# **Turbomachinery** Design and Theory



Rama S. R. Gorla Aijaz A. Khan

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## Turbomachinery

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## **Turbomachinery**

**Design and Theory** 

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To my parents, Tirupelamma and Subba Reddy Gorla, who encouraged me to strive for excellence in education

To my wife, Tahseen Ara, and to my daughters, Shumaila, Sheema, and Afifa

—A.A.K.

### Preface

*Turbomachinery: Design and Theory* offers an introduction to the subject of turbomachinery and is intended to be a text for a single-semester course for senior undergraduate and beginning graduate students in mechanical engineering, aerospace engineering, chemical engineering, design engineering, and manufacturing engineering. This book is also a valuable reference to practicing engineers in the fields of propulsion and turbomachinery.

A basic knowledge of thermodynamics, fluid dynamics, and heat transfer is assumed. We have introduced the relevant concepts from these topics and reviewed them as applied to turbomachines in more detail. An introduction to dimensional analysis is included. We applied the basic principles to the study of hydraulic pumps, hydraulic turbines, centrifugal compressors and fans, axial flow compressors and fans, steam turbines, and axial flow and radial flow gas turbines. A brief discussion of cavitation in hydraulic machinery is presented.

Each chapter includes a large number of solved illustrative and design example problems. An intuitive and systematic approach is used in the solution of these example problems, with particular attention to the proper use of units, which will help students understand the subject matter easily. In addition, we have provided several exercise problems at the end of each chapter, which will allow students to gain more experience. We urge students to take these exercise problems seriously: they are designed to help students fully grasp each topic and to lead them toward a more concrete understanding and mastery of the techniques presented.

This book has been written in a straightforward and systematic manner, without including irrelevant details. Our goal is to offer an engineering textbook on turbomachinery that will be read by students with enthusiasm and interest—we have made special efforts to touch students' minds and assist them in exploring the exciting subject matter.

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Rama S.R.Gorla Aijaz A.Khan

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## Turbomachinery

## Introduction: Dimensional Analysis—Basic Thermodynamics and Fluid Mechanics

#### **1.1 INTRODUCTION TO TURBOMACHINERY**

A turbomachine is a device in which energy transfer occurs between a flowing fluid and a rotating element due to dynamic action, and results in a change in pressure and momentum of the fluid. Mechanical energy transfer occurs inside or outside of the turbomachine, usually in a steady-flow process. Turbomachines include all those machines that produce power, such as turbines, as well as those types that produce a head or pressure, such as centrifugal pumps and compressors. The turbomachine extracts energy from or imparts energy to a continuously moving stream of fluid. However in a positive displacement machine, it is intermittent.

The turbomachine as described above covers a wide range of machines, such as gas turbines, steam turbines, centrifugal pumps, centrifugal and axial flow compressors, windmills, water wheels, and hydraulic turbines. In this text, we shall deal with incompressible and compressible fluid flow machines.

#### **1.2 TYPES OF TURBOMACHINES**

There are different types of turbomachines. They can be classified as:

1. Turbomachines in which (i) work is done by the fluid and (ii) work is done on the fluid.



**Figure 1.1** Types and shapes of turbomachines.

2. Turbomachines in which fluid moves through the rotating member in axial direction with no radial movement of the streamlines. Such machines are called axial flow machines whereas if the flow is essentially radial, it is called a radial flow or

centrifugal flow machine. Some of these machines are shown in Fig. 1.1, and photographs of actual machines are shown in Figs. 1.2–1.6. Two primary points will be observed: first, that the main element is a rotor or runner carrying blades or vanes; and secondly, that the path of the fluid in the rotor may be substantially axial, substantially radial, or in some cases a combination of both. Turbomachines can further be classified as follows:

*Turbines:* Machines that produce power by expansion of a continuously flowing fluid to a lower pressure or head.

Pumps: Machines that increase the pressure or head of flowing fluid.

*Fans:* Machines that impart only a small pressure-rise to a continuously flowing gas; usually the gas may be considered to be incompressible.



**Figure 1.2** Radial flow fan rotor. (Courtesy of the Buffalo Forge Corp.)



**Figure 1.3** Centrifugal compressor rotor (the large double-sided impellar on the right is the main compressor and the small single-sided impellar is an auxiliary for cooling purposes). (Courtesy of Rolls-Royce, Ltd.)



**Figure 1.4** Centrifugal pump rotor (open type impeller). (Courtesy of the Ingersoll-Rand Co.)



**Figure 1.5** Multi-stage axial flow compressor rotor. (Courtesy of the Westinghouse Electric Corp.)



**Figure 1.6** Axial flow pump rotor. (Courtesy of the Worthington Corp.)

*Compressors:* Machines that impart kinetic energy to a gas by compressing it and then allowing it to rapidly expand. Compressors can be axial flow, centrifugal, or a combination of both types, in order to produce the highly compressed air. In a dynamic compressor, this is achieved by imparting kinetic energy to the air in the impeller and then this kinetic energy is converted into pressure energy in the diffuser.

#### **1.3 DIMENSIONAL ANALYSIS**

To study the performance characteristics of turbomachines, a large number of variables are involved. The use of dimensional analysis reduces the variables to a number of manageable dimensional groups. Usually, the properties of interest in regard to turbomachine are the power output, the efficiency, and the head. The performance of turbomachines depends on one or more of several variables. A summary of the physical properties and dimensions is given in Table 1.1 for reference.

Dimensional analysis applied to turbomachines has two more important uses: (1) prediction of a prototype's performance from tests conducted on a scale

Property	Dimension
Surface	L <sup>2</sup>
Volume	$L^3$
Density	M/L <sup>3</sup>
Velocity	L/T
Acceleration	$L/T^2$
Momentum	ML/T
Force	ML/T <sup>2</sup>
Energy and work	$ML^2/T^2$
Power	$ML^2/T^3$
Moment of inertia	$ML^2$
Angular velocity	I/T
Angular acceleration	$I/T^2$
Angular momentum	$ML^2/T$
Torque	$ML^2/T^2$
Modules of elasticity	M/LT <sup>2</sup>
Surface tension	M/T <sup>2</sup>
Viscosity (absolute)	M/LT
Viscosity (kinematic)	L <sup>2</sup> /T

 Table 1.1 Physical Properties and Dimensions

model (similitude), and (2) determination of the most suitable type of machine, on the basis of maximum efficiency, for a specified range of head, speed, and flow rate. It is assumed here that the student has acquired the basic techniques of forming nondimensional groups.

#### **1.4 DIMENSIONS AND EQUATIONS**

The variables involved in engineering are expressed in terms of a limited number of basic dimensions. For most engineering problems, the basic dimensions are:

- 1. SI system: mass, length, temperature and time.
- 2. English system: mass, length, temperature, time and force.

The dimensions of pressure can be designated as follows

#### Turbomachinery 8

$$P = \frac{F}{L^2} \tag{1.1}$$

Equation (1.1) reads as follows: "The dimension of P equals force per length squared." In this case,  $L^2$  represents the dimensional characteristics of area. The left hand side of Eq. (1.1) must have the same dimensions as the right hand side.

#### **1.5 THE BUCKINGHAM П THEOREM**

In 1915, Buckingham showed that the number of independent dimensionless group of variables (dimensionless parameters) needed to correlate the unknown variables in a given process is equal to n-m, where n is the number of variables involved and m is the number of dimensionless parameters included in the variables. Suppose, for example, the drag force F of a flowing fluid past a sphere is known to be a function of the velocity (v) mass density  $(\rho)$  viscosity  $(\mu)$  and diameter (D). Then we have five variables  $(F, v, \rho, \mu, and D)$  and three basic dimensions (L, F, and T) involved. Then, there are 5–3=2 basic grouping of variables that can be used to correlate experimental results.

#### **1.6 HYDRAULIC MACHINES**

Consider a control volume around the pump through which an incompressible fluid of density  $\rho$  flows at a volume flow rate of Q.

Since the flow enters at one point and leaves at another point the volume flow rate Q can be independently adjusted by means of a throttle valve. The discharge Q of a pump is given by

$$Q=f(N,D,g,H,\mu,\rho)$$

where *H* is the head, *D* is the diameter of impeller, *g* is the acceleration due to gravity,  $\rho$  is the density of fluid, *N* is the revolution, and  $\mu$  is the viscosity of fluid.

In Eq. (1.2), primary dimensions are only four. Taking N, D, and  $\rho$  as repeating variables, we get

 $II_1 = (N)^2 (d)^{\overline{b}} (\rho)^c (Q)$  $M^0 L^0 T^0 = (T^{-1})^a (L)^b (ML^{-3})^c (L^3 T^{-1})$ 

For dimensional homogeneity, equating the powers of M, L, and T on both sides of the equation: for M, 0=c or c=0; for T, 0=-a-1 or a=-1; for L, 0=b-3c+3 or b=-3.

Therefore,

$$\Pi_1 = N^{-1} D^{-3} \rho^0 Q = \frac{Q}{ND^3}$$
(1.3)

Similarly,

 $II_2 = (N)^d (d)^e (\rho)^f (g)$ 

$$M^{0}L^{0}T^{0} = (T^{-1})^{d}(L)^{e}(ML^{-3})^{f}(LT^{-2})$$

Now, equating the exponents: for M, 0=f or f=0; for T, 0 =-d-2 or d =-2; for L, 0=e-3f+1 or e =-1.

Thus,

$$\Pi_2 = N^{-2} D^{-1} \rho^0 g = \frac{g}{N^2 D} \tag{1.4}$$

Similarly,

 $\underset{M^{0}L^{0}T^{0}=(T^{-1})^{g}(L)^{h}(\rho)^{i}(H) }{ M^{0}L^{0}T^{0}=(T^{-1})^{g}(L)^{h}(ML^{-3})^{i}(L) }$ 

Equating the exponents: for M, 0=i or i=0; for T, 0=-g or g=0; for L, 0=h-3i+1 or h=-1.

Thus,

$$\Pi_3 = N^0 D^{-1} \rho^0 H = \frac{H}{D}$$
(1.5)

and,

$$\Pi_4 = (N)^i (D)^k (\rho)^l (\mu)$$
  
$$M^0 L^0 T^0 = (T^{-1})^j (L)^k (ML^{-3})^l (ML^{-1}T^{-1})$$

Equating the exponents: for M, 0=l+1 or l=-1; for T, 0=-j-1 or j=-1; for L, 0=k-3l-1 or k=-2.

Thus,

$$\Pi_4 = N^{-1} D^{-2} \rho^{-1} \mu = \frac{\mu}{N D^2 \rho}$$
(1.6)

The functional relationship may be written as

$$f\left(\frac{Q}{ND^3}, \frac{g}{N^2D}, \frac{H}{D}, \frac{\mu}{ND^2\rho}\right) = 0$$

Since the product of two  $\Pi$  terms is dimensionless, therefore replace the terms  $\Pi_2$  and  $\Pi_3$  by  $gh/N^2D^2$ 

$$f\left(\frac{Q}{ND^3},\frac{gH}{N^2D^2},\frac{\mu}{ND^2\rho}\right) = 0$$

or

$$Q = ND^3 f\left(\frac{gH}{N^2 D^2}, \frac{\mu}{ND^2 \rho}\right) = 0$$
(1.7)

A dimensionless term of extremely great importance that may be obtained by manipulating the discharge and head coefficients is the specific speed, defined by the equation

$$N_{\rm s} = \sqrt{\frac{\text{Flow coefficient}}{\text{Head coefficient}}} = N\sqrt{Q}/(gH)^{3/4}$$
(1.8)

The following few dimensionless terms are useful in the analysis of incompressible fluid flow machines:

- 1. The flow coefficient and speed ratio: The term  $Q/(ND^3)$  is called the flow coefficient or specific capacity and indicates the volume flow rate of fluid through a turbomachine of unit diameter runner, operating at unit speed. It is constant for similar rotors.
- 2. The head coefficient: The term  $gH/N^2D^2$  is called the specific head. It is the kinetic energy of the fluid spouting under the head H divided by the kinetic energy of the fluid running at the rotor tangential speed. It is constant for similar impellers.

$$\psi = H/(U^2/g) = gH/(\pi^2 N^2 D^2)$$
(1.9)

- 3. *Power coefficient or specific power:* The dimensionless quantity  $P/(\rho N^2 D^2)$  is called the power coefficient or the specific power. It shows the relation between power, fluid density, speed and wheel diameter.
- 4. *Specific speed:* The most important parameter of incompressible fluid flow machinery is specific speed. It is the non-dimensional term. All turbomachineries operating under the same conditions of flow and head having the same specific speed, irrespective of the actual physical size of the machines. Specific speed can be expressed in this form

$$N_{\rm s} = N\sqrt{Q}/(gH)^{3/4} = N\sqrt{P}/[\rho^{1/2}(gH)^{5/4}]$$
(1.10)

The specific speed parameter expressing the variation of all the variables N, Q and H or N, P and H, which cause similar flows in turbomachines that are geometrically similar. The specific speed represented by Eq. (1.10) is a nondimensional quantity. It can also be expressed in alternate forms.

These are

$$N_{\rm s} = N\sqrt{Q}/H^{3/4} \tag{1.11}$$

and

$$N_{\rm s} = N\sqrt{P}/H^{5/4} \tag{1.12}$$

Equation (1.11) is used for specifying the specific speeds of pumps and Eq. (1.12) is used for the specific speeds of turbines. The turbine specific speed may be defined as the speed of a geometrically similar turbine, which develops 1 hp under a head of 1 meter of water.

It is clear that  $N_s$  is a dimensional quantity. In metric units, it varies between 4 (for very high head Pelton wheel) and 1000 (for the low-head propeller on Kaplan turbines).

#### **1.7 THE REYNOLDS NUMBER**

Reynolds number is represented by

$$Re = D^2 N/\nu$$

where v is the kinematic viscosity of the fluid. Since the quantity  $D^2N$  is proportional to DV for similar machines that have the same speed ratio. In flow through turbomachines, however, the dimensionless parameter  $D^2N/v$  is not as important since the viscous resistance alone does not determine the machine losses. Various other losses such as those due to shock at entry, impact, turbulence, and leakage affect the machine characteristics along with various friction losses.

Consider a control volume around a hydraulic turbine through which an incompressible fluid of density  $\rho$  flows at a volume flow rate of Q, which is controlled by a valve. The head difference across the control volume is H, and if the control volume represents a turbine of diameter D, the turbine develops a shaft power P at a speed of rotation N. The functional equation may be written as

$$P = f(\rho, N, \mu, D, Q, gH)$$
(1.13)

Equation (1.13) may be written as the product of all the variables raised to a power and a constant, such that

$$P = \text{const.}\left(\rho^a N^b \mu^c D^d Q^e (gH)^f\right)$$
(1.14)

Substituting the respective dimensions in the above Eq. (1.14),  $\left(ML^{2}/T^{3}\right) = \text{const.}\left(M/L^{3}\right)^{a}(1/T)^{b}(M/LT)^{c}(L)^{d}(L^{3}/T)^{e}(L^{2}/T^{2})^{f}$ (1.15)

Equating the powers of M, L, and T on both sides of the equation: for M, 1 = a+c; for L, 2=-3a-c+d+3e+2f; for T, -3=-b-c-e-2f.

There are six variables and only three equations. It is therefore necessary to solve for three of the indices in terms of the remaining three. Solving for a, b, and d in terms of c, e, and f we have:

a=1-cb=3-c-e-2fd=5-2c-3e-2f

Substituting the values of a, b, and d in Eq. (1.13), and collecting like indices into separate brackets,

$$P = \text{const.}\left[\left(\rho N^3 D^5\right), \left(\frac{\mu}{\rho N D^2}\right)^c, \left(\frac{Q}{N D^3}\right)^e, \left(\frac{gH}{N^2 D^2}\right)^f\right]$$
(1.16)

In Eq. (1.16), the second term in the brackets is the inverse of the Reynolds number. Since the value of c is unknown, this term can be inverted and Eq. (1.16) may be written as

$$P/\rho N^3 D^5 = \text{const.}\left[\left(\frac{\rho N D^2}{\mu}\right)^c, \left(\frac{Q}{N D^3}\right)^e, \left(\frac{gH}{N^2 D^2}\right)^f\right]$$
(1.17)

In Eq. (1.17) each group of variables is dimensionless and all are used in hydraulic turbomachinery practice, and are known by the following names: the power coefficient  $(P/\rho N^3 D^5 = \overline{P})_{\text{the flow coefficient } (Q/ND^3 = \emptyset);}$  and the head coefficient  $(gH/N^2D^2 = \psi)$ .

Equation (1.17) can be expressed in the following form:

$$\overline{P} = f(Re, \phi, \psi) \tag{1.18}$$

Equation (1.18) indicates that the power coefficient of a hydraulic machine is a function of Reynolds number, flow coefficient and head coefficient. In flow through hydraulic turbomachinery, Reynolds number is usually very high. Therefore the viscous action of the fluid has very little effect on the power output of the machine and the power coefficient remains only a function of  $\emptyset$  and  $\psi$ .



**Figure 1.7** Performance characteristics of hydraulic machines: (a) hydraulic turbine, (b) hydraulic pump.

Typical dimensionless characteristic curves for a hydraulic turbine and pump are shown in Fig. 1.7 (a) and (b), respectively. These characteristic curves are also the curves of any

other combination of *P*, *N*, *Q*, and *H* for a given machine or for any other geometrically similar machine.

#### **1.8 MODEL TESTING**

Some very large hydraulic machines are tested in a model form before making the fullsized machine. After the result is obtained from the model, one may transpose the results from the model to the full-sized machine. Therefore if the curves shown in Fig 1.7 have been obtained for a completely similar model, these same curves would apply to the fullsized prototype machine.

#### **1.9 GEOMETRIC SIMILARITY**

For geometric similarity to exist between the model and prototype, both of them should be identical in shape but differ only in size. Or, in other words, for geometric similarity between the model and the prototype, the ratios of all the corresponding linear dimensions should be equal.

Let  $L_p$  be the length of the prototype,  $B_p$ , the breadth of the prototype,  $D_p$  the depth of the prototype, and  $L_m$ ,  $B_m$  and  $D_m$  the corresponding dimensions of the model. For geometric similarity, linear ratio (or scale ratio) is given by

$$L_{\rm r} = \frac{L_{\rm p}}{L_{\rm m}} = \frac{B_{\rm p}}{B_{\rm m}} = \frac{D_{\rm p}}{D_{\rm m}} \tag{1.19}$$

Similarly, the area ratio between prototype and model is given by

$$A_{\rm r} = \left(\frac{L_{\rm p}}{L_{\rm m}}\right)^2 = \left(\frac{B_{\rm p}}{B_{\rm m}}\right)^2 = \left(\frac{D_{\rm p}}{D_{\rm m}}\right)^2 \tag{1.20}$$

and the volume ratio

$$V_{\rm r} = \left(\frac{L_{\rm p}}{L_{\rm m}}\right)^3 = \left(\frac{B_{\rm p}}{B_{\rm m}}\right)^3 = \left(\frac{D_{\rm p}}{D_{\rm m}}\right)^3 \tag{1.21}$$

#### **1.10 KINEMATIC SIMILARITY**

For kinematic similarity, both model and prototype have identical motions or velocities. If the ratio of the corresponding points is equal, then the velocity ratio of the prototype to the model is

$$V_{\rm r} = \frac{V_1}{v_1} = \frac{V_2}{v_2} \tag{1.22}$$

where  $V_1$  is the velocity of liquid in the prototype at point 1,  $V_2$ , the velocity of liquid in the prototype at point 2,  $v_1$ , the velocity of liquid in the model at point 1, and  $v_2$  is the velocity of liquid in the model at point 2.

#### 1.11 DYNAMIC SIMILARITY

If model and prototype have identical forces acting on them, then dynamic similarity will exist. Let  $F_1$  be the forces acting on the prototype at point 1, and  $F_2$  be the forces acting on the prototype at point 2. Then the force ratio to establish dynamic similarity between the prototype and the model is given by

$$F_{\rm r} = \frac{F_{\rm p1}}{F_{\rm m1}} = \frac{F_{\rm p2}}{F_{\rm m2}} \tag{1.23}$$

#### **1.12 PROTOTYPE AND MODEL EFFICIENCY**

Let us suppose that the similarity laws are satisfied,  $\eta_p$  and  $\eta_m$  are the prototype and model efficiencies, respectively. Now from similarity laws, representing the model and prototype by subscripts m and p respectively,

$$\frac{H_{\rm p}}{\left(N_{\rm p}D_{\rm p}\right)^2} = \frac{H_{\rm m}}{\left(N_{\rm m}D_{\rm m}\right)^2} \quad \text{or} \quad \frac{H_{\rm p}}{H_{\rm m}} = \left(\frac{N_{\rm p}}{N_{\rm m}}\right)^2 \left(\frac{D_{\rm p}}{D_{\rm m}}\right)^2$$
$$\frac{Q_{\rm p}}{N_{\rm p}D_{\rm p}^3} = \frac{Q_{\rm m}}{N_{\rm m}D_{\rm m}^3} \quad \text{or} \quad \frac{Q_{\rm p}}{Q_{\rm m}} = \left(\frac{N_{\rm p}}{N_{\rm m}}\right) \left(\frac{D_{\rm p}}{D_{\rm m}}\right)^3$$
$$\frac{P_{\rm p}}{N_{\rm p}^3 D_{\rm p}^5} = \frac{P_{\rm m}}{N_{\rm m}^3 D_{\rm m}^5} \quad \text{or} \quad \frac{P_{\rm p}}{P_{\rm m}} = \left(\frac{N_{\rm p}}{N_{\rm m}}\right)^3 \left(\frac{D_{\rm p}}{D_{\rm m}}\right)^5$$

Turbine efficiency is given by

$$\eta_t = \frac{\text{Power transferred from fluid}}{\text{Fluid power available.}} = \frac{P}{\rho g Q H}$$

 $\frac{\eta_{\rm m}}{\eta_{\rm p}} = \left(\frac{P_{\rm m}}{P_{\rm p}}\right) \left(\frac{Q_{\rm p}}{Q_{\rm m}}\right) \left(\frac{H_{\rm p}}{H_{\rm m}}\right) = 1$ 

Thus, the efficiencies of the model and prototype are the same providing the similarity laws are satisfied.

#### 1.13 PROPERTIES INVOLVING THE MASS OR WEIGHT OF THE FLUID

#### 1.13.1 Specific Weight (y)

The weight per unit volume is defined as specific weight and it is given the symbol  $\gamma$ (gamma). For the purpose of all calculations relating to hydraulics, fluid machines, the specific weight of water is taken as 1000 1/m<sup>3</sup>. In S.I. units, the specific weight of water is taken as 9.80 kN/m<sup>3</sup>.

#### 1.13.2 Mass Density (p)

The mass per unit volume is mass density. In S.I. systems, the units are kilograms per cubic meter or NS<sup>2</sup>/m<sup>4</sup>. Mass density, often simply called density, is given the greek symbol  $\rho$  (rho). The mass density of water at 15.5° is 1000 kg/m<sup>3</sup>.

#### 1.13.3 Specific Gravity (sp.gr.)

The ratio of the specific weight of a given liquid to the specific weight of water at a standard reference temperature is defined as specific gravity. The standard reference temperature for water is often taken as 4°C Because specific gravity is a ratio of specific weights, it is dimensionless and, of course, independent of system of units used.

#### **1.13.4** Viscosity (*µ*)

We define viscosity as the property of a fluid, which offers resistance to the relative motion of fluid molecules. The energy loss due to friction in a flowing fluid is due to the viscosity. When a fluid moves, a shearing stress develops in it. The magnitude of the shearing stress depends on the viscosity of the fluid. Shearing stress, denoted by the symbol  $\tau$  (tau) can be defined as the force required to slide on unit area layers of a substance over another. Thus  $\tau$  is a force divided by an area and can be measured in units  $N/m^2$  or Pa. In a fluid such as water, oil, alcohol, or other common liquids, we find that the magnitude of the shearing stress is directly proportional to the change of velocity between different positions in the fluid. This fact can be stated mathematically as

$$\tau = \mu \left(\frac{\Delta v}{\Delta y}\right) \tag{1.24}$$

where  $\frac{\Delta \nu}{\Delta y}$  is the velocity gradient and the constant of proportionality  $\mu$  is called the dynamic viscosity of fluid.

#### Units for Dynamic Viscosity

Solving for  $\mu$  gives

$$\mu = \frac{\tau}{\Delta \nu / \Delta y} = \tau \left( \frac{\Delta y}{\Delta \nu} \right)$$

Substituting the units only into this equation gives

$$\mu = \frac{N}{m^2} \times \frac{m}{m/s} = \frac{N \times s}{m^2}$$

Since Pa is a shorter symbol representing N/m<sup>2</sup>, we can also express  $\mu$  as  $\mu = \mathbf{Pa} \cdot \mathbf{s}$ 

#### 1.13.5 Kinematic Viscosity (v)

The ratio of the dynamic viscosity to the density of the fluid is called the kinematic viscosity v (nu). It is defined as

$$\nu = \frac{\mu}{\rho} = \mu(1/\rho) = \frac{\text{kg}}{\text{ms}} \times \frac{\text{m}^3}{\text{kg}} = \frac{\text{m}^2}{\text{s}}$$
 (1.25)

Any fluid that behaves in accordance with Eq. (1.25) is called a Newtonian fluid.

#### **1.14 COMPRESSIBLE FLOW MACHINES**

Compressible fluids are working substances in gas turbines, centrifugal and axial flow compressors. To include the compressibility of these types of fluids (gases), some new variables must be added to those already discussed in the case of hydraulic machines and changes must be made in some of the definitions used. The important parameters in compressible flow machines are pressure and temperature.



**Figure 1.8** Compression and expansion in compressible flow machines: (a) compression, (b) expansion.

In Fig. 1.8 T-s charts for compression and expansion processes are shown.

Isentropic compression and expansion processes are represented by s and the subscript 0 refers to stagnation or total conditions. 1 and 2 refer to the inlet and outlet states of the gas, respectively. The pressure at the outlet,  $P_{02}$ , can be expressed as follows

$$P_{02} = f(D, N, m, P_{01}, T_{01}, T_{02}, \rho_{01}, \rho_{02}, \mu)$$
(1.26)

The pressure ratio  $P_{02}/P_{01}$  replaces the head *H*, while the mass flow rate *m* (kg/s) replaces *Q*. Using the perfect gas equation, density may be written as  $\rho = P/RT$ . Now, deleting density and combining *R* with *T*, the functional relationship can be written as

$$P_{02} = f(P_{01}, RI_{01}, RI_{02}, m, N, D, \mu)$$
(1.27)

Substituting the basic dimensions and equating the indices, the following fundamental relationship may be obtained

$$\frac{P_{02}}{P_{01}} = f\left(\left(\frac{RT_{02}}{RT_{01}}\right), \left(\frac{\left(\frac{m}{RT_{01}}\right)^{1/2}}{P_{01}D^2}\right), \left(\frac{ND}{(RT_{01})^{1/2}}\right), \operatorname{Re}\right)$$
(1.28)

In Eq. (1.28), *R* is constant and may be eliminated. The Reynolds number in most cases is very high and the flow is turbulent and therefore changes in this parameter over the usual operating range may be neglected. However, due to



**Figure 1.9** Axial flow compressor characteristics: (a) pressure ratio, (b) efficiency.

large changes of density, a significant reduction in Re can occur which must be taken into consideration. For a constant diameter machine, the diameter D may be ignored, and hence Eq. (1.28) becomes

$$\frac{P_{02}}{P_{01}} = f\left(\left(\frac{T_{02}}{T_{01}}\right), \left(\frac{mT_{01}^{1/2}}{P_{01}}\right), \left(\frac{N}{T_{01}^{1/2}}\right)\right)$$
(1.29)

In Eq. (1.29) some of the terms are new and no longer dimensionless. For a particular machine, it is typical to plot  $P_{02}/P_{01}$  and  $T_{02}/T_{01}$  against the mass flow



**Figure 1.10** Axial flow gas turbine characteristics: (a) pressure ratio, (b) efficiency.

rate parameter  $mT_{01}^{1/2}/P_{01}$  for different values of the speed parameter  $N/T_{01}^{1/2}$ . Equation (1.28) must be used if it is required to change the size of the machine. The term  $ND/(RT_{01})^{1/2}$  indicates the Mach number effect. This occurs because the impeller velocity  $v \propto ND$  and the acoustic velocity  $a_{01} \propto RT_{01}$  while the Mach number

$$M = V/a_{01}$$
 (1.30)

The performance curves for an axial flow compressor and turbine are shown in Figs. 1.9 and 1.10.

#### 1.15 BASIC THERMODYNAMICS, FLUID MECHANICS, AND DEFINITIONS OF EFFICIENCY

In this section, the basic physical laws of fluid mechanics and thermodynamics will be discussed. These laws are:

- 1. The continuity equation.
- 2. The First Law of Thermodynamics.
- 3. Newton's Second Law of Motion.

4. The Second Law of Thermodynamics.

The above items are comprehensively dealt with in books on thermodynamics with engineering applications, so that much of the elementary discussion and analysis of these laws need not be repeated here.

#### **1.16 CONTINUITY EQUATION**

For steady flow through a turbomachine, *m* remains constant. If  $A_1$  and  $A_2$  are the flow areas at Secs. 1 and 2 along a passage respectively, then

$$\dot{m} = \rho_1 A_1 C_1 = \rho_2 A_2 C_2 = \text{constant}$$
(1.31)

where  $\rho_1$ , is the density at section 1,  $\rho_2$ , the density at section 2,  $C_1$ , the velocity at section 1, and  $C_2$ , is the velocity at section 2.

#### 1.17 THE FIRST LAW OF THERMODYNAMICS

According to the First Law of Thermodynamics, if a system is taken through a complete cycle during which heat is supplied and work is done, then

$$\oint (\delta Q - \delta W) = 0 \tag{1.32}$$

where  $\oint \delta Q$  represents the heat supplied to the system during this cycle and  $\oint \delta W$  the work done by the system during the cycle. The units of heat and work are taken to be the same. During a change of state from 1 to 2, there is a change in the internal energy of the system

$$U_2 - U_1 = \int_1^2 (\delta Q - \delta W)$$
 (1.33)

For an infinitesimal change of state

$$\mathrm{d}U = \delta Q - \delta W \tag{1.24}$$

(1.34)

#### 1.17.1 The Steady Flow Energy Equation

The First Law of Thermodynamics can be applied to a system to find the change in the energy of the system when it undergoes a change of state. The total energy of a system, E may be written as:

*E*=Internal Energy+Kinetic Energy+Potential Energy

$$E = U + K.E. + P.E. \tag{1.35}$$

where U is the internal energy. Since the terms comprising E are point functions, we can write Eq. (1.35) in the following form

$$dE = dU + d(K.E.) + d(P.E.)$$
 (1.36)

The First Law of Thermodynamics for a change of state of a system may therefore be written as follows

$$\delta Q = \mathrm{d}U + \mathrm{d}(\mathrm{KE}) + \mathrm{d}(\mathrm{PE}) + \delta W \tag{1.37}$$

Let subscript 1 represents the system in its initial state and 2 represents the system in its final state, the energy equation at the inlet and outlet of any device may be written

$$Q_{1-2} = U_2 - U_1 + \frac{m(C_2^2 - C_1^2)}{2} + mg(Z_2 - Z_1) + W_{1-2}$$
(1.38)

Equation (1.38) indicates that there are differences between, or changes in, similar forms of energy entering or leaving the unit. In many applications, these differences are insignificant and can be ignored. Most closed systems encountered in practice are stationary; i.e. they do not involve any changes in their velocity or the elevation of their centers of gravity during a process. Thus, for stationary closed systems, the changes in kinetic and potential energies are negligible (i.e.  $\Delta(K.E.)=\Delta(P.E.)=0$ ), and the first law relation reduces to

$$Q - W = \Delta E \tag{1.39}$$

If the initial and final states are specified the internal energies 1 and 2 can easily be determined from property tables or some thermodynamic relations.

#### 1.17.2 Other Forms of the First Law Relation

The first law can be written in various forms. For example, the first law relation on a unit-mass basis is

$$q - w = \Delta e(kJ/kg) \tag{1.40}$$

Dividing Eq. (1.39) by the time interval  $\Delta t$  and taking the limit as  $\Delta t \rightarrow 0$  yields the rate form of the first law

$$\dot{Q} - \dot{W} = \frac{\mathrm{d}E}{\mathrm{d}t} \tag{1.41}$$

where  $\dot{Q}$  is the rate of net heat transfer, the power, and  $\frac{dE}{dt}$  is the rate of change of total energy. Equations. (1.40) and (1.41) can be expressed in differential form

$$\delta Q - \delta W = \mathrm{d} E(\mathrm{kJ}) \tag{1.42}$$

$$\delta q - \delta w = \mathrm{d}e(\mathrm{kJ/kg}) \tag{1.43}$$

For a cyclic process, the initial and final states are identical; therefore,  $\Delta E = E_2 - E_1$ . Then the first law relation for a cycle simplifies to

$$Q - W = 0(kJ) \tag{1.44}$$

That is, the net heat transfer and the net work done during a cycle must be equal. Defining the stagnation enthalpy by:  $h_0 = h + \frac{1}{2}c^2$  and assuming  $g(Z_2-Z_1)$  is negligible, the steady flow energy equation becomes

$$\hat{Q} - \hat{W} = \hat{m}(h_{02} - h_{01}) \tag{1.45}$$

Most turbomachinery flow processes are adiabatic, and so  $\hat{Q}=0$ . For work producing machines,  $\hat{W}>0$ ; so that

$$W = \dot{m}(h_{01} - h_{02}) \tag{1.46}$$

For work absorbing machines (compressors) *W*<0; so that

 $\dot{W} \to -\dot{W} = \dot{m}(h_{02} - h_{01})$  (1.47)

#### **1.18 NEWTON'S SECOND LAW OF MOTION**

Newton's Second Law states that the sum of all the forces acting on a control volume in a particular direction is equal to the rate of change of momentum of the fluid across the control volume. For a control volume with fluid entering with uniform velocity  $C_1$  and leaving with uniform velocity  $C_2$ , then

$$\sum F = \dot{m}(C_2 - C_1) \tag{1.48}$$

Equation (1.48) is the one-dimensional form of the steady flow momentum equation, and applies for linear momentum. However, turbomachines have impellers that rotate, and the power output is expressed as the product of torque and angular velocity. Therefore, angular momentum is the most descriptive parameter for this system.

#### 1.19 THE SECOND LAW OF THERMODYNAMICS: ENTROPY

This law states that for a fluid passing through a cycle involving heat exchanges

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$$\oint \frac{\delta Q}{T} \le 0 \tag{1.49}$$

where  $\delta Q$  is an element of heat transferred to the system at an absolute temperature *T*. If all the processes in the cycle are reversible, so that  $\delta Q = \delta Q_R$ , then

$$\oint \frac{\delta Q_{\rm R}}{T} = 0 \tag{1.50}$$

The property called entropy, for a finite change of state, is then given by

$$S_2 - S_1 = \int_1^2 \frac{\delta Q_{\rm R}}{T}$$
 (1.51)

For an incremental change of state

$$\mathrm{d}S = m\mathrm{d}s = \frac{\delta Q_{\mathrm{R}}}{T} \tag{1.52}$$

where m is the mass of the fluid. For steady flow through a control volume in which the fluid experiences a change of state from inlet 1 to outlet 2,

$$\int_{-1}^{2} \frac{\delta Q}{T} \le \dot{m}(s_2 - s_1) \tag{1.53}$$

For adiabatic process,  $\delta Q=0$  so that

$$s_2 \ge s_1 \tag{1.54}$$

For reversible process

$$s_2 = s_1$$
 (1.55)

In the absence of motion, gravity and other effects, the first law of thermodynamics, Eq. (1.34) becomes

$$Tds = du + pdv \tag{1.56}$$

(1.57)

Putting h=u+pv and dh=du+pdv+vdp in Eq. (1.56) gives Tds = dh - vdp

#### **1.20 EFFICIENCY AND LOSSES**