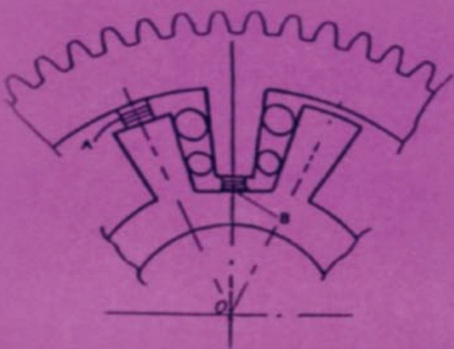
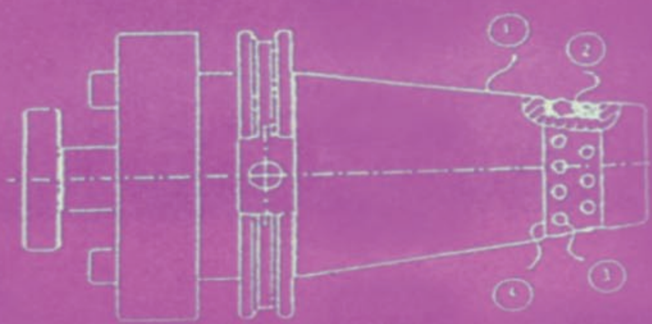
Stiffness and Damping in Mechanical Design



EUGENE I. RIVIN



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Preface

Computers are becoming more and more powerful tools for assisting in the design process. Finite element analysis and other software packages constituting computer-aided design (CAD) allow quick and realistic visualization and optimization of stresses and deformations inside the component of a structure. This computer technology frees designers from tedious drafting and computational chores: it not only allows them to concentrate on general, conceptual issues of design, but also forces them to do so. Some of these issues are so-called conceptual design, reliability, energy efficiency, accuracy, and use of advanced materials. Very important conceptual issues are stiffness of mechanical structures and their components and damping in mechanical systems sensitive to and/or generating vibrations.

Stiffness and strength are the most important criteria for many mechanical designs. However, although there are hundreds of books on various aspects of strength, and strength issues are heavily represented in all textbooks on machine elements, stiffness-related issues are practically neglected, with a few exceptions. Although dynamics and vibrations, both forced and self-excited, of mechanical systems are becoming increasingly important, damping and stiffness are usually considered separately. However, frequently damping and stiffness are closely interrelated, and efforts to improve one parameter while neglecting the other are generally ineffective or even counterproductive.

This book intends to correct this situation by addressing various aspects of structural stiffness and structural damping and their roles in design. Several typical cases in which stiffness is closely associated with damping are addressed. The basic conceptual issues related to stiffness and damping are accentuated. A more detailed analytical treatment is given in cases where the results were not previously published or were only published in hard-to-obtain sources (e.g., publications in languages other than English). Many of these concepts are illustrated by practical results and/or applications (practical case studies) either in the text or as appendices and articles. The articles, mostly authored or coauthored by the author of this book, are intended both to extend coverage of some important issues and to provide practical application examples.

This book originated from course notes prepared for the ''Stiffness in Design'' tutorial successfully presented at four Annual Meetings of the American Society for Precision Engineering (ASPE). The contents of the book are based to a substantial degree on the author's personal professional experiences and research results.

The two parameters covered in this book are treated differently. No monographs and few if any extended chapters on stiffness have recently been published in English. However, there are several books and handbook chapters available on damping. Accordingly, although an attempt was made here to provide a comprehensive picture of the role of stiffness in mechanical design, the treatment of damping is less exhaustive. Two main groups of the many damping-related issues are addressed: (1) damping properties of contacts (joints) and power transmission systems, which are addressed only scantily in other publications, and (2) the interrelationship between stiffness and damping parameters in mechanical systems and structural materials. Thus, the damping-related sections can be considered complementary to the currently available monographs and handbooks.

Many important stiffness- and damping-related issues were studied in depth in the former Soviet Union. The results were published in Russian and are practically unavailable to the engineering community in non-Russian-speaking countries. Several of these results are covered in the book.

A general introduction to the subject matter is given in Chapter 1. General performance characteristics are described for which the stiffness and damping criteria are critical. This chapter also lists a selection of structural materials for stiffness- and damping-critical applications. Information on the influence of the mode of loading and the component design on stiffness is provided in Chapter 2.

Chapter 3 is dedicated to an important subject of nonlinear and variable stiffness (and damping) systems. Specially addressed is the issue of preloading, which is very important for understanding and controlling stiffness and damping characteristics.

Design and performance information on various aspects of normal and tangential contact stiffness, as well as of damping associated with mechanical contacts, is given in Chapter 4. Information on these subjects is very scarce in the technical literature available in English. Stiffness of mechanical components is determined not only by their own structural properties, but also by their supporting conditions and devices. Influence of the latter on both static stiffness and dynamic characteristics is frequently not well understood. These issues, as well as some issues related to machine foundations, are addressed in Chapter 5.

Chapter 6 concentrates on very specific issues of stiffness (and damping) in power transmission and drive systems, which play a significant role in various mechanical systems. Several useful techniques, both passive and active, aimed at enhancing structural stiffness and damping characteristics (i.e., reduction of structural deformations and enhancement of dynamic stability) are described in Chapter 7. Special cases in which performance of stiffness-critical systems can be improved by reduction or a proper tuning of components' stiffness are described in Chapter 8.

The issues related to stiffness and damping in mechanical design are numerous and very diverse. This book does not pretend to be a handbook covering all of them, but it is the first attempt to provide illuminating coverage of some of these issues.

In addition to the body of the book, I have included Appendices 1–3 to provide more detailed treatments and derivations for some small but important subjects. I have also provided, in their entirety, several articles from previous publications, each of which gives an in-depth treatment of an important stiffness and/or damping critical area of mechanical design.

I am very grateful to the book reviewers, who made valuable suggestions. Especially helpful have been discussions with Professor Dan DeBra (Stanford University). These discussions resulted in important changes in the book's emphasis. Suggestions by Professor Vladimir Portman (Ben Gurion University of the Negev, Israel) were also very useful. I take full responsibility for all of the shortcomings of the book and will greatly appreciate readers' feedback.

Eugene I. Rivin

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1

Introduction and Definitions

1.1 BASIC NOTIONS

1.1.1 Stiffness

Stiffness is the capacity of a mechanical system to sustain loads without excessive changes of its geometry (deformations). It is one of the most important design criteria for mechanical components and systems. Although strength is considered the most important design criterion, there are many cases in which stresses in components and their connections are significantly below the allowable levels, and dimensions as well as performance characteristics of mechanical systems and their components are determined by stiffness requirements. Typical examples of such mechanical systems are aircraft wings, and frames/beds of production machinery (machine tools, presses, etc.), in which stresses frequently do not exceed 3–7 MPa (500–1,000 psi). Another stiffness-critical group of mechanical components is power transmission components, especially shafts, whose deformations may lead to failures of gears and belts while stresses in the shafts caused by the payload are relatively low.

Recently, great advances in improving strength of mechanical systems and components were achieved. The main reasons for such advances are development of high strength structural metals and other materials, better understanding of fracture/failure phenomena, and development of better techniques for stress analysis and computation, which resulted in the reduction of safety factors. These advances often result in reduction of cross sections of the structural components. Since the loads in the structures (unless they are weight-induced) do not change, structural deformations in the systems using high strength materials and/or designed with reduced safety factors are becoming more pronounced. It is important to note that while the strength of structural metals can be greatly improved by selection of alloying materials and of heat treatment procedures (as much as 5–7 times for steel and aluminum), modulus of elasticity (Young's modulus) is not very sensitive to alloying and to heat treatment. For example, the Young's modulus of stainless steels is even 5–15% lower than that of carbon steels (see Table 1.1). As a result, stiffness can be modified (enhanced) only by proper selection of the component geometry (shape and size) and its interaction with other components.

Stiffness effects on performance of mechanical systems are due to influence of deformations on static and fatigue strength, wear resistance, efficiency (friction losses), accuracy, dynamic/vibration stability, and manufacturability. The importance of the stiffness criterion is increasing due to:

1. Increasing accuracy requirements (especially due to increasing speeds and efficiency of machines and other mechanical systems)
2. Increasing use of high strength materials resulting in the reduced cross sections and, accordingly, in increasing structural deformations
3. Better analytical techniques resulting in smaller safety factors, which also result in the reduced cross sections and increasing deformations
4. Increasing importance of dynamic characteristics of machines since their increased speed and power, combined with lighter structures, may result in intense resonances and in the development of self-excited vibrations (chatter, stick-slip, etc.)

Factors 2–4 are especially pronounced for surface and flying vehicles (cars, airplanes, rockets, etc.) in which the strength resources of the materials are utilized to the maximum in order to reduce weight.

Stiffness is a complex parameter of a system. At each point, there are generally different values of stiffness k_{xx}, k_{yy}, k_{zz} in three orthogonal directions of a selected coordinate frame, three values of interaxial stiffness k_{xy}, k_{xz}, k_{yz} related to deformations along one axis (first subscript) caused by forces acting along an orthogonal axis (second subscript), and also three values of angular stiffness about the x, y, and z axes. If the interaxial stiffnesses vanish, $k_{xy} = k_{xz} = k_{yz} = 0$, then x, y, z are the *principal stiffness axes.* These definitions are important since in some cases several components of the stiffness tensor are important; in special cases, ratios of the stiffness values in the orthogonal directions determine dynamic stability of the system. Such is the case of chatter instability of some machining operations [1]. Chatter stability in these operations increases if the cutting and/or the friction force vector is oriented in a certain way relative to principal stiffness axes x and y. Another case is vibration isolation. Improper stiffness ratios in vibration isolators and machinery mounts may cause undesirable intermodal coupling in vibration isolation systems (see Article 1).

Main effects of an inadequate stiffness are absolute deformations of some

Table 1.1 Young's Modulus and Density of Structural Materials

Material	E (10^5 MPa)	γ (10^3 kg/m^3)	E/γ (10^7 m^2/s^2)
(a) Homogeneous Materials			
Graphite	7.5	2.25	33.4
Diamond	18.0	5.6	32
Boron carbide, BC	4.50	2.4	19
Silicon carbide, SiC	5.6	3.2	17.5
Carbon, C	3.6	2.25	16.0
Beryllium, Be	2.9	1.9	15.3
Boron, B	3.8	2.5	15.2
Sapphire	4.75	4.5	10.1
Alumina, Al_2O_3	3.9	4.0	9.8
Lockalloy (62% Be + 38% Al)	1.90	2.1	9.1
Kevlar 49	1.3	1.44	9.0
Titanium carbide, TiC	4.0–4.5	5.7–6.0	7.0–9.1
Silicone, Si	1.1	2.3	4.8
Tungsten carbide, WC	5.50	16.0	3.4
Aluminum/Lithium (97% Al + 3% Li)	0.82	2.75	3.0
Molybdenum, Mo	3.20	10.2	3.0
Glass	0.7	2.5	2.8
Steel, Fe	2.10	7.8	2.7
Titanium, Ti	1.16	4.4	2.6
Aluminum, Al (wrought)	0.72	2.8	2.6
Aluminum, Al (cast)	0.65	2.6	2.5
Steel, stainless (.08–0.2% C, 17% Cr, 7% Ni)	1.83	7.7	2.4
Magnesium, Mg	0.45	1.9	2.4
Wood (along fiber)	0.11–0.15	0.41–0.82	2.6–1.8
Marble	0.55	2.8	2.0
Tungsten (W + 2 to 4% Ni, Cu)	3.50	18.0	1.9
Granite	0.48	2.7	1.8
Beryllium copper	1.3	8.2	1.6
Polypropylene	0.08	0.9	0.9
Nylon	0.04	1.1	0.36
Paper	0.01–0.02	0.5	0.2–0.4
(b) Composite Materials			
HTS graphite/5208 epoxy	1.72	1.55	11.1
Boron/5505 epoxy	2.07	1.99	10.4
Boron/6601 Al	2.14	2.6	8.2
Lanxide NX − 6201 (Al + SiC)	2.0	2.95	6.8
T50 graphite/2011 Al	1.6	2.58	6.2
Kevlar 49/resin	0.76	1.38	5.5
80% Al + 20% Al_2O_3 powder	0.97	2.93	3.3
Melram (80% Mg, 6.5% Zn, 12% SiC)	0.64	2.02	3.2
E glass/1002 epoxy	0.39	1.8	2.2

components of the system and/or relative displacements between two or several components. Such deformations/displacements can cause:

Geometric distortions (inaccuracies)
Change of actual loads and friction conditions, which may lead to reduced efficiency, accelerated wear, and/or fretting corrosion
Dynamic instability (self-excited vibrations)
Increased amplitudes of forced vibrations

Inadequate stiffness of transmission shafts may cause some specific effects. The resulting linear and angular deformations determine behavior of bearings (angular deformations cause stress concentrations and increased vibrations in antifriction bearings and may distort lubrication and friction conditions in sliding bearings); gears and worm transmissions (angular and linear deformations lead to distortions of the meshing process resulting in stress concentrations and variations in the instantaneous transmission ratios causing increasing dynamic loads); and traction drives (angular deformations cause stress concentrations and changing friction conditions).

It is worthwhile to introduce some more definitions related to stiffness:

Structural stiffness due to deformations of a part or a component considered as beam, plate, shell, etc.
Contact stiffness due to deformations in a connection between two components (contact deformations may exceed structural deformations in precision systems)
Compliance $e = 1/k$, defined as a reciprocal parameter to stiffness k (ratio of deformation to force causing this deformation)
Linear stiffness vs. *nonlinear stiffness* (see Ch. 3)
Hardening vs. *softening* nonlinear stiffness (see Ch. 3)
Static stiffness k_{st} (stiffness measured during a very slow loading process, such as a periodic loading with a frequency less than 0.5 Hz) vs. *dynamic stiffness* k_{dyn}, which is measured under faster changing loads. Dynamic stiffness is characterized by a *dynamic stiffness coefficient* $K_{dyn} = k_{dyn}/k_{st}$. Usually $K_{dyn} > 1$ and depends on frequency and/or amplitude of load and/or amplitude of vibration displacement (see Ch. 3). In many cases, especially for fibrous and elastomeric materials K_{dyn} is inversely correlated with damping, e.g., see Fig. 3.2 and Table 1 in Article 1.

1.1.2 Damping

Damping is the capacity of a mechanical system to reduce intensity of a vibratory process. The damping capacity can be due to interactions with outside systems, or due to internal performance-related interactions. The damping effect for a vibratory process is achieved by transforming (dissipating) mechanical energy of the vibratory motion into other types of energy, most frequently heat, which can be evacuated from the system. If the vibratory process represents self-excited vibrations (e.g., chatter), the advent of the vibratory process can be prevented by an adequate damping capacity of the system.

In the equations of motion to vibratory systems (e.g., see Appendix 1), both intensity and character of energy dissipation are characterized by coefficients at the first derivative (by time) of vibratory displacements. These coefficients can be constant (linear or viscous damping) or dependent on amplitude and/or frequency of the vibratory motion (nonlinear damping). There are various mechanisms of vibratory energy dissipation which can be present in mechanical systems, some of which are briefly explored in Appendix 1.

Since the constant coefficient at the time-derivative of the vibratory displacement term results in a linear differential equation, which is easy to solve and to analyze, such systems are very popular in textbooks on vibration. However, the constant damping coefficient describes a so-called viscous mechanism of energy dissipation that can be realized, for example, by a piston moving with a relatively slow velocity inside a conforming cylinder with a relatively large clearance between the piston and the cylinder walls, so that the resistance force due to viscous friction has a direction opposite to the velocity vector and is proportional to the relative velocity between the cylinder and the piston. In real-life applications such schematic and conditions are not often materialized. The most frequently observed energy dissipation mechanisms are hysteretic behavior or structural materials; friction conditions similar to coulomb (dry) friction whereas the friction (resistance) force is directionally opposed to the velocity vector but does not depend (or depends weakly) on the vibratory velocity magnitude; damping in joints where the vibratory force is directed perpendicularly to the joint surface and causes squeezing of the lubricating oil through the very thin clearance between the contacting surfaces (thus, with a very high velocity) during one-half of the vibratory cycle and sucking it back during the other half of the cycle; and damping due to impact interactions between the contacting surfaces. Some of these mechanisms are analytically described in Appendix 1.

Effects of damping on performance of mechanical systems are due to reduction of intensity of undesirable resonances; acceleration of decay (settling) of transient vibration excited by abrupt changes in motion parameters of mechanical components (start/stop conditions of moving tables in machine tools and of robot links, engagement/disengagement between a cutting tool and the machined part, etc.); prevention or alleviation of self-excited vibrations; prevention of impacts between vibrating parts when their amplitudes are reduced by damping; potential for reduction of heat generation, and thus for increase in efficiency due to reduced peak vibratory velocities of components having frictional or microimpacting interactions; reduction of noise generation and of harmful vibrations transmitted to human operators; and more.

It is important to note that while damping is associated with transforming mechanical energy of the vibratory component into heat, increase of damping capacity of mechanical system does not necessarily result in a greater heat generation. Damping enhancement is, first of all, changing the dynamic status of the system and, unless the displacement amplitude is specified (for example, like inside a compensating coupling connecting misaligned shafts; see Section 8.5.2), most probably would cause a *reduction in the heat generation.* This somewhat paradoxical statement is definitely true in application to mechanical systems prone to development of self-excited vibrations, since enhancement of damping in the system would prevent starting of the vibratory process, and thus the heat generation, which is usually caused by vibratory displacements. This statement is also true for a system subjected to transient vibration. Since the initial displacement of mass m in Fig. A.1.1 and the natural frequency of the system do not significantly depend on damping in the system, a higher damping would result in smaller second, third, etc. amplitudes of the decaying vibrations, and thus in a lower energy dissipation.

Less obvious is the case of forced vibration when force $F = F_0 \sin \omega t$ is applied to mass m in Fig. A.1.1. Let's consider the system in which mass m is attached to the frame by a rubber flexible element combining both stiffness and damping properties (hysteresis damping, $r = 1$; see Appendix 1). If amplitude of mass m is A, then the maximum potential energy of deformation of the flexible element is

$$V = k\frac{A^2}{2} \tag{1.1}$$

The amount of energy dissipated (transformed into heat) in the damper c or in the rubber flexible element is

$$\Delta V = \Psi V = \Psi k \frac{A^2}{2} \tag{1.2}$$

At the resonance, amplitude A_{res} of mass m is, from formula (A.1.19b) at $\omega = \omega_0$ and from (A1.18)

$$A_{res} = \frac{F_0}{k\sqrt{\left(\frac{\alpha}{\pi k}\right)^2}} = \frac{F_0}{k}\frac{\pi k}{\alpha} = A_0 \frac{\pi}{\delta} \approx A_0 \frac{2\pi}{\Psi} \tag{1.3a}$$

where $A_0 = F_0/k$ = static ($\omega = 0$) deflection of the flexible element, δ = logarithmic decrement, and for not very high damping

$$\Psi \approx 2\delta \tag{1.3b}$$

Thus, the *energy dissipation at resonance* (or the maximum energy dissipation in the system) *is decreasing with increasing damping capacity* (increasing Y).

This result, although at the first sight paradoxical, does not depend on the character (mechanism) of damping in the system and can be easily explained. The resonance amplitude is inversely proportional to the damping parameter (Ψ, δ, etc.) because the increasing damping shifts an equilibrium inside the dynamic system between the excitation (given, constant amplitude), elastic (displacement-proportional), inertia (acceleration-proportional), and damping (velocity-proportional) forces. The amount of energy dissipation is a secondary effect of this equilibrium; the energy dissipation is directly proportional to the square of the vibration amplitude. Although this effect of decreasing energy dissipation with increasing damping is especially important at the resonance where vibratory amplitudes are the greatest and energy dissipation is most pronounced, it is not as significant in the areas outside of the resonance where the amplitudes are not strongly dependent on the damping magnitude (see Fig. A.1.3).

Effects of damping on performance of mechanical system are somewhat similar to the effects of stiffness, as presented in Section 1.1.1. Damping influences, directly or indirectly, the following parameters of mechanical systems, among others:

1. *Fatigue strength.* Increasing damping leads to reduction of strain and stress amplitudes if the loading regime is close to a resonance. It is even more important for high-frequency components of strain/stress processes, which are frequently intensified due to resonances of inevitable high frequency components of the excitation force(s) and/or nonlinear responses of the system with higher natural frequencies of the system.

2. *Wear resistance.* High (resonance) vibratory velocities, especially associated with high-frequency parasitic microvibrations, may significantly accelerate the wear process. High damping in the system alleviates these effects.

3. *Efficiency (friction losses).* Depending on vibration parameters (amplitudes, frequencies, and, especially, directivity), vibrations can increase or reduce friction. In the former case, increasing damping can improve efficiency.

4. *Accuracy and surface finish* of parts machined on machined tools. Although surface finish of the machined surface is directly affected by vibrations, accuracy (both dimensions and macrogeometry) may be directly influenced by low-frequency vibrations, e.g., transmitted from the environment (see Article 1) or may be indirectly affected by changing geometry of the cutting tool whose sharp edge(s) are fast wearing out under chatter- or microvibrations. The latter

are especially dangerous for brittle cutting materials such as ceramic and diamond tools.

5. *Dynamic/vibration stability* of mechanical systems can be radically enhanced by introducing damping into the system.

6. *Manufacturability*, especially of low-stiffness parts, can be limited by their dynamic instability, chatter, and resonance vibrations during processing. Damping enhancement of the part and/or of the fixtures used in its processing can significantly improve manufacturability.

Importance of the damping criterion is increasing with the increasing importance of the stiffness criterion as discussed in Section 1.1.1 due to:

a. Increasing accuracy requirements
b, c. Increasing use of high strength materials and decreasing safety factors, which result in lower stiffness and thus higher probability of vibration excitation
d. Increasing importance of dynamic characteristics
e. Increasing awareness of noise and vibration pollution

Main sources of damping in mechanical systems are:

a. Energy dissipation in structural materials
b. Energy dissipation in joints/contacts between components (both in moving joints, such as guideways, and in stationary joints)
c. Energy dissipation in special damping devices (couplings, vibration isolators, dampers, dynamic vibration absorbers, etc.). These devices may employ viscous (or electromagnetic) dampers in which relative vibratory motion between component generates a viscous (velocity dependent) resistance force; special high-damping materials, such as elastomers or ''shape memory metals'' (see Table 1.2); specially designed (''vibroimpact'') mechanisms in which coimpacting between two surfaces results in dissipation of vibratory energy (see Appendix 1); etc.

1.2 INFLUENCE OF STIFFNESS ON STRENGTH AND LENGTH OF SERVICE

This influence can materialize in several ways:

Inadequate or excessive stiffness of parts may lead to overloading of associated parts or to a nonuniform stress distribution

Inadequate stiffness may significantly influence strength if loss of stability (buckling) of some component occurs

Impact/vibratory loads are significantly dependent on stiffness

Excessive stiffness of some elements in statically indeterminate systems may lead to overloading of the associated elements

It is known that fatigue life of a component depends on a high power (5–9) of maximum (peak) stresses. Thus, uniformity of the stress distribution is very important.

Fig. 1.1 [2] shows the influence of the stiffness of rims of meshing gears on load distribution in their teeth. In Fig. 1.1a, left sides of both gear rims have higher stiffness than their right sides due to positioning of the stiffening disc/spokes on their hubs. This leads to concentration of the loading in the stiff area so that the peak contact stresses in this area are about two times higher than the average stress between the meshing profiles. In Fig. 1.1b, the gear hubs are symmetrical, but again the stiff areas of both rims work against each other. Although the stress distribution diagram is different, the peak stress is still about twice as high as the average stress. The design shown in Fig. 1.1c results in a more uniform stiffness along the tooth width and, accordingly, in much smaller peak stresses—about equal to the average stress magnitude. The diagrams in Fig. 1.1 are constructed with an assumption of absolutely stiff shafts. If shaft deformations are significant, they can sustantially modify the stress distributions and even reverse the characteristic effects shown in Fig. 1.1.

Another example of influence of stiffness on load distribution is shown in Fig. 1.2. It is a schematic model of threaded connection between bolt 1 and nut 2. Since compliances of the thread coils are commensurate with compliances of bolt and nut bodies, bending deformations of the most loaded lower coils are larger than deformations of the upper coils by the amount of bolt elongation between these coils. This leads to a very nonuniform load distribution between the coils. Theoretically, for a 10-coil thread, the first coil takes 30–35% of the total axial load on the bolt, while the eighth coil takes only 4% of the load [3]. In real threaded connections, the load distribution may be more uniform due to

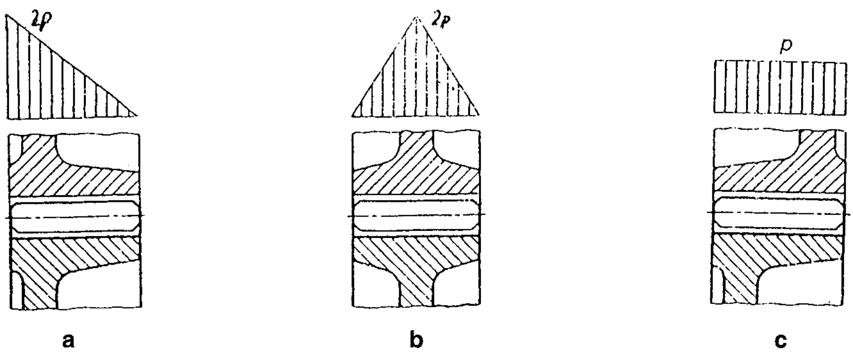

Figure 1.1 Contact pressure distribution in meshing gears as influenced by design of gears.

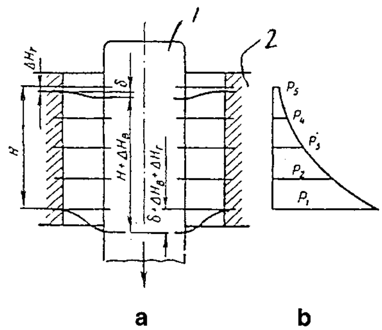

Figure 1.2 Contact pressure distribution (b) in threaded connection (a).

possible yielding of the highest loaded coils, contact deformations in the thread, and higher compliances of the contacting coils because of their inaccuracies and less than perfect contact. Thus, the first coil may take only 25–30% of the total load instead of 34%. However, it is still a very dramatic nonuniformity that can cause excessive plastic deformations of the most loaded coils and/or their fatigue failure. Such a failure may cause a chain reaction of failures in the threaded connection.

Such redistribution and concentration of loading influencing the overall deformations and the effective stiffness of the system can be observed in various mechanical systems. Fig. 1.3 [2] shows a pin connection of a rod with a tube. Since the tube is much stiffer than the rod, a large fraction of the axial load P is acting on the upper pin, which can be overloaded (Fig. 1.3a). The simplest way to equalize loading of the pins is by loosening the hole for the upper pin (Fig. 1.3b). This leads to the load being applied initially to the lower pin only.

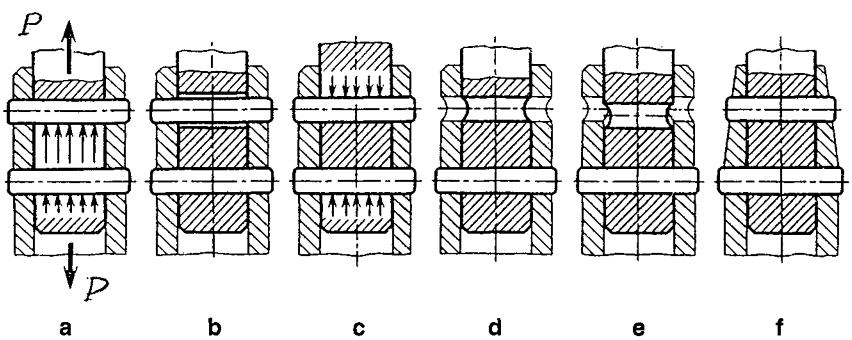

Figure 1.3 Influence of component deformations on load distribution.

The upper pin takes the load only after some stretching of the rod has occurred. Another way to achieve the same effect is by prestressing (preloading) the system by creating an initial loading (during assembly) in order to counteract the loading by force P (Fig. 1.3c). This effect can be achieved, for example, by simultaneously drilling holes in the rod and in the tube (Fig. 1.3d) and then inserting the pins while the rod is heated to the specified temperature. After the rod cools down, it shrinks (Fig. 1.3e) and the system becomes prestressed. The load equalization effect can also be achieved by local reduction of the tube stiffness (Fig. 1.3f).

1.3 INFLUENCE OF STIFFNESS AND DAMPING ON VIBRATION AND DYNAMICS

This effect of stiffness can be due to several mechanisms.

At an impact, kinetic energy of the impacting mass is transformed into potential energy of elastic deformation; accordingly, dynamic overloads are stiffness-dependent. For a simple model in Fig. 1.4, kinetic energy of mass m impacting a structure having stiffness k is

$$E = \tfrac{1}{2}mv^2 \tag{1.1}$$

After the impact, this kinetic energy transforms into potential energy of the structural impact-induced deformation x

$$V = \tfrac{1}{2}kx^2 = E = \tfrac{1}{2}mv^2 \tag{1.2}$$

Since the impact force $F = kx$, from (1.2) we find that

$$x = v\sqrt{\frac{m}{k}} \quad \text{and} \quad F = v\sqrt{km} \tag{1.3}$$

Thus, in the first approximation the impact force is proportional to the square root of stiffness.

For forced vibrations, a resonance can cause significant overloads. The resonance frequency can be shifted by a proper choice of stiffness and mass values and distribution. While shifting of the resonance frequencies may help to avoid the excessive resonance displacement amplitudes and overloads, this can help only if the forcing frequencies are determined and cannot shift. In many cases this is not a realistic assumption. For example, the forcing (excitation) frequencies acting on a machine tool during milling operation are changing with the change of the number of cutting inserts in the milling cutter and with the changing spindle

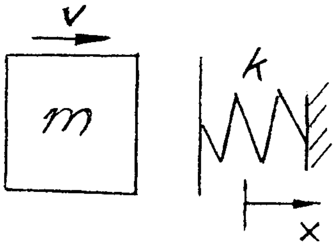

Figure 1.4 Impact interaction between moving mass and stationary spring.

speed (rpm). A much more effective way to reduce resonance amplitudes is by *enhancing damping* in the vibrating system. The best results can be achieved if the stiffness and damping changes are considered simultaneously (see Article 2 and discussion on loudspeaker cones in Section 1.6).

Variable stiffness of shafts, bearings, and mechanisms (in which stiffness may be orientation-dependent) may cause quasi-harmonic (parametric) vibrations and overloads. While variability of the stiffness can be reduced by design modifications, the best results are achieved when these modifications are combined with damping enhancement.

Chatter resistance (stability in relation to self-excited vibrations) of machine tools and other processing machines is determined by the criterion $K\delta$ (K = effective stiffness and δ = damping, e.g., logarithmic decrement). Since in many cases dynamic stiffness and damping are interrelated, such as in mechanical joints (see Ch. 4) and materials (see Ch. 3 and Article 1), the stiffness increase can be counterproductive if it is accompanied by reduction of damping. In some cases, stiffness reduction can be beneficial if it is accompanied by a greater increase in damping (see the case study on influence of mount characteristics on chatter resistance of machine tools and Ch. 8).

Deviation of the vector of cutting (or friction) forces from a principal stiffness axis may cause self-exciting vibrations (coordinate coupling) [1].

Low stiffness of the drive system may cause stick-slip vibration of the driven unit on its guideways.

1.4 INFLUENCE OF MACHINING SYSTEM STIFFNESS AND DAMPING ON ACCURACY AND PRODUCTIVITY

1.4.1 Introduction

Elastic deformations of the production (machining) system, machine tool–fixture–tool–machined part, under cutting forces are responsible for a significant fraction of the part inaccuracy. These deformations also influence productivity of the machining system, either directly by slowing the process of achieving the desired geometry or indirectly by causing self-excited chatter vibrations.

In a process of machining a precision part from a roughly shaped blank, there is the task to reduce deviation Δb of the blank surface from the desired geometry to a smaller allowable deviation Δp of the part surface (Fig. 1.5). This process can be modeled by introduction of an *accuracy enhancement factor* ζ

$$\zeta = \Delta b/\Delta p = t_1 - t_2/y_1 - y_2 \tag{1.4}$$

where t_1 and t_2 are the maximum and minimum depth of cut; and y_1 and y_2 are the cutter displacements normal to machined surface due to structural deformations caused by the cutting forces. If the cutting force is

$$P_y = C_m t s^q \tag{1.5}$$

then

$$\zeta = (k/C_m)/s^q \tag{1.6}$$

where C_m = material coefficient; k = stiffness of the machining system; t = depth of cut; s = feed; and q = 0.6–0.75. For the process of turning medium-hardness steel with s = 0.1–0.75 mm/rev on a lathe with k = 20 $N/\mu m$,

$$\zeta = 150 - 30 \tag{1.7}$$

Knowing shape deviations of the blanks and the required accuracy, the above formula for ζ allows us to estimate the required k and allowable s, or to decide on the number of passes required to achieve the desired accuracy.

Inadequate stiffness of the machining system may result in various distortions of the machining process. Some examples of such distortions are shown in

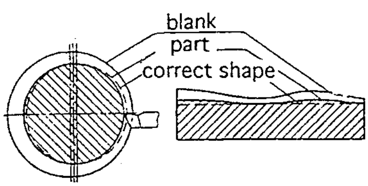

Figure 1.5 Evolution of geometry of machined parts when machining system has finite stiffness.

Fig. 1.6. The total cross sectional area of the cut is smaller during the transient phases of cutting (when the tool enters into and exits from the machined part) than during the steady cutting. As a result, deflection of the blank part is smaller during the transient phases thus resulting in deeper cuts (Fig. 1.6a, b).

Turning of a part supported between two centers requires driving of the part by a driving yoke clamped to the part (Fig. 1.6c). Asymmetry of the driving system results in an eccentricity (runout) of the part with the magnitude

$$\delta = P_z d / k_c R \tag{1.8}$$

where k_c = stiffness of the supporting center closest to the driving yoke.

Heavy traveling tables supporting parts on milling machines, surface grinders, etc., may change their angular orientation due to changing contact deformations in the guideways caused by shifting of the center of gravity during the travel (Fig. 1.6d). This also results in geometrical distortions of the part surface.

A surface deviation Δ caused by a variable stiffness of the machining system

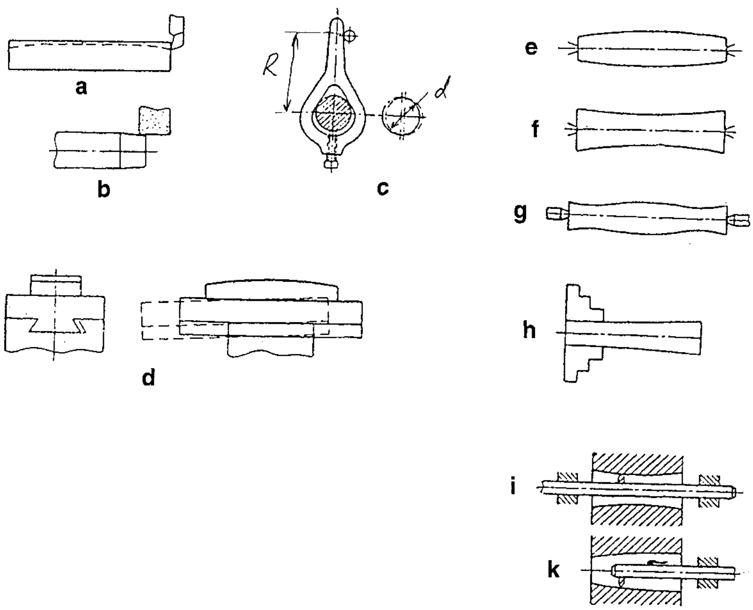

Figure 1.6 Influence of compliances in the machining system on geometry of machined parts.

can be expressed as

$$\Delta = P_y(1/k_{min} - 1/k_{max}) \tag{1.9}$$

where k_{min} and k_{max} = low and high stiffness of the machining system and P_y = cutting force.

Figure 1.6e shows a barrel shape generated in the process of turning a slender elongated part between the rigid supporting centers; Fig. 1.6f shows a ''corset'' shape when a rigid part is supported by compliant centers. The part in Fig. 1.6g is slender and was supported by compliant centers. Fig. 1.6h shows the shape of a cantilever part clamped during machining in a nonrigid chuck. Fig. 1.6i shows shape of the hole bored by a slender boring bar guided by two stationary rigid supports, while Fig. 1.6k illustrates shape of the hole machined by a cantilever boring bar guided by one stationary support.

The role of stiffness enhancement is to reduce these distortions. When they are repeatable, corrections that would compensate for these errors can be commanded to a machine by its controller. However, the highest accuracy is still obtained when the error is small and it is always preferable to avoid the complications of this compensation procedure, which appropriate stiffness can accomplish.

Manufacturing requirements for stiffness of parts often determine the possibility of their fabrication with high productivity (especially for mass production). Sometimes, shaft diameters for mass-produced machines are determined not by the required strength but by a possibility of productive multicutter machining of the shafts and/or of the associated components (e.g., gears). Machining of a low-stiffness shaft leads to chatter, to a need to reduce regimes, and to copying of inaccuracies of the original blank.

Stiffness of the production equipment influences not only its accuracy and productivity. For example, stiffness characteristics of a stamping press also influence its energy efficiency (since deformation of a low-stiffness frame absorbs a significant fraction of energy contained in one stroke of the moving ram); dynamic loads and noise generation (due to the same reasons); product quality (since large deformations of the frame cause misalignments between the punch and the die and thus, distortions of the stamping); and die life (due to the same reasons). In crank presses developing the maximum force at the end of the stroke, the amount of energy spent on the elastic structural deformations can be greater than the amount of useful energy (e.g., spent on the punching operation). Abrupt unloading of the frame after the breakthrough event causes dangerous dynamic loads/noise, which increase with increasing structural deformations.

In mechanical measuring instruments/fixtures, a higher stiffness is sometimes needed to reduce deformations from the measuring (contact) force.

Deformations at the tool end caused by the cutting forces result in geometric inaccuracies and in a reduced dynamic stability of the machining process. It is important to understand that there are many factors causing deflections at the tool end. For example, in a typical boring mill, deformation of the tool itself represents only 11% of the total deflection while deformation of the spindle and its bearings is responsible for 37%, and the tapered interface between the tool-holder and the spindle hole is responsible for 52% of the total deflection [4], [5].

1.4.2 Stiffness and Damping of the Cutting Process

Background

Deformations in the machining system are not only due to the finite stiffness of the structural components, but also due to finite stiffness of the cutting process itself. The cutting process can be modeled as a spring representing effective cutting stiffness and a damper representing effective cutting damping. The stiffness and damping parameters can be derived from the expression describing the dynamic cutting force. Various expressions for dynamic cutting forces were suggested. The most convenient expression for deriving the stiffness and damping parameters of the cutting process is one given in Tobias [1]. The dynamic increment of the cutting force dP_z in the z-direction for turning operation can be written as

$$dP_z = K_1[z(t) - \mu z(t - T)] + K_2\dot{z}(t) \tag{1.10}$$

Here z = vibratory displacement between the tool and the workpiece, whose direction is perpendicular to the axis of the workpiece and also to the cutting speed direction in the horizontal plane; μ = overlap factor between the two subsequent tool passes in the z-direction; K_1 = cutting stiffness coefficient in the z-direction; K_2 = *penetration rate coefficient* due to the tool penetrating the workpiece in the z-direction; and $T = 2\pi/\Omega$, where Ω rev/sec is the rotating speed of the workpiece.

By assuming displacement z as

$$z(t) = A \cos \omega t \tag{1.11}$$

where A = an indefinite amplitude constant and ω = chatter frequency, Eq. (1.10) can be rearranged as

$$dP_z = K_{cz}z + C_{cz}\frac{dz}{dt}(t) \tag{1.12}$$

where

$$K_{cz} = K_1[1 - \mu \cos 2\pi(\omega/\Omega)] \tag{1.13}$$

$$C_{cz} = K_1(\mu/\omega)\sin 2\pi(\omega/\Omega) + K_2 \tag{1.14}$$

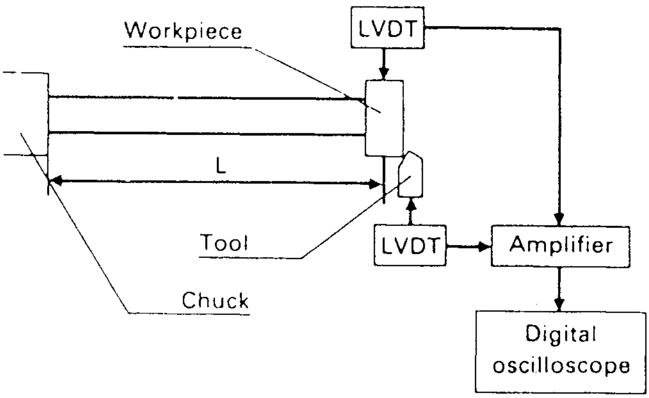

Figure 1.7 Cantilever workpiece for measuring cutting process stiffness and measurement setup.

K_{cz} and C_{cz} can be defined as effective cutting stiffness and effective cutting damping, respectively (since only the z-dircection is considered, the subscript z is further omitted). The effective cutting stiffness and cutting damping are functions not only of the cutting conditions but also of the structural parameters of the machining system (stiffness and mass), which enter Eq. (1.13) and (1.14) via frequency ω. The dynamic cutting force P_z depends not only on displacement $z(t)$ but also on velocity $\dot{z}(t) = dz/dt$. The velocity-dependent term may bring the system instability when effective cutting damping $C_{cz} < 0$, and the magnitude of C_{cz} is so large that it cannot be compensated by positive structural damping.

Experimental Determination of Effective Cutting Stiffness

Experimental determination of the cutting process stiffness can be illustrated on the example of a cantilever workpiece [6]. A cantilever workpiece with a larger diameter segment at the end (Fig. 1.7) can be modeled as a single degree of freedom system with stiffness K_w without cutting and with stiffness $K_w + K_t$ during cutting, where K_w is the stiffness of the workpiece at the end and K_t is the effective cutting stiffness. Since stiffness of the cantilever workpiece is relatively small as compared with structural stiffness of the machine tool (lathe) and of the clamping chuck, chatter conditions are determined by the workpiece and the cutting process only. Thus, if the natural frequency f_w of the workpiece (without cutting) and the frequency f_c of the tool or workpiece vibration at the chatter threshold were measured, then the effective cutting stiffness can be determined using the following equation:

$$K_c = \left(\frac{f_c^2}{f_w^2} - 1 \right) K_w \tag{1.15}$$

The frequency f_w can be measured using an accelerometer, while the chatter frequency f_c can be measured on the workpiece or on the tool using a linear variable differential transformer (LVDT) during cutting as shown in Fig. 1.7.

A cantilever bar with overhang L = 127 mm (5 in.) having stiffness (as measured) K_w = 10,416 lb/in. was used for the tests. The natural frequency $f_w \approx 200$ Hz and the equivalent mass is about 0.0065 lb-sec^2/in. The values of the effective cutting stiffness and vibration amplitude under different cutting conditions are given in Fig. 1.8a–c. It can be seen that smaller vibration amplitudes are correlated with higher effective cutting stiffness values. This validates representation of the effective cutting stiffness as a spring.

1.5 GENERAL COMMENTS ON STIFFNESS IN DESIGN

In most of the structures, their structural stiffness depends on the following factors:

- Elastic moduli of structural material(s)
- Geometry of the deforming segments (cross-sectional area A for tension/compression/shear, cross-sectional moment of inertia $I_{x,y}$ for bending, and polar moment of inertia J_p for torsion)
- Linear dimensions (e.g., length L, width B, height H)
- Character and magnitude of variation of the above parameters across the structure
- Character of loading and supporting conditions of the structural components
- In structures having slender, thin-walled segments, stiffness can depend on elastic stability of these segments
- Joints between substructures and/or components frequently contribute the dominant structural deformations (e.g., see data on the breakdown of tool-end deflections above in Section 1.4).

While for most machine components a stiffness increase is desirable, there are many cases where stiffness values should be limited or even reduced. The following are some examples:

- Perfectly rigid bodies are usually more brittle and cannot accommodate shock loads
- Many structures are designed as statically indeterminate systems, but if the connections in such a system are very rigid, it would not function prop-

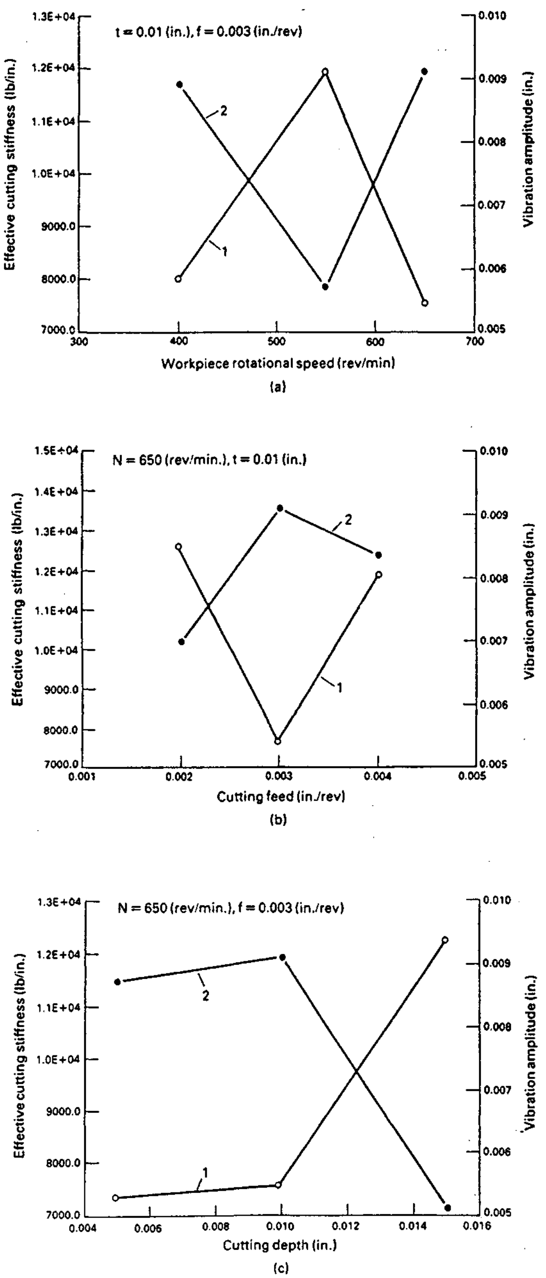

Figure 1.8 Effective cutting stiffness (line 1) and workpiece vibration amplitude (line 2) vs. (a) cutting speed, (b) feed, and (c) depth of cut.

erly since some connections might be overloaded. If the most highly loaded connection fails, others would fail one after another

Huge peak loads (stress concentrations) may develop in contacts between very rigid bodies due to presence of surface asperities

Stiffness adjustment/tuning by preloading would not be possible for very rigid components

High stiffness may result in undesirable values for the structural natural frequencies

1.6 STRUCTURAL CHARACTERISTICS OF SOME WIDELY USED MATERIALS

Stiffness of a structural material is characterized by its elastic (Young's) modulus E for tension/compression. However, there are many cases when knowledge of just Young's modulus is not enough for a judicious selection of the structural material. Another important material parameter is shear modulus G. For most metals, $G = \sim 0.4E$.

Frequently, stiffer materials (materials with higher E) are heavier. Thus, use of such materials would result in structures having smaller cross sections but heavier weight, which is undesirable. In cases when the structural deflections are caused by inertia forces, like in a revolute robot arm, use of a stiffer but heavier material can be of no benefit or even counterproductive if its weight increases more than its stiffness and specific stiffness E/γ is the more important parameter (see Section 7.5 for ways to overcome this problem).

Very frequently, stiffer materials are used to increase natural frequencies of the system. This case can be illustrated on the example of two single-degree-of-freedom dynamic systems in Fig. 1.9. In these sketches, γ, A_1, l_1, are density, cross-sectional area, and length, respectively, of the inertia element (mass m); A_2, l_2, h, and b are cross-sectional area, length, thickness, width, respectively, of the elastic elements (stiffness k). For the system in Fig. 1.9a (tension/compression elastic element) the natural frequency is

$$\sqrt{\frac{k}{m}} = \sqrt{\frac{EA_2/l_2}{\gamma A_1 l_1}} = \sqrt{\frac{E}{\gamma}}\sqrt{\frac{A_2}{A_1 l_1 l_2}} \tag{1.16}$$

For the system in Fig. 1.9b (elastic element loaded in bending)

$$m = \gamma A_1 l_1, \qquad k = 3EI/l_2^3 = (3/12)\,(Ebh^3/l_2^3) \tag{1.17}$$

thus the natural frequency is

$$\sqrt{\frac{k}{m}} = \sqrt{\frac{3}{12}\frac{Ebh^3}{l_2^3}\Big/\gamma A_1 l_1} = \sqrt{\frac{E}{\gamma}}\sqrt{\frac{1}{4}\frac{bh^3}{l_2^3 A_1 l_1}} \tag{1.18}$$

In both cases, the natural frequency depends on the criterion E/γ. A similar criterion can be used for selecting structural materials for many nonvibratory applications.

To provide a comprehensive information, Table 1.1 lists data on E, γ, E/γ for various structural materials (see page 3). It is interesting to note that for the most widely used structural materials (steel, titanium, aluminum, and magnesium), values of E/γ are very close.

While graphite has the second highest Young's modulus and the highest ratio E/γ in Table 1.1, it does not necessarily mean that the graphite fiber–based composites can realize such high performance characteristics. First of all, the fibers in a composite material are held together by a relatively low modulus matrix (epoxy resin or a low E metal such as magnesium or aluminum). Second of all, the fibers realize their superior elastic properties only in one direction (in tension). Since mechanical structures are frequently rated in a three-dimensional stress-strain environment, the fibers have to be placed in several directions, and this weakens the overall performance characteristics of the composite structures.

Fig. 1.10 illustrates this statement on an example of a propeller shaft for a surface vehicle [7]. Although in a steel shaft (Fig. 1.10a) steel resists loads in

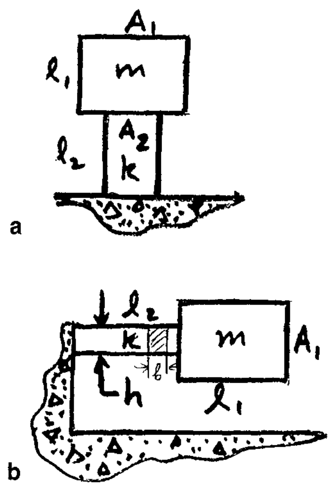

Figure 1.9 (a) Tension-compression and (b) bending single degree of freedom vibratory systems.

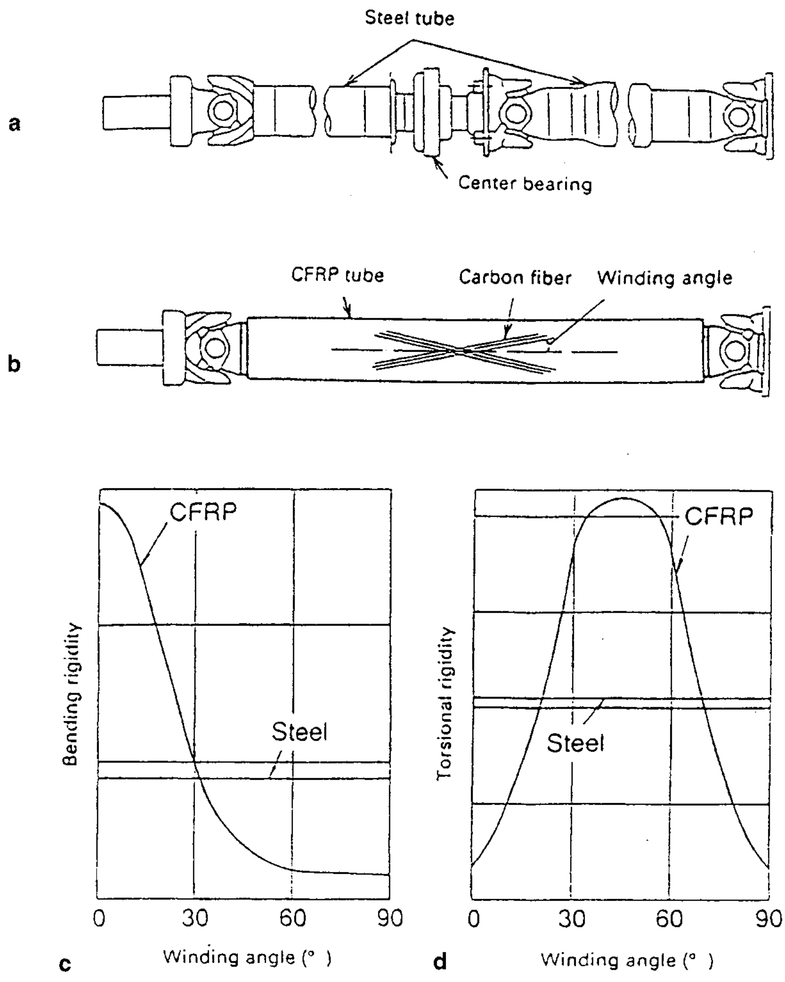

Figure 1.10 (a) Steel and (b) composite propeller shafts for automotive transmissions and comparison of their (c) bending and (d) torsional rigidity.

all directions, in a shaft made of carbon fiber reinforced plastic (CFRP) (Fig. 1.10b) there is a need to place several layers of fiber at different winding angles. Fig. 1.10c,d show how bending and torsional rigidity of the composite shaft depend on the winding angles. While it is easy to design bending or torsional stiffness of the composite shaft to be much higher than these characteristics of the steel shaft, a combination of both stiffnesses can be made superior to the steel shaft only marginally (at the winding angle, ~25 degrees).

Another example of a stiffness-critical and natural-frequency-critical components are cones and diaphragms for loudspeakers [8]. Three important material properties for loudspeaker diaphragms are:

Large specific modulus E/γ (resulting in high natural frequencies) in order to get a wider frequency range of the speaker
High flexural rigidity EI in order to reduce harmonic distortions
Large internal energy dissipation (damping) characterized by the "loss factor" $\eta = \tan\beta$ (β = "loss angle" of the material; log decrement $\delta = \pi \tan\beta$) to suppress breakups of the diaphragms at resonances

Although paper (a natural fiber-reinforced composite material) and synthetic fiber-reinforced diaphragms were originally used, their stiffness values were not adequate due to the softening influence of the matrix. Yamamoto and Tsukagoshi [8] demonstrated that use of beryllium and boronized titanium (25 μm thick titanium substrate coated on both sides with 5 μm thick boron layers) resulted in significant improvement of the frequency range for high frequency and midrange speakers.

As with loudspeaker cones and diaphragms described earlier, damping of a material is an important consideration in many applications. Frequently, performance of a component or a structure is determined by combination of its stiffness and damping. Such a combination is convenient to express in the format of a criterion. For the important problems of dynamic stability of structures or processes (e.g., chatter resistance of a cutting process, settling time of a decelerating revolute link such as a robot arm, wind-induced self-excited vibrations of smoke stack, and some vibration isolation problems as in Article 1) the criterion is $K\delta$, where K is effective stiffness of the component/structure and δ is its log decrement. For such applications, Table 1.2 can be of some use. Table 1.2 lists Young's

Table 1.2 Damping (Loss Factor) and Young's Modulus of Some Materials

Material	η	E (MPa)	$E\eta$
Tinel	6.5×10^{-2}	4×10^4	2600
Polysulfide rubber (Thiokol H-5)	5.0	30	150
Tin	2×10^{-3}	6.7×10^4	134
Steel	$1 - 6 \times 10^{-4}$	21×10^4	20–120
Neoprene (type CG-1)	0.6	86.7	52
Zinc	3×10^{-4}	8×10^4	24
Aluminum	10^{-4}	6.7×10^4	6.7

modulus E, determining the effective stiffness of a component, loss factor $\eta = \tan \beta$, and product $E\eta$, the so-called loss modulus for some structural and energy absorbing materials. It can be seen that the best (highest) value of $E\eta$ is for a nickel titanium ''shape memory'' alloy Tinel (~50% Ni + ~50% Ti), and the lowest value is for aluminum.

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2

Stiffness of Structural Components: Modes of Loading

2.1 INFLUENCE OF MODE OF LOADING ON STIFFNESS [1]

There are four principal types of structural loading: tension, compression, bending, and torsion. Parts experiencing tension-compression demonstrate much smaller deflections for similar loading intensities and therefore usually are not stiffness-critical. Figure 2.1a shows a rod of length L having a uniform cross-sectional area A along its length and loaded in tension by its own weight W and by force P. Fig. 2.1b shows the same rod loaded in bending by the same force P or by distributed weight $w = W/L$ as a cantilever built-in beam, and Fig. 2.1c shows the same rod as a double-supported beam.

Deflections of the rod in tension are

$$f_P^{te} = PL/EA; \quad f_W^{te} = WL/2EA \tag{2.1}$$

Bending deflections for cases b and c, respectively, are

$$f_P^{bb} = PL^3/3EI; \quad f_W^{bb} = WL^3/8EI \tag{2.2}$$

$$f_P^{bc} = PL^3/48EI \quad f_W^{bc} = 5WL^3/384EI \tag{2.3}$$

where I = cross-sectional moment of inertia. For a round cross section (diameter d, $A = \pi d^2/4$, $I = \pi d^4/64$, and $I/A = d^2/16$)

$$f^b/f^{te} = kL^2/d^2 \tag{2.4}$$

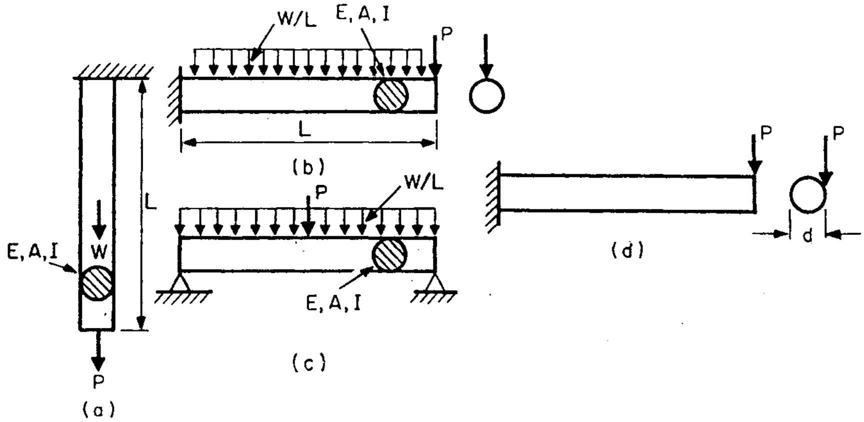


Figure 2.1 Various modes of loading of a rod-like structure: (a) tension; (b) bending in a cantilever mode; (c) bending in a double-supported mode; and (d) bending with an out-of-center load.

where coefficient k depends on loading and supporting conditions. For example, for a cantilever beam with $L/d = 20$, $(f^{bb}/f^{te})_F = 2{,}130$ and $(f^{bb}/f^{te})_W = 1{,}600$; for a double-supported beam with $L/d = 20$, $(f^{bc}/f^{te})_F \cong 133$ and $(f^{bb}/f^{te})_W \cong 167$. Thus, bending deflections are exceeding tension-compression deflections by several decimal orders of magnitude.

Figure 2.1d shows the same rod whose supporting conditions are as in Fig. 2.1b, but which is loaded in bending with an eccentricity, thus causing bending [as described by the first expression in Eq. (2.2)] and torsion, with the translational deflection on the rod periphery (which is caused by the torsional deformation) equal to

$$f^{to} = PLd^2/4GJ_p \tag{2.5}$$

where J_p = polar moment of inertia and G = shear modulus of the material. Since $J_p = \pi d^4/32$ for a circular cross section then

$$f^{to}/f^{te} = d^2/4(EA/GJ_p) = 2E/G \cong 5 \tag{2.6}$$

since for structural metals $E \cong 2.5G$. Thus, the torsion of bars with solid cross sections is also associated with deflections substantially larger than those under tension/compression.

These simple calculations help to explain why bending and/or torsional compliance is in many cases critical for the structural deformations.

Many stiffness-critical mechanical components are loaded in bending. It was shown earlier that bending is associated with much larger deformations than tension/compression of similar-size structures under the same loads. Because of this, engineers have been trying to replace bending with tension/compression. The most successful designs of this kind are trusses and arches.

Advantages of truss structures are illustrated by a simple case in Fig. 2.2 [2], where a cantilever truss having overhang l is compared with cantilever beams of the same length and loaded by the same load P. If the beam has the same cross section as links of the truss (case a) then its weight G_p is 0.35 of the truss weight G_t, but its deflection is 9,000 times larger while stresses are 550 times higher. To achieve the same deflection (case c), diameter of the beam has to be increased by the factor of 10, thus the beam becomes 35 times heavier than the truss. The stresses are equalized (case b) if the diameter of the beam is increased by 8.25 times; the weight of such beam is 25 times that of the truss. Ratio of the beam deflection f_b to the truss deflection f_t is expressed as

$$f_b/f_t \approx 10.5(l/d)^2 \sin^2 \alpha \cos \alpha \qquad (2.7)$$

Deflection ratio f_b/f_t and maximum stress ratio σ_b/σ_t are plotted in Fig. 2.3 as functions of l/d and α.

Similar effects are observed if a double-supported beam loaded in the middle

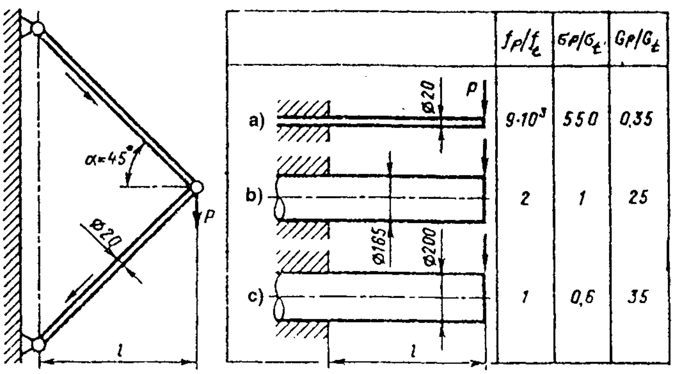


Figure 2.2 Comparison of structural characteristics of a truss bracket and cantilever beams.

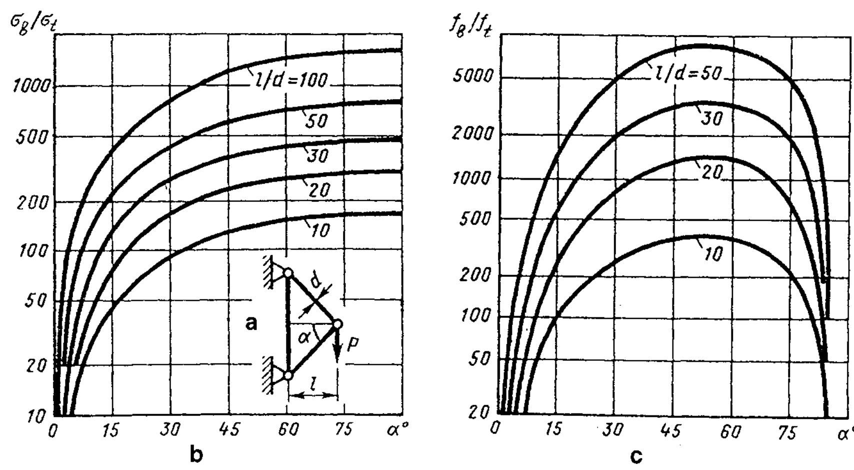


Figure 2.3 Ratios of (b) stresses and (c) deflections between a cantilever beam (diameter d, length l) and (a) a truss bracket.

of its span (as shown in Fig. 2.4a) is replaced by a truss (Fig. 2.4b). In this case

$$f_b/f_t = \sim 1.3(1/d)^3 \sin^2 \alpha \cos \alpha \tag{2.8}$$

Deflection ratio f_b/f_t and maximum stress ratio σ_b/σ_t are plotted in Fig. 2.5 as functions of l/d and α. A similar effect can be achieved if the truss is transformed into an arch (Fig. 2.4c).

These principles of transforming the bending mode of loading into the tension/compression mode of loading can be utilized in a somewhat "disguised" way in designs of basic mechanical components, such as brackets (Fig. 2.6). The

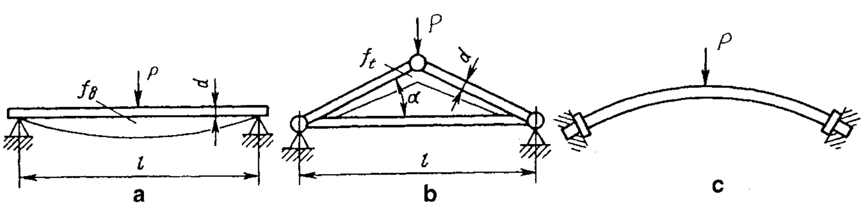


Figure 2.4 Typical load-carrying structures: (a) double-supported beam; (b) truss bridge; (c) arch.

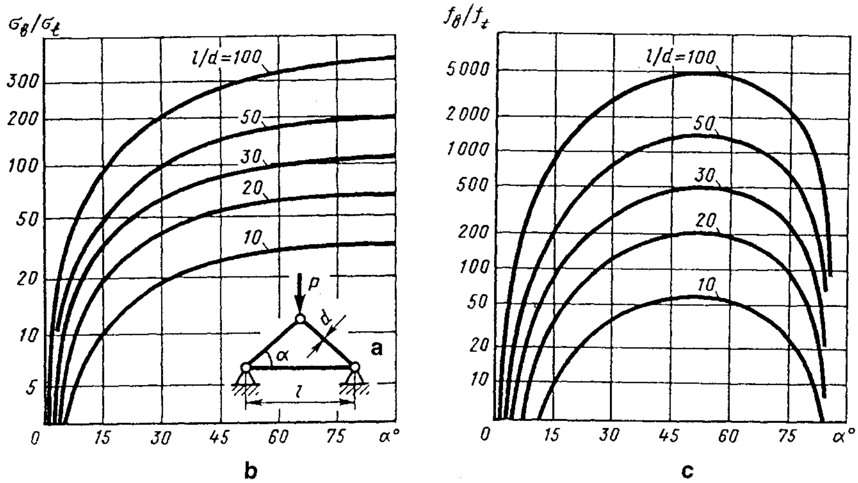


Figure 2.5 Ratios of (b) stresses and (c) deflections between (a) a double-supported beam in Fig. 2.4a and a truss bridge in Fig. 2.4b.

bracket in Fig. 2.6a(I) is loaded in bending. An inclination of the lower wall of the bracket, as in Fig. 2.6a(II), reduces deflection and stresses, but the upper wall does not contribute much to the load accommodation. Design in Fig. 2.6a(III) provides a much more uniform loading of the upper and lower walls, which allows one to significantly reduce size and weight of the bracket.

Even further modification of the "truss concept" is illustrated in Fig. 2.6b.

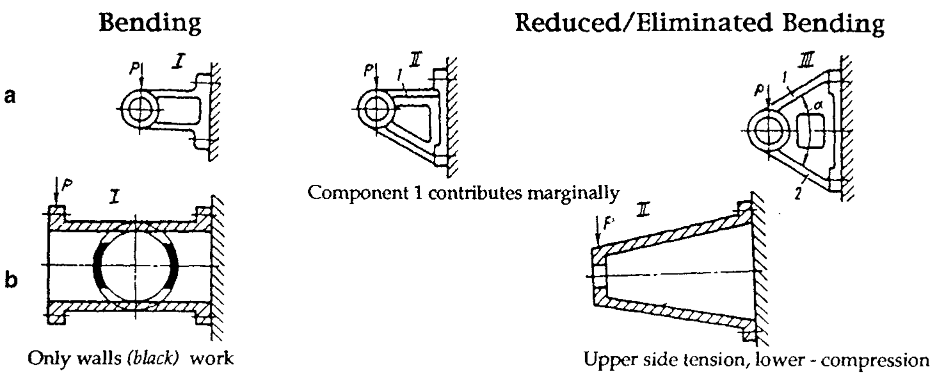


Figure 2.6 Use of tension/compression instead of bending for structural components.

Load P in case 2.6b(I) (cylindrical bracket) is largely accommodated by segments of the side walls, which are shown in black. Tapering the bracket, as in Fig. 2.6b(II), allows one to distribute stresses more evenly. Face wall f is an important feature of the system since it prevents distortion of the cross section into an elliptical one and it is necessary for achieving optimal performance.

There are many other design techniques aimed at reduction or elimination of bending in favor of tension/compression. Some of them are illustrated in Fig. 2.7. Fig. 2.7a(I) shows a mounting foot of a machine bed. Horizontal forces on the bed cause bending of the wall and result in a reduced stiffness. ''Pocketing'' of the foot as in Fig. 2.7a(II) aligns the anchoring bolt with the wall and thus reduces the bending moment; it also increases the effective cross section of the foot area, which resists bending. The disc-like hub of a helical gear in Fig. 2.7b(I) bends under the axial force component of the gear mesh. Inclination of the hub as in Fig. 2.7b(II) enhances stiffness by introducing the ''arch concept.'' Vertical load on the block bearing in Fig. 2.7c(I) causes bending of its frame, while in Fig. 2.7c(II) it is accommodated by compression of the added central support. Bending of the structural member under tension in Fig. 2.7d(I) is caused by its asymmetry. After slight modifications as shown in Fig. 2.7d(II), its effective cross section can be reduced due to total elimination of bending.

Some structural materials, such as cast iron, are better suited to accommodate compressive than tensile stress. While it is more important for strength, stiffness can also be influenced if some microcracks which can open under tension, are present. Fig. 2.8 gives some directions for modifying components loaded in bend-

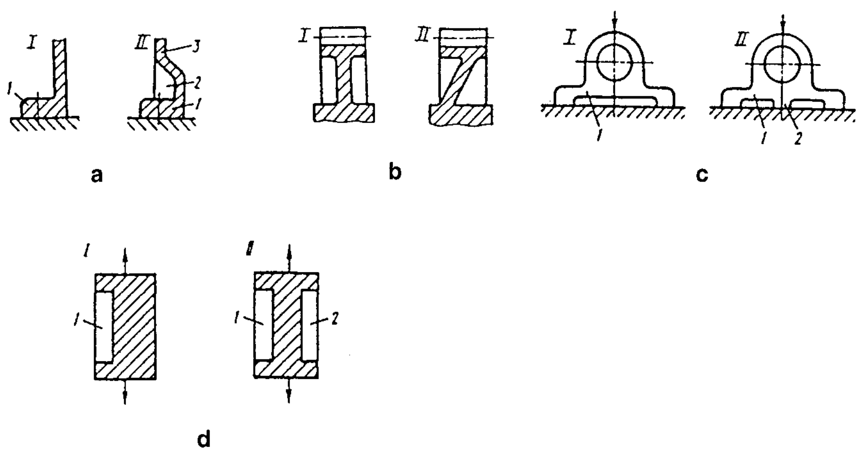


Figure 2.7 Reduction of bending deformations in structural components.

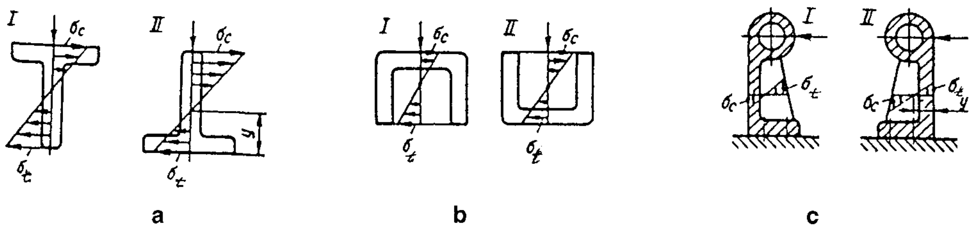


Figure 2.8 Increasing compressive stresses at the expense of tensile stresses.

ing so that maximum stresses are compressive rather than tensile. While the maximum stresses in the beam whose cross section is shown in Fig. 2.8a(I) are tensile (in the bottom section), turning this beam upside down as in Fig. 2.8a(II) brings maximum stresses to the compressed side (top). Same is true for Fig. 2.8b. A similar principle is used in transition from the bracket with the stiffening wall shown in Fig. 2.8c(I) to the identical but opposedly mounted bracket in Fig. 2.8c(II).

2.1.1 Practical Case 1: Tension/Compression Machine Tool Structure

While use of tension/compression mode of loading in structures is achieved by using trusses and arches, there are also mechanisms providing up to six degrees-of-freedom positioning and orientation of objects by using only tension/compression actuators. The most popular of such mechanisms is the so-called Stewart Platform [3]. First attempts to use the Stewart Platform for machine tools (machining centers) were made in the former Soviet Union in the mid-1980s [4].

Figure 2.9 shows the design schematic of the Russian machining center based on application of the Stewart Platform mechanism. Positioning and orientation of the platform 1 holding the spindle unit 2 which carries a tool machining part 3 is achieved by cooperative motions of six independent tension/compression actuators 4, which are pivotably engaged via spherical joints 5 and 6 with platform 1 and base plate 7, respectively.

Cooperation between the actuators is realized by using a rather complex controlling software which commands each actuator to participate in the programmed motion of the platform. One shortcoming of such a machining center is a limited range of motion along each coordinate, which results in a rather complex shape of the work zone as illustrated in Fig. 2.10.

However, there are several advantages that make such designs promising for many applications. Astanin and Sergienko [4] claim that while stiffness along

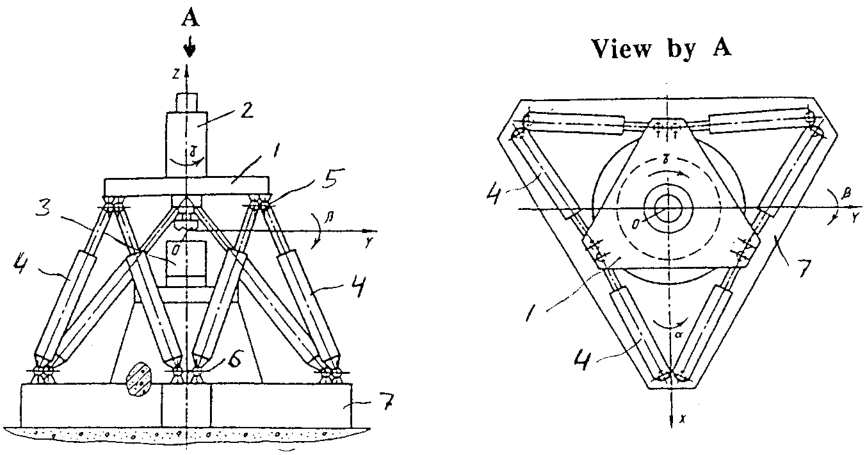


Figure 2.9 Design schematic and coordinate axes of Russian machining center based on the Stewart Platform kinematics.

the y-axis (k_y) is about the same as for conventional machining centers, stiffness k_z is about 1.7 times higher. The overall stiffness is largely determined by deformations in spherical joints 5 and 6, by platform deformations, and by spindle stiffness, and can be enhanced 50–80% by increasing platform stiffness in the x-y plane and by improving the spindle unit. The machine weighs 3–4 times less

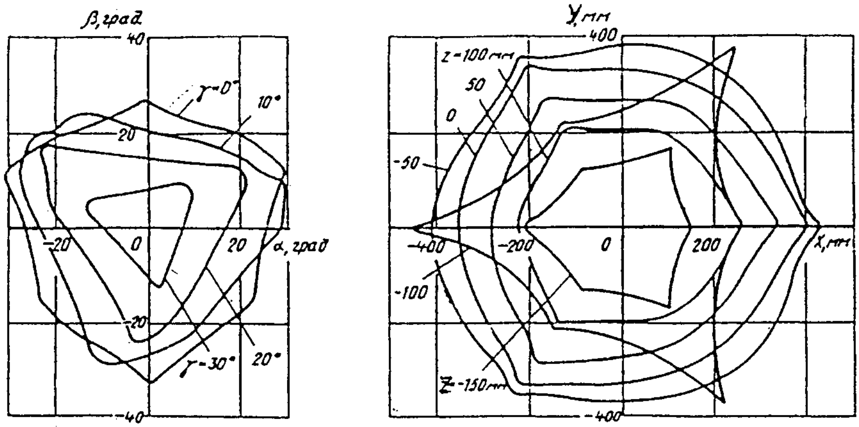


Figure 2.10 Work zone of machining center in Fig. 2.9.

than a conventional machining center and is much smaller (2–3 times smaller footprint). It costs 3–4 times less due to use of standard identical and not very complex actuating units and has 3–5 times higher feed force.

Similar machining centers were developed in the late 1980s and early 1990s by Ingersol Milling Machines Co. (Octahedral–Hexapod) and by Giddings and Lewis Co. (Variax). Popularity of this concept and its modifications for CNC machining centers and milling machines has recently been increasing [5], [6].

2.1.2 Practical Case 2: Tension/Compression Robot Manipulator

Tension/compression actuators also found application in robots. Fig. 2.11 shows schematics and work zone of a manipulating robot from NEOS Robotics Co. While conventional robots are extremely heavy in relation to their rated payload (weight-to-payload ratios 15–25 [1]), the NEOS robot has extremely high performance characteristics for its weight (about 300 kg), as listed in Table 2.1.

2.2 OPTIMIZATION OF CROSS-SECTIONAL SHAPE

2.2.1 Background

Significant gains in stiffness and/or weight of structural components loaded in bending can be achieved by a judicious selection of their cross-sectional shape. Importance of the cross-section optimization can be illustrated on the example of robotic links, which have to comply with numerous, frequently contradictory, constraints. Some of the constraints are as follows:

The links should have an internal hollow area to provide conduits for electric power and communication cables, hoses, power-transmitting components, control rods, etc.

At the same time, their external dimensions are limited in order to extend the usable workspace.

Links have to be as light as possible to reduce inertia forces and to allow for the largest payload per given size of motors and actuators.

For a given weight, links have to possess the highest possible bending (and in some cases torsional) stiffness.

One of the parameters that can be modified to comply better with these constraints is the shape of the cross section. The two basic cross sections are hollow round (Fig. 2.12a) and hollow rectangular (Fig. 2.12b). There can be various approaches to the comparison of these cross sections. Two cases are analyzed below [1]:

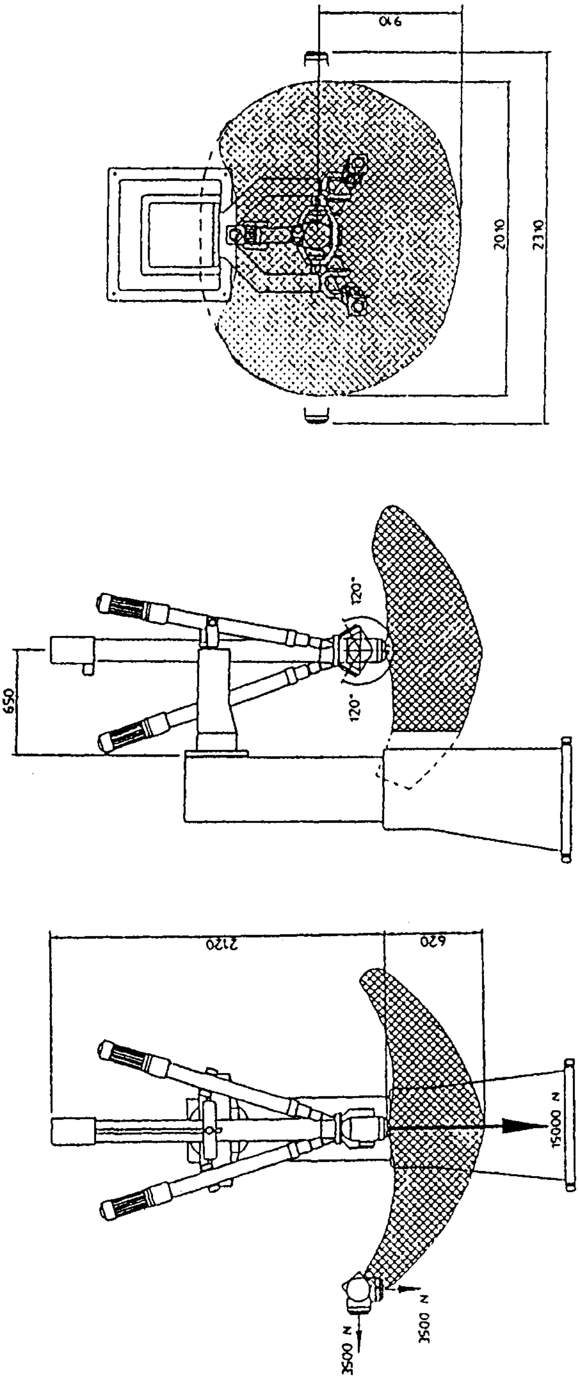


Figure 2.11 Design schematic and work zone of NEOS Robotics robot utilizing tension/compression links.

Table 2.1 Specifications of NEOS Robot

Load capacity	Handling payload	150 kg
	Turning torque	200 Nm
	Pressing, maximum	15,000 N
	Lifting, maximum	500 kg
Accuracy	Repeatability (ISO 9283)	$\leq \pm$ 0.02 mm
	Positioning	$\leq \pm$ 0.20 mm
	Path following at 0.2 m/s	$\leq \pm$ 0.10 mm
	Incremental motion	$\leq$ 0.01 mm
Stiffness	Static bending deflection (ISO 9283.10)	
	X and Y directions	0.0003 mm/N
	Z direction	0.0001 mm/N

1. The wall thickness of both cross sections is the same.
2. The cross-sectional areas (i.e., weight) of both links are the same.

In both cases, the rectangular cross section is assumed to be a square whose external width is equal to the external diameter of the round cross section.

The bending stiffness of a beam is characterized by its cross-sectional moment of inertia I, and its weight is characterized by the cross-sectional area A. For the round cross section in Fig. 2.12a

$$I_{rd} = \pi(D_0^4 - D_i^4)/64 = \pi[D_0^4 - (D_0 - 2t)^4]/64 \cong \pi(D_0^3 t/8)(1 - 3t/D_0 + 4t^2/D_0^2) \quad (2.9)$$

$$A_{rd} = \pi(D_0^2 - D_i^2)/4 = \pi D_0 t(1 - t/D_0) \quad (2.10)$$

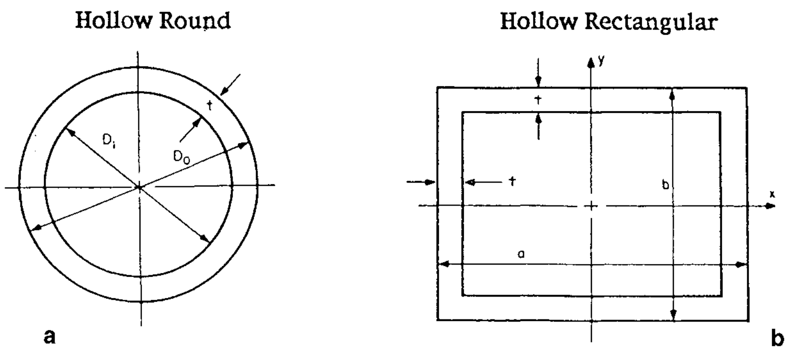


Figure 2.12 Typical cross sections of a manipulator link: (a) hollow round (ring-like); (b) hollow rectangular.

For the rectangular cross section in Fig. 2.12b, the value of I depends on the direction of the neutral axis in relation to which the moment of inertia is computed. Thus

$$I_{re,x} = ab^3/12 - (a - 2t)(b - 2t)^3/12; \quad I_{re,y} = a^3b/12 - (a - 2t)^3(b - 2t)/12 \quad (2.11a)$$

For the square cross section

$$I_{sq} = a^4/12 - (a - 2t)^4/12 \cong 2/3\, a^3t(1 - 3t/a + 4t^2/a^2) \quad (2.11b)$$

The cross-sectional areas for the rectangular and square cross sections, respectively, are

$$A_{re} = ab - (a - 2t)(b - 2t) = 2t(a + b) - 4t^2; \quad A_{sq} = 4at(1 - t/a) \quad (2.12)$$

For case 1, $D_0 = a$, and t is the same for both cross sections. Thus,

$$I_{sq}/I_{rd} = (2/3)/(\pi/8) = 1.7; \quad A_{sq}/A_{rd} = 4/\pi = 1.27 \quad (2.13)$$

or a square cross section provides a 70% increase in rigidity with only a 27% increase in weight; or a 34% increase in rigidity for the same weight.

For case 2 ($D_0 = a$, $A_{rd} = A_{sq}$, and $t_{rd} \neq t_{sq}$), if $t_{rd} = 0.2D_0$, then $t_{1sq} = 0.147D_0 = 0.147a$ and

$$I_{rd} = 0.0405D_0^4; \quad I_{sq} = 0.0632a^4; \quad I_{sq}/I_{rd} = 1.56 \quad (2.14a)$$

If $t_{2rd} = 0.1D_0$, then $t_{2sq} = 0.0765D_0 = 0.0765a$, and

$$I_{rd} = 0.029D_0^4; \quad I_{sq} = 0.0404a^4; \quad I_{sq}/I_{rd} = 1.40 \quad (2.14b)$$

Thus, for the same weight, a beam with the thin-walled square cross section would have 34–40% higher stiffness than a beam with the hollow round cross section. In addition, the internal cross-sectional area of the square beam is significantly larger than that for the round beam of the same weight (the thicker the wall, the more pronounced is the difference).

From the design standpoint, links of the square cross section have also an advantage of being naturally suited for using roller guideways. The round links have to be specially machined when used in prismatic joints. On the other hand, round links are easier to fit together (e.g., if telescopic links with sliding connections are used).

Both stiffness and strength of structural components loaded in bending (beams) can be significantly enhanced if a solid cross section is replaced with the cross-sectional shape in which the material is concentrated farther from the neutral line of bending. Fig. 2.13 [2] shows comparisons of both stiffness (cross-sectional moment of inertia I_0) and strength (cross-sectional modulus W) for round cross sections and for solid square vs. standard I-beam profile for the same cross-sectional area (weight).

2.2.2 Composite/Honeycomb Beams

Bending resistance of beams is largely determined by the parts of their cross sections, which are farthest removed from the neutral plane. Thus, enhancement of bending stiffness-to-weight ratio for a beam can be achieved by designing its cross section to be of such shape that the load-bearing parts are relatively thin strips on the upper and lower sides of the cross section. However, there is a need for some structural members maintaining stability of the cross section so that the

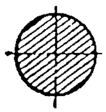
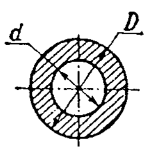
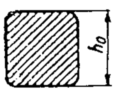
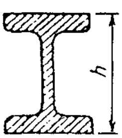
Section	Ratios d/D ,h/ho	I/I_0	W/W_0
	0	1	1
	0.6 0.8 0.9	2.1 4.5 10	1.7 2.7 4.1
	—	1	1
	1.5 2.5 3.0	4.3 11.5 21.5	2.7 4.5 7.0

Figure 2.13 Relative stiffness (cross-sectional moment of inertia I) and strength (section modulus W) of various cross sections having same weight (cross-sectional area A).

positions of the load-bearing strips are not noticeably changed by loading of the beam. Rolling or casting of an integral beam (e.g., I-beams and channel beams in which an elongated wall holds the load-bearing strips) can achieve this. Another approach is by using composite beams in which the load-bearing strips are separated by an intermediate filler (core) made of a light material or by a honeycomb structure made from the same material as the load-bearing strips or from some lighter metal or synthetic material. The composite beams can be lighter than the standard profiles such as I-beams or channels, and they are frequently more convenient for the applications. For example, it is not difficult to make composite beams of any width (*composite plates*), to provide the working surfaces with smooth or threaded holes for attaching necessary components (''*breadboard*'' *optical tables*), or to use high damping materials for the middle layer (or to use damping fillers for honeycomb structures).

It is important to realize that there are significant differences in the character of deformation between solid beams (plates) and composite beams (plates). Bending deformation of a beam comprises two components: moment-induced deformations and shear-induced deformations [7]. For beams with solid cross sections made from a uniform material, the shear deformation can be neglected for $L/h \geq 10$. For example, for a double-supported beam loaded with a uniformly distributed force with intensity q per unit length, deflection at the mid-span is [7]

$$f_{ms} = \frac{5qL^4}{384EI}\left(1 + \frac{48\alpha_{sh}EI}{5GFL^2}\right) \tag{2.15a}$$

where E = Young's modulus, G = shear modulus, F = cross-sectional area, and α_{sh} is the so-called shear factor ($\alpha_{sh} \approx 1.2$ for rectangular cross sections, $\alpha_{sh} \approx 1.1$ for round cross sections). If the material has $E/G = 2.5$ (e.g., steel), then for a rectangular cross section ($I/F = h^2/12$)

$$f_{ms} = \frac{5qL^4}{384EI}\left(1 + 2.4\frac{h^2}{L^2}\right) \tag{2.15b}$$

For $L/h = 10$, the second (shear) term in brackets in Eq. (2.15) is 0.024, less than 2.5%.

For a double-supported beam loaded with a concentrated force P in the middle, deformation under the force is [7]

$$f_{ms} = \frac{PL^3}{48EI}\left(1 + \frac{12\alpha_{sh}EI}{GFL^2}\right) \tag{2.16a}$$