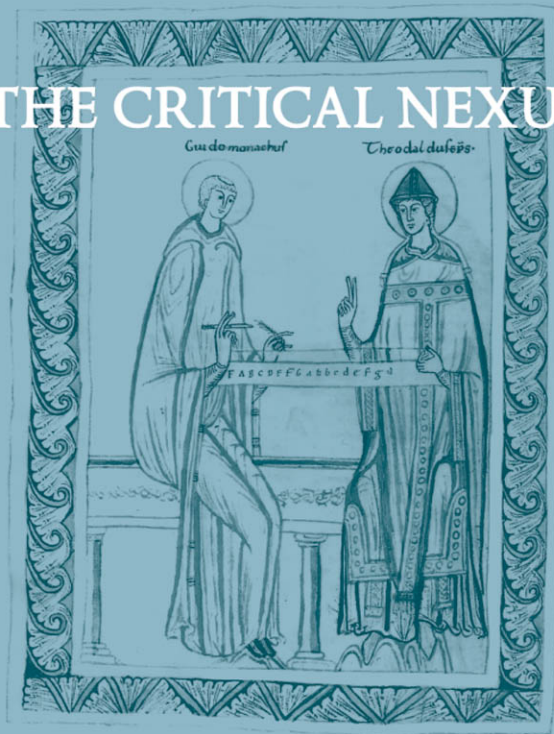


AMS Studies in Music

THE CRITICAL NEXUS



Tone-System, Mode, and Notation in Early Medieval Music

Charles M. Atkinson

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OXFORD
UNIVERSITY PRESS

2009

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Oxford University Press, Inc., publishes works that further
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Oxford New York

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Kuala Lumpur Madrid Melbourne Mexico City Nairobi
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Argentina Austria Brazil Chile Czech Republic France Greece
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Published by Oxford University Press, Inc.
198 Madison Avenue, New York, New York 10016
www.oup.com

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Library of Congress Cataloging-in-Publication Data
Atkinson, Charles M. (Charles Mercer), 1941–

The critical nexus: tone-system, mode, and notation in early
medieval music / Charles M. Atkinson
p. cm. — (AMS studies in music)

Includes bibliographical references and index.

ISBN 978-0-19-514888-6

1. Music theory—History—500–1400. 2. Music
theory—To 500—Influence. I. Title.

ML174.A85 2008

781.2'63—dc22 2007011368

2 4 6 8 9 7 5 3 1

Printed in the United States of America
on acid-free paper

In memoriam Fritz Reckow

The original impetus for this study came from my friend Fritz Reckow (1940–1998), who invited me to participate in a symposium in Kiel in 1985. Held under the co-sponsorship of the Universität Kiel and the Schleswig-Holsteinische Landesbibliothek, the symposium dealt with the creation of a European musical culture in the Middle Ages (“Die Formung einer europäischen musikalischen Kultur im Mittelalter”). My presentation at the symposium was originally titled “On the Formation of a Medieval Theory of Mode.” As I continued to work on the topic, with a view toward publishing the essay in the conference report, I realized that the substance of my study encompassed far more than the original title conveyed. That realization notwithstanding, and given the restrictions of page limits and deadlines, the version I submitted for publication was essentially the paper I had presented at the symposium. Unfortunately, the proceedings of the original symposium had not been published at the time of Fritz’s untimely death on August 30, 1998, and the plan for publishing the proceedings had to be abandoned. I hope that publishing this essay in its expanded form may serve to complete one small part of Fritz Reckow’s legacy to the fields of musicology and medieval studies.

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ACKNOWLEDGMENTS

As its title intends to suggest, this is a book with a broad scope but a narrow focus. It is simultaneously both an attempt to provide a satisfactory answer to a specific question and a compendium of much of my work during the past three decades. Although its immediate impetus was the invitation from Fritz Reckow to participate in the 1985 Kiel symposium mentioned in the dedication, its roots ultimately extend back to a paper I wrote on the *parapteres* for a seminar on medieval theory with Calvin Bower at the University of North Carolina. As one might expect of a work with such a long gestation period, this one owes much to many. I cannot possibly name all the people and institutions that have played a role in its realization, but I should like to name at least a few.

The person who was present at the inception of this work, and who continues to be a source of information and inspiration to me, is Calvin Bower. His insights inform this study in ways too numerous to catalogue, but they are especially important to the sections on Boethius in chapter 1 and on the *Alia musica* in chapter 5.

I owe a debt of a special kind to Leo Treitler. His work on the advent of musical notation in the West has been of seminal importance for me, as it has for many others. Beyond that, though, his interest in the subject of this book and his reading and commenting on parts of the manuscript itself have made it a far better work than it would otherwise have been.

This study has profited additionally from intensive discussions with colleagues in the United States and Europe, who have afforded me the opportunity to present my work on this topic in seminars and colloquia at various stages of its development. The present study took its initial shape in a seminar I presented at Harvard University in 1984, at the invitation of David Hughes. I was able to present more fully developed versions in seminars at the University of North Carolina, where I had the honor of being a guest professor in the spring of 1987, and at the Graduate Center of the City University of New York in 1994, upon the invitation of Raymond Erickson. The work took something close to its present form in Paris in 2001, thanks to Marie-Noël Colette's having arranged for me to present it as a course at the École Pratique des Hautes Études of the Sorbonne. Her searching questions and

observations as we translated the text into French were of great help in clarifying a number of issues, especially those concerning notation. The seminars and colloquia I was subsequently asked to present by Susan Patrick (University of New Mexico), Wulf Arlt (Universität Basel), and Andreas Haug (Universität Erlangen-Nürnberg) likewise contributed substantially to giving the work its present shape and substance. My profound thanks go to these friends and colleagues and to their students.

Of course, the people who have been most immediately involved in this work are my colleagues and students in musicology at Ohio State University. Arved Ashby, Daniel Avorgbedor, Graeme Boone, Burdette Green, Herbert Livingston (deceased), Martha Maas, Margarita Mazo, Lois Rosow, and Udo Will have been constant sources of inspiration and encouragement. I have been fortunate to have had a number of fine students in the various courses and seminars I offered on topics related to this study. Two of them, Cynthia Cyrus and Jane Warburton, contributed directly to this work by writing excellent papers on the *Alia musica* and on species theory, respectively. I take special pleasure in acknowledging their contributions here.

The following institutions provided support for this work in different ways, for which I am extremely grateful: The Ohio State University and its libraries; Bamberg, Staatsbibliothek; Bern, Burgerbibliothek; Besançon, Bibliothèque Municipale; Brussels, Bibliothèque Royale; Düsseldorf, Universitäts- und Landesbibliothek; Erlangen, Universitätsbibliothek; Karlsruhe, Badische Landesbibliothek; Leiden, Universiteitsbibliotheek; London, The British Library; Munich, Bayerische Staatsbibliothek; Orléans, Bibliothèque Municipale; Oxford, Bodleian Library; Paris, Bibliothèque nationale de France; Reims, Bibliothèque Municipale; Rochester, Sibley Music Library, Eastman School of Music of the University of Rochester; and St. Gall, Stiftsbibliothek. I should especially like to acknowledge the Österreichische Nationalbibliothek in Vienna for its permission to reproduce on the cover of this book the picture of Guido d'Arezzo and Bishop Theodaldus that appears on folio 35v of the manuscript cpv 51 in their collection.

The excursus on Aurelian and the Paleofrankish script in Chapter 3 is drawn from my article, "*De accentibus toni*," which appeared in *Essays on Medieval Music in Honor of David G. Hughes*. I wish to thank the Department of Music of Harvard University, publisher of the volume, and its editor, Graeme Boone, for their permission to republish that material here.

A special acknowledgment is due the National Endowment for the Humanities, whose fellowship for the academic year 2003/04 enabled me to complete this book and prepare the final manuscript.

I cannot close this section of acknowledgments without expressing my deep gratitude to the members of the Publications Committee of the American Musicological Society, who serve as the editorial board for the AMS Studies in Music. I am especially grateful to the two readers who reviewed the final manuscript for the Publications Committee. They raised questions about various parts of the study and made a number of valuable comments and suggestions. I have tried to respond to their questions and incorporate their suggestions in every case, but, should mistakes or shortcomings still remain, I accept full responsibility for them.

I should especially like to thank the series editor for this volume, Lawrence Bernstein, not only for his wonderful editorial work but also for his unflagging support in seeing the book through to publication.

Finally, I must express my profound gratitude to my family, Gretchen, Karen, and David, for their constant encouragement despite the fact that they so often had to compete with a manuscript, a book, or an article for my attention. This project would not have been possible without their support.

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NOTE ON ABBREVIATIONS AND NOMENCLATURE FOR PITCH

ABBREVIATIONS

General bibliographical abbreviations are explicated at the beginning of the bibliography. Throughout the text, manuscripts are cited by the city of their current provenance and the numeric part of their call number (or a portion thereof). More complete citations are given in the list of manuscripts in the bibliography.

PITCH

Letter names are used to identify pitches. Gamma (Γ) is equivalent to the *G* on the first line of the staff in bass clef. *A* represents the note a second above that; *a* the note an octave higher; and *aa* the note yet another octave higher.

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THE CRITICAL NEXUS

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PROLOGUE

In the sixth chapter of his *Dialogus de musica* (ca. 1000 A.D.), the anonymous author usually known as Pseudo-Odo of Cluny tries to explain to his student the effect that the placement of tones and semitones in a chant has on the determination of its mode. He gives as examples several chants that were difficult to classify, along with various solutions to the problems they present, and he concludes his discussion as follows.

From this it is understood that the musician who lightly and presumptuously emends many melodies is ignorant unless he first goes through all the modes to determine whether the melody may perhaps not stand in one or another, nor should he care as much for its similarity to other melodies as for its fidelity to the rules. But if it conforms to no mode, let it be emended according to the one with which it least disagrees. This also should be observed: that the emended melody either sound better or depart little from its previous likeness.¹

This statement raises a number of interesting questions, but perhaps the most fundamental for a modern-day reader is why the melodies of these chants, represented as having been divinely inspired,² should have had to be “emended” at all!

Providing the answer to that and to several other questions raised by Pseudo-Odo’s statement will be the task of this study. As will become apparent, the prob-

1. “Ex quo comprehenditur, quia imperitus musicus est, qui facile ac praesumptuose plures cantus emendat, nisi prius per omnes modos investigaverit, si forsitan in aliquo stare possit; nec magnopere de similitudine aliorum cantuum, sed de regulari veritate curet. Quodsi nulli tono placet, secundum eum tonum emendetur, in quo minus dissonat. Atque hoc observari debet, ut emendatus cantus aut decen-
tius sonet, aut a priori similitudine parum discrepet” (GS 1: 256–57; transl. in McKinnon, *The Early Christian Period and the Latin Middle Ages*, 96). As just one hint of the kinds of questions to be treated here, one might note that Pseudo-Odo uses two different Latin words, *tonus* and *modus*, for what McKinnon appropriately translates as “mode.”

2. One of the most enduring images of the divine origin of the chant is that of the Holy Spirit in the form of a dove, perched on the shoulder of Gregory the Great and singing the chants into his ear (see Matt. 3: 16–17). On the history of this image and some of its implications for the subject under investigation here, see Treitler, “Homer and Gregory.”

lem addressed by Pseudo-Odo concerns a complex of issues in the areas of tone-system or scale, mode or tone, and musical notation. Obviously, each of these topics is vast in its own right, each has been investigated extensively, and each still deserves further studies of its own.³ Rather than examining tone-system, mode, and notation as separate entities, however, I shall, in this book, treat them as interwoven with each other, as a web or nexus, so to speak.⁴ For the sake of clarity, each will be discussed separately at various points, but the reader should bear in mind their inherent interconnectedness.

As far as possible, the sources will be allowed to speak for themselves.⁵ There will, however, be no attempt to account for the evidence brought forth by every theorist or in every manuscript. Instead, the focus will be on those sources that appear to offer the most telling treatment of the topics under consideration. The organization of this book's narrative will be chronological for the most part, although, of course, the testimony of contemporaneous witnesses will have to be presented sequentially, rather than simultaneously.

We shall begin in chapter 1 with an examination of the concepts of tone-system, mode, and notation that were a legacy to the Middle Ages from Antiquity. The

3. Among the studies that proved to be of seminal importance for my own work in these areas are Brambach, *Das Tonsystem und die Tonarten des christlichen Abendlandes im Mittelalter*; Jacobsthal, *Die chromatische Alteration im liturgischen Gesang der abendländischen Kirche*; and Markovits, *Das Tonsystem der abendländischen Musik im frühen Mittelalter*. On the subject of tone-system, I cite especially Sachs, *Mensura fistularum*; and Sachs, "Musikalische Elementarlehre im Mittelalter." Important contributions on the subject of mode include Gombosi, "Studien zur Tonartenlehre des frühen Mittelalters"; Huglo, *Les Tónaires*, along with many other articles by the same author; Powers, "Mode," *NG*; and Ferarri-Barassi, "I modi ecclesiastici." Finally, on the early history of notation, see Stäblein, *Schriftbild der einstimmigen Musik*; and various articles by Leo Treitler, but particularly his studies "The Early History of Music Writing in the West" and "Reading and Singing: On the Genesis of Occidental Music-Writing." See also a number of studies by Kenneth Levy now brought together in *Gregorian Chant and the Carolingians*; but note especially "On the Origin of Neumes."

Perhaps the single most important primary source for all three of these aspects of this study is Boethius's *De institutione musica*. The penetrating insights of Boethius's translator and interpreter, Calvin Bower (in *Boethius: Fundamentals of Music* and other studies) have informed this work from its very inception.

4. It would be disingenuous of me not to acknowledge that several of my own essays have formed the foundation for this study. The most directly related are "Parapter" (in *HmT*); "'Harmonia' and the 'Modi, quos abusive tonos dicimus'"; "From 'Vitium' to 'Tonus acquisitus'"; "*De accentibus toni oritur nota quae dicitur neumā*"; "Modus" (in *HmT*); and "Das Tonsystem des Choral im Spiegel mittelalterlicher Musiktraktate."

I should also like to acknowledge that some of the issues I treat here have been explored quite effectively by Marie-Elizabeth Duchez in several of her articles listed in the bibliography. Unfortunately, I was unable to consult her dissertation, "*Imago mundi, naissance de la théorie musicale occidentale dans les commentaires carolingiens de Martianus Capella*," which she had announced in earlier publications. I hope, in any event, that my own work may prove to be a worthwhile complement to hers.

5. I realize that, strictly speaking, this is impossible. The sources are written in Latin and Greek, which means that, at the very least, they must be translated, and hence interpreted, at some level. The order of presentation of the witnesses and of their testimony also constitute a component of the narrative that must be guided, rather than simply expressed. These factors notwithstanding, the principle set forth here—that of attempting to allow the evidence of the sources to resonate directly—is one I have attempted to maintain throughout the course of this work.

next two chapters form a pair that spans the eighth and ninth centuries. Chapter 2 will consider the ways ancient texts treating these topics were received and taught in the Carolingian era, and chapter 3 will proceed to examine the traditions and practices of the Christian church and some of the early attempts to develop a rational system of classification for its music. In the next section of the book—its last three chapters—I shall examine the ways various components of ancient Greek theory were grafted onto medieval practice and were themselves modified, leading to a theory of both tone-system and mode, and a concomitant system of notation, that is uniquely medieval. Chapter 4 addresses the writings of Hucbald of St. Amand and Regino of Prüm; chapter 5 the *Alia musica*; and chapter 6 the relevant contributions of Pseudo-Bernelinus, Bern of Reichenau, Pseudo-Odo, and Guido d'Arezzo. In the epilogue, I examine some of the difficulties that arose from this synthesis, concluding with an exploration of some of the ways theory moves to accommodate practice in the later Middle Ages. We shall see that what resulted from this accommodation was a theory of tone-system and mode that would remain viable until it was supplanted by Glareanus's theory of twelve modes in the sixteenth century.

THE HERITAGE OF ANTIQUITY

Interim parabantur exsequiae . . . cum subito raptus in spiritu ad tribunal iudicis pertrahor, . . . interrogatus condicionem Christianum me esse respondi. et ille, qui residebat: "mentiris," ait, "Ciceronianus es, non Christianus; ubi thesaurus tuus, ibi et cor tuum."

—Jerome, letter to Eustochium

It has often been said that Western European intellectual life in the Middle Ages rested on two bases—the heritage of Antiquity and the traditions and practices of the Christian church. The bifurcate nature of medieval intellectual history is, to my mind, nowhere better exemplified than in the formation of a theory of melodic classification into tones or modes and the concomitant establishment of a tone-system or scalar matrix for medieval music. Whereas in some areas the two main sources of influence stood in conflict with each other (as the words of Jerome above suggest),¹ the formation of a theory of both mode and tone-system in the medieval Latin West represents not so much a conflict as a construct of ideas from both Antiquity and the Christian church. Let us begin, then, with a brief look at two fundamental aspects of ancient Greek music that are particularly relevant to this study, and then turn our attention to the ways the knowledge of Greek music was transmitted to the Middle Ages.

A number of excellent studies have reminded us recently that ancient Greece had a long, rich musical tradition that extended from pre-Homeric times (ca. ninth century B.C.E.) up to the fifth century C.E. and beyond.² A serious problem in gain-

1. Sancti Hieronymi Epistula XXII: 30, 3, *Sancti Evsepii Hieronymi epistulae*, ed. Hilberg, 1: 190. In Mierow's translation (cited below), the passage reads: "Meanwhile, preparations for my funeral were being made. . . . Suddenly I was caught up in the spirit and dragged before the tribunal of the Judge. . . . Upon being asked my status, I replied that I was a Christian. And He who sat upon the judgment seat said: 'Thou liest. Thou art a Ciceronian, not a Christian. Where thy treasure is, there is thy heart also'" (Matt. 6:21). *The Letters of St. Jerome*, transl. Mierow, 1: 166. This letter is addressed "ad Eustochium" and was probably written in 384.

2. I refer, in particular, to the works of Mathiesen (*Apollo's Lyre*, as well as his introductory "Greece,"

ing an overview of this tradition is that there are only about thirty surviving examples of actual Greek music; most of those are mere fragments on stone or papyrus, and fairly late.³ What we do have in relative abundance are (1) depictions of musicians and music-making in works of plastic art, such as vase paintings;⁴ (2) references to music in literary and philosophical writings, such as those of Homer (e.g., *Iliad*, *Odyssey*), Plato (e.g., *Laws*, *Republic*, *Timaeus*), and Aristotle (e.g., *Politics*, *Metaphysics*, *De anima*);⁵ and (3) a relatively small group of technical works that treat music as a manifestation of harmonic and acoustic theories.⁶

One of the features of ancient Greek music that will prove to be especially important for the subject under investigation here is the integral relationship between the disciplines of music and grammar. As Frieder Zaminer points out, the discipline of μουσική [Lat. *musica*] originally included poetry, music, and dance, but it was then subdivided into poetry and music.⁷ He says that at the time of Plato and Aristotle the discipline of grammar (γραμματική) included the theory of speech-sounds (vowels, consonants, etc.) and letters, as well as prosody. With the latter, however, it extended into the areas of meter, rhythm, and melody,⁸ and whoever taught it could with equal validity be designated γραμματικός (grammarian) or μουσικός (musician).⁹ To the knowledge of the μουσικός also belonged, as Plato expressly mentions in *Philebos*,¹⁰ the knowledge of the varying qualities, number, and names of the intervals (διαστήματα). With the increasing specialization in the fourth–third centuries B.C.E., the areas designated μουσική and γραμματική gradually moved apart, although they are united in works such as Aristides Quintilianus's *De musica* (late third–early fourth centuries C.E.),¹¹ Augustine's *De*

pt. 1, "Ancient," in NG 2, 10: 327–48); West (*Ancient Greek Music*); Anderson (*Music and Musicians in Ancient Greece*); and Barker (*Greek Musical Writings*). Earlier studies of equal moment include: Winnington-Ingram ("Greece, Ancient," in NG, 7: 659–72); and Henderson ("Ancient Greek Music," NOHM, 1: 336–403). Many additional studies might be mentioned. In *Apollo's Lyre* (13–16), Mathiesen provides a concise survey of recent scholarship that is complemented by a comprehensive bibliography (669–783). Although there is archaeological evidence for ancient Greek music as early as 2700 B.C.E., Mathiesen adopts the so-called Archaic Period (eighth–sixth centuries B.C.E.) as his terminus a quo, and the fifth century C.E., marked by the fall of Rome and the collapse of the Western empire, as a reasonable terminus ante quem (*Apollo's Lyre*, 17–18).

3. These are collected, edited, and transcribed in Pöhlmann, ed., *Denkmäler altgriechischer Musik*.

4. On the importance of the three categories named here, see Barker, *Greek Musical Writings*, 1: 1–2. For further information and bibliography on ancient Greek music and musical instruments in the plastic arts, see 1: 4–17; Maas and Snyder, *Stringed Instruments of Ancient Greece*; and Wegner, *Griechenland*.

5. A selection of these has been translated into English in Barker, *Greek Musical Writings*, vol. 1; and Mathiesen, *Greek Views of Music*.

6. The most important of these are translated in Barker, *Greek Musical Writings*, vol. 2; and Mathiesen, *Greek Views of Music*.

7. Zaminer, "Über Grammatica und Musica," 255–57. See also Laum, *Das Alexandrinische Akzentuationssystem*, esp. 21–26, 103–9, 119–25. On the broader Greek concept of music, μουσική, see Mathiesen, *Apollo's Lyre*, 6–7.

8. See Plato, *Hippias maior*, 285d; Aristotle, *Poetica*, 1456b–1459a.

9. See Quintilian, *Institutio oratoria*, 1: 10, 17–22.

10. See *Philebos*, 55e–56c.

11. On the date of Aristides' treatise, see Mathiesen, *Aristides Quintilianus on Music*, 14; Barker, *Greek*

musica libri sex (ca. 387–89),¹² and Martianus Capella's *De nuptiis Philologiae et Mercurii* (ca. 437 C.E.).¹³

Ancient Greek music was important to the Middle Ages not only, however, because of its relationship with grammar. Even more important was the role of music as another discipline within the liberal arts, namely harmonics. This was the discipline that provided the system of nomenclature, principles, and procedures through which the abstract concept of ἁρμονία [Lat. *harmonia*]¹⁴—the “well-fittedness” of things, the “divine ordering of the universe”—could be discussed.¹⁴ As a harmonic discipline, music was grouped among the mathematical arts, those that treated of number in its various manifestations: arithmetic (number as static quantity), geometry (number in static spatial relationships), music (number as quantity in motion), and astronomy (number in moving spatial relationships).¹⁵

Based solidly on the disciplines of grammar and mathematics, the theoretical foundation on which the Middle Ages could construct its own theories of tone-system, mode, and notation was thus rather substantial. Indeed, virtually every Greek writer on harmonics included a treatment of the theory of tone-system and mode, the latter most often designated with the terms τόνος [Lat. *tonus*] or τρόπος [Lat. *tropus*].¹⁶ Several of these writings became the subjects of translations or commentaries by Roman authors, thereby making them available to medieval Europeans in a language they could understand.¹⁷ In his study of the transmission of ancient music theory to the Latin West, Michael Bernhard provides a list of Roman writers who treat of music, along with a description of their influence on the Middle Ages, as measured by manuscript transmission and citation by later writers (a summary of his conclusions appears in table 1.1).¹⁸

Musical Writings, 2: 392; and Winnington-Ingram, ed., *Aristidis Quintiliani: De musica libri tres*, xxiii–xxiv. Both Mathiesen and Barker provide characterizations of the treatise itself.

12. See Finaert and Thonnard, eds., *De musica libri sex*.

13. See note 19 below for bibliography and information on the dating of this work.

14. On ἁρμονία, see Mathiesen, “Problems of Terminology in Ancient Greek Theory: ἈΡΜΟΝΙΑ.” Mathiesen points out that the meanings of ἁρμονία, ἁρμονική (harmonics), and related terms shift somewhat between writers of the Hellenic period (e.g., Plato, Aristotle, Aristoxenus, and Pseudo-Plutarch), and those of the later, Greco-Roman tradition (e.g., Alypius, Cleonides, and Gaudentios).

15. On the importance of number as a key to the understanding of the universe, see Barker, *Greek Musical Writings*, 2: 28–29; and Burkert, *Weisheit und Wissenschaft*, 14–45 and 348–64. As both writers make clear, this view was one associated most closely with Pythagoras and the so-called Pythagorean School.

16. Tone-system and mode are two of the seven categories of harmonics set forth by Aristoxenos, who wrote in the late fourth century B.C.E. (*Elementa harmonica*, Book II, secs. 35–38): genera (γέννη), intervals (διαστήματα), notes (φθόγγοι), tone-systems (συστήματα), modes (τόνοι), modulation (μετάβολη), and melic composition (μελοποιία). (See da Rios, ed., *Aristoxeni Elementa harmonica*, 44–48.) On the terms τόνος and τρόπος themselves and their ranges of meaning in Greek Antiquity, see in particular the entries for them in Michaelides, *The Music of Ancient Greece*. See also the discussion of the *tonoi* in Barker, *Greek Musical Writings*, 2: 17–27, and Atkinson, “Tonos/tonus.”

17. Knowledge of Greek in the medieval Latin West was not widespread, but it did not die out entirely. For an excellent study of the topic see Berschin, *Griechisch-Lateinisches Mittelalter*, transl. Frakes, *Greek Letters and the Latin Middle Ages*. On the subject of ancient Latin translations from Greek originals, see, in particular, Wille, *Musica romana*, 406–42; 594–715; and Manitius, *Geschichte der lateinischen Literatur im Mittelalter*, 1: 1–36.

18. Bernhard, “Überlieferung und Fortleben der antiken lateinischen Musiktheorie im Mittelalter.”

TABLE 1.1. Treatises from Roman Antiquity that deal with music (from Bernhard, “Überlieferung und Fortleben der antiken lateinischen Musiktheorie im Mittelalter,” 7–35)

1. Vitruvius, *De architectura* (ca. 27 B.C.E.). Disseminated in 55 MSS, but had little impact on the Middle Ages.
2. Quintilianus, *Institutio oratoria*. (2nd c., C.E.). Virtually unknown in the Middle Ages.
3. Censorinus, *De die natali* (238 C.E.) and Fragmentum Censorini. 3 MSS from the early Middle Ages (one in 8th-c MS., Cologne 166; and a number from the 15th and 16th cc. The *Musica enchiriadis* (9th c.) cites this, probably from the Cologne MS.
4. Calcidius, translation of and commentary on Plato's *Timaeus* (4th c. C.E.). Disseminated in ca. 150 MSS, but exerted very little influence on medieval musical writing. *Musica enchiriadis* (9th c.) begins with Calcidius's definition of *vox* (cf. *Timaeus a Calcidio translatus*, XLIV [ed., Waszink, 92]).
5. Augustine, *De musica* (387–89 C.E.). Transmitted widely in MSS. Consists of 6 books, treating music as part of metrics. John Scottus (d. 877) cites it for *numerus*, but it does not become truly important until the 12th–13th cc. Its definition of music, *Musica est scientia bene modulandi* (probably from Varro), finds its way into Cassiodorus, thence into the Middle Ages.
6. Macrobius, Commentary on Cicero's *Somnium scipionis* (ca. 400 C.E.). Disseminated in ca. 230 MSS. Transmitted astronomical, mathematical, musical, and cosmological knowledge of Antiquity into the Middle Ages. Especially important as a source for harmonic theory, but reception in musical circles begins fairly late: 12th c. and later.
7. Favonius Eulogius. Preserved in 1 MS. Another commentary on Cicero's *Somnium scipionis*, perhaps originating before Macrobius's.
8. Martianus Capella, *De nuptiis Philologiae et Mercurii* (before 439 C.E.). Transmitted in 241 MSS. Very important school text from the 9th c. and later, with important commentaries by 9th-c. figures such as John Scottus Eriugena and Remigius of Auxerre, but with surprisingly little resonance in writings dealing specifically with music. (Hucbald for names of notes; Regino of Prüm for numbering of planets; *Dulce ingenium* for designations of notes, intervals; Engelbert of Admont cites it with Remigius's commentary). Its 9th book., “De Armonia,” is drawn in part from Quintilianus, *De musica*.
9. Fulgentius, *Mitologiae* (5th–6th cc. C.E.). Disseminated in a reasonably large number of MSS. Important as a source for the study of ancient poets. Its version of the Orpheus legend is the one used by John Scottus and Remigius of Auxerre in their commentaries on Martianus Capella, and by the *Musica enchiriadis* and Regino of Prüm.
10. Boethius (ca. 480–524 C.E.), *De institutione musica libri V* (ca. 500). Preserved in more MSS than almost any other music treatise except Guido's *Micrologus*.
11. Cassiodorus, *Institutiones* (after 540 C.E.). Treatment of music in the second book. Quite widespread in the Middle Ages. Mynors lists 109 MSS and states that he has not tried to enumerate those mentioned in medieval library catalogues from the continent.
12. Isidore of Seville, *Etymologiae* (ca. 627–36 C.E.). Chapters on music in the third book. Quite widely disseminated in the Middle Ages. Lindsay's edition is based on 35 MSS.

As one can see (table 1.1), there are relatively few authors whose treatises transmit ancient Greek harmonic theories to the Middle Ages, and only two who present those theories in a manner that could be considered extensive: Martianus Capella, whose *De nuptiis Philologiae et Mercurii*¹⁹ became one of the favorite handbooks on the liberal arts among medieval readers, and Anicius Manlius Severinus Boethius, whose *De institutione musica* and *De arithmetica* became the prime sources for Greek harmonic theory in the Middle Ages.²⁰ Because Boethius provides the more complete theory of the two, his treatment of mode and tone-system will better serve as the starting point for our investigation.

Boethius follows in the tradition of several of the Greek treatises on harmonics (especially that of Ptolemy, which he translates in part),²¹ in that he restricts himself to “musica . . . quae in quibusdam constituta est instrumentis,” that is to say, music that is “constituted,” “arranged or disposed,” or “fixed” in instruments such as the kithara, tibia, organ, and bells, and whose principles can be demonstrated on the monochord.²² For Boethius, as for his Greek predecessors, this type of music is preferred not only because it embodies the principles of *harmonia* found in all music²³

Bernhard traces the influence of ancient Latin writers on music on medieval music theory, from Vitruvius (*De architectura*, before 27 B.C.E.) through Isidore of Seville (*Etymologiae*, 627–36 C.E.). His method is to examine the transmission of their works in manuscript sources and citations of them by later writers. He mentions (10), for example, that although Vitruvius’s work was well known in the Middle Ages, having been preserved in fifty-five manuscripts, the music-theoretical portion of *De architectura* had no impact on the medieval world.

19. Edited most recently by Willis, *Martianus Capella*; it is available in English translation in Stahl, *Martianus Capella and the Seven Liberal Arts*. The treatise has been dated as early as ca. 410–39 C.E. and as late as the 470s–480s. The earlier dating was suggested initially by Cappuyns, “Martianus Capella,” and has been adopted more recently by both Bernhard, “Überlieferung und Fortleben der antiken lateinischen Musiktheorie im Mittelalter,” 20; and Grebe, “Die Musiktheorie des Martianus Capella,” 23. Shanzer opts for the later date in *Martianus Capella’s De nuptiis Philologiae et Mercurii*, Book 1, 28.

20. For modern editions of *De arithmetica* and *De musica*, see Friedlein, ed., *Anicii Manlii Torquati Severini Boetii: De institutione arithmetica libri duo; De institutione musica libri quinque*. *De arithmetica* has been translated into English by Masi in *Boethian Number Theory*; for an English translation of *De musica*, see Bower, *Boethius: Fundamentals of Music*. Both treatises were written ca. 500 C.E. (see Bower, *Fundamentals*, xix–xx; Bernhard, “Überlieferung und Fortleben der antiken lateinischen Musiktheorie im Mittelalter,” 24–31). The medieval glosses on Boethius’s *De musica* are edited in Bernhard and Bower, eds., *Glossa maior in institutionem musicam Boethii*.

21. On the relationship of Boethius’s *De musica* to Ptolemy, see Bower, *Boethius: Fundamentals*, xxvi, xxviii–xxix; Bower, “Boethius and Nichomachus,” 5, 28–38, 41–45; Pizzani, “Studi sulle fonti del ‘De institutione Musica’ di Boezio,” 126–36, 139–56; and Gushee, “Questions of Genre in Medieval Treatises on Music,” 376–82. Both Bower and Pizzani agree that Book V of Boethius is a paraphrased translation of Ptolemy. While Pizzani believes that the last chapters of Book IV are also translated (albeit poorly) from Ptolemy, Bower maintains that a translation of Nichomachus’s now-lost *Fundamentals of Music* served as the basis for all of the first four books. It should be pointed out that all of the material from Boethius presented in this study is drawn from Books I–IV. That Nichomachus might have provided the model for Boethius is important, because Boethius’s treatment of tone-system differs in approach from that of Ptolemy. In substance, however, the two are very closely related.

22. Boethius, *De musica*, Book I, chap. 2 (ed. Friedlein, 189). For the monochord division, see *De musica*, Book IV, chaps. 5–12 (ed. Friedlein, 314–35); as well as Meyer, *Mensura monochordi*, xxvi–xxix; Adkins, “The Theory and Practice of the Monochord,” 95–108; and Wantzloebe, *Das Monochord*, 35–40.

23. Boethius defines *harmonia* as follows: “Est enim armonia plurimorum adunatio et dissidentium

but also—and more important—because it makes possible the precise definition and determination of these principles governing music as the expression of abstract, proportionate, quantitative relationships.²⁴

Using these proportionate relationships, Boethius ultimately derives the Pythagorean consonances (diatessaron, diapente, octave) and the remaining notes of the Greater and Lesser Perfect systems in all three genera of their constituent tetrachords: diatonic (ST-T-T, reading upward in pitch), chromatic (ST-ST-m₃), and enharmonic (1/4T, 1/4T, M₃).²⁵ Boethius's diagrams for deriving species and explicating the modes, however, use the diatonic genus only (possibly because the proportions for determining the chromatic and enharmonic genera made use of large numbers). As a result, and perhaps also because its division of tonal space was perceived to be closest to that of the chant repertoire to which it was eventually applied, the diatonic genus was the one taken over from Boethius into the medieval theoretical tradition.²⁶

The two ancient Greek tone-systems presented by Boethius may be described as shown in examples 1.1–2. The Greater Perfect System (ex. 1.1), referred to by Boethius as the bis-diapason system,²⁷ consists of two pairs of conjunct tetrachords, separated in the middle by a point of disjunction between the mese and paramese. This results in a two-octave scale of fifteen notes that may be represented as *A–a*¹ in modern pitch nomenclature.²⁸ The Lesser Perfect System (ex. 1.2), which Boe-

consensio" ("Harmonia is the uniting of the many and the agreement of the disagreeing"); *De arithmetica*, Book II, chap. 32; ed. Friedlein, 126. When not otherwise indicated, the translations in this study are my own.

24. On the relationship between instruments and the quantitative theory found in Boethius, see Reckow, "Organum-Begriff und frühe Mehrstimmigkeit," 56–62.

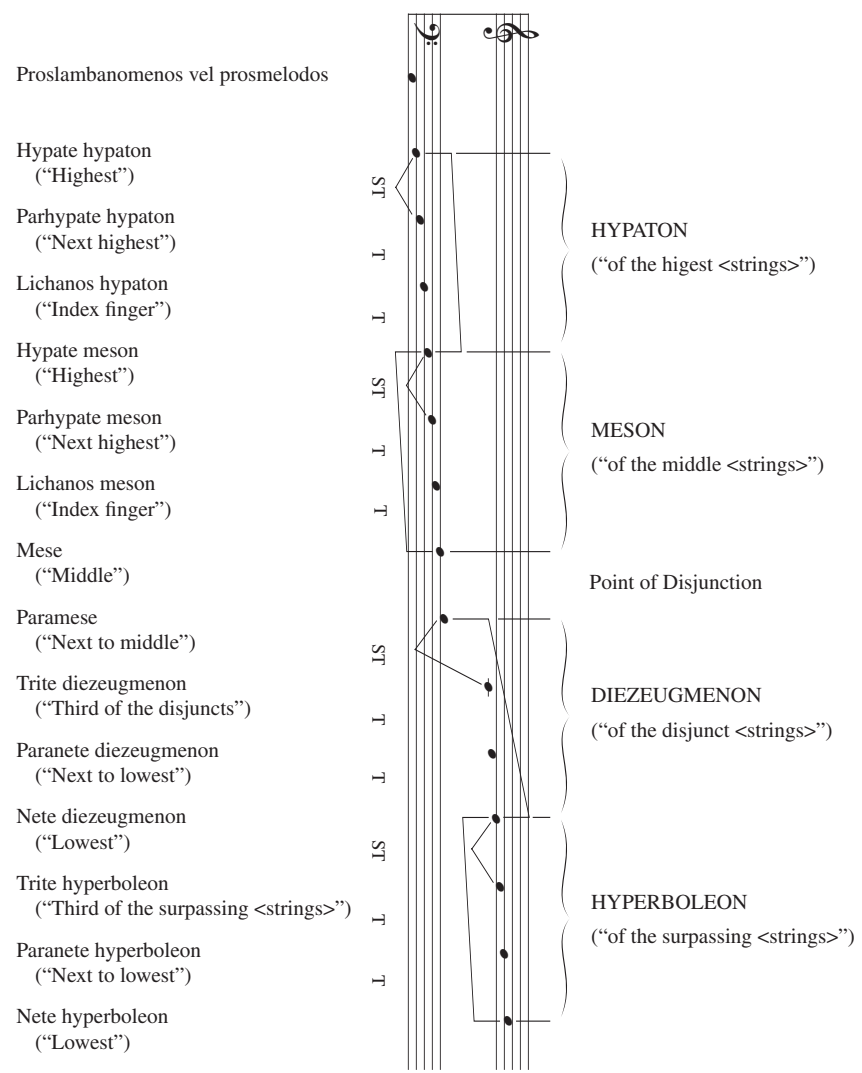
25. In Book IV, chaps. 6–12 of *De musica* (ed. Friedlein, 318–35; Bower, *Boethius: Fundamentals*, 131–46). Boethius presents an arithmetic division of the monochord in all three genera, using numbers that parallel those used by Aristides Quintilianus (Book III, chap. 2, ed. Winnington-Ingram, 97; cf. Mathiesen, *Aristides Quintilianus on Music*, 162; Barker, *Greek Musical Writings*, 2: 497–98).

26. Accordingly, the following discussion will treat only the diatonic genus. Boethius's own motivation—or that of his Greek source—for focusing on the diatonic genus in his diagrams for species and the modes may have been that, by his day, the diatonic was the genus most commonly in use. Aristides Quintilianus (fourth century C.E.?), for example, says: "Of these [genera], the diatonic is the more natural, for it is singable by everyone, even by those altogether uneducated. The color is the more artistic, for it is sung only by men of education; and the enharmonic is the more precise, for it has gained approval by those most distinguished in music; but for the multitude, it is impossible" (*De musica* Book I, chap. 9, ed. Winnington-Ingram, 16; transl. Mathiesen, *Aristides Quintilianus on Music*, 84; cf. Barker, *Greek Musical Writings*, 2: 418).

27. *De institutione musica*, Book IV, chap. 15 (ed. Friedlein, 341–42; Bower, *Boethius: Fundamentals*, 153). This two-octave system is called the "perfect" or "complete system" (*systema teleion*) by Ptolemy (*Harmonika*, Book II, chap. 4, ed. Düring, 50–51), in contrast to systems of smaller ambitus, such as the diapason, the diapason-plus-diatessaron, or the diapason-plus-diapente, which do not contain all the possible species of octave. (Further on this matter, see ex. 1.7 and the associated discussion.) For the sake of clarity, I shall maintain the traditional designations for the Greater and Lesser Perfect Systems as found, for example, in Cleonides, *Harmonica introductio* (ed. Jan, *Musici scriptores graeci*, 199–201; Solomon, "Cleonides," 136–37; Mathiesen, *Greek Views of Music*, 43–44). On the systemic nomenclature in Greek music treatises, see Barker, *Greek Musical Writings*, 2: 11–17.

28. I shall use the words "note" or "pitch" to designate what Boethius refers to as either *chorda* (lit.

EXAMPLE 1.1. The Ancient Greek Greater Perfect System, as presented in Boethius, *De institutione musica*, Book I, chap. 20 (ed. Friedlein, 212; Bower, *Boethius: Fundamentals*, 39)

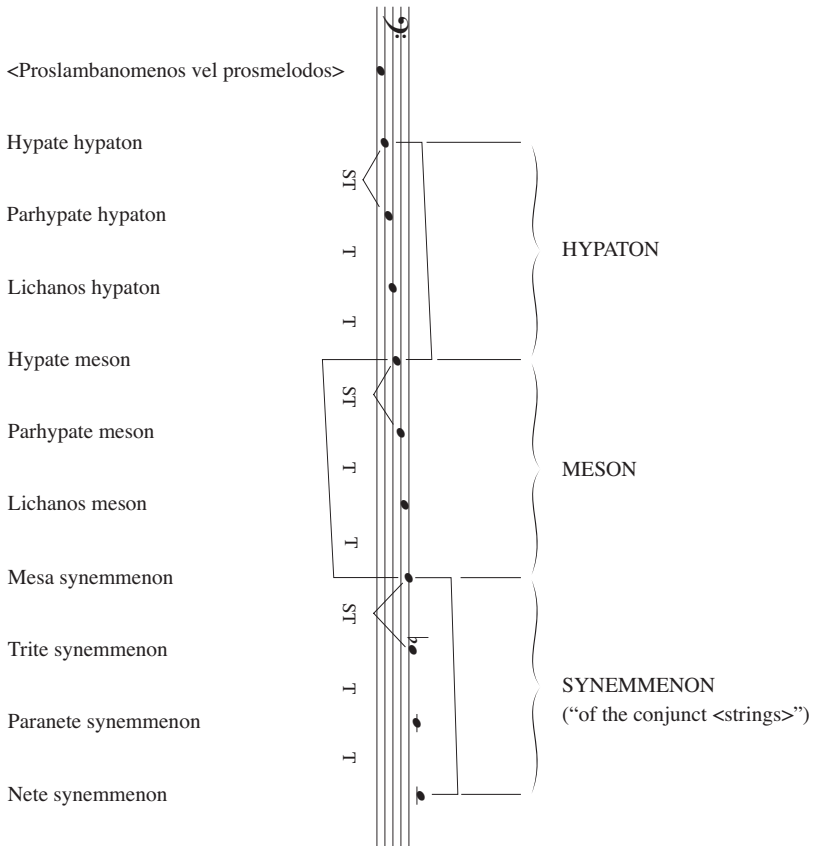


thius refers to as the diapason-plus-diatessaron or synemmenon system,²⁹ consists exclusively of conjunct tetrachords, joining the synemmenon tetrachord to the hypaton and meson tetrachords at the mese.

"string") or vox ("voice," "pitch"). *Nota* ("graphic sign"), Boethius's term for the notational symbols of the notes, I shall either leave in Latin or translate as "graphic sign" or "notational symbol." I do this because the semantic field of "note" in English includes the meanings of both pitch and graphic symbol.

29. *De institutione musica*, Book IV, chap. 15 (ed. Friedlein, 342; Bower, *Boethius: Fundamentals*, 153).

EXAMPLE 1.2. The Ancient Greek Lesser Perfect System, as presented in Boethius, *De institutione musica*, Book I, chap. 20 (ed. Friedlein, 210; Bower, *Boethius: Fundamentals*, 37)

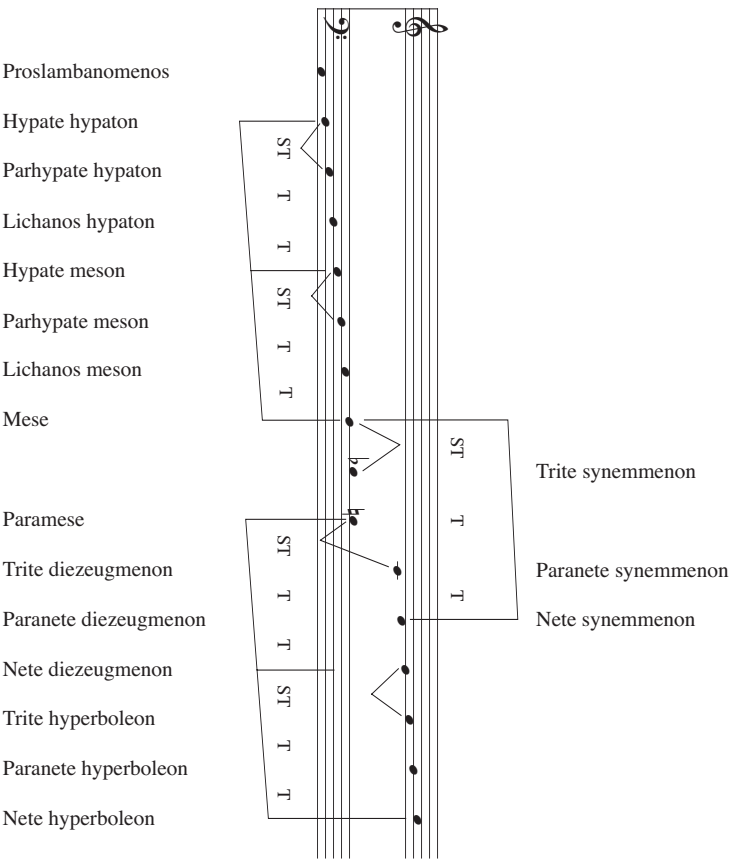


This system has three note names not in the Greater Perfect System, namely the trite, nete, and paranete synemmenon; in terms of pitches; however, it has only one degree that differs from the two-octave system, the trite synemmenon (b^b). It is thus usually represented as an additional tetrachord in the Greater Perfect System, which results in a combined eighteen-note system, referred to by several Greek writers as the *ametabolon systema* or Immutable System.³⁰ This can also be repre-

Ptolemy also refers to this system either as the diapason-plus-diatessaron or "conjunct system" (*systema synemmenon*; *Harmonika*, Book II, chap. 4, ed. Düring, 50–51, and Book II, chap. 6, ed. Düring, 54).

30. This system carries no designation in Boethius, but it is called *ametabolon systema* by later Greek writers including Thrasyllus (d. 36 C.E.), as quoted in Theon of Smyrna (fl. 115–40 C.E.), *Expositio rerum mathematicarum ad legendum Platonem vtilium* ("Exposition of the Mathematics Useful for Reading Plato"), ed. Hiller, p. 90, l. 22–p. 93, l. 9; Cleonides (second century C.E.?), in his *Harmonica introductio* (sec. 10; Jan, *Scriptores*, p. 201, ll. 8–11; Solomon, "Cleonides," p. 137) and Bacchius Geron (fourth century C.E. or later) in his *Introductio artis musicae* (Jan, *Scriptores*, p. 308, l. 3).

EXAMPLE 1.3. The Ancient Greek Immutable System, as presented in Boethius, *De institutione musica*, Book I, chap. 22 (ed. Friedlein, 215–16; Bower, *Boethius: Fundamentals*, 44)



sented as a fifteen-note system with one alternative pitch, the trite synemmenon (ex. 1.3). As we shall see, Boethius utilizes the latter method in one of the most famous diagrams of the treatise, that in Book IV, chap. 16, which presents the eight modes (see ex. 1.8 later).

As is apparent in the discussion and examples just cited, Boethius uses the ancient Greek names (e.g., mese, paramese, synemmenon) to designate both individual pitches and tetrachords. This nomenclature is based on the position of these notes as strings on an instrument, the kithara, not on their positions relative to each other within acoustic space. Hence, the system is presented “upside down” with reference to pitch. The hypate hypaton, the “highest of the high” tetrachord, has that name because it occupies the highest position on the kithara and, accordingly, appears at the top of the diagrams of the system; its pitch is actually the lowest. The nete hyperboleon (“lowest of the ‘surpassing’” tetrachord) is at the bottom of the

EXAMPLE 1.4. Boethius's first tone-system for the determination of species, from *De institutione musica*, Book IV, chap. 14 (ed. Friedlein, 341; Bower, *Boethius: Fundamentals*, 152)

A	—	Hypate hypaton
B	—	Parhypate hypaton
C	—	Hypaton lichanos
D	—	Hypate meson
E	—	Parhypate meson
F	—	Lichanos meson
G	—	Mese
H	—	Paramese
I	—	Trite diezeugmenon
K	—	Paranete diezeugmenon
L	—	Nete diezeugmenon
M	—	Trite hyperboleon
N	—	Paranete hyperboleon
O	—	Nete hyperboleon.

system, even though it is the highest pitch (cf. ex. 1.1).³¹ In addition to this nomenclature, however, Boethius also attaches Latin letter names to the degrees of the system for the purposes of dividing the monochord (Book IV, chaps. 5–11) and deriving the species of consonances (Book IV, chap. 14). For the latter, he uses two different letter arrays, one presenting fourteen notes beginning with the hypate hypaton as “A” and extending from A to O (ex. 1.4), the other presenting fifteen degrees lettered A to P, but without designation of string names (see ex. 1.9 later).³² These letters have sometimes been referred to as a kind of musical notation.³³ They are best not characterized in this way, however, but viewed, rather, as convenient designators of mathematic or geometric points, as in a Euclidean proof.³⁴

31. It is clear from Boethius's discussion of tone-system that he was well aware of the distinction between the “physical” or “instrumental” nomenclature for the strings and the acoustic basis for their functioning. As is represented by the orientation of the diagrams in Bower's translation, the tone-system is set out vertically in the manuscripts, with the lowest pitch, the proslambanomenos, at the top and the highest, the nete hyperboleon, at the bottom. The diagrams therefore project a visual image of the physical, not the acoustic, nomenclature for the strings. See, for example, the diagrams in Book I, chap. 20 (ed. Friedlein, 205–12; Bower, *Boethius: Fundamentals*, 29–39) and Book IV, chaps. 4 and 14 (ed. Friedlein, 312–14, 341; Bower, *Boethius: Fundamentals*, 127, 152).

32. Boethius, *De musica*, Book IV, chap. 17 (ed. Friedlein, 347; Bower, *Boethius: Fundamentals*, 159).

33. See, for example, Vogel, “Die Entstehung der Kirchentonarten.”

34. This point was made by Hans Schmid in the discussion following Vogel's presentation (“Die Entstehung der Kirchentonarten”) at the 1962 meeting of the Gesellschaft für Musikforschung. As Schmid pointed out, the letters used by Boethius for designating species do not correspond to those used in dividing the monochord, and in the monochord divisions themselves, the assignment of letters to notes varies according to genus. Cf. Bernhard, “Traditionen im mittelalterlichen Tonsystem,” 11–12.

Boethius does provide a discussion and diagram of actual Greek musical notation; he does so in his fourth book, in preparation for the derivation of species and the explication of the modes that appear at the end of that book. The relevant chapter (chap. 3) is titled *Musicarum notarum per graecas ac latinas litteras nuncupatio* (The naming of musical notes in Greek and Latin scholarship). As this title suggests, the pitches are named in Greek, but they are now also given their equivalent names in Latin.³⁵ More important: Boethius provides in this chapter the Alypian notational signs both for singing and for the playing of instruments in all three genera in the Lydian mode (ex. 1.5).³⁶

These signs are constructed of the letters of the Greek alphabet, modified and manipulated in various ways, as may be seen, for example, in the following quotation from the Bower translation of *De musica*: “Proslambanomenos, which can be called *adquisitus* [added]; an incomplete zeta and a tau lying on its side: ζ . Hypate hypaton, which is the *principalis principalium* [principal of the principal tetrachord]; a backward gamma and a normal gamma: Γ .”³⁷ Once the notational signs for the pitches have been introduced, Boethius can assign actual pitch content to them by converting the proportional ratios they represent into sound. His tool for doing this is the monochord.³⁸

The division of the monochord itself is one of the most crucial components of Boethius’s treatise, since it provided the means by which the mathematical theory of consonances and systems could be demonstrated precisely and translated into actual sound.³⁹ Two different divisions are presented—the first, a fairly straightforward, geometric one in the diatonic genus, which is presented in example 1.6; the second, a more complicated, arithmetic one in all three genera.⁴⁰

The manuscript transmission for both divisions is problematic. The first is incomplete; the second is marred by a number of inconsistencies and omissions.⁴¹ As

To Bernhard’s remarks I would add that Boethius uses letters in this way throughout the treatise, not just for the division of the monochord and for the determination of species in Book IV. A classic case may be found in Book III, chap. 1, Boethius’s proof that the tone cannot be divided into two equal parts.

35. The English translation of the title from chap. 3 is from Bower, *Boethius: Fundamentals*, 122. Note that Boethius’s term for the graphic signs of musical notation is *nota*, the standard Latin term for such signs in Antiquity and the early Middle Ages. As Bower points out (122), the Latin names for the pitches closely parallel those given in Martianus Capella, *De nuptiis*, Book IX, sec. 931.

36. Boethius, *De musica*, Book IV, chap. 3 (ed. Friedlein, 308–14; Bower, *Boethius: Fundamentals*, 122–27). For the Alypian signs themselves, see the edition of Alypius’s *Eisagoge* in Jan, ed., *Musici scriptores graeci*, 367–406. Other sources for the Greek notational symbols are Gaudentius, *Harmonica introductio* (ed., Jan, *Musici scriptores graeci*, 319–56; transl. Mathiesen, *Greek Views of Music*, 66–85); and the Beller-mann Anonymous III (ed. Najock, *Anonyma de musica scripta Bellermanniana*, 19–21).

37. Bower, *Boethius: Fundamentals*, 123. The diagram on p. 127 of Bower’s translation offers a more accurate rendering of the signs than that in Friedlein’s edition, 312–14.

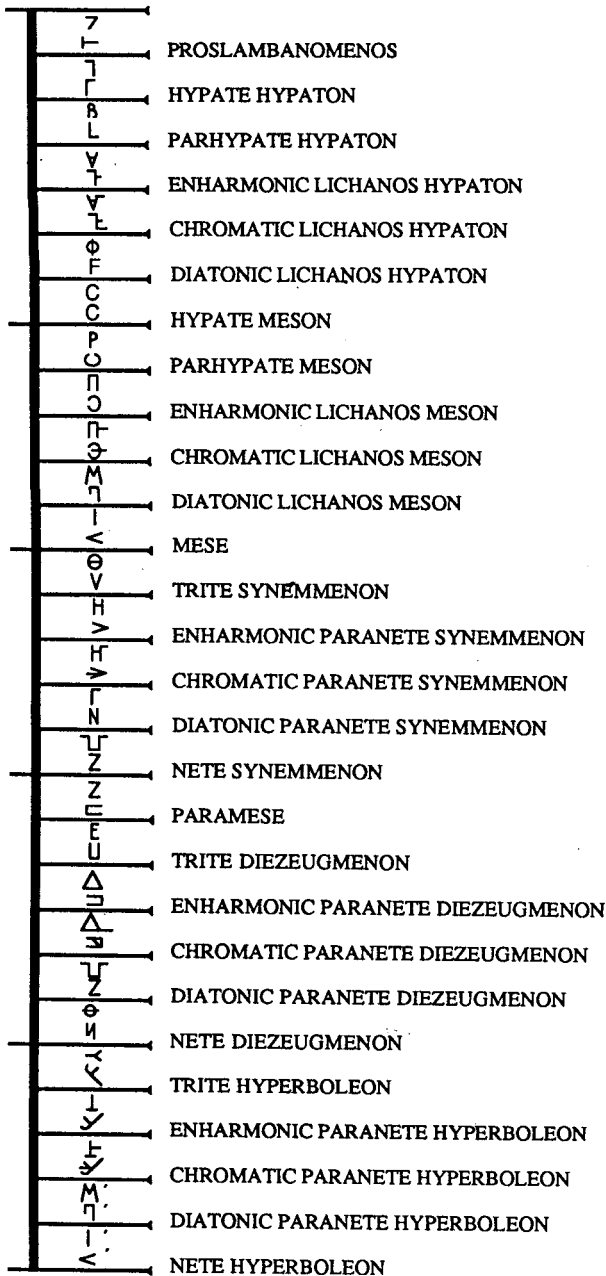
38. Since there is no standard length for the monochord, the sounding pitches produced by dividing it are relative, not absolute.

39. See Sachs, “Musikalische Elementarlehre im Mittelalter,” 152–61; and Sachs, *Mensura fistularum*, 2: 132–34, 144–46.

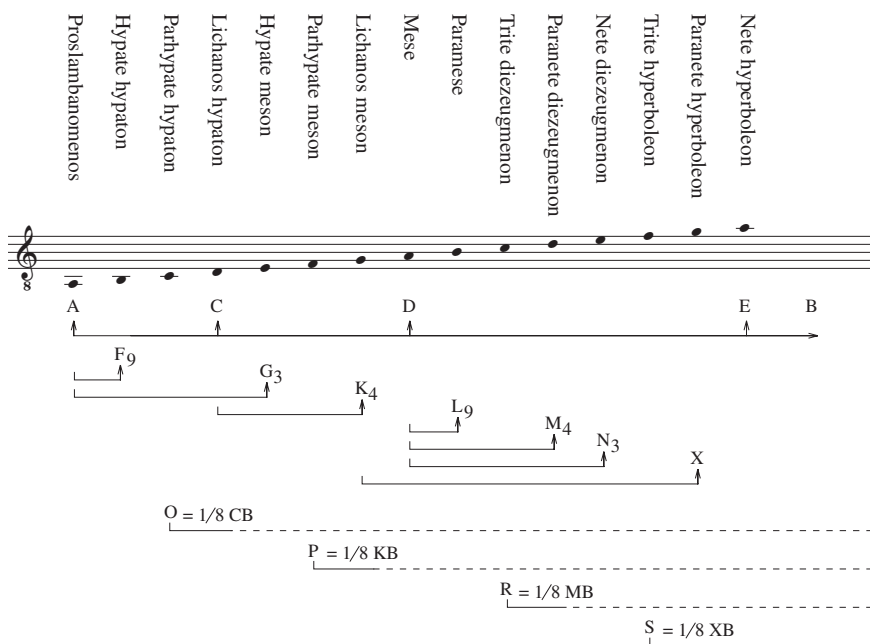
40. Boethius, *De musica*, Book IV, chaps. 5–12 (ed. Friedlein, 314–35). See Sachs, “Musikalische Elementarlehre im Mittelalter,” 152–54; and Sachs, *Mensura fistularum*, 2: 132–43. See also Bower, *Boethius: Fundamentals*, 126–46.

41. For discussion of these divisions, see Bower, *Boethius: Fundamentals*, 126, 130.

EXAMPLE 1.5. The Alypian notes for the pitches of the combined Greater and Lesser Perfect System in all three genera in the Lydian mode (from Boethius, *De institutione musica*, Book IV, chap. 4 (ed. Friedlein, 312–14; Bower, *Boethius: Fundamentals*, 127—reprinted from Bower)



EXAMPLE 1.6. Geometric division of the monochord in the diatonic genus, from Boethius, *De institutione musica*, Book IV, chap. 5 (ed. Friedlein, 314–18; Bower, *Boethius: Fundamentals*, 126–31)*



*The steps indicated by dotted lines, which are necessary to complete the division, have been supplied from Bower, *Boethius: Fundamentals*. 130–31.

mentioned above, the diatonic division is the one that had the greatest resonance in the Middle Ages. In it (ex. 1.6), the string, AB, is first divided into four equal sections, which locates the proslambanomenos, the lichanos hypaton, the mese, and the nete hyperboleon. Then the string is shortened by a ninth part, yielding the hypate hypaton. Following this, it is shortened by a third to yield the hypate meson. In subsequent steps, it is divided into proportionate segments (thirds, fourths, ninths) to yield the paramese, nete synemmenon, nete diezeugmenon, and paranete hyperboleon. Although the steps necessary to derive the parhypatai and tritai are not present in this division, these degrees could be located by adding eighth parts to the points already set out.⁴² For the Middle Ages, however, the procedural model this monochord division provided was its most important feature. Later writers would devise any number of variants on Boethius's original scheme.⁴³

42. See Bower, *Boethius: Fundamentals*, 130–31; Bower, "The Transmission of Ancient Music Theory into the Middle Ages"; and Sachs, "Musikalische Elementarlehre im Mittelalter," 154 n. 185.

43. For discussion of these variants, see Markovits, *Das Tonsystem der abendländischen Musik im frühen Mittelalter*, 37–42; Meyer, *Mensura monochordi*, xxvi–xxxvii; Smits van Waesberghe, *De musico-paedagogico et theoretico Guidone*, 156–72, nos. 1–37; and Adkins, "The Theory and Practice of the Monochord," 108–37.