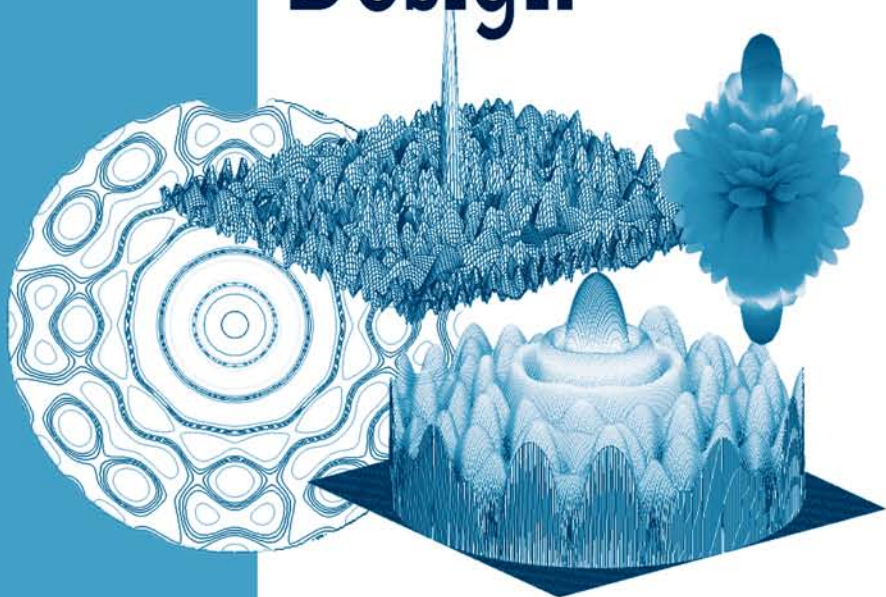


MATLAB[®]

Simulations for Radar Systems Design



Bassem R. Mahafza
Atef Z. Elsherbeni

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Radar Systems
Design

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*To: My wife and four sons;
Wayne and Shirley;
and
in the memory of my parents*

Bassem R. Mahafza

*To: My wife and children;
my mother;
and
in the memory of my father*

Atef Z. Elsherbeni

Preface

The emphasis of “*MATLAB Simulations for Radar Systems Design*” is on radar systems design. However, a strong presentation of the theory is provided so that the reader will be equipped with the necessary background to perform radar systems analysis. The organization of this book is intended to teach a conceptual design process of radars and related trade-off analysis and calculations. It is intended to serve as an engineering reference for radar engineers working in the field of radar systems. The MATLAB[®]¹ code provided in this book is designed to provide the user with hands-on experience in radar systems, analysis and design.

A radar design case study is introduced in Chapter 1 and carried throughout the text, where the authors’ view of how to design this radar is detailed and analyzed. Trade off analyses and calculations are performed. Additionally, several mini design case studies are scattered throughout the book.

“*MATLAB Simulations for Radar Systems Design*” is divided into two parts: Part I provides a comprehensive description of radar systems, analyses and design. A design case study, which is carried throughout the text, is introduced in Chapter 1. In each chapter the authors’ view of how to design the case-study radar is presented based on the theory covered up to that point in the book. As the material coverage progresses through the book, and new theory is discussed, the design case-study requirements are changed and/or updated, and of course the design level of complexity is also increased. This design process is supported by a comprehensive set of MATLAB 6 simulations developed for this purpose. This part will serve as a valuable tool to students and radar engineers in helping them understand radar systems, design process. This includes 1) learning how to go about selecting different radar parameters to meet the design requirements; 2) performing detailed trade-off analysis in the context of radar sizing, modes of operations, frequency selection, waveforms and signal processing; 3) establishing and developing loss and error budgets associated with the design; and 4) generating an in-depth understanding of radar operations and design philosophy. Additionally, Part I includes several mini design case studies pertinent to different chapters in order to help enhance understanding of radar design in the context of the material presented in different chapters.

Part II includes few chapters that cover specialized radar topics, some of which is authored and/or coauthored by other experts in the field. The material

1. MATLAB is a registered trademark of the The MathWorks, Inc. For product information, please contact: The MathWorks, Inc., 3 Apple Hill Drive, Natick, MA 01760-2098 USA. Web: www.mathworks.com.

included in Part II is intended to further enhance the understanding of radar system analysis by providing detailed and comprehensive coverage of these radar related topics. For this purpose, MATLAB 6 code has also been developed and made available.

All MATLAB programs and functions provided in this book can be downloaded from the CRC Press Web site (www.crcpress.com). For this purpose, follow this procedure: 1) from your Web browser type "<http://www.crcpress.com>", 2) click on "*Electronic Products*", 3) click on "*Download & Updates*", and finally 4) follow instructions of how to download a certain set of code off that Web page. Furthermore, this MATLAB code can also be downloaded from The MathWorks Web site by following these steps: 1) from your Web browser type: "<http://mathworks.com/matlabcentral/fileexchange/>", 2) place the cursor on "*Companion Software for Books*" and click on "*Communications*". The MATLAB functions and programs developed in this book include all forms of the radar equation: pulse compression, stretch processing, matched filter, probability of detection calculations with all Swerling models, High Range Resolution (HRR), stepped frequency waveform analysis, ghk tracking filter, Kalman filter, phased array antennas, clutter calculations, radar ambiguity functions, ECM, chaff, and many more.

Chapter 1 describes the most common terms used in radar systems, such as range, range resolution, and Doppler frequency. This chapter develops the radar range equation. Finally, a radar design case study entitled "*MyRadar Design Case Study*" is introduced. Chapter 2 is intended to provide an overview of the radar probability of detection calculations and related topics. Detection of fluctuating targets including Swerling I, II, III, and IV models is presented and analyzed. Coherent and non-coherent integration are also introduced. Cumulative probability of detection analysis is in this chapter. Visit 2 of the design case study "*MyRadar*" is introduced.

Chapter 3 reviews radar waveforms, including CW, pulsed, and LFM. High Range Resolution (HRR) waveforms and stepped frequency waveforms are also analyzed. The concept of the Matched Filter (MF) is introduced and analyzed. Chapter 4 presents in detail the principles associated with the radar ambiguity function. This includes the ambiguity function for single pulse, Linear Frequency Modulated pulses, train of unmodulated pulses, Barker codes, and PRN codes. Pulse compression is introduced in Chapter 5. Both the MF and the stretch processors are analyzed.

Chapter 6 contains treatment of the concepts of clutter. This includes both surface and volume clutter. Chapter 7 presents clutter mitigation using Moving Target Indicator (MTI). Delay line cancelers implementation to mitigate the effects of clutter is analyzed.

Chapter 8 presents detailed analysis of Phased Arrays. Linear arrays are investigated and detailed and MATLAB code is developed to calculate and plot

the associated array patterns. Planar arrays, with various grid configurations, are also presented.

Chapter 9 discusses target tracking radar systems. The first part of this chapter covers the subject of single target tracking. Topics such as sequential lobing, conical scan, monopulse, and range tracking are discussed in detail. The second part of this chapter introduces multiple target tracking techniques. Fixed gain tracking filters such as the $\alpha\beta$ and the $\alpha\beta\gamma$ filters are presented in detail. The concept of the Kalman filter is introduced. Special cases of the Kalman filter are analyzed in depth.

Chapter 10 is coauthored with Mr. J. Michael Madewell from the US Army Space and Missile Defense Command, in Huntsville, Alabama. This chapter presents an overview of Electronic Counter Measures (ECM) techniques. Topics such as self screening and stand off jammers are presented. Radar chaff is also analyzed and a chaff mitigation technique for Ballistic Missile Defense (BMD) is introduced.

Chapter 11 is concerned with the Radar Cross Section (RCS). RCS dependency on aspect angle, frequency, and polarization is discussed. The target scattering matrix is developed. RCS formulas for many simple objects are presented. Complex object RCS is discussed, and target fluctuation models are introduced. Chapter 12 is coauthored with Dr. Brian Smith from the US Army Aviation and Missile Command (AMCOM), Redstone Arsenal in Alabama. This chapter presents the topic of Tactical Synthetic Aperture Radar (SAR). The topics of this chapter include: SAR signal processing, SAR design considerations, and the SAR radar equation. Finally Chapter 13 presents an overview of signal processing.

Using the material presented in this book and the MATLAB code designed by the authors by any entity or person is strictly at will. The authors and the publisher are neither liable nor responsible for any material or non-material losses, loss of wages, personal or property damages of any kind, or for any other type of damages of any and all types that may be incurred by using this book.

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1.1. Radar Classifications

The word radar is an abbreviation for RADio Detection And Ranging. In general, radar systems use modulated waveforms and directive antennas to transmit electromagnetic energy into a specific volume in space to search for targets. Objects (targets) within a search volume will reflect portions of this energy (radar returns or echoes) back to the radar. These echoes are then processed by the radar receiver to extract target information such as range, velocity, angular position, and other target identifying characteristics.

Radars can be classified as ground based, airborne, spaceborne, or ship based radar systems. They can also be classified into numerous categories based on the specific radar characteristics, such as the frequency band, antenna type, and waveforms utilized. Another classification is concerned with the mission and/or the functionality of the radar. This includes: weather, acquisition and search, tracking, track-while-scan, fire control, early warning, over the horizon, terrain following, and terrain avoidance radars. Phased array radars utilize phased array antennas, and are often called multifunction (multi-mode) radars. A phased array is a composite antenna formed from two or more basic radiators. Array antennas synthesize narrow directive beams that may be steered mechanically or electronically. Electronic steering is achieved by controlling the phase of the electric current feeding the array elements, and thus the name phased array is adopted.

Radars are most often classified by the types of waveforms they use, or by their operating frequency. Considering the waveforms first, radars can be Con-

tinuous Wave (CW) or Pulsed Radars (PR).¹ CW radars are those that continuously emit electromagnetic energy, and use separate transmit and receive antennas. Unmodulated CW radars can accurately measure target radial velocity (Doppler shift) and angular position. Target range information cannot be extracted without utilizing some form of modulation. The primary use of unmodulated CW radars is in target velocity search and track, and in missile guidance. Pulsed radars use a train of pulsed waveforms (mainly with modulation). In this category, radar systems can be classified on the basis of the Pulse Repetition Frequency (PRF) as low PRF, medium PRF, and high PRF radars. Low PRF radars are primarily used for ranging where target velocity (Doppler shift) is not of interest. High PRF radars are mainly used to measure target velocity. Continuous wave as well as pulsed radars can measure both target range and radial velocity by utilizing different modulation schemes.

Table 1.1 has the radar classifications based on the operating frequency.

TABLE 1.1. Radar frequency bands.

Letter designation	Frequency (GHz)	New band designation (GHz)
<i>HF</i>	<i>0.003 - 0.03</i>	<i>A</i>
<i>VHF</i>	<i>0.03 - 0.3</i>	<i>A<0.25; B>0.25</i>
<i>UHF</i>	<i>0.3 - 1.0</i>	<i>B<0.5; C>0.5</i>
<i>L-band</i>	<i>1.0 - 2.0</i>	<i>D</i>
<i>S-band</i>	<i>2.0 - 4.0</i>	<i>E<3.0; F>3.0</i>
<i>C-band</i>	<i>4.0 - 8.0</i>	<i>G<6.0; H>6.0</i>
<i>X-band</i>	<i>8.0 - 12.5</i>	<i>I<10.0; J>10.0</i>
<i>Ku-band</i>	<i>12.5 - 18.0</i>	<i>J</i>
<i>K-band</i>	<i>18.0 - 26.5</i>	<i>J<20.0; K>20.0</i>
<i>Ka-band</i>	<i>26.5 - 40.0</i>	<i>K</i>
<i>MMW</i>	<i>Normally >34.0</i>	<i>L<60.0; M>60.0</i>

High Frequency (HF) radars utilize the electromagnetic waves' reflection off the ionosphere to detect targets beyond the horizon. Very High Frequency (VHF) and Ultra High Frequency (UHF) bands are used for very long range Early Warning Radars (EWR). Because of the very large wavelength and the sensitivity requirements for very long range measurements, large apertures are needed in such radar systems.

1. See Appendix 1A.

Radars in the L-band are primarily ground based and ship based systems that are used in long range military and air traffic control search operations. Most ground and ship based medium range radars operate in the S-band. Most weather detection radar systems are C-band radars. Medium range search and fire control military radars and metric instrumentation radars are also C-band.

The X-band is used for radar systems where the size of the antenna constitutes a physical limitation; this includes most military multimode airborne radars. Radar systems that require fine target detection capabilities and yet cannot tolerate the atmospheric attenuation of higher frequency bands may also be X-band. The higher frequency bands (Ku, K, and Ka) suffer severe weather and atmospheric attenuation. Therefore, radars utilizing these frequency bands are limited to short range applications, such as police traffic radar, short range terrain avoidance, and terrain following radar. Milli-Meter Wave (MMW) radars are mainly limited to very short range Radio Frequency (RF) seekers and experimental radar systems.

1.2. Range

Figure 1.1 shows a simplified pulsed radar block diagram. The time control box generates the synchronization timing signals required throughout the system. A modulated signal is generated and sent to the antenna by the modulator/transmitter block. Switching the antenna between the transmitting and receiving modes is controlled by the duplexer. The duplexer allows one antenna to be used to both transmit and receive. During transmission it directs the radar electromagnetic energy towards the antenna. Alternatively, on reception, it directs the received radar echoes to the receiver. The receiver amplifies the radar returns and prepares them for signal processing. Extraction of target information is performed by the signal processor block. The target's range, R , is computed by measuring the time delay, Δt , it takes a pulse to travel the two-way path between the radar and the target. Since electromagnetic waves travel at the speed of light, $c = 3 \times 10^8 \text{ m/sec}$, then

$$R = \frac{c\Delta t}{2} \quad (1.1)$$

where R is in meters and Δt is in seconds. The factor of $\frac{1}{2}$ is needed to account for the two-way time delay.

In general, a pulsed radar transmits and receives a train of pulses, as illustrated by Fig. 1.2. The Inter Pulse Period (IPP) is T , and the pulsewidth is τ . The IPP is often referred to as the Pulse Repetition Interval (PRI). The inverse of the PRI is the PRF, which is denoted by f_r ,

$$f_r = \frac{1}{PRI} = \frac{1}{T} \quad (1.2)$$

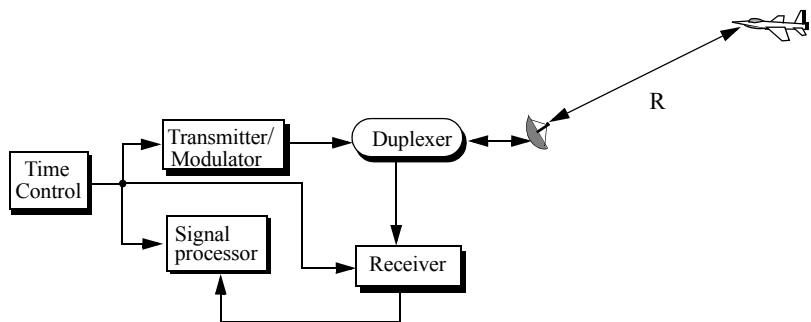


Figure 1.1. A simplified pulsed radar block diagram.

During each PRI the radar radiates energy only for τ seconds and listens for target returns for the rest of the PRI. The radar transmitting duty cycle (factor) d_t is defined as the ratio $d_t = \tau/T$. The radar average transmitted power is

$$P_{av} = P_t \times d_t, \quad (1.3)$$

where P_t denotes the radar peak transmitted power. The pulse energy is $E_p = P_t \tau = P_{av} T = P_{av} / f_r$.

The range corresponding to the two-way time delay T is known as the radar unambiguous range, R_u . Consider the case shown in Fig. 1.3. Echo 1 represents the radar return from a target at range $R_1 = c\Delta t/2$ due to pulse 1. Echo 2 could be interpreted as the return from the same target due to pulse 2, or it may be the return from a faraway target at range R_2 due to pulse 1 again. In this case,

$$R_2 = \frac{c\Delta t}{2} \quad \text{or} \quad R_2 = \frac{c(T + \Delta t)}{2} \quad (1.4)$$

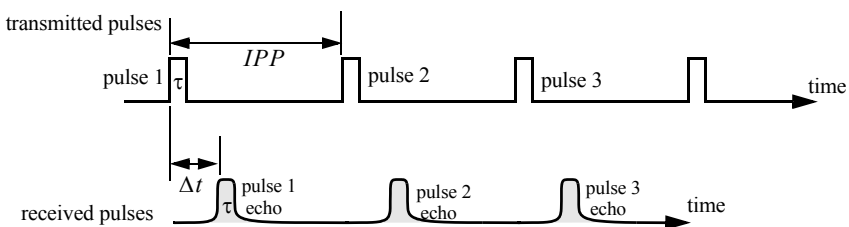


Figure 1.2. Train of transmitted and received pulses.

Clearly, range ambiguity is associated with echo 2. Therefore, once a pulse is transmitted the radar must wait a sufficient length of time so that returns from targets at maximum range are back before the next pulse is emitted. It follows that the maximum unambiguous range must correspond to half of the PRI,

$$R_u = c \frac{T}{2} = \frac{c}{2f_r} \quad (1.5)$$

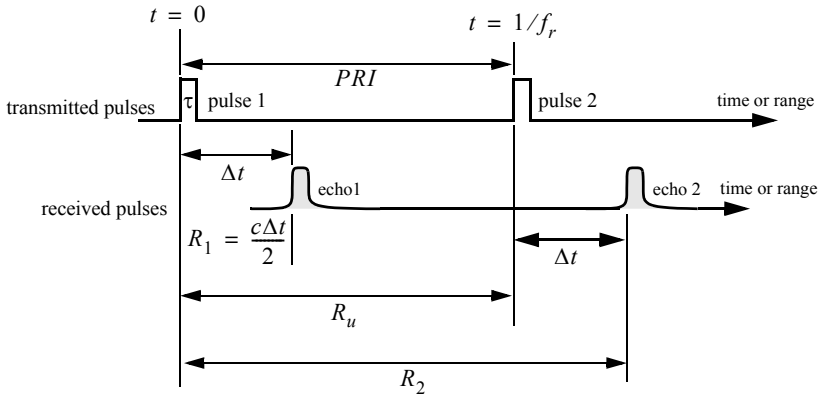


Figure 1.3. Illustrating range ambiguity.

1.3. Range Resolution

Range resolution, denoted as ΔR , is a radar metric that describes its ability to detect targets in close proximity to each other as distinct objects. Radar systems are normally designed to operate between a minimum range R_{min} , and maximum range R_{max} . The distance between R_{min} and R_{max} is divided into M range bins (gates), each of width ΔR ,

$$M = (R_{max} - R_{min}) / \Delta R \quad (1.6)$$

Targets separated by at least ΔR will be completely resolved in range. Targets within the same range bin can be resolved in cross range (azimuth) utilizing signal processing techniques. Consider two targets located at ranges R_1 and R_2 , corresponding to time delays t_1 and t_2 , respectively. Denote the difference between those two ranges as ΔR :

$$\Delta R = R_2 - R_1 = c \frac{(t_2 - t_1)}{2} = c \frac{\delta t}{2} \quad (1.7)$$

Now, try to answer the following question: What is the minimum δt such that target 1 at R_1 and target 2 at R_2 will appear completely resolved in range (different range bins)? In other words, what is the minimum ΔR ?

First, assume that the two targets are separated by $c\tau/4$, where τ is the pulsewidth. In this case, when the pulse trailing edge strikes target 2 the leading edge would have traveled backwards a distance $c\tau$, and the returned pulse would be composed of returns from both targets (i.e., unresolved return), as shown in Fig. 1.4a. However, if the two targets are at least $c\tau/2$ apart, then as the pulse trailing edge strikes the first target the leading edge will start to return from target 2, and two distinct returned pulses will be produced, as illustrated by Fig. 1.4b. Thus, ΔR should be greater or equal to $c\tau/2$. And since the radar bandwidth B is equal to $1/\tau$, then

$$\Delta R = \frac{c\tau}{2} = \frac{c}{2B} \tag{1.8}$$

In general, radar users and designers alike seek to minimize ΔR in order to enhance the radar performance. As suggested by Eq. (1.8), in order to achieve fine range resolution one must minimize the pulsewidth. However, this will reduce the average transmitted power and increase the operating bandwidth. Achieving fine range resolution while maintaining adequate average transmitted power can be accomplished by using pulse compression techniques.

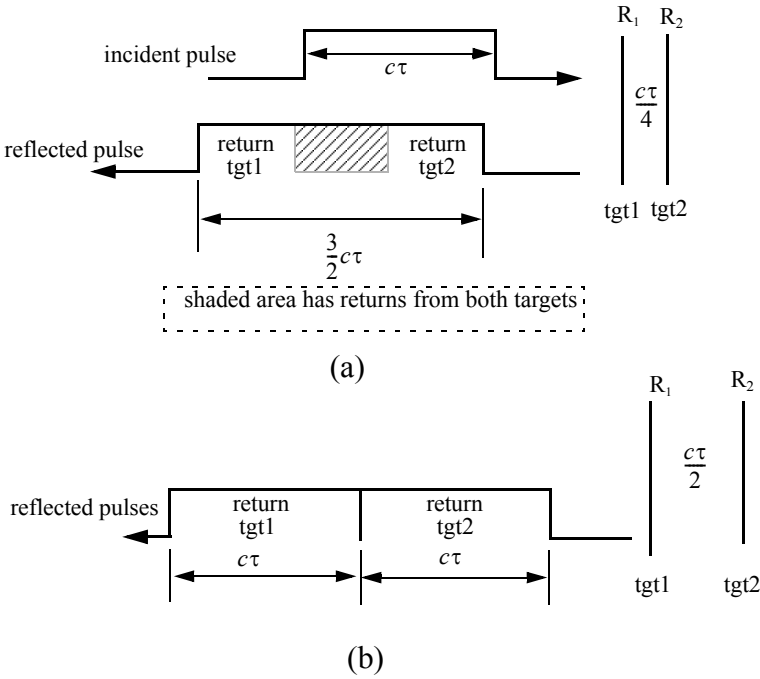


Figure 1.4. (a) Two unresolved targets. (b) Two resolved targets.

1.4. Doppler Frequency

Radars use Doppler frequency to extract target radial velocity (range rate), as well as to distinguish between moving and stationary targets or objects such as clutter. The Doppler phenomenon describes the shift in the center frequency of an incident waveform due to the target motion with respect to the source of radiation. Depending on the direction of the target's motion, this frequency shift may be positive or negative. A waveform incident on a target has equiphase wavefronts separated by λ , the wavelength. A closing target will cause the reflected equiphase wavefronts to get closer to each other (smaller wavelength). Alternatively, an opening or receding target (moving away from the radar) will cause the reflected equiphase wavefronts to expand (larger wavelength), as illustrated in Fig. 1.5.

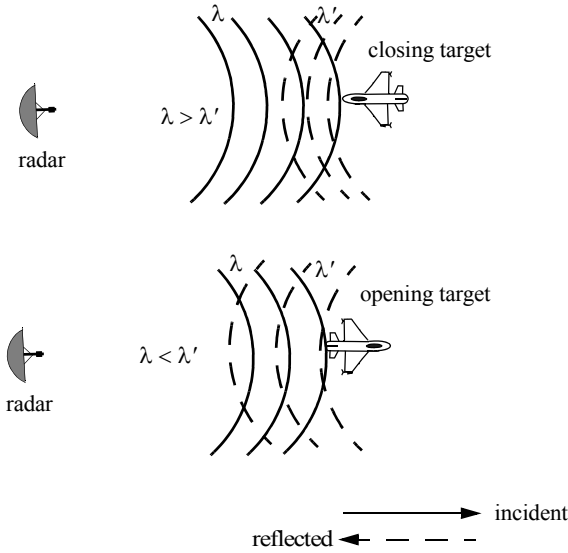


Figure 1.5. Effect of target motion on the reflected equiphase waveforms.

Consider a pulse of width τ (seconds) incident on a target which is moving towards the radar at velocity v , as shown in Fig. 1.6. Define d as the distance (in meters) that the target moves into the pulse during the interval Δt ,

$$d = v\Delta t \quad (1.9)$$

where Δt is equal to the time between the pulse leading edge striking the target and the trailing edge striking the target. Since the pulse is moving at the speed of light and the trailing edge has moved distance $c\tau - d$, then

$$c\tau = c\Delta t + v\Delta t \quad (1.10)$$

and

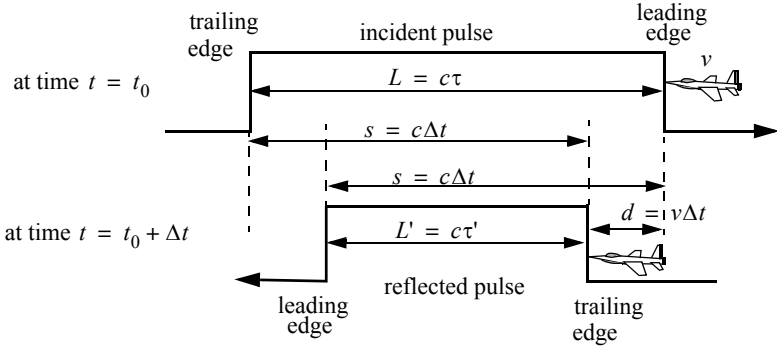


Figure 1.6. Illustrating the impact of target velocity on a single pulse.

$$c\tau' = c\Delta t - v\Delta t \quad (1.11)$$

Dividing Eq. (1.11) by Eq. (1.10) yields,

$$\frac{c\tau'}{c\tau} = \frac{c\Delta t - v\Delta t}{c\Delta t + v\Delta t} \quad (1.12)$$

which after canceling the terms c and Δt from the left and right side of Eq. (1.12) respectively, one establishes the relationship between the incident and reflected pulses widths as

$$\tau' = \frac{c - v}{c + v} \tau \quad (1.13)$$

In practice, the factor $(c - v)/(c + v)$ is often referred to as the time dilation factor. Notice that if $v = 0$, then $\tau' = \tau$. In a similar fashion, one can compute τ' for an opening target. In this case,

$$\tau' = \frac{v + c}{c - v} \tau \quad (1.14)$$

To derive an expression for Doppler frequency, consider the illustration shown in Fig. 1.7. It takes the leading edge of pulse 2 Δt seconds to travel a distance $(c/f_r) - d$ to strike the target. Over the same time interval, the leading edge of pulse 1 travels the same distance $c\Delta t$. More precisely,

$$d = v\Delta t \quad (1.15)$$

$$\frac{c}{f_r} - d = c\Delta t \quad (1.16)$$

solving for Δt yields

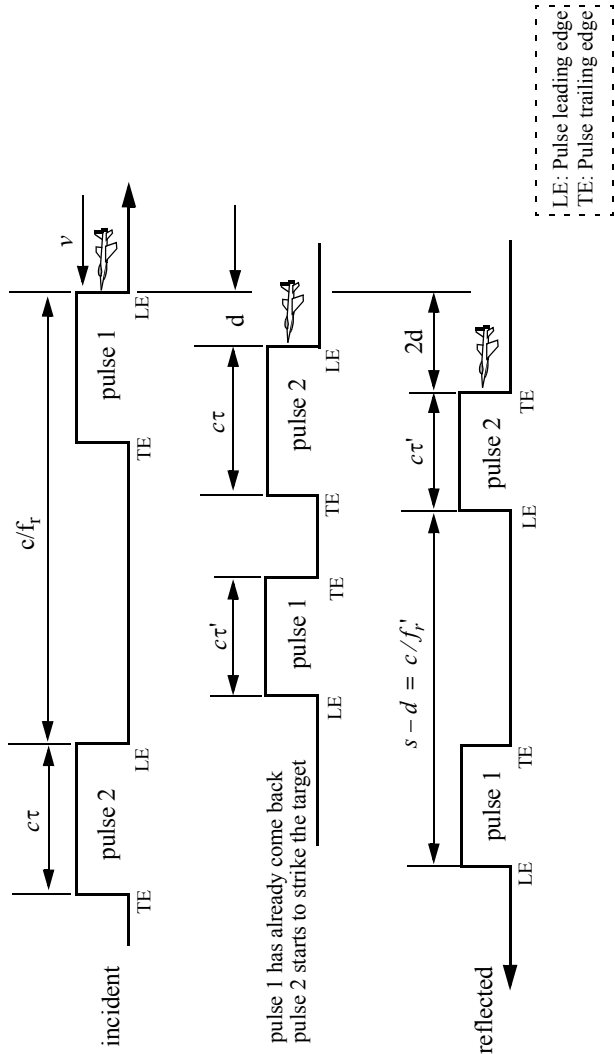


Figure 1.7. Illustration of target motion effects on the radar pulses.

$$\Delta t = \frac{c/f_r}{c+v} \quad (1.17)$$

$$d = \frac{cv/f_r}{c+v} \quad (1.18)$$

The reflected pulse spacing is now $s - d$ and the new PRF is f_r' , where

$$s - d = \frac{c}{f_r'} = c\Delta t - \frac{cv/f_r}{c+v} \quad (1.19)$$

It follows that the new PRF is related to the original PRF by

$$f_r' = \frac{c+v}{c-v} f_r \quad (1.20)$$

However, since the number of cycles does not change, the frequency of the reflected signal will go up by the same factor. Denoting the new frequency by f_0' , it follows

$$f_0' = \frac{c+v}{c-v} f_0 \quad (1.21)$$

where f_0 is the carrier frequency of the incident signal. The Doppler frequency f_d is defined as the difference $f_0' - f_0$. More precisely,

$$f_d = f_0' - f_0 = \frac{c+v}{c-v} f_0 - f_0 = \frac{2v}{c-v} f_0 \quad (1.22)$$

but since $v \ll c$ and $c = \lambda f_0$, then

$$f_d \approx \frac{2v}{c} f_0 = \frac{2v}{\lambda} \quad (1.23)$$

Eq. (1.23) indicates that the Doppler shift is proportional to the target velocity, and, thus, one can extract f_d from range rate and vice versa.

The result in Eq. (1.23) can also be derived using the following approach: Fig. 1.8 shows a closing target with velocity v . Let R_0 refer to the range at time t_0 (time reference); then the range to the target at any time t is

$$R(t) = R_0 - v(t - t_0) \quad (1.24)$$

The signal received by the radar is then given by

$$x_r(t) = x(t - \psi(t)) \quad (1.25)$$

where $x(t)$ is the transmitted signal, and

$$\psi(t) = \frac{2}{c}(R_0 - vt + vt_0) \quad (1.26)$$

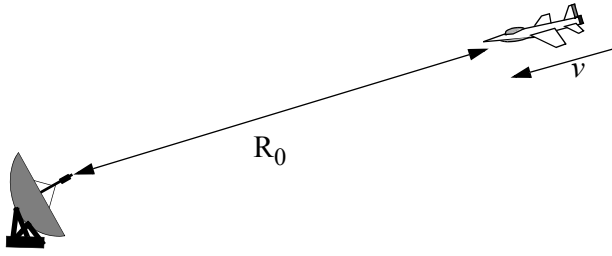


Figure 1.8. Closing target with velocity v .

Substituting Eq. (1.26) into Eq. (1.25) and collecting terms yield

$$x_r(t) = x\left(\left(1 + \frac{2v}{c}\right)t - \psi_0\right) \quad (1.27)$$

where the constant phase ψ_0 is

$$\psi_0 = \frac{2R_0}{c} + \frac{2v}{c} t_0 \quad (1.28)$$

Define the compression or scaling factor γ by

$$\gamma = 1 + \frac{2v}{c} \quad (1.29)$$

Note that for a receding target the scaling factor is $\gamma = 1 - (2v/c)$. Utilizing Eq. (1.29) we can rewrite Eq. (1.27) as

$$x_r(t) = x(\gamma t - \psi_0) \quad (1.30)$$

Eq. (1.30) is a time-compressed version of the return signal from a stationary target ($v = 0$). Hence, based on the scaling property of the Fourier transform, the spectrum of the received signal will be expanded in frequency to a factor of γ .

Consider the special case when

$$x(t) = y(t) \cos \omega_0 t \quad (1.31)$$

where ω_0 is the radar center frequency in radians per second. The received signal $x_r(t)$ is then given by

$$x_r(t) = y(\gamma t - \psi_0) \cos(\gamma \omega_0 t - \psi_0) \quad (1.32)$$

The Fourier transform of Eq. (1.32) is

$$X_r(\omega) = \frac{1}{2\gamma} \left(Y\left(\frac{\omega}{\gamma} - \omega_0\right) + Y\left(\frac{\omega}{\gamma} + \omega_0\right) \right) \quad (1.33)$$

where for simplicity the effects of the constant phase ψ_0 have been ignored in Eq. (1.33). Therefore, the bandpass spectrum of the received signal is now centered at $\gamma\omega_0$ instead of ω_0 . The difference between the two values corresponds to the amount of Doppler shift incurred due to the target motion,

$$\omega_d = \omega_0 - \gamma\omega_0 \quad (1.34)$$

ω_d is the Doppler frequency in radians per second. Substituting the value of γ in Eq. (1.34) and using $2\pi f = \omega$ yield

$$f_d = \frac{2v}{c} f_0 = \frac{2v}{\lambda} \quad (1.35)$$

which is the same as Eq. (1.23). It can be shown that for a receding target the Doppler shift is $f_d = -2v/\lambda$. This is illustrated in Fig. 1.9.

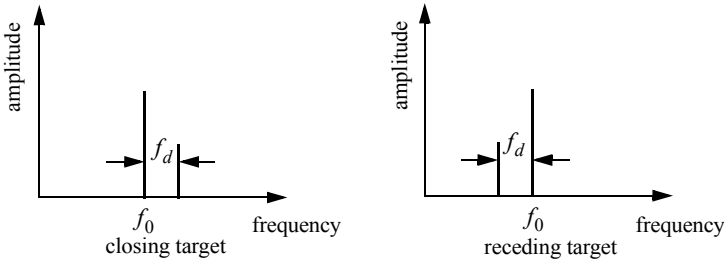


Figure 1.9. Spectra of received signal showing Doppler shift.

In both Eq. (1.35) and Eq. (1.23) the target radial velocity with respect to the radar is equal to v , but this is not always the case. In fact, the amount of Doppler frequency depends on the target velocity component in the direction of the radar (radial velocity). Fig. 1.10 shows three targets all having velocity v : target 1 has zero Doppler shift; target 2 has maximum Doppler frequency as defined in Eq. (1.35). The amount of Doppler frequency of target 3 is $f_d = 2v\cos\theta/\lambda$, where $v\cos\theta$ is the radial velocity; and θ is the total angle between the radar line of sight and the target.

Thus, a more general expression for f_d that accounts for the total angle between the radar and the target is

$$f_d = \frac{2v}{\lambda} \cos\theta \quad (1.36)$$

and for an opening target

$$f_d = \frac{-2v}{\lambda} \cos\theta \quad (1.37)$$

where $\cos\theta = \cos\theta_e \cos\theta_a$. The angles θ_e and θ_a are, respectively, the elevation and azimuth angles; see Fig. 1.11.

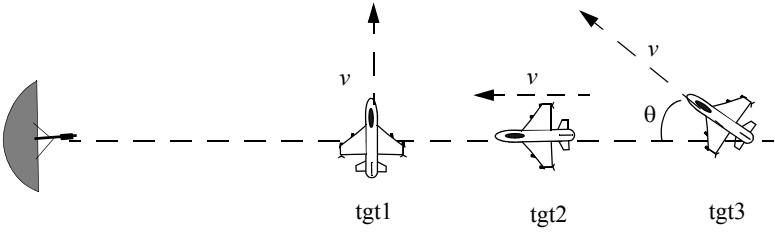


Figure 1.10. Target 1 generates zero Doppler. Target 2 generates maximum Doppler. Target 3 is in between.

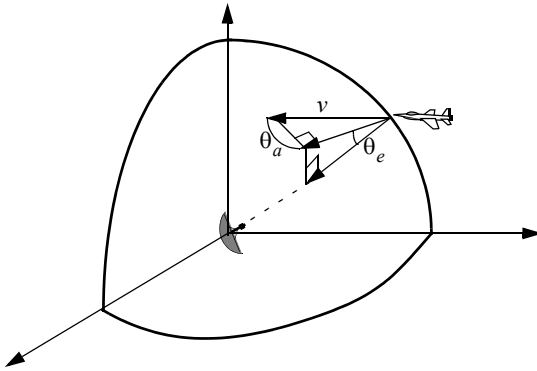


Figure 1.11. Radial velocity is proportional to the azimuth and elevation angles.

1.5. The Radar Equation

Consider a radar with an omni directional antenna (one that radiates energy equally in all directions). Since these kinds of antennas have a spherical radiation pattern, we can define the peak power density (power per unit area) at any point in space as

$$P_D = \frac{\text{Peak transmitted power}}{\text{area of a sphere}} \quad \frac{\text{watts}}{\text{m}^2} \quad (1.38)$$

The power density at range R away from the radar (assuming a lossless propagation medium) is

$$P_D = \frac{P_t}{4\pi R^2} \quad (1.39)$$

where P_t is the peak transmitted power and $4\pi R^2$ is the surface area of a sphere of radius R . Radar systems utilize directional antennas in order to

increase the power density in a certain direction. Directional antennas are usually characterized by the antenna gain G and the antenna effective aperture A_e . They are related by

$$G = \frac{4\pi A_e}{\lambda^2} \quad (1.40)$$

where λ is the wavelength. The relationship between the antenna's effective aperture A_e and the physical aperture A is

$$\begin{aligned} A_e &= \rho A \\ 0 &\leq \rho \leq 1 \end{aligned} \quad (1.41)$$

ρ is referred to as the aperture efficiency, and good antennas require $\rho \rightarrow 1$. In this book we will assume, unless otherwise noted, that A and A_e are the same. We will also assume that antennas have the same gain in the transmitting and receiving modes. In practice, $\rho = 0.7$ is widely accepted.

The gain is also related to the antenna's azimuth and elevation beamwidths by

$$G = k \frac{4\pi}{\theta_e \theta_a} \quad (1.42)$$

where $k \leq 1$ and depends on the physical aperture shape; the angles θ_e and θ_a are the antenna's elevation and azimuth beamwidths, respectively, in radians. An excellent approximation of Eq. (1.42) introduced by Stutzman and reported by Skolnik is

$$G \approx \frac{26000}{\theta_e \theta_a} \quad (1.43)$$

where in this case the azimuth and elevation beamwidths are given in degrees.

The power density at a distance R away from a radar using a directive antenna of gain G is then given by

$$P_D = \frac{P_t G}{4\pi R^2} \quad (1.44)$$

When the radar radiated energy impinges on a target, the induced surface currents on that target radiate electromagnetic energy in all directions. The amount of the radiated energy is proportional to the target size, orientation, physical shape, and material, which are all lumped together in one target-specific parameter called the Radar Cross Section (RCS) denoted by σ .

The radar cross section is defined as the ratio of the power reflected back to the radar to the power density incident on the target,

$$\sigma = \frac{P_r}{P_D} m^2 \quad (1.45)$$

where P_r is the power reflected from the target. Thus, the total power delivered to the radar signal processor by the antenna is

$$P_{Dr} = \frac{P_t G \sigma}{(4\pi R^2)^2} A_e \quad (1.46)$$

Substituting the value of A_e from Eq. (1.40) into Eq. (1.46) yields

$$P_{Dr} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \quad (1.47)$$

Let S_{min} denote the minimum detectable signal power. It follows that the maximum radar range R_{max} is

$$R_{max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}} \right)^{1/4} \quad (1.48)$$

Eq. (1.48) suggests that in order to double the radar maximum range one must increase the peak transmitted power P_t sixteen times; or equivalently, one must increase the effective aperture four times.

In practical situations the returned signals received by the radar will be corrupted with noise, which introduces unwanted voltages at all radar frequencies. Noise is random in nature and can be described by its Power Spectral Density (PSD) function. The noise power N is a function of the radar operating bandwidth, B . More precisely

$$N = \text{Noise PSD} \times B \quad (1.49)$$

The input noise power to a lossless antenna is

$$N_i = k T_e B \quad (1.50)$$

where $k = 1.38 \times 10^{-23}$ joule/degree Kelvin is Boltzman's constant, and T_e is the effective noise temperature in degrees Kelvin. It is always desirable that the minimum detectable signal (S_{min}) be greater than the noise power. The fidelity of a radar receiver is normally described by a figure of merit called the noise figure F (see Appendix 1B for details). The noise figure is defined as

$$F = \frac{(SNR)_i}{(SNR)_o} = \frac{S_i/N_i}{S_o/N_o} \quad (1.51)$$

$(SNR)_i$ and $(SNR)_o$ are, respectively, the Signal to Noise Ratios (SNR) at the input and output of the receiver. S_i is the input signal power; N_i is the input

noise power. S_o and N_o are, respectively, the output signal and noise power. Substituting Eq. (1.50) into Eq. (1.51) and rearranging terms yields

$$S_i = kT_e BF(SNR)_o \quad (1.52)$$

Thus, the minimum detectable signal power can be written as

$$S_{min} = kT_e BF(SNR)_{o_{min}} \quad (1.53)$$

The radar detection threshold is set equal to the minimum output SNR, $(SNR)_{o_{min}}$. Substituting Eq. (1.53) in Eq. (1.48) gives

$$R_{max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_e BF(SNR)_{o_{min}}} \right)^{1/4} \quad (1.54)$$

or equivalently,

$$(SNR)_{o_{min}} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_e BF R_{max}^4} \quad (1.55)$$

In general, radar losses denoted as L reduce the overall SNR, and hence

$$(SNR)_o = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_e BF L R^4} \quad (1.56)$$

Although it may take on many different forms, Eq. (1.56) is what is widely known as the Radar Equation. It is a common practice to perform calculations associated with the radar equation using decibel (dB) arithmetic. A review is presented in Appendix A.

MATLAB Function “radar_eq.m”

The function “*radar_eq.m*” implements Eq. (1.56); it is given in Listing 1.1 in Section 1.10. The syntax is as follows:

$$[snr] = \text{radar_eq}(pt, freq, g, sigma, te, b, nf, loss, range)$$

where

Symbol	Description	Units	Status
<i>pt</i>	<i>peak power</i>	<i>Watts</i>	<i>input</i>
<i>freq</i>	<i>radar center frequency</i>	<i>Hz</i>	<i>input</i>
<i>g</i>	<i>antenna gain</i>	<i>dB</i>	<i>input</i>
<i>sigma</i>	<i>target cross section</i>	<i>m²</i>	<i>input</i>
<i>te</i>	<i>effective noise temperature</i>	<i>Kelvin</i>	<i>input</i>

Symbol	Description	Units	Status
b	<i>bandwidth</i>	<i>Hz</i>	<i>input</i>
nf	<i>noise figure</i>	<i>dB</i>	<i>input</i>
$loss$	<i>radar losses</i>	<i>dB</i>	<i>input</i>
$range$	<i>target range (can be either a single value or a vector)</i>	<i>meters</i>	<i>input</i>
snr	<i>SNR (single value or a vector, depending on the input range)</i>	<i>dB</i>	<i>output</i>

The function “*radar_eq.m*” is designed such that it can accept a single value for the input “*range*”, or a vector containing many range values. Figure 1.12 shows some typical plots generated using MATLAB program “*fig1_12.m*” which is listed in Listing 1.2 in Section 1.10. This program uses the function “*radar_eq.m*”, with the following default inputs: Peak power $P_t = 1.5MW$, operating frequency $f_0 = 5.6GHz$, antenna gain $G = 45dB$, effective temperature $T_e = 290K$, radar losses $L = 6dB$, noise figure $F = 3dB$. The radar bandwidth is $B = 5MHz$. The radar minimum and maximum detection range are $R_{min} = 25Km$ and $R_{max} = 165Km$. Assume target cross section $\sigma = 0.1m^2$.

Note that one can easily modify the MATLAB function “*radar_eq.m*” so that it solves Eq. (1.54) for the maximum detection range as a function of the minimum required SNR for a given set of radar parameters. Alternatively, the radar equation can be modified to compute the pulsewidth required to achieve a certain SNR for a given detection range. In this case the radar equation can be written as

$$\tau = \frac{(4\pi)^3 k T_e F L R^4 SNR}{P_t G^2 \lambda^2 \sigma} \quad (1.57)$$

Figure 1.13 shows an implementation of Eq. (1.57) for three different detection range values, using the radar parameters used in MATLAB program “*fig1_13.m*”. It is given in Listing 1.3 in Section 1.10.

When developing radar simulations, Eq. (1.57) can be very useful in the following sense. Radar systems often utilize a finite number of pulsewidths (waveforms) to accomplish all designated modes of operations. Some of these waveforms are used for search and detection, others may be used for tracking, while a limited number of wideband waveforms may be used for discrimination purposes. During the search mode of operation, for example, detection of a certain target with a specific RCS value is established based on a pre-determined probability of detection P_D . The probability of detection, P_D , is used to calculate the required detection SNR (this will be addressed in Chapter 2).

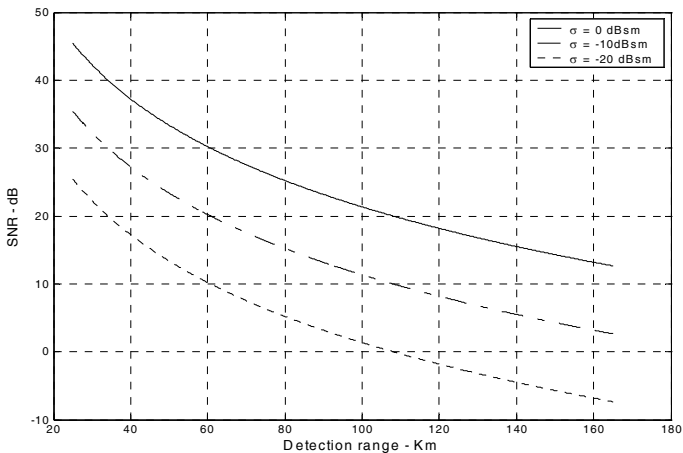


Figure 1.12a. SNR versus detection range for three different values of RCS.

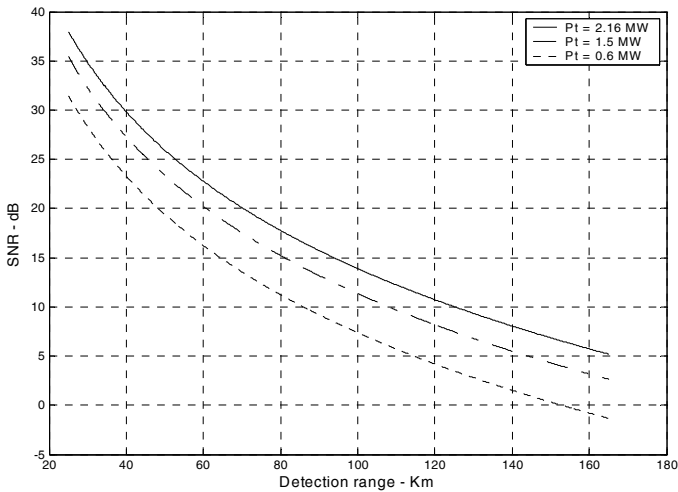


Figure 1.12b. SNR versus detection range for three different values of radar peak power.

Once the required SNR is computed, Eq. (1.57) can then be used to find the most suitable pulse (or waveform) that achieves the required SNR (or equivalently the required P_D). Often, it may be the case that none of the available radar waveforms may be able to guarantee the minimum required SNR for a particular RCS value at a particular detection range. In this case, the radar has to wait until the target is close enough in range to establish detection, otherwise pulse integration (coherent or non-coherent) can be used. Alternatively, cumulative probability of detection can be used. All these issues will be addressed in Chapter 2.

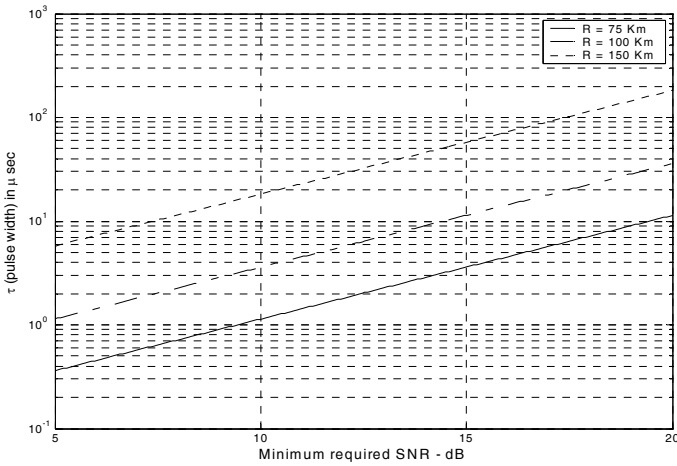


Figure 1.13. Pulsewidth versus required SNR for three different detection range values.

1.5.1. Radar Reference Range

Many radar design issues can be derived or computed based on the radar reference range R_{ref} which is often provided by the radar end user. It simply describes that range at which a certain SNR value, referred to as SNR_{ref} , has to be achieved using a specific reference pulsewidth τ_{ref} for a pre-determined target cross section, σ_{ref} . Radar reference range calculations assume that the target is on the line defined by the maximum antenna gain within a beam (broad side to the antenna). This is often referred to as the radar line of sight, as illustrated in Fig. 1.14.

The radar equation at the reference range is

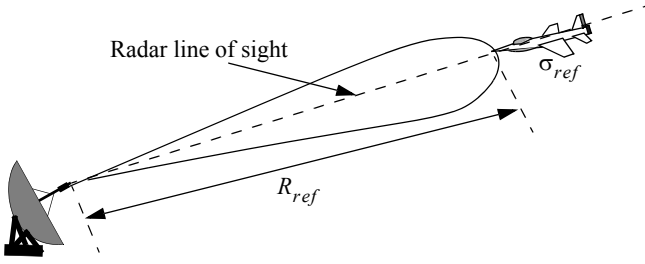


Figure 1.14. Definition of radar line of sight and radar reference range.

$$R_{ref} = \left(\frac{P_t G^2 \lambda^2 \sigma_{ref} \tau_{ref}}{(4\pi)^3 k T_e F L (SNR)_{ref}} \right)^{1/4} \quad (1.58)$$

The radar equation at any other detection range for any other combination of SNR, RCS, and pulsewidth can be given as

$$R = R_{ref} \left(\frac{\tau}{\tau_{ref}} \frac{\sigma}{\sigma_{ref}} \frac{SNR_{ref}}{SNR} \frac{1}{L_p} \right)^{1/4} \quad (1.59)$$

where the additional loss term L_p is introduced to account for the possibility that the non-reference target may not be on the radar line of sight, and to account for other losses associated with the specific scenario. Other forms of Eq. (1.59) can be in terms of the SNR. More precisely,

$$SNR = SNR_{ref} \frac{\tau}{\tau_{ref}} \frac{1}{L_p} \frac{\sigma}{\sigma_{ref}} \left(\frac{R_{ref}}{R} \right)^4 \quad (1.60)$$

As an example, consider the radar described in the previous section, in this case, define $\sigma_{ref} = 0.1 m^2$, $R_{ref} = 86 Km$, and $SNR_{ref} = 20 dB$. The reference pulsewidth is $\tau_{ref} = 0.1 \mu sec$. Using Eq. (1.60) we compute the SNR at $R = 120 Km$ for a target whose RCS is $\sigma = 0.2 m^2$. Assume that $L_p = 2 dB$ to be equal to $(SNR)_{120 Km} = 15.2 dB$. For this purpose, the MATLAB program “ref_snr.m” has been developed; it is given in Listing 1.4 in Section 1.10.

1.6. Search (Surveillance)

The first task a certain radar system has to accomplish is to continuously scan a specified volume in space searching for targets of interest. Once detection is established, target information such as range, angular position, and possibly target velocity are extracted by the radar signal and data processors. Depending on the radar design and antenna, different search patterns can be

adopted. A two-dimensional (2-D) fan beam search pattern is shown in Fig.1.15a. In this case, the beamwidth is wide enough in elevation to cover the desired search volume along that coordinate; however, it has to be steered in azimuth. Figure 1.15b shows a stacked beam search pattern; here the beam has to be steered in azimuth and elevation. This latter kind of search pattern is normally employed by phased array radars.

Search volumes are normally specified by a search solid angle Ω in steradians. Define the radar search volume extent for both azimuth and elevation as Θ_A and Θ_E . Consequently, the search volume is computed as

$$\Omega = (\Theta_A \Theta_E) / (57.296)^2 \text{ steradians} \quad (1.61)$$

where both Θ_A and Θ_E are given in degrees. The radar antenna 3dB beamwidth can be expressed in terms of its azimuth and elevation beamwidths θ_a and θ_e , respectively. It follows that the antenna solid angle coverage is $\theta_a \theta_e$ and, thus, the number of antenna beam positions n_B required to cover a solid angle Ω is

$$n_B = \frac{\Omega}{(\theta_a \theta_e) / (57.296)^2} \quad (1.62)$$

In order to develop the search radar equation, start with Eq. (1.56) which is repeated here, for convenience, as Eq. (1.63)

$$SNR = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_e B F L R^4} \quad (1.63)$$

Using the relations $\tau = 1/B$ and $P_t = P_{av} T / \tau$, where T is the PRI and τ is the pulsewidth, yields

$$SNR = \frac{T}{\tau} \frac{P_{av} G^2 \lambda^2 \sigma \tau}{(4\pi)^3 k T_e F L R^4} \quad (1.64)$$



Figure 1.15. (a) 2-D fan search pattern; (b) stacked search pattern.

Define the time it takes the radar to scan a volume defined by the solid angle Ω as the scan time T_{sc} . The time on target can then be expressed in terms of T_{sc} as

$$T_i = \frac{T_{sc}}{n_B} = \frac{T_{sc}}{\Omega} \theta_a \theta_e \quad (1.65)$$

Assume that during a single scan only one pulse per beam per PRI illuminates the target. It follows that $T_i = T$ and, thus, Eq. (1.64) can be written as

$$SNR = \frac{P_{av} G^2 \lambda^2 \sigma}{(4\pi)^3 k T_e FLR^4} \frac{T_{sc}}{\Omega} \theta_a \theta_e \quad (1.66)$$

Substituting Eqs. (1.40) and (1.42) into Eq. (1.66) and collecting terms yield the search radar equation (based on a single pulse per beam per PRI) as

$$SNR = \frac{P_{av} A_e \sigma}{4\pi k T_e FLR^4} \frac{T_{sc}}{\Omega} \quad (1.67)$$

The quantity $P_{av} A_e$ in Eq. (1.67) is known as the power aperture product. In practice, the power aperture product is widely used to categorize the radar's ability to fulfill its search mission. Normally, a power aperture product is computed to meet a predetermined SNR and radar cross section for a given search volume defined by Ω .

As a special case, assume a radar using a circular aperture (antenna) with diameter D . The 3-dB antenna beamwidth θ_{3dB} is

$$\theta_{3dB} \approx \frac{\lambda}{D} \quad (1.68)$$

and when aperture tapering is used, $\theta_{3dB} \approx 1.25\lambda/D$. Substituting Eq. (1.68) into Eq. (1.62) yields

$$n_B = \frac{D^2}{\lambda^2} \Omega \quad (1.69)$$

For this case, the scan time T_{sc} is related to the time-on-target by

$$T_i = \frac{T_{sc}}{n_B} = \frac{T_{sc} \lambda^2}{D^2 \Omega} \quad (1.70)$$

Substitute Eq. (1.70) into Eq. (1.64) to get

$$SNR = \frac{P_{av} G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_e FL D^2 \Omega} \frac{T_{sc} \lambda^2}{D^2 \Omega} \quad (1.71)$$

and by using Eq. (1.40) in Eq. (1.71) we can define the search radar equation for a circular aperture as

$$SNR = \frac{P_{av} A \sigma}{16 R^4 k T_e L F \Omega} \frac{T_{sc}}{\Omega} \quad (1.72)$$

where the relation $A = \pi D^2 / 4$ (aperture area) is used.

MATLAB Function “power_aperture.m”

The function “power_aperture.m” implements the search radar equation given in Eq. (1.67); it is given in Listing 1.5 in Section 1.10. The syntax is as follows:

$PAP = \text{power_aperture}(\text{snr}, \text{tsc}, \text{sigma}, \text{range}, \text{te}, \text{nf}, \text{loss}, \text{az_angle}, \text{el_angle})$

where

Symbol	Description	Units	Status
<i>snr</i>	<i>sensitivity snr</i>	<i>dB</i>	<i>input</i>
<i>tsc</i>	<i>scan time</i>	<i>seconds</i>	<i>input</i>
<i>sigma</i>	<i>target cross section</i>	<i>m²</i>	<i>input</i>
<i>range</i>	<i>target range (can be either single value or a vector)</i>	<i>meters</i>	<i>input</i>
<i>te</i>	<i>effective temperature</i>	<i>Kelvin</i>	<i>input</i>
<i>nf</i>	<i>noise figure</i>	<i>dB</i>	<i>input</i>
<i>loss</i>	<i>radar losses</i>	<i>dB</i>	<i>input</i>
<i>az_angle</i>	<i>search volume azimuth extent</i>	<i>degrees</i>	<i>input</i>
<i>el_angle</i>	<i>search volume elevation extent</i>	<i>degrees</i>	<i>input</i>
<i>PAP</i>	<i>power aperture product</i>	<i>dB</i>	<i>output</i>

Plots of the power aperture product versus range and plots of the average power versus aperture area for three RCS choices are shown in Figure 1.16. MATLAB program “fig1_16.m” was used to produce these figures. It is given in Listing 1.6 in Section 1.10. In this case, the following radar parameters were used

σ	T_{sc}	$\theta_e = \theta_a$	R	T_e	$nf \times loss$	snr
0.1 m^2	2.5 sec	2°	250 Km	900 K	13 dB	15 dB

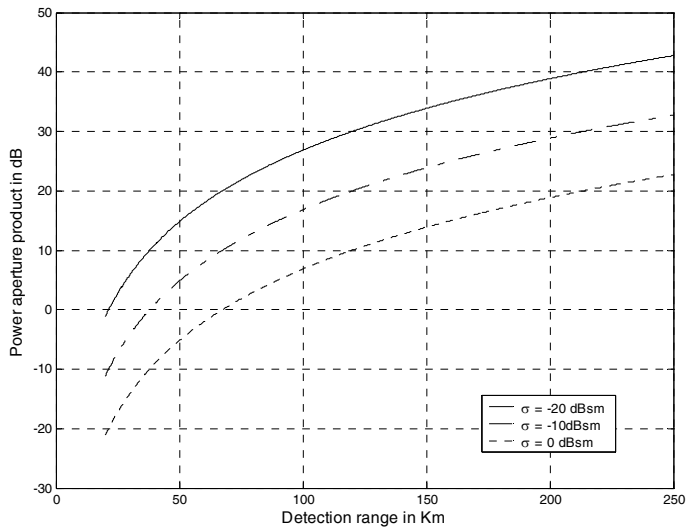


Figure 1.16a. Power aperture product versus detection range.

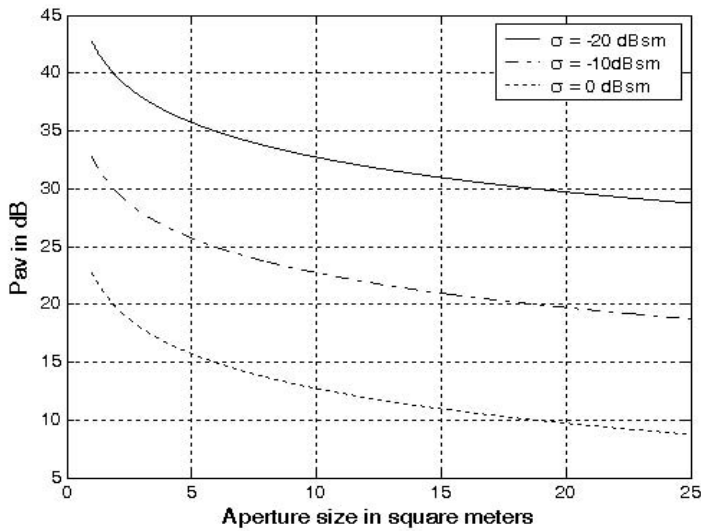


Figure 1.16b. Radar average power versus power aperture product.

Example:

Compute the power aperture product corresponding to the radar that has the following parameters: Scan time $T_{sc} = 2\text{sec}$, Noise figure $F = 8\text{dB}$, losses $L = 6\text{dB}$, search volume $\Omega = 7.4$ steradians, range of interest is $R = 75\text{Km}$, and the required SNR is 20dB . Assume that $T_e = 290\text{Kelvin}$ and $\sigma = 3.162\text{m}^2$.

Solution:

Note that $\Omega = 7.4$ steradians corresponds to a search sector that is three fourths of a hemisphere. Thus, using Eq. (1.61) we conclude that $\theta_a = 180^\circ$ and $\theta_e = 135^\circ$. Using the MATLAB function “power_aperture.m” with the following syntax:

$\text{PAP} = \text{power_aperture}(20, 2, 3.162, 75\text{e}3, 290, 8, 6, 180, 135)$

we compute the power aperture product as 36.7 dB .

1.6.1. Mini Design Case Study 1.1**Problem Statement:**

Design a ground based radar that is capable of detecting aircraft and missiles at 10 Km and 2 Km altitudes, respectively. The maximum detection range for either target type is 60 Km . Assume that an aircraft average RCS is 6 dBsm , and that a missile average RCS is -10 dBsm . The radar azimuth and elevation search extents are respectively $\Theta_A = 360^\circ$ and $\Theta_E = 10^\circ$. The required scan rate is 2 seconds and the range resolution is 150 meters . Assume a noise figure $F = 8\text{ dB}$, and total receiver noise $L = 10\text{ dB}$. Use a fan beam with azimuth beamwidth less than 3 degrees . The SNR is 15 dB .

A Design:

The range resolution requirement is $\Delta R = 150\text{m}$; thus by using Eq. (1.8) we calculate the required pulsewidth $\tau = 1\mu\text{sec}$, or equivalently require the bandwidth $B = 1\text{MHz}$. The statement of the problem lends itself to radar sizing in terms of power aperture product. For this purpose, one must first compute the maximum search volume at the detection range that satisfies the design requirements. The radar search volume is

$$\Omega = \frac{\Theta_A \Theta_E}{(57.296)^2} = \frac{360 \times 10}{(57.296)^2} = 1.097 \text{ steradians} \quad (1.73)$$

At this point, the designer is ready to use the radar search equation (Eq. (1.67)) to compute the power aperture product. For this purpose, one can mod-

ify the MATLAB function “power_aperture.m” to compute and plot the power aperture product for both target types. To this end, the MATLAB program “casestudy1_1.m”, which is given in Listing 1.7 in Section 1.10, was developed. Use the parameters in Table 1.2 as inputs for this program. Note that the selection of $T_e = 290\text{Kelvin}$ is arbitrary.

TABLE 1.2: Input parameters to MATLAB program “casestudy1_1.m”.

Symbol	Description	Units	Value
<i>snr</i>	<i>sensitivity snr</i>	<i>dB</i>	<i>15</i>
<i>tsc</i>	<i>scan time</i>	<i>seconds</i>	<i>2</i>
<i>sigma_tgtm</i>	<i>missile radar cross section</i>	<i>dBsm</i>	<i>-10</i>
<i>sigma_tgta</i>	<i>aircraft radar cross section</i>	<i>dBsm</i>	<i>6</i>
<i>rangem</i>	<i>missile detection range</i>	<i>Km</i>	<i>60</i>
<i>rangea</i>	<i>aircraft detection range</i>	<i>Km</i>	<i>60</i>
<i>te</i>	<i>effective temperature</i>	<i>Kelvin</i>	<i>290</i>
<i>nf</i>	<i>noise figure</i>	<i>dB</i>	<i>8</i>
<i>loss</i>	<i>radar losses</i>	<i>dB</i>	<i>10</i>
<i>az_angle</i>	<i>search volume azimuth extent</i>	<i>degrees</i>	<i>360</i>
<i>el_angle</i>	<i>search volume elevation extent</i>	<i>degrees</i>	<i>10</i>

Figure 1.17 shows a plot of the output produced by this program. The same program also calculates the corresponding power aperture product for both the missile and aircraft cases, which can also be read from the plot,

$$\begin{aligned} PAP_{\text{missile}} &= 38.53\text{dB} \\ PAP_{\text{aircraft}} &= 22.53\text{dB} \end{aligned} \quad (1.74)$$

Choosing the more stressing case for the design baseline (i.e., select the power-aperture-product resulting from the missile analysis) yields

$$P_{av} \times A_e = 10^{3.853} = 7128.53 \Rightarrow A_e = \frac{7128.53}{P_{av}} \quad (1.75)$$

Choose $A_e = 1.75\text{m}^2$ to calculate the average power as

$$P_{av} = \frac{7128.53}{1.75} = 4.073\text{KW} \quad (1.76)$$

and assuming an aperture efficiency of $\rho = 0.8$ yields the physical aperture area. More precisely,

$$A = \frac{A_e}{\rho} = \frac{1.75}{0.8} = 2.1875\text{m}^2 \quad (1.77)$$

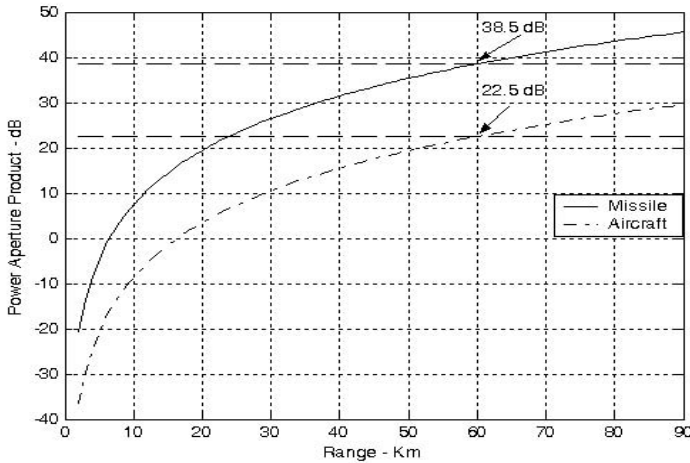


Figure 1.17. Power aperture product versus detection range for radar in mini design case study 1.1.

Use $f_0 = 2.0\text{GHz}$ as the radar operating frequency. Then by using $A_e = 1.75\text{m}^2$ we calculate using Eq. (1.40) $G = 29.9\text{dB}$. Now one must determine the antenna azimuth beamwidth. Recall that the antenna gain is also related to the antenna 3-dB beamwidth by the relation

$$G = \frac{26000}{\theta_e \theta_a} \quad (1.78)$$

where (θ_a, θ_e) are the antenna 3-dB azimuth and elevation beamwidths, respectively. Assume a fan beam with $\theta_e = \Theta_E = 15^\circ$. It follows that

$$\theta_a = \frac{26000}{\theta_e G} = \frac{26000}{10 \times 977.38} = 2.66^\circ \Rightarrow \theta_a = 46.43\text{mrad} \quad (1.79)$$

1.7. Pulse Integration

When a target is located within the radar beam during a single scan it may reflect several pulses. By adding the returns from all pulses returned by a given target during a single scan, the radar sensitivity (SNR) can be increased. The number of returned pulses depends on the antenna scan rate and the radar PRF. More precisely, the number of pulses returned from a given target is given by

$$n_P = \frac{\theta_a T_{sr} f_r}{2\pi} \quad (1.80)$$

where θ_a is the azimuth antenna beamwidth, T_{sc} is the scan time, and f_r is the radar PRF. The number of reflected pulses may also be expressed as

$$n_p = \frac{\theta_a f_r}{\dot{\theta}_{scan}} \quad (1.81)$$

where $\dot{\theta}_{scan}$ is the antenna scan rate in degrees per second. Note that when using Eq. (1.80), θ_a is expressed in radians, while when using Eq. (1.81) it is expressed in degrees. As an example, consider a radar with an azimuth antenna beamwidth $\theta_a = 3^\circ$, antenna scan rate $\dot{\theta}_{scan} = 45^\circ/\text{sec}$ (antenna scan time, $T_{sc} = 8 \text{ seconds}$), and a PRF $f_r = 300 \text{ Hz}$. Using either Eq.s (1.80) or (1.81) yields $n_p = 20$ pulses.

The process of adding radar returns from many pulses is called radar pulse integration. Pulse integration can be performed on the quadrature components prior to the envelope detector. This is called coherent integration or pre-detection integration. Coherent integration preserves the phase relationship between the received pulses. Thus a build up in the signal amplitude is achieved. Alternatively, pulse integration performed after the envelope detector (where the phase relation is destroyed) is called non-coherent or post-detection integration.

Radar designers should exercise caution when utilizing pulse integration for the following reasons. First, during a scan a given target will not always be located at the center of the radar beam (i.e., have maximum gain). In fact, during a scan a given target will first enter the antenna beam at the 3-dB point, reach maximum gain, and finally leave the beam at the 3-dB point again. Thus, the returns do not have the same amplitude even though the target RCS may be constant and all other factors which may introduce signal loss remain the same. This is illustrated in Fig. 1.18, and is normally referred to as antenna beam-shape loss.

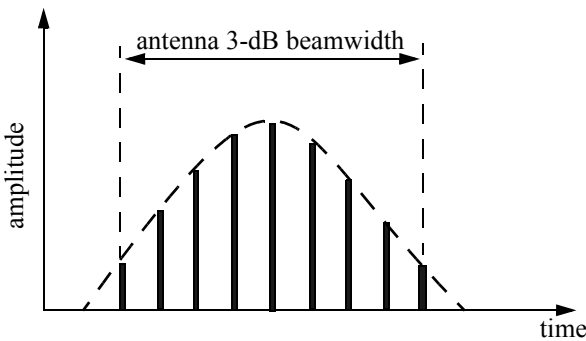


Figure 1.18. Pulse returns from a point target using a rotating (scanning) antenna

Other factors that may introduce further variation to the amplitude of the returned pulses include target RCS and propagation path fluctuations. Additionally, when the radar employs a very fast scan rate, an additional loss term is introduced due to the motion of the beam between transmission and reception. This is referred to as scan loss. A distinction should be made between scan loss due to a rotating antenna (which is described here) and the term scan loss that is normally associated with phased array antennas (which takes on a different meaning in that context). These topics will be discussed in more detail in other chapters.

Finally, since coherent integration utilizes the phase information from all integrated pulses, it is critical that any phase variation between all integrated pulses be known with a great level of confidence. Consequently, target dynamics (such as target range, range rate, tumble rate, RCS fluctuation, etc.) must be estimated or computed accurately so that coherent integration can be meaningful. In fact, if a radar coherently integrates pulses from targets without proper knowledge of the target dynamics it suffers a loss in SNR rather than the expected SNR build up. Knowledge of target dynamics is not as critical when employing non-coherent integration; nonetheless, target range rate must be estimated so that only the returns from a given target within a specific range bin are integrated. In other words, one must avoid range walk (i.e., avoid having a target cross between adjacent range bins during a single scan).

A comprehensive analysis of pulse integration should take into account issues such as the probability of detection P_D , probability of false alarm P_{fa} , the target statistical fluctuation model, and the noise or interference statistical models. These topics will be discussed in Chapter 2. However, in this section an overview of pulse integration is introduced in the context of radar measurements as it applies to the radar equation. The basic conclusions presented in this chapter concerning pulse integration will still be valid, in the general sense, when a more comprehensive analysis of pulse integration is presented; however, the exact implementation, the mathematical formulation, and /or the numerical values used will vary.

1.7.1. Coherent Integration

In coherent integration, when a perfect integrator is used (100% efficiency), to integrate n_p pulses the SNR is improved by the same factor. Otherwise, integration loss occurs, which is always the case for non-coherent integration. Coherent integration loss occurs when the integration process is not optimum. This could be due to target fluctuation, instability in the radar local oscillator, or propagation path changes.

Denote the single pulse SNR required to produce a given probability of detection as $(SNR)_1$. The SNR resulting from coherently integrating n_p pulses is then given by

$$(SNR)_{CI} = n_p(SNR)_1 \quad (1.82)$$

Coherent integration cannot be applied over a large number of pulses, particularly if the target RCS is varying rapidly. If the target radial velocity is known and no acceleration is assumed, the maximum coherent integration time is limited to

$$t_{CI} = \sqrt{\lambda/2a_r} \quad (1.83)$$

where λ is the radar wavelength and a_r is the target radial acceleration. Coherent integration time can be extended if the target radial acceleration can be compensated for by the radar.

1.7.2. Non-Coherent Integration

Non-coherent integration is often implemented after the envelope detector, also known as the quadratic detector. Non-coherent integration is less efficient than coherent integration. Actually, the non-coherent integration gain is always smaller than the number of non-coherently integrated pulses. This loss in integration is referred to as post detection or square law detector loss. Marcum and Swerling showed that this loss is somewhere between $\sqrt{n_p}$ and n_p . DiFranco and Rubin presented an approximation of this loss as

$$L_{NCI} = 10\log(\sqrt{n_p}) - 5.5 \text{ dB} \quad (1.84)$$

Note that as n_p becomes very large, the integration loss approaches $\sqrt{n_p}$.

The subject of integration loss is treated in great levels of detail in the literature. Different authors use different approximations for the integration loss associated with non-coherent integration. However, all these different approximations yield very comparable results. Therefore, in the opinion of these authors the use of one formula or another to approximate integration loss becomes somewhat subjective. In this book, the integration loss approximation reported by Barton and used by Curry will be adopted. In this case, the non-coherent integration loss which can be used in the radar equation is

$$L_{NCI} = \frac{1 + (SNR)_1}{(SNR)_1} \quad (1.85)$$

It follows that the SNR when n_p pulses are integrated non-coherently is

$$(SNR)_{NCI} = \frac{n_p(SNR)_1}{L_{NCI}} = n_p(SNR)_1 \times \frac{(SNR)_1}{1 + (SNR)_1} \quad (1.86)$$

1.7.3. Detection Range with Pulse Integration

The process of determining the radar sensitivity or equivalently the maximum detection range when pulse integration is used is as follows: First, decide

whether to use coherent or non-coherent integration. Keep in mind the issues discussed in the beginning of this section when deciding whether to use coherent or non-coherent integration.

Second, determine the minimum required $(SNR)_{CI}$ or $(SNR)_{NCI}$ required for adequate detection and track. Typically, for ground based surveillance radars that can be on the order of 13 to 15 dB. The third step is to determine how many pulses should be integrated. The choice of n_p is affected by the radar scan rate, the radar PRF, the azimuth antenna beamwidth, and of course by the target dynamics (remember that range walk should be avoided or compensated for, so that proper integration is feasible). Once n_p and the required SNR are known one can compute the single pulse SNR (i.e., the reduction in SNR). For this purpose use Eq. (1.82) in the case of coherent integration. In the non-coherent integration case, Curry presents an attractive formula for this calculation, as follows

$$(SNR)_1 = \frac{(SNR)_{NCI}}{2n_p} + \sqrt{\frac{(SNR)_{NCI}^2}{4n_p^2} + \frac{(SNR)_{NCI}}{n_p}} \quad (1.87)$$

Finally, use $(SNR)_1$ from Eq. (1.87) in the radar equation to calculate the radar detection range. Observe that due to the integration reduction in SNR the radar detection range is now larger than that for the single pulse when the same SNR value is used. This is illustrated using the following mini design case study.

1.7.4. Mini Design Case Study 1.2

Problem Statement:

A MMW radar has the following specifications: Center frequency $f = 94\text{GHz}$, pulsewidth $\tau = 50 \times 10^{-9}\text{sec}$, peak power $P_t = 4\text{W}$, azimuth coverage $\Delta\alpha = \pm 120^\circ$, Pulse repetition frequency $\text{PRF} = 10\text{KHz}$, noise figure $F = 7\text{dB}$; antenna diameter $D = 12\text{in}$; antenna gain $G = 47\text{dB}$; radar cross section of target is $\sigma = 20\text{m}^2$; system losses $L = 10\text{dB}$; radar scan time $T_{sc} = 3\text{sec}$. Calculate: The wavelength λ ; range resolution ΔR ; bandwidth B ; antenna half power beamwidth; antenna scan rate; time on target. Compute the range that corresponds to 10 dB SNR. Plot the SNR as a function of range. Finally, compute the number of pulses on the target that can be used for integration and the corresponding new detection range when pulse integration is used, assuming that the SNR stays unchanged (i.e., the same as in the case of a single pulse). Assume $T_e = 290\text{ Kelvin}$.

A Design:

The wavelength λ is

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{94 \times 10^9} = 0.00319m$$

The range resolution ΔR is

$$\Delta R = \frac{c\tau}{2} = \frac{(3 \times 10^8)(50 \times 10^{-9})}{2} = 7.5m$$

Radar operating bandwidth B is

$$B = \frac{1}{\tau} = \frac{1}{50 \times 10^{-9}} = 20MHz$$

The antenna 3-dB beamwidth is

$$\theta_{3dB} = 1.25 \frac{\lambda}{D} = 0.7499^\circ$$

Time on target is

$$T_i = \frac{\theta_{3dB}}{\dot{\theta}_{scan}} = \frac{0.7499^\circ}{80^\circ/\text{sec}} = 9.38msec$$

It follows that the number of pulses available for integration is calculated using Eq. (1.81),

$$n_p = \frac{\theta_{3dB}}{\dot{\theta}_{scan}} f_r = 9.38 \times 10^{-3} \times 10 \times 10^3 \Rightarrow 94 \text{ pulses}$$

Coherent Integration case:

Using the radar equation given in Eq. (1.58) yields $R_{ref} = 2.245Km$. The SNR improvement due to coherently integrating 94 pulses is 19.73dB. However, since it is requested that the SNR remains at 10dB, we can calculate the new detection range using Eq. (1.59) as

$$R_{CI}|_{n_p=94} = 2.245 \times (94)^{1/4} = 6.99Km$$

Using the MATLAB Function “radar_eq.m” with the following syntax

$$[snr] = \text{radar_eq}(4, 94e9, 47, 20, 290, 20e6, 7, 10, 6.99e3)$$

yields SNR = -9.68 dB. This means that using 94 pulses integrated coherently at 6.99 Km where each pulse has a SNR of -9.68 dB provides the same detection criteria as using a single pulse with SNR = 10dB at 2.245Km. This is illustrated in Fig. 1.19, using the MATLAB program “fig1_19.m”, which is given in Listing 1.8 in Section 1.10. Figure 1.19 shows the improvement of the detection range if the SNR is kept constant before and after integration.

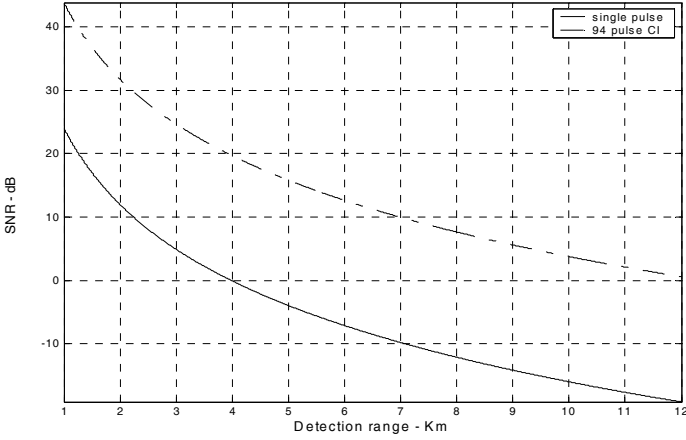


Figure 1.19. SNR versus detection range, using parameters from example.

Non-coherent Integration case:

Start with Eq. (1.87) with $(SNR)_{NCI} = 10\text{dB}$ and $n_p = 94$,

$$(SNR)_1 = \frac{10}{2 \times 94} + \sqrt{\frac{(10)^2}{4 \times 94^2} + \frac{10}{94}} = 0.38366 \Rightarrow -4.16\text{dB}$$

Therefore, the single pulse SNR when 94 pulses are integrated non-coherently is -4.16dB . You can verify this result by using Eq. (1.86). The integration loss L_{NCI} is calculated using Eq. (1.85). It is

$$L_{NCI} = \frac{1 + 0.38366}{0.38366} = 3.6065 \Rightarrow 5.571\text{dB}$$

Therefore, the net non-coherent integration gain is

$$10 \times \log(94) - 5.571 = 14.16\text{dB} \Rightarrow 26.06422$$

and, consequently, the maximum detection range is

$$R_{NCI}|_{n_p=94} = 2.245 \times (26.06422)^{1/4} = 5.073\text{Km}$$

This means that using 94 pulses integrated non-coherently at 5.073 Km where each pulse has SNR of -4.16dB provides the same detection criterion as using a single pulse with $SNR = 10\text{dB}$ at 2.245Km. This is illustrated in Fig. 1.20, using the MATLAB program “fig1_19.m”.

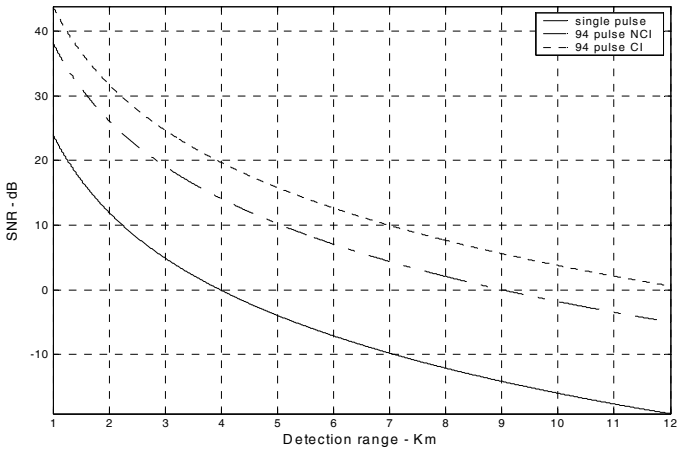


Figure 1.20. SNR versus detection range, for the same example.

MATLAB Function “pulse_integration.m”

Figure 1.21 shows the SNR gain versus the number of integrated pulses for both coherent and non-coherent integration. This figure corresponds to parameters from the previous example at $R = 5.01\text{Km}$. Figure 1.22 shows the general case SNR improvement versus number of integrated pulses. Both figures were generated using MATLAB program “fig1_21.m” which is given in Listing 1.9 in Section 1.10. For this purpose the MATLAB function “pulse_integration.m” was developed. It is given in Listing 1.10 in Section 1.10. This function calculates the radar equation given in Eq. (1.56) with pulse integration. The syntax for MATLAB function “pulse_integration.m” is as follows

$[snr] = pulse_integration (pt, freq, g, sigma, te, b, nf, loss, range, np, ci_nci)$

where

Symbol	Description	Units	Status
<i>pt</i>	<i>peak power</i>	<i>Watts</i>	<i>input</i>
<i>freq</i>	<i>radar center frequency</i>	<i>Hz</i>	<i>input</i>
<i>g</i>	<i>antenna gain</i>	<i>dB</i>	<i>input</i>
<i>sigma</i>	<i>target cross section</i>	<i>m²</i>	<i>input</i>
<i>te</i>	<i>effective noise temperature</i>	<i>Kelvin</i>	<i>input</i>
<i>b</i>	<i>bandwidth</i>	<i>Hz</i>	<i>input</i>

Symbol	Description	Units	Status
<i>nf</i>	<i>noise figure</i>	<i>dB</i>	<i>input</i>
<i>loss</i>	<i>radar losses</i>	<i>dB</i>	<i>input</i>
<i>range</i>	<i>target range (can be either a single value or a vector)</i>	<i>meters</i>	<i>input</i>
<i>np</i>	<i>number of integrated pulses</i>	<i>none</i>	<i>input</i>
<i>ci_nci</i>	<i>1 for CI; 2 for NCI</i>	<i>none</i>	<i>input</i>
<i>snr</i>	<i>SNR (single value or a vector, depending on the input range)</i>	<i>dB</i>	<i>output</i>

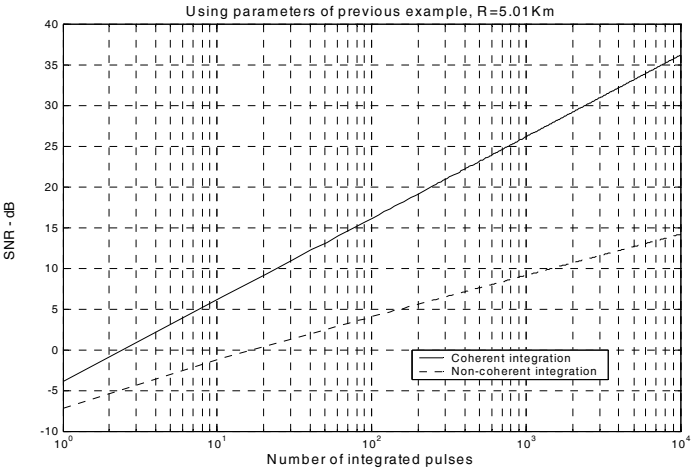


Figure 1.21. SNR improvement when integration is utilized.

1.8. Radar Losses

As indicated by the radar equation, the receiver SNR is inversely proportional to the radar losses. Hence, any increase in radar losses causes a drop in the SNR, thus decreasing the probability of detection, as it is a function of the SNR. Often, the principal difference between a good radar design and a poor radar design is the radar losses. Radar losses include ohmic (resistance) losses and statistical losses. In this section we will briefly summarize radar losses.

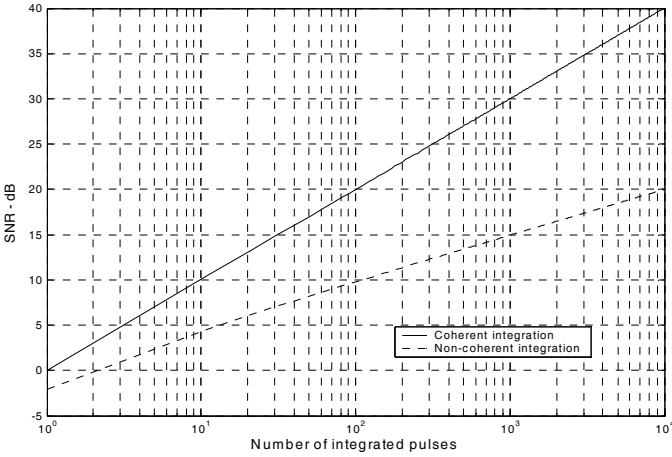


Figure 1.22. SNR improvement when integration is utilized.

1.8.1. Transmit and Receive Losses

Transmit and receive losses occur between the radar transmitter and antenna input port, and between the antenna output port and the receiver front end, respectively. Such losses are often called plumbing losses. Typically, plumbing losses are on the order of 1 to 2 dB.

1.8.2. Antenna Pattern Loss and Scan Loss

So far, when we used the radar equation we assumed maximum antenna gain. This is true only if the target is located along the antenna's boresight axis. However, as the radar scans across a target the antenna gain in the direction of the target is less than maximum, as defined by the antenna's radiation pattern. The loss in SNR due to not having maximum antenna gain on the target at all times is called the antenna pattern (shape) loss. Once an antenna has been selected for a given radar, the amount of antenna pattern loss can be mathematically computed.

For example, consider a $\sin x/x$ antenna radiation pattern as shown in Fig. 1.23. It follows that the average antenna gain over an angular region of $\pm\theta/2$ about the boresight axis is

$$G_{av} \approx 1 - \left(\frac{\pi r}{\lambda}\right)^2 \frac{\theta^2}{36} \quad (1.88)$$

where r is the aperture radius and λ is the wavelength. In practice, Gaussian antenna patterns are often adopted. In this case, if θ_{3dB} denotes the antenna 3dB beamwidth, then the antenna gain can be approximated by

$$G(\theta) = \exp\left(-\frac{2.776\theta^2}{\theta_{3dB}^2}\right) \quad (1.89)$$

If the antenna scanning rate is so fast that the gain on receive is not the same as on transmit, additional scan loss has to be calculated and added to the beam shape loss. Scan loss can be computed in a similar fashion to beam shape loss. Phased array radars are often prime candidates for both beam shape and scan losses.

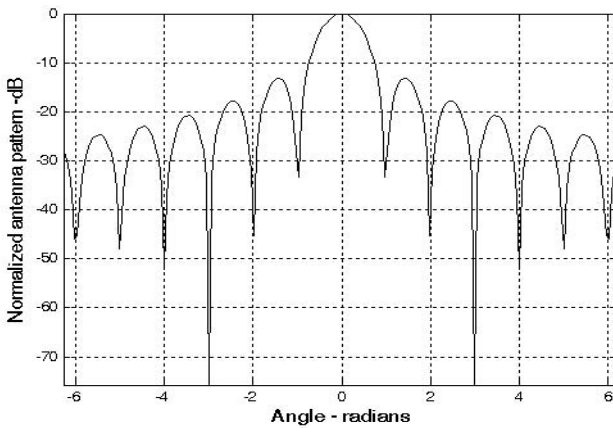


Figure 1.23. Normalized $(\sin x / x)$ antenna pattern.

1.8.3. Atmospheric Loss

Detailed discussion of atmospheric loss and propagation effects is in a later chapter. Atmospheric attenuation is a function of the radar operating frequency, target range, and elevation angle. Atmospheric attenuation can be as high as a few dB.

1.8.4. Collapsing Loss

When the number of integrated returned noise pulses is larger than the target returned pulses, a drop in the SNR occurs. This is called collapsing loss. The collapsing loss factor is defined as

$$\rho_c = \frac{n+m}{n} \quad (1.90)$$

where n is the number of pulses containing both signal and noise, while m is the number of pulses containing noise only. Radars detect targets in azimuth, range, and Doppler. When target returns are displayed in one coordinate, such as range, noise sources from azimuth cells adjacent to the actual target return converge in the target vicinity and cause a drop in the SNR. This is illustrated in Fig. 1.24.

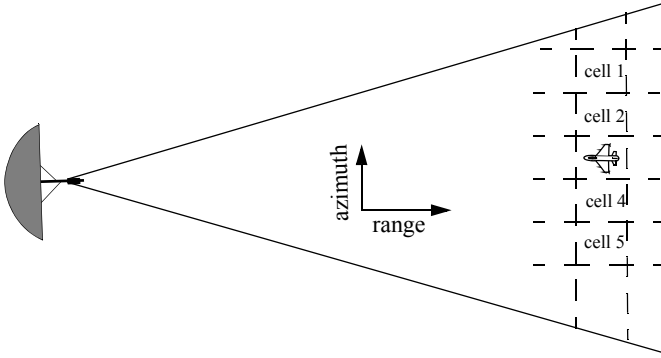


Figure 1.24. Illustration of collapsing loss. Noise sources in cells 1, 2, 4, and 5 converge to increase the noise level in cell 3.

1.8.5. Processing Losses

a. Detector Approximation:

The output voltage signal of a radar receiver that utilizes a linear detector is

$$v(t) = \sqrt{v_I^2(t) + v_Q^2(t)} \quad (1.91)$$

where (v_I, v_Q) are the in-phase and quadrature components. For a radar using a square law detector, we have $v^2(t) = v_I^2(t) + v_Q^2(t)$.

Since in real hardware the operations of squares and square roots are time consuming, many algorithms have been developed for detector approximation. This approximation results in a loss of the signal power, typically 0.5 to 1 dB.

b. Constant False Alarm Rate (CFAR) Loss:

In many cases the radar detection threshold is constantly adjusted as a function of the receiver noise level in order to maintain a constant false alarm rate. For this purpose, Constant False Alarm Rate (CFAR) processors are utilized in

order to keep the number of false alarms under control in a changing and unknown background of interference. CFAR processing can cause a loss in the SNR level on the order of 1 dB.

Three different types of CFAR processors are primarily used. They are adaptive threshold CFAR, nonparametric CFAR, and nonlinear receiver techniques. Adaptive CFAR assumes that the interference distribution is known and approximates the unknown parameters associated with these distributions. Nonparametric CFAR processors tend to accommodate unknown interference distributions. Nonlinear receiver techniques attempt to normalize the root mean square amplitude of the interference.

c. Quantization Loss:

Finite word length (number of bits) and quantization noise cause an increase in the noise power density at the output of the Analog to Digital (A/D) converter. The A/D noise level is $q^2/12$, where q is the quantization level.

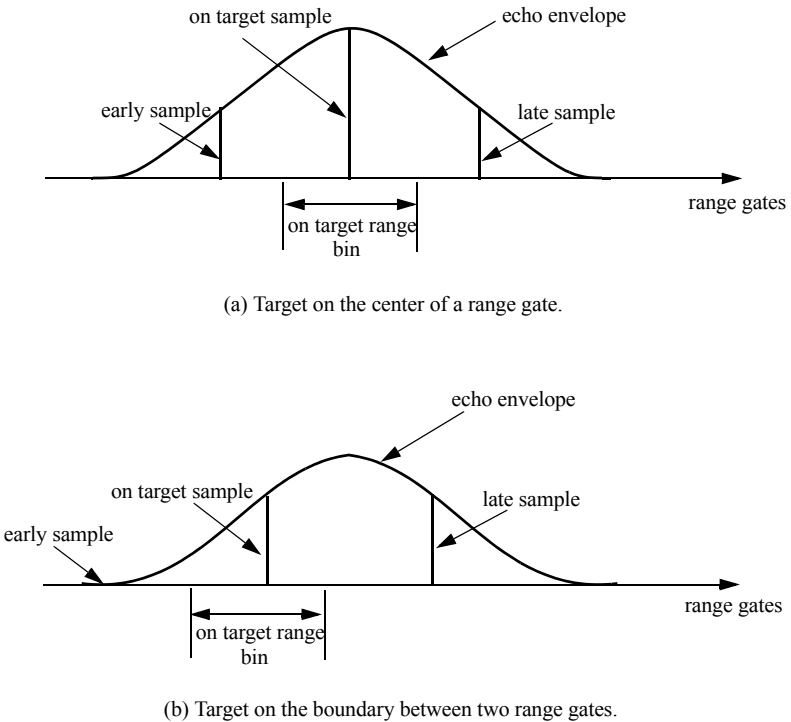


Figure 1.25. Illustration of range gate straddling.

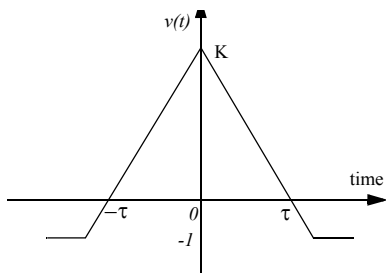
d. Range Gate Straddle:

The radar receiver is normally mechanized as a series of contiguous range gates (bins). Each range bin is implemented as an integrator matched to the transmitted pulsewidth. Since the radar receiver acts as a filter that smears (smooths), the received target echoes. The smoothed target return envelope is normally straddled to cover more than one range gate.

Typically, three gates are affected; they are called the early, on, and late gates. If a point target is located exactly at the center of a range gate, then the early and late samples are equal. However, as the target starts to move into the next gate, the late sample becomes larger while the early sample gets smaller. In any case, the amplitudes of all three samples should always roughly add up to the same value. Fig. 1.25 illustrates the concept of range straddling. The envelope of the smoothed target echo is likely to be Gaussian shaped. In practice, triangular shaped envelopes may be easier and faster to implement. Since the target is likely to fall anywhere between two adjacent range bins, a loss in the SNR occurs (per range gate). More specifically, a target's returned energy is split between three range bins. Typically, straddle loss of about 2 to 3 dB is not unusual.

Example:

Consider the smoothed target echo voltage shown below. Assume 1Ω resistance. Find the power loss due to range gate straddling over the interval $\{0, \tau\}$.



Solution:

The smoothed voltage can be written as

$$v(t) = \begin{cases} K + \left(\frac{K+1}{\tau}\right)t & ; t < 0 \\ K - \left(\frac{K+1}{\tau}\right)t & ; t \geq 0 \end{cases}$$

The power loss due to straddle over the interval $\{0, \tau\}$ is

$$L_s = \frac{v^2}{K^2} = 1 - 2\left(\frac{K+1}{K\tau}\right)t + \left(\frac{K+1}{K\tau}\right)^2 t^2$$

The average power loss is then

$$\begin{aligned} \bar{L}_s &= \frac{2}{\tau} \int_0^{\tau/2} \left(1 - 2\left(\frac{K+1}{K\tau}\right)t + \left(\frac{K+1}{K\tau}\right)^2 t^2\right) dt \\ &= 1 - \frac{K+1}{2K} + \frac{(K+1)^2}{12K^2} \end{aligned}$$

and, for example, if $K = 15$, then $\bar{L}_s = 2.5dB$.

e. Doppler Filter Straddle:

Doppler filter straddle is similar to range gate straddle. However, in this case the Doppler filter spectrum is spread (widened) due to weighting functions. Weighting functions are normally used to reduce the sidelobe levels. Since the target Doppler frequency can fall anywhere between two Doppler filters, signal loss occurs.

1.8.6. Other Losses

Other losses may include equipment losses due to aging radar hardware, matched filter loss, and antenna efficiency loss. Tracking radars suffer from crossover (squint) loss.

1.9. “MyRadar” Design Case Study - Visit 1

In this section, a design case study, referred to as “MyRadar” design case study, is introduced. For this purpose, only the theory introduced in this chapter is used to fulfill the design requirements. Note that since only a limited amount of information has been introduced in this chapter, the design process may seem illogical to some readers. However, as new material is introduced in subsequent chapters, the design requirements are updated and/or new design requirements are introduced based on the particular material of that chapter. Consequently, the design process will also be updated to accommodate the new theory and techniques learned in that chapter.

1.9.1. Authors and Publisher Disclaimer

The design case study “MyRadar” is a ground based air defense radar derived and based on Brookner’s¹ open literature source. However, the design approach introduced in this book is based on the authors’ point of view of how to design such radar. Thus, the design process takes on a different flavor than

that introduced by Brookner. Additionally, any and all design alternatives presented in this book are based on and can be easily traced to open literature sources.

Furthermore, the design approach adopted in this book is based on modeling many of the radar system components with no regards to any hardware constraints nor to any practical limitations. The design presented in this book is intended to be tutorial and academic in nature and does not adhere to any other requirements. Finally, the MATLAB code presented in this book is intended to be illustrative and academic and is not designed nor intended for any other uses.

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1.9.2. Problem Statement

You are to design a ground based radar to fulfill the following mission: Search and Detection. The threat consists of aircraft with an average RCS of 6 dBsm ($\sigma_a = 4m^2$), and missiles with an average RCS of -3 dBsm ($\sigma_m = 0.5m^2$). The missile altitude is 2Km, and the aircraft altitude is about 7 Km. Assume a scanning radar with 360 degrees azimuth coverage. The scan rate is less than or equal to 1 revolution every 2 seconds. Assume L to X band. We need range resolution of 150 m. No angular resolution is specified at this time. Also assume that only one missile and one aircraft constitute the whole threat. Assume a noise figure $F = 6$ dB, and total receiver loss $L = 8$ dB. For now use a fan beam with azimuth beamwidth of less than 3 degrees. Assume that 13 dB SNR is a reasonable detection threshold. Finally, assume flat earth.

1.9.3. A Design

The desired range resolution is $\Delta R = 150m$. Thus, using Eq. (1.8) one calculates the required pulsewidth as $\tau = 1\mu\text{sec}$, or equivalently the required bandwidth $B = 1\text{MHz}$. At this point a few preliminary decisions must be made. This includes the selection of the radar operating frequency, the aperture size, and the single pulse peak power.

1. Brookner, Eli, Editor, *Practical Phased Array Antenna Systems*, Artech House, 1991, Chapter 7.

The choice of an operating frequency that can fulfill the design requirements is driven by many factors, such as aperture size, antenna gain, clutter, atmospheric attenuation, and the maximum peak power, to name a few. In this design, an operating frequency $f = 3\text{ GHz}$ is selected. This choice is somewhat arbitrary at this point; however, as we proceed with the design process this choice will be better clarified.

Second, the transportability (mobility) of the radar drives the designer in the direction of a smaller aperture type. A good choice would be less than 5 meters squared. For now choose $A_e = 2.25\text{ m}^2$. The last issue that one must consider is the energy required per pulse. Note that this design approach assumes that the minimum detection SNR (13 dB) requirement is based on pulse integration. This condition is true because the target is illuminated with several pulses during a single scan, provided that the antenna azimuth beamwidth and the PRF choice satisfy Eq. (1.81).

The single pulse energy is $E = P_t \tau$. Typically, a given radar must be designed such that it has a handful of pulsewidths (waveforms) to choose from. Different waveforms (pulsewidths) are used for definite modes of operations (search, track, etc.). However, for now only a single pulse which satisfies the range resolution requirement is considered. To calculate the minimum single pulse energy required for proper detection, use Eq. (1.57). More precisely,

$$E = P_t \tau = \frac{(4\pi)^3 k T_e F L R^4 \text{SNR}_1}{G^2 \lambda^2 \sigma} \quad (1.92)$$

All parameters in Eq. (1.92) are known, except for the antenna gain, the detection range, and the single pulse SNR. The antenna gain is calculated from

$$\left(G = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \times 2.25}{(0.1)^2} = 2827.4 \right) \Rightarrow G = 34.5\text{ dB} \quad (1.93)$$

where the relation ($\lambda = c/f$) was used.

In order to estimate the detection range, consider the following argument. Since an aircraft has a larger RCS than a missile, one would expect an aircraft to be detected at a much longer range than that of a missile. This is depicted in Fig. 1.26, where R_a refers to the aircraft detection range and R_m denotes the missile detection range. As illustrated in this figure, the minimum search elevation angle θ_1 is driven by the missile detection range, assuming that the missiles are detected, with the proper SNR, as soon as they enter the radar beam. Alternatively, the maximum search elevation angle θ_2 is driven the aircraft's position along with the range that corresponds to the defense's last chance to intercept the threat (both aircraft and missile). This range is often called “keep-out minimum range” and is denoted by R_{min} . In this design approach,

The minimum and maximum elevation angles are, respectively, calculated as

$$\theta_1 = \text{atan}\left(\frac{2}{55}\right) = 2.08^\circ \quad (1.98)$$

$$\theta_2 = \text{atan}\left(\frac{7}{30}\right) = 13.13^\circ \quad (1.99)$$

These angles are then used to compute the elevation search extent (remember that the azimuth search extent is equal to 360°). More precisely, the search volume Ω (in steradians) is given by

$$\Omega = \frac{\theta_2 - \theta_1}{(57.296)^2} \times 360 \quad (1.100)$$

Consequently, the search volume is

$$\Omega = 360 \times \frac{\theta_2 - \theta_1}{(57.296)^2} = 360 \times \frac{13.13 - 2.08}{(57.296)^2} = 1.212 \text{ steradians} \quad (1.101)$$

The desired antenna must have a fan beam; thus using a parabolic rectangular antenna will meet the design requirements. Select $A_e = 2.25\text{m}^2$; the corresponding antenna 3-dB elevation and azimuth beamwidths are denoted as θ_e, θ_a , respectively. Select

$$\theta_e = \theta_2 - \theta_1 = 13.13 - 2.08 = 11.05^\circ \quad (1.102)$$

The azimuth 3-dB antenna beamwidth is calculated using Eq. (1.42) as

$$\theta_a = \frac{4\pi}{G\theta_e} = \frac{4 \times \pi \times 180^2}{2827.4 \times \pi^2 \times 11} = 1.33^\circ \quad (1.103)$$

It follows that the number of pulses that strikes a target during a single scan is calculated using Eq. (1.81) as

$$n_p \leq \frac{\theta_a f_r}{\dot{\theta}_{scan}} = \frac{1.33 \times 1000}{180} = 7.39 \Rightarrow n_p = 7 \quad (1.104)$$

The design approach presented in this book will only assume non-coherent integration (the reader is advised to re-calculate all results by assuming coherent integration, instead). The design requirement mandates a 13 dB SNR for detection. By using Eq. (1.87) one calculates the required single pulse SNR,

$$(SNR)_1 = \frac{10^{1.3}}{2 \times 7} + \sqrt{\frac{(10^{1.3})^2}{4 \times 7^2} + \frac{10^{1.3}}{7}} = 3.635 \Rightarrow (SNR)_1 = 5.6\text{dB} \quad (1.105)$$

Furthermore the non-coherent integration loss associated with this case is computed from Eq. (1.85),

$$L_{NCI} = \frac{1 + 3.635}{3.635} = 1.27 \Rightarrow L_{NCI} = 1.056 \text{ dB} \quad (1.106)$$

It follows that the corresponding **single pulse** energy for the missile and the aircraft cases are respectively given by

$$E_m = \frac{(4\pi)^3 k T_e F L R_m^4 (SNR)_1}{G^2 \lambda^2 \sigma_m} \Rightarrow \quad (1.107)$$

$$E_m = \frac{(4\pi)^3 (1.38 \times 10^{-23}) (290) (10^{0.8}) (10^{0.6}) (55 \times 10^3)^4 10^{0.56}}{(2827.4)^2 (0.1)^2 (0.5)} = 0.1658 \text{ Joules}$$

$$E_a = \frac{(4\pi)^3 k T_e F L R_a^4 (SNR)_1}{G^2 \lambda^2 \sigma_a} \Rightarrow \quad (1.108)$$

$$E_a = \frac{(4\pi)^3 (1.38 \times 10^{-23}) (290) (10^{0.8}) (10^{0.6}) (90 \times 10^3)^4 10^{0.56}}{(2827.4)^2 (0.1)^2 (4)} = 0.1487 \text{ Joules}$$

Hence, the peak power that satisfies the single pulse detection requirement for both target types is

$$P_t = \frac{E}{\tau} = \frac{0.1658}{1 \times 10^{-6}} = 165.8 \text{ KW} \quad (1.109)$$

The radar equation with pulse integration is

$$SNR = \frac{P_t^1 G^2 \lambda^2 \sigma}{(4\pi)^3 k T_e B F L R^4} \frac{n_p}{L_{NCI}} \quad (1.110)$$

Figure 1.27 shows the SNR versus detection range for both target-types with and without integration. To reproduce this figure use MATLAB program “fig1_27.m” which is given in Listing 1.12 in Section 1.10.

1.9.4. A Design Alternative

One could have elected not to reduce the single pulse peak power, but rather keep the single pulse peak power as computed in Eq. (1.109) and increase the radar detection range. For example, integrating 7 pulses coherently would improve the radar detection range by a factor of

$$R_{imp} = (7)^{0.25} = 1.63 \quad (1.111)$$

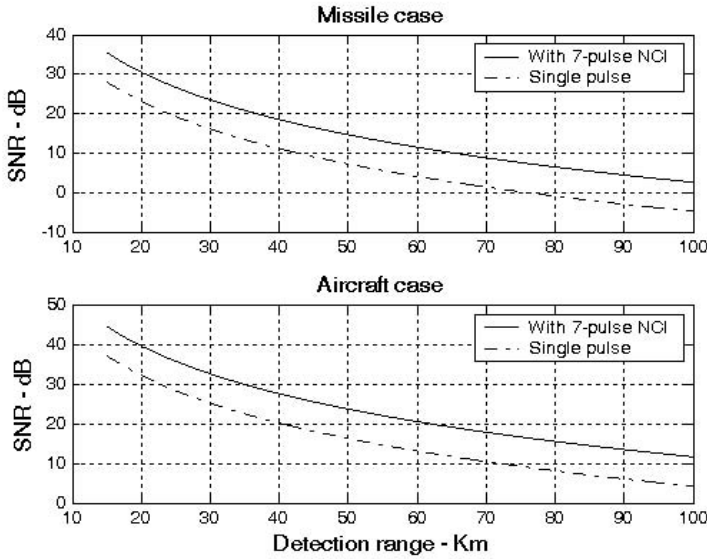


Figure 1.27. SNR versus detection range for both target types with and without pulse integration.

It follows that the new missile and aircraft detection ranges are

$$\begin{aligned} R_a &= 78 \times 1.63 = 126.9 \text{ Km} \\ R_m &= 48 \times 1.63 = 78.08 \text{ Km} \end{aligned} \quad (1.112)$$

Note that extending the minimum detection range for a missile to $R_m = 78 \text{ Km}$ would increase the size of the extent of the elevation search volume. More precisely,

$$\theta_1 = \text{atan}\left(\frac{2}{78}\right) = 1.47^\circ \quad (1.113)$$

It follows that the search volume Ω (in steradians) is now

$$\Omega = 360 \times \frac{\theta_2 - \theta_1}{(57.296)^2} = 360 \times \frac{13.13 - 1.47}{(57.296)^2} = 1.279 \text{ steradians} \quad (1.114)$$

Alternatively, integrating 7 pulses non-coherently with $(SNR)_{NCI} = 13 \text{ dB}$ yields

$$(SNR)_1 = 5.6 \text{ dB} \quad (1.115)$$

and the integration loss is