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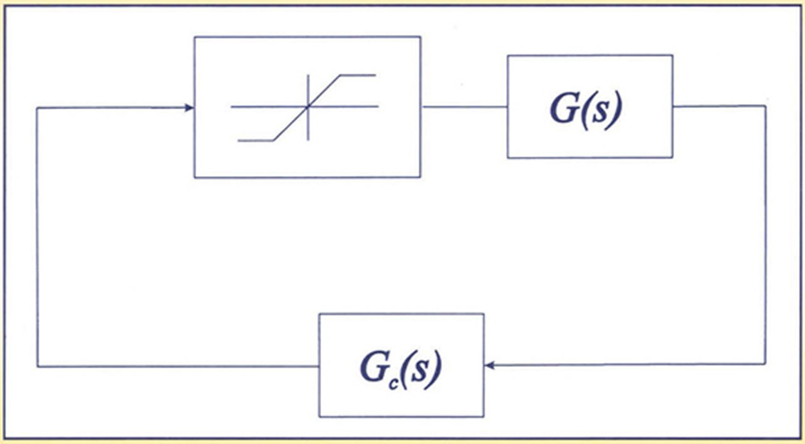

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ACTUATOR SATURATION CONTROL

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Series Introduction

Many textbooks have been written on control engineering, describing new techniques for controlling systems, or new and better ways of mathematically formulating existing methods to solve the ever-increasing complex problems faced by practicing engineers. However, few of these books fully address the applications aspects of control engineering. It is the intention of this new series to redress this situation.

The series will stress applications issues, and not just the mathematics of control engineering. It will provide texts that present not only both new and well-established techniques, but also detailed examples of the application of these methods to the solution of real-world problems. The authors will be drawn from both the academic world and the relevant applications sectors.

There are already many exciting examples of the application of control techniques in the established fields of electrical, mechanical (including aerospace), and chemical engineering. We have only to look around in today's highly automated society to see the use of advanced robotics techniques in the manufacturing industries; the use of automated control and navigation systems in air and surface transport systems; the increasing use of intelligent control systems in the many artifacts available to the domestic consumer market; and the reliable supply of water, gas, and electrical power to the domestic consumer and to industry. However, there are currently many challenging problems that could benefit from wider exposure to the applicability of control methodologies, and the systematic systems-oriented basis inherent in the application of control techniques.

This series presents books that draw on expertise from both the academic world and the applications domains, and will be useful not only as academically recommended course texts but also as handbooks for practitioners in many applications domains. *Actuator Saturation Control* is another outstanding entry to Dekker's Control Engineering series.

Neil Munro

Preface

All real-world applications of feedback control involve control actuators with amplitude and rate limitations. In particular, any physical electromechanical device can provide only a limited force, torque, stroke, flow capacity, or linear/angular rate. The control design techniques that ignore these actuator limits may cause undesirable transient response, degrade the closed-loop performance, and may even cause closed-loop instability. For example, in advanced tactical fighter aircraft with high maneuverability requirements, actuator amplitude and rate saturation in the control surfaces may cause pilot-induced oscillations leading to degraded flight performance or even catastrophic failure. Thus, actuator saturation constitutes a fundamental limitation of many linear (and even nonlinear) control design techniques and has attracted the attention of numerous researchers, especially in the last decade.

In prior research, the control saturation problem has been examined via the extensions of optimal control theory, anti-windup compensation, supervisory error governor approach, Riccati and Lyapunov-based local and semi-global stabilization, and bounded-real, positive-real, and absolute stabilization frameworks. This prior research literature and the currently developing research directions provide a rich variety of techniques to account for actuator saturation. Furthermore, tremendous strides are currently being made to advance the saturation control design techniques to address important issues of performance degradation, disturbance attenuation, robustness to uncertainty/time delays, domain of attraction estimation, and control rate saturation.

The scope of this edited volume includes advanced analysis and synthesis methodologies for systems with actuator saturation, an area of intense current research activity. This volume covers some of the significant research advancements made in this field over the past decade. It emphasizes the issue of rigorous, non-conservative, mathematical formulations of actuator saturation control along with the development of efficient computational algorithms for this class of problems. The volume is intended for researchers and graduate students in engineering and applied mathematics with interest in control systems analysis and design.

This edited volume provides a unified forum to address various novel aspects of actuator saturation control. The contributors of this edited volume include some nationally and internationally recognized researchers who have made or continue to make significant contributions to this important field of research in our discipline. Below we highlight the key issues addressed by each contributor.

Chapter 1 by Barbu *et al.* considers the design of anti-windup control for linear systems with exponentially unstable modes in the presence of input magnitude and rate saturation. The chapter builds on prior work by these authors on uniting local and global controllers. Specifically, the anti-windup design of this chapter enables exponentially unstable saturated linear systems to perform satisfactorily in a large operating region. In addition, the chapter provides sufficient conditions for this class of systems to achieve local performance and global stability. Finally, via a manual flight control example involving an unstable aircraft with saturating actuators, it illustrates the efficacy of the proposed control design methodology in facilitating aggressive maneuvers while preserving stability.

Chapter 2 by Eun *et al.* focuses on selecting the actuator saturation level for small performance degradation in linear designs. A novel application of a general stochastic linearization methodology, which approximates the saturation nonlinearity with a quasi-linear gain, is brought to bear on this problem. Specifically, to determine the allowable actuator saturation level, standard deviations of performance and control in the presence of saturation are obtained using stochastic linearization. The resulting expression for the allowable actuator saturation level is shown to be a function of performance degradation, a positive real number based on the Nyquist plot of the linear part of the system, and the standard deviation of controller output. Numerical examples show that by choosing performance degradation of 10 percent, the actuator saturation level is a weak function of a system intrinsic parameter, *viz.*, the positive real number based on the Nyquist plot of the linear part of the system.

Chapter 3 by Hu *et al.* is motivated by the issue of asymmetric actuators, a problem of considerable practical concern. In previous research, the authors studied the problem of null controllable regions and stabilizability of exponentially unstable linear systems in the presence of actuator saturation. However, this earlier attempt was restricted to symmetric actuator saturation and hence excluded a large class of real-world problems with asymmetric actuator saturation. This chapter addresses the characterization of null controllable regions and stabilization on the null controllable region, for linear, exponentially unstable systems with asymmetrically saturating actuators. First, it is shown that the trajectories produced by extremal control inputs of linear low-order systems have explicit reachable boundaries. Next, under certain conditions, a closed-trajectory is demonstrated to be the boundary of the domain of attraction under saturated linear state feedback. Finally, it is proven that the domain of attraction of second order anti-stable systems under the influence of linear quadratic control can be enlarged arbitrarily close to the null controllability region by using high gain feedback.

Chapter 4 by Iwasaki and Fu is concerned with regional H_2 performance synthesis of dynamic output feedback controllers for linear time-invariant systems subject to known bounds on control input magnitude. In order to guarantee closed-loop stability and H_2 performance, this chapter utilizes the circle and linear analysis techniques. Whereas the circle analysis is applicable to a state space region in which the actuator may saturate, the linear analysis is restricted to a state space region in which the saturation is not activated. It is shown that the circle criterion based control design does not enhance the domain of performance for a specified performance level *vis-a-vis* the linear design. Finally, since the performance overbound is inherently conservative, it is illustrated that the circle criterion based control design can indeed lead to improved performance *vis-a-vis* the linear design. Both fixed-gain and switching control design are addressed.

Chapter 5 by Jabbari employs a linear parameter varying (LPV) approach to handle the inevitable limitations in actuator capacity in a disturbance attenuation setting. The chapter begins by converting a saturating control problem to an unconstrained LPV problem. Next, a fixed Lyapunov function based approach is considered to address an output feedback control design problem for polytopic LPV system. To overcome the conservatism of LPV control designs based on fixed Lyapunov function, a parameter-dependent LPV control methodology is presented. It is shown that the LPV control design framework is capable of handling input magnitude and rate saturation. A scheduling control design approach to deal with actuator saturation is also considered. Two numerical examples illustrate the effectiveness of the proposed control methodologies.

Chapter 6 by Pan and Kapila is focused on the control of discrete-time systems with actuator saturation. It is noted that a majority of the previous research effort in the literature has focused on the control of continuous-time systems with control signal saturation. Nevertheless, in actual practical applications of feedback control, it is the overwhelming trend to implement controllers digitally. Thus, this chapter develops linear matrix inequality (LMI) formulations for the state feedback and dynamic, output feedback control designs for discrete-time systems with simultaneous actuator amplitude and rate saturation. Furthermore, it provides a direct methodology to determine the stability multipliers that are essential for reducing the conservatism of the weighted circle criterion-based saturation control design. The chapter closes with two illustrative numerical examples which demonstrate the efficacy of the proposed control design framework.

Chapter 7 by Pare *et al.* addresses the design of feedback controllers for local stabilization and local performance synthesis of saturated feedback systems. In particular, the chapter formulates optimal control designs for saturated feedback systems by considering three different performance objectives: region of attraction, disturbance rejection, and $\mathcal{L}_2$-gain. The Popov stability theory and a sector model of the saturation nonlinearity are brought to bear on these optimal control design problems. The bilinear matrix inequality (BMI) and LMI optimization frameworks are exploited to characterize the resulting optimal control laws. Commercially available LMI software facilitates efficient numerical computation of the controller matrices. A linearized inverted pendulum example illustrates the proposed local $\mathcal{L}_2$-gain design.

Chapter 8 by Saberi *et al.* focuses on output regulation of linear systems in the presence of state and input constraints. A recently developed novel nonlinear operator captures the simultaneous amplitude and rate constraints on system states and input. The notion of a constraint output is developed to handle both the state and input constraints. A taxonomy of constraints is developed to characterize conditions under which various constraint output regulation problems are solvable. Low-gain and low-high gain control designs including a scheduled low-gain control design are developed for linear systems with amplitude and rate saturating actuators. Finally, output regulation problems in the presence of right invertible and non-right-invertible constraints are also considered.

Chapter 9 by Soroush and Daoutidis begins by surveying the notions of directionality and windup and recent directionality and windup compensation schemes that account for and negate the degrading influence of constrained actuators. The principal focus of the chapter is on stability and performance issues for input-constrained multi-input multi-output (MIMO) nonlinear systems subject to directionality and integrator windup. In particular, the chapter poses the optimal directionality compensation problem as a finite-time horizon, state dependent, constrained quadratic optimization problem with an objective to minimize the distance between the output of the unsaturated plant with an ideal controller and the output of the saturated plant with directionality compensator. Simulation results for a MIMO linear time invariant system and a nonlinear bioreactor subject to input constraints illustrate that the optimal directionality compensation improves system performance *vis-a-vis* traditional clipping and direction preservation algorithms. Finally, the chapter proposes an input-output linearizing control algorithm with integral action and optimal directionality compensation to handle input-constrained MIMO nonlinear systems affected by integrator windup. This windup compensation methodology is illustrated to be effective on a simulated nonlinear chemical reactor.

Chapter 10 by Tarbouriech and Garcia develops Riccati- and LMI-based approaches to design robust output feedback controllers for uncertain systems with position and rate bounded actuators. The proposed controllers ensure robust stability and performance in the presence of norm-bounded time-varying parametric uncertainty. In addition, this control design methodology is applicable to local stabilization of open-loop unstable systems. It is noted that in this chapter, the authors present yet another novel approach, *viz.*, polytopic representation of saturation nonlinearities, to address the actuator saturation problem. Two numerical examples illustrate the efficacy of the proposed saturation control designs.

Chapter 11 by Wu and Grigoriadis addresses the problem of feedback control design in the presence of actuator amplitude saturation. Specifically, by exploiting the LPV design framework, this chapter develops a systematic anti-windup control design methodology for systems with actuator saturation. In contrast to the conventional two-step anti-windup design approaches, the proposed scheme involving induced $\mathcal{L}_2$ gain control schedules the parameter-varying controller by using a saturation indicator parameter. The LPV control law is characterized via LMIs that can be solved efficiently using interior-point optimization algorithms. The resulting gain-scheduled controller is nonlinear in general and would lead to graceful performance degradation in the presence of actuator saturation nonlinearities and linear performance recovery. An aircraft longitudinal dynamics control problem with two input saturation nonlinearities is used to demonstrate the effectiveness of the proposed LPV anti-windup scheme.

We believe that this edited volume is a unique addition to the growing literature on actuator saturation control, in that it provides coverage to competing actuator saturation control methodologies in a single volume. Furthermore, it includes major new control paradigms proposed within the last two to three years for actuator saturation control. Several common themes emerge in these 11 chapters. Specifically, actuator amplitude and rate saturation control is considered in Chapters 1, 5, 6, 8, and 10. LMI-based tools for actuator saturation control are employed in Chapters 4, 5, 6, 7, 10, and 11. Furthermore, an LPV approach is used to handle input saturation in Chapters 5 and 11. Finally, scheduled/switching control designs for saturating systems are treated in Chapters 4, 5, and 8.

We thank all the authors who made this volume possible by their contributions and by providing timely revisions. We also thank the anonymous reviewers who reviewed an early version of this manuscript and provided valuable feedback. We thank B. J. Clark, Executive Acquisitions Editor, Marcel Dekker, Inc., who encouraged this project from its inception. Last but not least, we thank Dana Bigelow, Production Editor, Marcel Dekker, Inc., who patiently worked with us to ensure timely completion of this endeavor.

Vikram Kapila

Karolos M. Grigoriadis

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ACTUATOR SATURATION CONTROL

Chapter 1

Anti-windup for Exponentially Unstable Linear Systems with Rate and Magnitude Input Limits

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1.1. Introduction

Virtually all control actuation devices are subject to magnitude and/or rate limits and this typically leads to degradation of the nominal performance and even to instability. Historically, this phenomenon has been called "windup" and it has been addressed since the 1950's (see, e.g., [25]). To deal with the "windup" phenomenon, "anti-windup" constructions correspond to introducing control modifications when the system saturates, aiming to prevent instability and performance degradations. Early developments of anti-windup employed ad-hoc methods (see, e.g., [6, 9, 18] and surveys in [1,19,30]). In the late 1980's, the increasing complexity of control systems led to the necessity for more rigorous solutions to the anti-windup problem (see, e.g., [8]) and in the last decade new approaches have been proposed with the aim of allowing for general designs with stability and performance guarantees [12, 17, 29, 31, 36, 39, 42, 47].

In many applications, actuator magnitude saturation is one of the main sources of performance limitation. On the other hand, rate saturation is

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particularly problematic in some applications, such as modern flight control systems, where it has been shown to contribute to the onset of pilot-induced oscillations (PIO) and has been the cause of many airplane crashes [7, 16, 33]. The combination of magnitude and rate limits is, in general, a very challenging problem and has been considered less in the anti-windup framework. Some results have been obtained for specific applications [2, 27, 40], and some results can be adapted to this problem (see, e.g., [12, 15, 36]). Additional results on stabilization of systems with inputs bounded in magnitude and rate can be found in [11, 23, 24, 37, 38]. On the other hand, these last results don't directly address the anti-windup problem, where the performance induced by a nominal predesigned controller needs to be recovered by means of the anti-windup design.

Additional concerns arise when the plant contains exponentially unstable modes. In this case, the operating region for the closed loop system has to be restricted in the directions of the unstable modes, a fact that is especially important when large state excursion is required as in tracking problems with large reference inputs. This problem has been addressed in the literature, especially in the discrete-time case, in the context of the reference governor approach [12, 15, 26, 27] (see also [14, 35]). In [15] and [12], the reference of the closed-loop system is modified to guarantee invariance of output admissible sets, corresponding to the control signal remaining within certain limits (see [13] for details), and set-point regulation for feasible references. Additional results, following a more general approach labeled "measurement governor" are given in [36]. The main drawback of these approaches is that the output admissible sets are controller dependent and the results hold only for initial conditions in these sets; in particular, in most applications, the more aggressive the predesigned controller is, the smaller the operating region becomes. This limitation is even more severe when disturbances are taken into account; for example, nothing can be guaranteed when impulsive disturbances propel the state of the system out of the output admissible set. In the continuous-time setting, invariant sets independent of the nominal design are exploited in [39]; for unstable systems, in [27] and [39], the system is allowed to reach the saturation limits during the transients, but the reference value is constrained to be within the steady-state feasibility limits at all times. More recent results [2,3,26] allow the reference to exceed the steady-state feasibility limits during transients.

In recent years, among other approaches, a number of results on anti-windup for linear systems have been achieved by addressing the problem with the aim of blending a local controller that guarantees a certain desired performance, but only local stability, with a global controller that guarantees stability disregarding the local performance. The combination of these two ingredients (according to the approach first proposed in [43]) is attained by augmenting the local design with extra dynamics in a scheme that retains the local controller when trajectories are small enough and activates the global controller when trajectories become too large, thus requiring its stabilizing action. Such an approach has been specialized for anti-windup designs for linear systems [42,45] and has been shown to be successful in a number of case studies [21,40,41,44,46]. The main advantage in adopting the local/global scheme for anti-windup synthesis is that, by identifying the local design with a (typically linear) controller designed disregarding the input limitation, the corresponding unsaturated closed-loop behavior can be recovered (as long as it is attainable within the input constraints) on the saturated system by means of an extra (typically nonlinear) stabilizing controller (the global controller) designed *without any performance* requirement. This decoupled design greatly simplifies, in some cases, the synthesis of the nonlinear controller for the saturated plant.

In this chapter, the uniting technique introduced in the companion papers [42,43] is revisited to design anti-windup compensation for linear systems with exponentially unstable modes in a non-local way. In particular, we address the problem of guaranteeing a large operating region for linear systems with exponentially unstable modes (thus improving the local design given in [42]) and give sufficient conditions for achieving local performance and global stability with large operating regions. Preliminary results in this direction are published in [3].

As compared to the results in [12,15,36], we want to guarantee stability and performance recovery in a region that is not dependent on the nominal controller design. To this aim, instead of focusing our attention on forward invariant regions for the nominal closed-loop system, we consider the null-controllability region of the saturated plant and modify the trajectories of the nominal closed-loop system only when they hit the boundaries of (a conservative estimate of) this last region. The resulting anti-windup design is appealing in the sense that the resulting operating region is typically obtained by shrinking the null-controllability region of the saturated system; [1] since the null-controllability region is unbounded in the marginally unstable directions, [2] it can be extended to infinity in these directions; whereas, in the exponentially unstable directions it needs to be bounded.

Subsequently, as an example, the proposed scheme is applied to the linearized short-period longitudinal dynamics of an unstable fighter air-

[1] Shrinking the null-controllability region is desirable to allow a robustness margin toward disturbances and to avoid the stickiness effect described, e.g., in [27].

[2] Results on null-controllability of linear systems with bounded controls can be found, e.g., in [22,34].

craft subject to rate and magnitude limits on the elevator deflection. The anti-windup design applied to this unstable linear system allows to achieve prototypical military specifications for small to moderate pitch rate pilot commands, while guaranteeing aircraft stability for all pitch rate pilot commands. Due to the large operating region achieved by the anti-windup scheme, the controlled aircraft allows the pilot to maneuver aggressively via large pitch rates during transients.

1.2. The Anti-windup Construction

1.2.1. Problem Statement

Consider a linear system with exponentially unstable modes having state $x \in \mathbf{R}^n$, control input $\delta \in \mathbf{R}^m$, measurable output $y \in \mathbf{R}^p$, and performance output $z \in \mathbf{R}^q$. Let the state x be partitioned as $x =: \begin{bmatrix} x_s \\ x_u \end{bmatrix} \in \mathbf{R}^n$, where the vector $x_u \in \mathbf{R}^{n_u}$ contains all of the exponentially unstable states and $x_s \in \mathbf{R}^{n_s}$ contains all of the other states. The state space representation of the system, consistent with the partition of x, is:

$$\begin{array}{c}\text{Linear}\\ \text{plant}\end{array} \left\{ \begin{array}{rcl} \dot{x} & = & A\,x + B\,\delta = \begin{bmatrix} A_s & A_{12} \\ 0 & A_u \end{bmatrix} x + \begin{bmatrix} B_s \\ B_u \end{bmatrix} \delta \\ z & = & C_z\,x + D_z\,\delta \\ y & = & C_y\,x + D_y\,\delta, \end{array} \right. \tag{1.1}$$

where all the eigenvalues of A_u have strictly positive real part (A_s can possibly have eigenvalues on the imaginary axis).

For system (1.1), assume a (possibly nonlinear) dynamic controller has been previously designed to achieve certain performance specifications in the case where the input is not limited. Let this controller (called *"nominal controller"*) be given in the form:

$$\begin{array}{c}\text{Nominal}\\ \text{controller}\end{array} \left\{ \begin{array}{rcl} \dot{x}_c & = & g(x_c, u_c, r) \\ y_c & = & k(x_c, u_c, r), \end{array} \right. \tag{1.2}$$

where $x_c \in \mathbf{R}^{n_c}$ is the controller state, $r \in \mathbf{R}^q$ is the reference input, and $u_c \in \mathbf{R}^p$, $y_c \in \mathbf{R}^m$ are its input and output, respectively. For the sake of generality, we allow the nominal controller to be nonlinear, although it frequently turns out to be linear. We assume that the design of the nominal controller (1.2) is such that the closed loop system (1.1), (1.2) with the feedback interconnection

$$\delta = y_c, \qquad u_c = y, \tag{1.3}$$

is well-posed (i.e., solutions exist and are unique) and internally stable, and provides asymptotic set-point regulation of the performance output,

$$\lim_{t \to +\infty} z(t) = r.$$

We also assume that, for each constant reference r, there exists an equilibrium (x^*, x_c^*) for (1.1), (1.2), (1.3), that is globally asymptotically stable and we define

$$(x^*, y_c^*) =: E(r), \tag{1.4}$$

as the corresponding state-input pair.

Notice that the internal stability assumption implies that the plant (1.1) is stabilizable and detectable. Throughout the chapter we refer to the closed-loop system (1.1), (1.2), (1.3) as the *"nominal closed-loop system"*.

We address the problem that arises when the actuators' response is limited both in magnitude and rate. The rate and magnitude saturation effect can be modeled (similarly as in [40] and [2]) by augmenting the plant dynamics with extra states $\delta \in \mathbf{R}^m$ satisfying the equation:

$$\dot{\delta} = R \operatorname{sgn}\left(M \operatorname{sat}\left(\frac{u}{M}\right) - \delta\right), \tag{1.5}$$

where the functions sgn($\cdot$) and sat($\cdot$) are the standard decentralized unit sign and saturation functions, M and R are positive numbers, and $u \in \mathbf{R}^m$ is the input to the actuators before saturation.

Since the design of the nominal controller disregards the magnitude and rate limits, instability can arise if that controller is connected in feedback with the actual plant (1.1), (1.5), especially because the plant contains exponentially unstable modes. On the other hand, by assumption, the performance induced by the nominal controller is desired for the actual plant (1.1), (1.2) and should be recovered whenever possible. Thus, our anti-windup design problem is to accommodate the requirements of respecting as much as possible the performance induced by the local controller, while guaranteeing stability of the closed-loop system in the presence of magnitude and rate limits, without restricting the magnitude of the reference signal a priori.

In the next sections we recall the state of the art for the particular anti-windup approach initiated in [43] and make further contributions to that design methodology especially suited for MIMO exponentially unstable linear systems subject to magnitude and rate limits.

1.2.2. The Anti-windup Compensator

In recent years, a number of results on anti-windup design for linear systems have been achieved following the guidelines in [43]. The underlying strategy is to augment the nominal controller with the dynamical system (called anti-windup compensator)

$$\begin{array}{l}\text{Anti-windup}\\ \text{compensator}\end{array} \left\{ \begin{array}{rcl} \dot{\xi} & = & \phi(\xi, y_c, x_u, \delta) \\ v & = & h(\xi, y_c, x_u, \delta), \end{array} \right. \tag{1.6}$$

where $\xi = [\xi_s^T \ \xi_u^T]^T \in \mathbf{R}^{n_s} \times \mathbf{R}^{n_u}$, $v = [v_1^T \ v_2^T]^T \in \mathbf{R}^m \times \mathbf{R}^p$, and x_u, y_c as in equations (1.1), (1.2), and to consider the system resulting from (1.1), (1.2), (1.5), (1.6) with the interconnection conditions

$$u = y_c + v_1, \qquad u_c = y + v_2. \tag{1.7}$$

The anti-windup design described in this chapter relies on the availability for measurement of the exponentially unstable modes, although to provide such information full state measurement might be required. Nevertheless, if the state of the plant is not available for measurement and the disturbances are small, a fast observer can be used.

Throughout the chapter, we will refer to the system (1.1), (1.2), (1.5), (1.6), (1.7) as the *"anti-windup closed-loop system"*. Figure 1 shows the block diagram of the anti-windup closed-loop system, which can be recognized as a natural extension of [39] for the case when the substate x_u is available for measurement and both magnitude and rate limits are present.

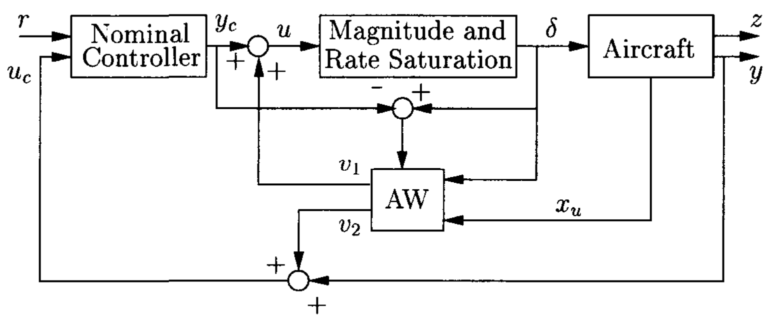

Figure 1: Block diagram of the anti-windup scheme.

Some of the critical issues that arise in the design of the anti-windup compensator are briefly discussed in the following.

Exponentially unstable systems. A basic issue arising with exponentially unstable plants is that global asymptotic stability cannot be achieved, because the null-controllability region of the plant is bounded in the directions of the exponentially unstable modes. Hence, the results are non-global and the goal is to obtain a large operating region for the closed-loop system without significantly sacrificing performance. The results in [42] apply to exponentially unstable linear systems but only for the solution of the local anti-windup problem, thus not computing explicitly the operating region and possibly resulting in conservative designs. More recently, based on [43], a more explicit construction for exponentially unstable plants with only magnitude saturation was given in [39].

Magnitude and rate saturation. The early anti-windup developments illustrate the importance of magnitude saturation in control applications. On the other hand, rate saturation plays a similar role in terms of the effects introduced in the system. For instance, in flight control problems, it has been remarked in [4, 27, 28] how the instabilities and/or performance losses due to windup are generated more frequently by the rate limits than by the magnitude limits. The combination of magnitude and rate limits or even general state constraints is a more challenging problem and has been addressed more in the discrete-time setting (see, for instance, [12, 26, 27]) than in the continuous-time case. The case of both magnitude and rate saturation is addressed in continuous time in [43] and applied to asymptotically stable plants in [40] and [41].

Reference values. Usually (see, e.g., [12, 39]), when the plant contains exponentially unstable modes, the reference signal is not allowed to take large values, although these would generate (at least for a limited amount of time) feasible trajectories for the saturated system. In [26], the problem of allowing large references during transients has been addressed in the context of the reference governor. As pointed out in [26], by allowing the reference to be arbitrarily large, better transient performance for the closed-loop system may be achieved. In [2], arbitrarily large references are allowed during the transients for a particular exponentially unstable plant. It is shown there that the performance of the saturated system is improved adding this extra degree of freedom (nevertheless, due to boundedness of the null-controllability region, the reference cannot be arbitrarily large at the steady state).

The main contribution of the approach described in this chapter is in the fact that the resulting anti-windup compensation allows for arbitrarily large references (at least during the transients) for exponentially unstable linear systems when both rate and magnitude saturation are present at the plant's input.

1.2.3. Main Result

Given a nominal controller and a plant with input magnitude and rate saturation, in this section we give a design algorithm that, on the basis of a desired operating region for the closed-loop system, and for a given stabilizing static feedback that satisfies certain assumptions, provides an anti-windup compensator that achieves stability and allows arbitrarily large references for the saturated system, guaranteeing restricted regulation for any reference outside the operating region. The following definition will be useful in the rest of the chapter.

Definition 1.1. The *null-controllability region* $\mathcal{V}$ for system (1.1), (1.5) is the subset $\mathcal{V} \subset \mathbf{R}^n \times \mathbf{R}^m$ of the state space such that for any initial condition in $\mathcal{V}$, there exists a measurable function $u : \mathbf{R}_{\geq 0} \to \mathbf{R}$ that drives the state of the system asymptotically to the origin.

Remark 1.1. A desirable property of the closed loop system is to have an operating region as large as possible. However, it is not always desirable to get very close to the boundary of the null-controllability region. Indeed, assume that there exists a locally Lipschitz controller that renders the null-controllability region forward invariant. Then, necessarily, the boundary of the null-controllability region is an invariant set and, by continuity of solutions with respect to initial conditions on compact time intervals, the closer the plant state gets to this boundary, the longer it will take to move away from it. We refer to this behavior as the "stickiness effect".[3] It is desirable then to define an "anti-sticking coefficient" and tune the anti-windup compensator using a conservative estimate of the null-controllability region, which guarantees that the trajectories of the system stay far enough from the boundary of the null-controllability region, thus improving the resulting performance.

We first specify a region [4] $\mathcal{U} \subset \mathbf{R}^{n_u} \times \mathbf{R}^m$ where we want the exponentially unstable modes and the inputs of the closed-loop system to operate (accordingly to anti-sticking requirements and/or performance specifications). We specify this region to be a compact set.

Then, we assume that a stabilizing static nonlinear state feedback γ is given that guarantees the first or both of the following properties to hold:

1. positive invariance of the set $\mathcal{U}$ for the plant with input magnitude and rate saturation.

[3] This effect has been noticed in a number of applications (see, e.g., [27]).

[4] The null-controllability region is bounded only in the subspace of the exponentially unstable modes [22], so we only need to specify the operating region in that subspace.

2. convergence to a set-point in $\mathcal{U}$;

After $\mathcal{U}$ and γ are chosen, the last ingredient for the design of the anti-windup compensator is the policy to follow when the closed-loop system is driven by a reference whose steady state value corresponds to an infeasible equilibrium for the saturated system (namely, a value r corresponding to a state-input pair $(x_s^*, x_u^*, \delta^*) = E(r)$ such that $(x_u^*, \delta^*) \notin \mathcal{U}$). To this aim, a function $\mathcal{P}$ that maps the infeasible set-point to a feasible one will be defined. A typical choice for $\mathcal{P}$ is to "project" the infeasible set-point to a feasible point that is, in some sense, "close" to the infeasible one. However, the reference limiting action achieved by $\mathcal{P}$ is not used during the transients but only at the steady state. This strategy allows to completely recover, on the saturated system, the nominal responses (even to infeasible references) for the maximal time interval allowable within the specified operating region and the saturation limits.

The following statements formally define the requirements described above.

Definition 1.2. Define the equilibrium manifold $\mathcal{E} \subset \mathbf{R}^n \times \mathbf{R}^m$ as the set of all the state-input pairs (x, δ) of the linear system (1.1) associated with an equilibrium of the nominal closed-loop system, [5] i.e. (with reference to equation (1.4)),

$$\mathcal{E} := \{(x, \delta) \in \mathbf{R}^n \times \mathbf{R}^m : \quad \exists r \in \mathbf{R}^q \text{ s.t. } (x, \delta) = E(r)\} . \tag{1.8}$$

Let the pair β, $\mathcal{F}$ be such that $\mathcal{F}$ is a compact strict subset of $\mathcal{U}$ and $\beta : \mathbf{R}^{n_u} \times \mathbf{R}^m \to [0, 1]$ is a continuous function satisfying [6]

$$\beta(x_u, \delta) := \begin{cases} 1, & \text{if} \quad (x_u, \delta) \in \mathcal{F} \\ 0, & \text{if} \quad (x_u, \delta) \in \overline{\mathcal{U}^c}. \end{cases} \tag{1.9}$$

Let $\mathcal{F}_u$ be the projection of $\mathcal{F}$ in the x_u direction, i.e.,

$$\mathcal{F}_u := \{x_u \in \mathbf{R}^{n_u} : \quad \exists \delta \in \mathbf{R}^m \text{ s.t. } (x_u, \delta) \in \mathcal{F}\} . \tag{1.10}$$

The role played by β and $\mathcal{F}$ is to guarantee that the nominal performance is preserved only when $(x_u, \delta) \in \mathcal{F}$. Outside $\mathcal{F}$, the anti-windup scheme modifies the nominal control action to guarantee forward invariance of $\mathcal{U}$. A possible choice for the function β, when $\mathcal{F}$ is a given compact

[5] In general, $\mathcal{E}$ is a subset of the set of the equilibria for the open-loop plant.

[6] Given a set $\mathcal{A}$, denote with $\mathcal{A}^c$ the complement of $\mathcal{A}$ and with $\overline{\mathcal{A}}$ the closure of $\mathcal{A}$.