known as shoot-through fault.

Thyristors will only conduct when they are forward biased and a gate signal is applied to its gate terminal. Therefore, in a positive half cycle, even though thyristors T1 and T2 are forward biased, they will not conduct unless they are fired. As soon as a gate signal is applied, T1 and T2 start conducting. At the same time, T3 and T4 are reverse biased and they will not conduct. In the operating stage diagram, T1 and T2 are shown as closed switches and T3 and T4 are open switches. Current starts flowing through the load. In other words, load is being supplied and source voltage will appear across it. The load current will be the source voltage divided by the load resistance. When source voltage becomes zero and, thus, load current also reaches zero, both thyristors T1 and T2 are reverse biased and stop conducting. At the same time, thyristors T3 and T4 are forward biased and ready to conduct, but not conducting. When gate signal is applied to them, they start conducting and load current starts flowing through them. Or in other words load is being supplied and a negative source voltage will appear across the load.



(a)

Figure 2.9 Circuit configuration: (a) T1, T2 are conducting (b) T3, T4 are conducting.



(b)

Figure 2.9 (Continued)

Figure 2.10 depicts input and output waveforms for a single-phase, full-wave, controlled rectifier when operated with purely resistive load. Here, α is the firing angle. This is given in degrees. The firing angle can be defined as the angle or instant at which the gate signal is applied to the thyristor. As the load is purely resistive and there is not any internal source, the operation of this converter is solely defined by the source voltage and firing angle. As the source voltage always encounters zero magnitude twice in each complete cycle, this configuration always operates in a discontinuous mode.

For an inductive load, we have one energy storage device. Therefore, circuit operation will also depend on the state of the inductor. When current though this converter is zero during the converter operation, the converter is in discontinuous conduction mode (DCM) of operation. If during the operation of the converter, inductor current never reaches zero, converter is in continuous conduction mode (CCM) of operation.

In CCM operation, during the positive half cycle of the source voltage, thyristors T1 and T2 are forward biased and ready to conduct. As soon as the gate signal is applied, they start conducting and source voltage appears across the load. When the source voltage becomes zero, thyristors are about to be reversed biased, but the load is inductive. Current through the inductor cannot change instantaneously and, thus, current is still positive, not zero. Therefore, after π, unless the load current becomes zero, source voltage will appear across the load. Gate signals are applied to thyristors T3 and T4 at $\alpha + \pi$. As soon as

Thyristors T3 and T4 are fired, they start conducting and T1 and T2 are forced to be off. Now, negative of the source voltage will appear across the load.

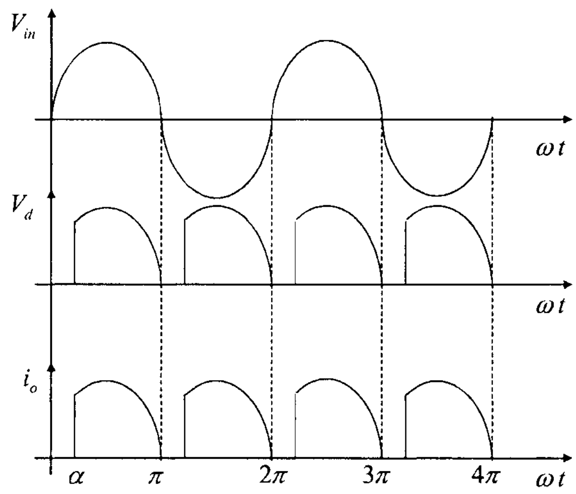


Figure 2.10 Input and output waveforms.

In DCM operation, current through the inductor reaches zero after π and before $\alpha + \pi$. When inductor current reaches zero, output voltage will be zero.

Output voltage for this rectifier in continuous conduction mode of operation is as follows:

$$
\begin{aligned}
v_s &= \sqrt{2}V_s Sin(\omega t) \\
V_d &= \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}V_s Sin(\omega t)d(\omega t) = \frac{2\sqrt{2}}{\pi}V_s Cos(\alpha)
\end{aligned}
\tag{2.1}
$$

If the firing angle is greater than 90 degrees, the output voltage is negative. Current is still in the same direction. Thus, this rectifier is a two-quadrant AC/DC converter. In fact, the full bridge controlled converter has the ability to operate as an inverter.

2.2 DC/DC Converters

Power electronic converters which change the level of DC source to a different level of DC, keeping regulation in consideration, are known as DC/DC converters. They are also popularly known as choppers. These converters can be regarded as DC transformers. But they are much more efficient with less volume, cost, and size.

2.2.1 Buck Converters

A DC/DC Buck converter steps down the input voltage source. Figure 2.11 depicts the power circuit of this converter. As discussed for rectifiers, here also DC/DC converter operation is discussed in both continuous and discontinuous conduction modes of operation.

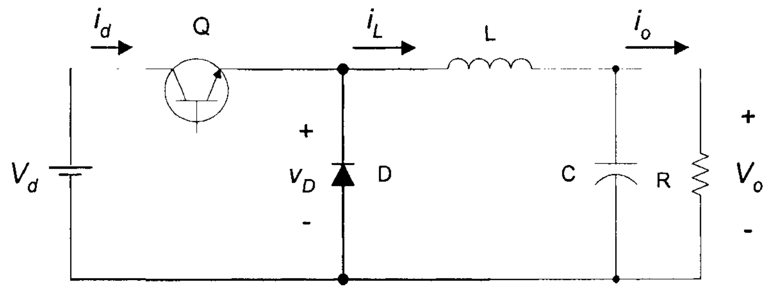


Figure 2.11 DC/DC Buck converter.

From the diagram shown above, it can be seen that there are two variables which should be monitored to have a complete idea about the performance of this converter. They are inductor current and capacitor voltage (output voltage). Opening and closing of the semiconductor switch changes the stage of the power circuit. Therefore, circuit performance is discussed in relation to the semiconductor switch. The switch can be a transistor, MOSFET, IGBT, or any other full-controlled power electronic switch.

In CCM operation, when switch is on ($0 < t < DT$), the circuit diagram for the converter is as shown in Figure 2.12. As soon as the switch is turned on, the current starts flowing through it. As there is an inductor in the path, the current cannot have a step change, but it starts increasing linearly. It is also seen that negative of input voltage appears across the diode and, thus, it will get reverse biased. Looking to the flow of the current, one can say that during this period, output capacitor is being charged by source and load is supplied from the source. The other variable, the inductor current, also increases linearly and energy is stored into it. In this interval, diode current is zero, as it is reverse biased. As there are two voltages (input and output) on either end of the inductor, the difference of them will appear across it. If we look into the state equations, the following observations can be made.

$$0 < t < DT$$

$$V_d = v_L + V_o \tag{2.2a}$$

$$\frac{di_L}{dt} = \frac{V_d - V_o}{L}$$

$$i_L(t) = \frac{V_d - V_o}{L}t + I_{L,\min.}$$
$$i_L(t = DT) = I_{L,\max.} \tag{2.2b}$$
$$\Delta I_L = I_{L,\max.} - I_{L,\min.} = \frac{V_d - V_o}{L}DT$$

In the above equations, V_d is source voltage, V_o is output voltage, i_L is inductor current, and D is the duty.

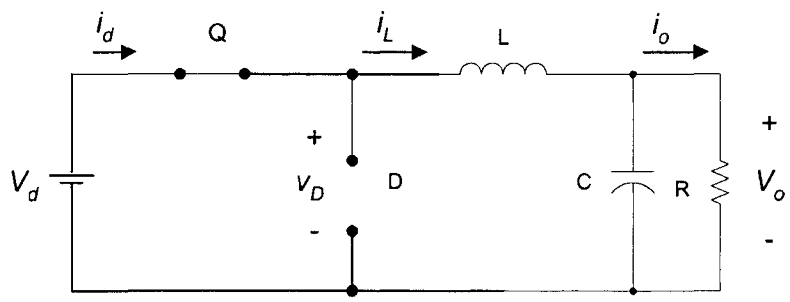


Figure 2.12 DC/DC Buck converter when switch is on.

When switch is off ($DT < t < T$), the circuit diagram for the converter is as shown in Figure 2.13. As can be seen from this circuit diagram, the path to the load current is provided by the diode. The diode is forward biased. The energy stored in the inductor is released and will supply the load. The job of the capacitor in this interval is to keep the output voltage within the ripple limit set at the time of design. Thus, the amount of load not supplied from the inductor is maintained by the output capacitor. But, as there is not any source and energy stored in the inductor is depleting, the current through the inductor decreases linearly in this duration. The capacitor is discharged in this interval. The only voltage now appearing across the inductor is a negative output voltage. Below are the mathematical equations for this interval.

$$DT < t < T$$
$$v_L = -V_o$$
$$\frac{di_L}{dt} = \frac{-V_o}{L} \tag{2.3a}$$
$$i_L(t) = \frac{-V_o}{L}(t - DT) + I_{L,\max.}$$

$$i_L(t = T) = I_{L,\min.}$$

$$\Delta I_L = I_{L,\max.} - I_{L,\min.} = \frac{V_o}{L}(1 - D)T \tag{2.3b}$$

From (2.2) and (2.3), we have

$$\frac{V_d - V_o}{L} DT = \frac{V_o}{L}(1 - D)T \tag{2.4}$$

$$V_o = DV_d, \; D = \frac{t_{on}}{T} \tag{2.5}$$

Duty ratio D varies between 0 and 1. Thus, one can easily say that the output voltage will always be less than the input voltage. Figure 2.14 depicts the inductor voltage and current waveforms in CCM operation. We can see that output voltage is not plotted below. The reason for that is we assume here that output voltage is essentially kept constant by good design.

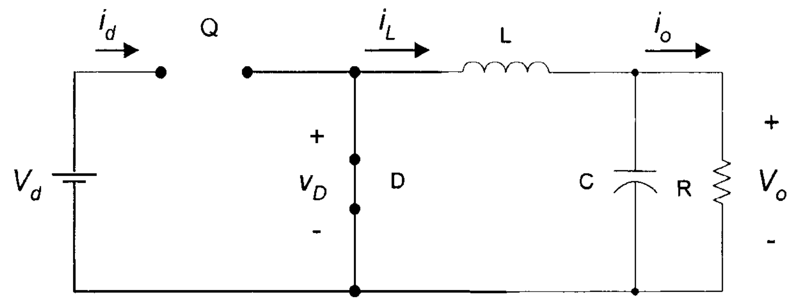


Figure 2.13 DC/DC Buck converter when switch is off.

Linear increment and decrement in the inductor can be observed from the waveform above, which justifies our description. It is also seen that the inductor current does not reach zero and, thus, the converter operates in CCM operation. One justifiable reason for the CCM operation is the large value of the output inductor. Obviously, load will always play its role in deciding the mode of operation. Therefore, one can say that if operating load range, input voltage, required output voltage, and switching frequency are given, it is possible to design the value of both duty ratio and inductor, to operate the converter in CCM operation. It is also possible to design the output capacitor, if it is given how much ripple voltage in output voltage is allowed.

The critically discontinuous conduction mode (CDCM) of operation is the boundary between continuous and discontinuous conduction modes of operation. Figure 2.15 shows the inductor voltage and current waveforms for CDCM operation.

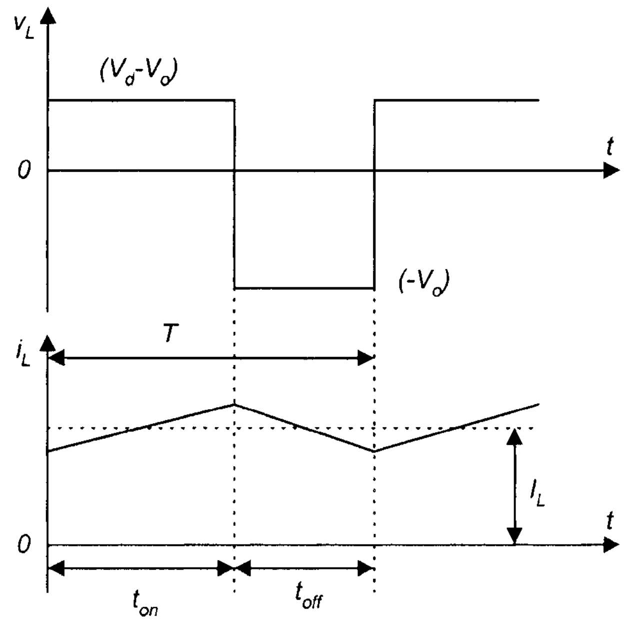


Figure 2.14 Inductor voltage and current waveforms in CCM operation.

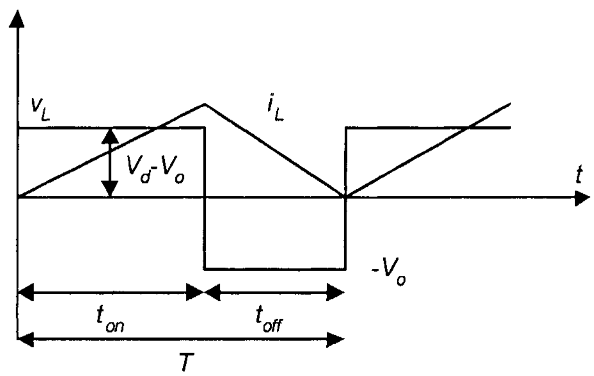


Figure 2.15 DC/DC Buck converter waveforms in CDCM operation.

In CDCM operation, the inductor current reaches zero, but as soon as it reaches zero, switch is immediately turned on and current again becomes non-zero. Thus, it is said that the converter is operating in boundary condition.

Figure 2.16 depicts the DC/DC Buck converter waveforms in DCM operation. It is clear that the duration for which the inductor current stays at zero is comparatively much larger than the CDCM. In fact, in CDCM, the inductor current just hits the zero line and increases. But, in this case, the current stays at zero. Here also minimum instantaneous inductor current is zero: it reaches zero before *T*, while in the case of CDCM, it reaches zero exactly at *T*.

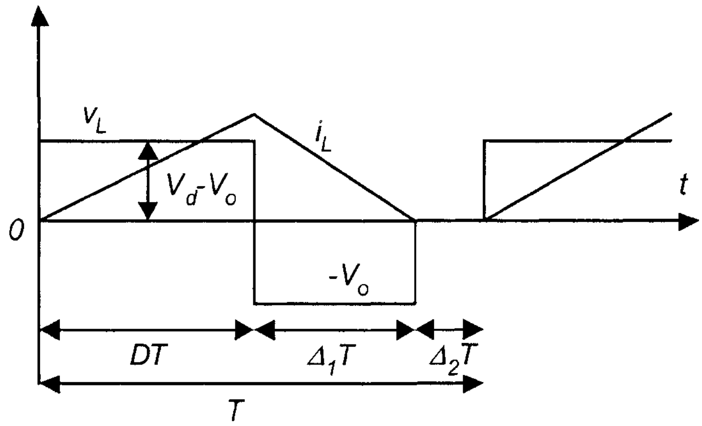


Figure 2.16 DC/DC Buck converter waveforms in DCM operation.

2.2.2 Boost Converters

A DC/DC Boost converter steps up the input voltage source. Figure 2.17 depicts the power circuit of this converter. This converter increases the level of the DC voltage source and can operate in both continuous and discontinuous conduction modes of operation, per the designed system parameters.

In this converter, there is a switch in parallel with the source, instead of series as it was in Buck converter. Therefore, drive is not floating and common ground is available to the semiconductor switch. One other difference, which can be seen from the power stage, is that there is an inductor immediately after the source. This inductor can be output of a rectifier with DC link. This makes it possible to have small size of EMI filter. The problem is that natural short circuit protection is not available, as switch is in parallel.

In CCM operation, when switch is on ($0 < t < DT$), the circuit diagram for the converter is as shown in Figure 2.18.

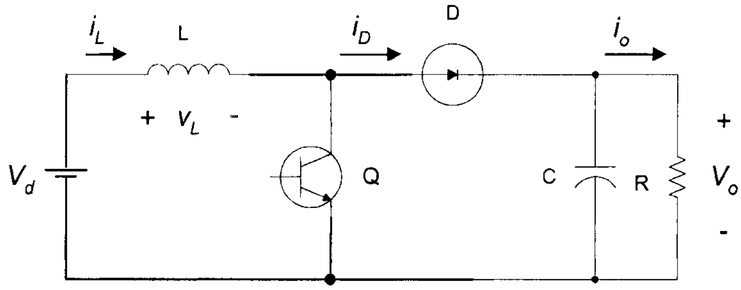


Figure 2.17 DC/DC Boost converter.

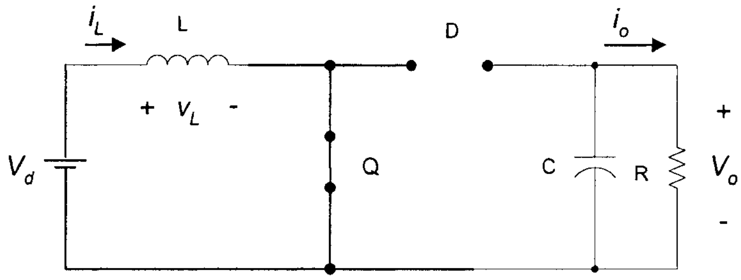


Figure 2.18 DC/DC Boost converter when switch is on.

As can be seen in Figure 2.18, due to closure of the switch, the negative voltage will appear across the diode and it will reverse biased. During this time, the switch is conducting. Inductor current is equal to the source current and the same current flows through the switch. Source voltage appears across the inductor. It is obvious that diode current is zero and, thus, the load current comes from the capacitor only. Energy is being stored in the inductor. Capacitor is being discharged through the load. Below are the state variable equations during this period.

$0 < t < DT$

$$V_d = v_L$$

$$\frac{di_L}{dt} = \frac{V_d}{L} \tag{2.6a}$$

$$i_L(t) = \frac{V_d}{L}t + I_{L,\min}.$$

$$i_L(t = DT) = I_{L,\max}.$$