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Leila Haaparanta

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Edited by
Leila Haaparanta

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Preface

This volume is the result of a long project. My work started sometime in the 1990s, when Professor Simo Knuuttila urged me to edit, together with a few colleagues, a volume on the history of logic from ancient times to the end of the twentieth century. Even if the project was not realized in that form, I continued with the plan and started to gather together scholars for a book project titled *The Development of Modern Logic*, thus making a reference to the famous book by William and Martha Kneale. Unlike that work, the new volume was meant to be written by a number of scholars *almost as if* it had been written by one scholar only. I decided to start with thirteenth-century logic and come up with quite recent themes up to 2000, hence, to continue the history written in *The Development of Logic*. My intention was to find a balance between the chronological exposition and thematic considerations. The philosophy of modern logic was also planned to be included; indeed, at the beginning the book had the subtitle "A Philosophical Perspective," which was deleted at the end, as the volume reached far beyond that perspective. The collection of articles is directed to philosophers, even if some chapters include a number of technical details. Therefore, when it is used as a textbook in advanced courses, for which it is also planned, those details are recommended reading to students who wish to develop their skills in mathematical logic.

In 1998, we had a workshop of the project with most of the contributors present. It was a fine beginning, organized by the Department of Philosophy at the University of Helsinki and by the Philosophical Society of Finland. We got financial support from the Academy of Finland and from the Finnish Cultural Foundation, which I wish to acknowledge. I moved to the University of Tampere in the fall of 1998. Unlike logic perhaps, life sometimes turns out to be chaotic. As we were a large group, it was no surprise that various personal and professional matters influenced the process of writing and editing. Still, we

happily completed the volume, which became even larger than was originally intended.

I wish to thank the contributors, from whom I have learned a great deal during the editorial process. It has been a pleasure to cooperate with them. Renne Pesonen and Risto Vilkko kindly assisted me with the editorial work. I am very grateful to my colleagues for useful pieces of advice. There are so many who have been helpful that it is impossible to name them all. My special thanks are due to Auli Kaipainen and Jarmo Niemelä, who prepared the camera-ready text for publication. Jarmo Niemelä also assisted me with compiling the index. I wish to thank Peter Ohlin, editor at Oxford University Press, who has been extremely helpful during the process. I have benefited considerably from the help of my editors, Stephanie Attia and Molly Wagener, of Oxford University Press. The financial support given by the Academy of Finland is gratefully acknowledged. I have done the editorial work at the University of Tampere, first at the Department of Mathematics, Statistics and Philosophy and then at the Department of History and Philosophy. Finally, I wish to express my deep gratitude to my mother and to my husband, whose support and encouragement have been invaluable.

L. H.

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1

Introduction

Leila Haaparanta

1. On the Concept of Logic

When we state in everyday language that a person's logic fails, we normally mean that the rules of valid reasoning, which ought to guide our thinking, are not in action for some reason. The word "logic" of our everyday language can usually be analyzed as "the collection of rules that guide correct thinking or reasoning." That collection is assumed to be known naturally; a rational human being follows those rules in normal circumstances, even if he or she could not formulate them, that is, express them in language. When the word "logic" (in Greek *logos* "word," "reason") refers to one subfield of philosophy or of mathematics, it usually means the discipline concerning valid reasoning or the science that studies that kind of reasoning.

In his logical studies, Aristotle (384–322 B.C.) considered inferences, which are called syllogisms. They consisted of two premises and a conclusion, and the validity of the argument of a syllogistic form was determined by the structure of the argument. If the premises of a syllogism were true, the conclusion was also true. According to Aristotle, the basic form of a judgment is "A is B," where "A" is a subject and "B" is a predicate. Forms of judgments include "Every A is B," "No A is B," "Some A is B," and "Some A is not B." Unlike Aristotelian logic, modern formal logic is called symbolic or mathematical, as it studies valid reasoning in artificial languages.

Until the nineteenth century logic was mainly Aristotelian. Following Aristotle, the main focus was on judgments that consisted of a subject and a predicate and that included such words as "every," "some," and "is" in addition to letters corresponding to the subject and the predicate. The Stoics, for their part, were

interested in what is nowadays called propositional logic, in which the focus is on such words as "not," "and," "or," and "if–then."

It was not until the nineteenth century that symbolic logic, which had its model in mathematics, became a serious rival of Aristotelian logic. The grammatical analysis of judgments was challenged in the late nineteenth century by logicians who took the model of analysis from mathematics. The words "function" and "argument" became part of the vocabulary of logic, and predicates that expressed relations as well as quantifiers were included in that vocabulary. In the new logic, which was mostly developed by Gottlob Frege (1848–1925) and Charles Peirce (1939–1914) and which was codified in *Principia Mathematica* (1910, 1912, 1913), written by A. N. Whitehead (1861–1947) and Bertrand Russell (1872–1970), the rules of logical inference received a new treatment, as the pioneers of modern logic tried to give an exact formulation of those rules in an artificial language.

Except for the collection of the rules of valid reasoning and the discipline or the science that focuses on those rules, the word "logic" means a specific language that fulfills certain requirements of preciseness. It also means a field of research that focuses on such a language or such languages. Since the seventeenth century, it has been typical of the field called logic to construct and study a formal language or formal languages called logic or logics.

The old Aristotelian logic heavily relied on natural language. Aristotle and his followers thought that natural language reflects the forms of logical inference and other logical relations, even the form of reality. The pioneers of modern logic sought to construct an artificial language that would be more precise than natural languages. In the twentieth century those languages called logics have been used as models of natural languages; hence, modern logic that rejected the grammatical analysis of judgments has, among other things, served as a tool in linguistic research. It is important to note that the pioneers of modern logic, such as G. W. Leibniz (1646–1716) and Frege, did not intend to present any tools of studying natural languages; they wished to construct a symbolic language that would overcome natural language as a medium of thought in being more precise and lacking ambiguities that are typical of natural language.

As the views of the tasks and the aims of logic have varied in history, we may wonder whether Aristotle and the representatives of modern logic, for example, Frege, were at all interested in the same object of research and whether it is possible to talk about the same field of research. In spite of differences, we may name a few common interests whose existence justifies the talk about research called logic and the history of that field. In each period in the history of logic, researchers called logicians have been interested in concepts or terms that are not empirical, that is, whose meanings are not, or at least not incontestably, based on sensuous experience, and that can be called logical concepts or terms. What concepts or terms have been regarded as logical has varied in the history, but interest in them unites Aristotle, William of Ockham, Immanuel Kant, and Frege as well as logicians in the twentieth and twenty-first centuries. Other

points of interest have been the so-called laws of thought, for example, the law of noncontradiction and the law of excluded middle. A third theme that unites logicians of different times is the question of the validity of reasoning.

In several chapters of the present volume, the question concerning the nature and the scope of logic is discussed in view of the period and the logicians that are introduced to the reader.

2. What Is Modern Logic?

The starting point of modern logic is presented in textbooks in various ways depending on what features are regarded as the characteristics of modernity. Some say modern logic started together with modern philosophy in the late Middle Ages, while others think that it started in the seventeenth century with Leibniz's logic. Still others argue that the beginning of modern logic was 1879, when Frege's *Begriffsschrift* appeared.

If the beginning of modern logic is dated to the seventeenth century, its pioneers include Leibniz, Bernard Bolzano (1781–1848), Augustus De Morgan (1806–1871), George Boole (1815–1864), John Venn (1834–1923), William Stanley Jevons (1835–1882), Frege, Peirce, Ernst Schröder (1841–1902), Giuseppe Peano (1858–1932), and Whitehead and Russell. Unlike many contemporary logicians, modern logicians believed that there is one and only one true logic. Leibniz was the most important of those thinkers who argued that the terms of our natural language do not correspond to the objects of the world in a proper way and that therefore we have to construct a new language, which mirrors the world correctly. Following Leibniz, modern logicians sought to construct an artificial language that would be better than natural languages. If we think that this kind of effort is an important feature of modern logic, then we may say that modern logic started with Leibniz. The idea of calculus has also been an important feature of modern logic. Logic has been considered a system which consists of logical and nonlogical vocabulary, formation rules, and transformation rules; the formation rules tell us what kind of sequences of symbols are well formed, and the transformation rules are the basis on which logical reasoning is performed like calculating.

Many early pioneers of modern logic relied on the grammatical subject-predicate analysis in analyzing sentences that was also part of traditional logic, as mentioned above. It was not until Frege's logic that this division was rejected. The division between arguments and functions thus became central in logic. Frege also stressed that it was the distinction between individuals and concepts that he wants to respect. If we stress that feature, we may say that the philosophical ideas of modern logic can be found in medieval nominalists, but that they did not become codified in formal languages until the latter half of the nineteenth century in Frege's and Peirce's discoveries. Those two logicians also made quantifiers into the basic elements of logic. As modern thinkers, many late medieval philosophers were interested in individuals, but

the distinction between an individual and a concept was not taken into account in logic until Frege's and Peirce's discoveries. Frege regarded his logic as an axiomatic theory. That feature can also be considered a typical feature of modern logic.

As was said before, it is often thought that Frege's *Begriffsschrift* gave birth to modern logic. In that book there were many logical discoveries, such as the theory of quantification and the argument-function analysis. Frege's book was both philosophical and mathematical. Later, in the first volume of his *Grundgesetze der Arithmetik* (1893), Frege states that he is likely to have few readers; all those mathematicians stop reading who, when seeing the words "concept," "relation," and "judgment" think: "It is metaphysics, we do not read it," and those philosophers stop reading, who, when seeing a formula, shout: "It is mathematics, we do not read it" (p. xii).

Charles Peirce discovered the logic of relatives in the 1870s. That logic was inspired by Boole's algebra of logic and De Morgan's theory of relations. Peirce's articles "The Logic of Relatives" (1883) and "On the Algebra of Logic: A Contribution to the Philosophy of Notation" (1885) contain the first formulation of his theory of quantification that he calls his general algebra of logic. Peirce's algebra differed from that of Boole's especially in that Peirce introduced signs that refer to individuals in addition to signs that signify relations. Second, he introduced the quantifiers "all" and "some." Frege only used the sign for generality and defined existence by means of generality and negation. Both the logicians rejected Boole's idea that judgments are formed by combining subjects and predicates. Frege and Peirce, who made their important discoveries independently of each other, Peirce maybe with his group of students and Frege alone, had common features. They were both philosophers and mathematicians and could combine philosophical ideas with technical novelties in their logical thought.

Frege and Peirce both invented a notation for quantifiers and quantification theory almost simultaneously, independently of each other. Therefore they can be regarded as the principal founders of modern logic. However, as many scholars have emphasized, most notably Jean van Heijenoort in his paper "Logic as Calculus and Logic as Language" (1967), Jaakko Hintikka in his papers "Frege's Hidden Semantics" (1979) and "Semantics: A Revolt Against Frege" (1981), and Warren Goldfarb in his article "Logic in the Twenties: The Nature of the Quantifier" (1979), the two logicians seem to be far apart philosophically. The division between the two traditions to which the logicians belong has also been emphasized by a number of authors of the present volume. The distinction between the two conceptions of logic, namely, seeing logic as language versus seeing it as calculus, has been suggested from the perspective of twentieth-century developments, but the origin of the division has been located in nineteenth-century logic. Different interpretations of the history of logic follow depending on how the distinction is understood. According to van Heijenoort, Hintikka, and Goldfarb, those who stressed the idea of logic as language thought that logic speaks about one single world. It is certain

that Frege held that position. He thought that there is one single domain of discourse for all quantifiers, as he assumed that any object can be the value of an individual variable and any function must be defined for all objects. On the other hand, those who supported the view that logic is a calculus gave various interpretations or models for their formal systems. That was Boole's and his followers' standpoint. Several other features of the two traditions are mentioned in the chapters of the present volume.

The volume titled *Studies in the Logic of Charles Sanders Peirce* (1997) introduces another pair of traditions, which are mathematical logic and algebraic logic and which are also touched upon in the present collection of articles. Ivor Grattan-Guinness states in his contribution to the volume on Peirce that the phrase "mathematical logic" was introduced by De Morgan in 1858 but that it served to distinguish logic using mathematics from "philosophical logic," which was also a term used by De Morgan. However, in Grattan-Guinness's terminology, De Morgan's logic was part of the algebraic tradition; using algebraic methods in logic would be typical of what he calls algebraic logic. The most common phrase used in the nineteenth century was "the algebra of logic" or sometimes "logical algebra."

In the figure which Grattan-Guinness presents to us, Boole, De Morgan, Peirce, and Schröder belong to the tradition of algebraic logic, while Peano and Russell belong to the tradition of mathematical logic. It seems that many of those who belong to the tradition of logic as calculus belong to the tradition of algebraic logic in Grattan-Guinness's division, and that many of those who think that logic is a language belong to what Grattan-Guinness calls the tradition of mathematical logic. Grattan-Guinness gives us a few typical features of the two traditions that he discusses. In algebraic logic, laws were stressed, while in mathematical logic axioms were emphasized. Moreover, he states that in mathematical logic, especially in the logicist version represented by Russell, logic was held to contain all mathematics, while in algebraic logic it was maintained that logic had some relationship with mathematics. In Grattan-Guinness's view, algebraic logic used part-whole theory and relied on a basically extensionalist conception of a collection, while in mathematical logic the theory of collections was based on Cantor's *Mengenlehre*. In addition, there was, in his view, an important difference between the traditions concerning quantification; the interpretation of the universal and existential cases as infinite conjunctions and disjunctions with the algebraic analogies of infinite products and sums was typical of the algebraic tradition. Grattan-Guinness also notes that the questions addressed in mathematical logic were more specific than those addressed in algebraic logic.

Frege's and Peirce's logical views are discussed in several chapters of the present volume. Many contributors also touch on the more general question concerning the borderline between traditional and modern logic, the divisions between the traditions of modern logic, and the shift from the modern logic of the late nineteenth century and the early twentieth century till twentieth-century logic. The periods of Western logic that are studied in the present

collection of articles extend from the thirteenth century to the end of the twentieth century. Unlike the rest of the contributions, the chapter on Indian logic covers several schools whose history reaches far back in the history but which are also living traditions in contemporary Indian logic.

3. Logic and the Philosophy of Logic

Besides the term "logic," the terms "philosophical logic" and "philosophy of logic" have various uses. Philosophy of logic can be understood as a subfield of philosophy that studies the philosophical problems raised by logic, including the problem concerning the nature and the scope of logic. Those problems also include metaphysical, or ontological, and epistemological questions of logic, problems related to the specific features of logical formal systems (e.g., related to the basic vocabulary of logic) and logical validity, questions concerning the nature of propositions, judgments, and sentences, as well as theories of truth and truth-functions, and the questions concerning modal concepts and the alternatives of classical logic, which some call by the name "deviant logics." The term "philosophical logic" is often used as a synonym of "philosophy of logic"; occasionally it means the same as "intensional logic," or it is used as an opposite to "mathematical logic." By metalogic, one normally means the study of the formal properties of logical systems, such as consistency and completeness, and thus distinguishes it from the philosophy of logic, which studies their philosophical aspects.

The present volume deals with the history of modern logic and pays attention both to the core area of logic and to the philosophy of logic. Such terms as "classical logic," "modal logic," "alternative logics," and "inductive logic" are also used and explained in the chapters of the volume. The variety of logics raises the problem of demarcation that is essential to the philosophy of logic: which formal systems belong to the objects of logical research, and which ought one to exclude from the field of logic? For example, the program of logicism, which was supported by Frege, among others, was a position taken in the discussion concerning the demarcation of logic.

Logic and philosophy have complicated relations. Nowadays logical tools are often used as the methods of philosophy. Logical discoveries have also been motivated by philosophical views, and philosophers have changed their opinions because of logical discoveries. Logic can be said to have a philosophical basis, and likewise there are philosophical doctrines that rely on developments of logic. The present collection of articles studies some of those relations. To some extent, it also pays attention to the relations between logic and mathematics and logic and linguistics.

Logic and rationality are often tied together, but the concept of rationality has many uses in everyday language and in philosophical discussion. We talk about logical or argumentative rationality and refer to one's ability to reason or to give arguments, and we also think that one who is rational is able to

evaluate various views critically and independently of authorities; in this latter meaning, logic is considered to play a significant role. Moreover, rationality is both theoretical and practical, the latter form of rationality being related to a person's actions, and philosophers also tend to regard one's ability to control one's volitional and emotional impulses as a sign of rationality. There is no one concept or "essence" of reason that can be detected in philosophical or in everyday discussion. However, what we can find in most uses of the concept is the general idea of control (control of thought, actions, passions, etc.), which is also central in logical rationality.

Even if rationality as control or as rule-following seems to be crucially important, rationality as a faculty of judgment is also in everyday use in the practice of logicians as in all science. In the tradition of logic, it has been important both to be able to follow rules or repeat patterns and to be able to evaluate the commands and prohibitions. It is important both to be able to think inside a given system and to be able to evaluate the very system from the outside. The history of modern logic is a history of these two huge projects.

Philosophers and logicians have used the volume titled *The Development of Logic* by William and Martha Kneale (1962) for decades. The ambitious idea behind the present work was to write a book on the development of modern logic that would bring the history of modern logic till the end of the twentieth century and would also pay attention to the philosophy of logic and philosophical logic in modern times. The idea was not to bring about a handbook but a volume that would be as close as possible to a one-author volume, that is, a balanced whole without serious gaps or overlaps. It was taken for granted in the very beginning that that goal cannot be reached in all respects. Each author has chosen his or her style, some wish to give detailed references, others are happier with drawing the main lines of development with fewer details; some express their ideas in many words, while others prefer a concise manner of writing. However, what has been reached is a story that covers a number of themes in the development of modern logic. The history begins with late medieval logic and continues with logic and philosophy of logic from humanism to Kant, that is, with two chapters whose scope is chronologically determined. Chapters 4–7 cover the nineteenth century and early twentieth century in certain respects, namely, they focus on the emergence of symbolic logic in two ways, first, by paying attention to the relations between logic and mathematics, second, by emphasizing the connections between logic and philosophy. That discussion is completed by a chapter that focuses on the themes of judgment and inference from 1837 to 1936.

The volume contains an extensive chapter of the development of mathematical logic 1900–1935, which is continued by a discussion on main trends in mathematical logic after the 1930s. The subfields of logic that are called modal logic and philosophical logic are discussed in two separate chapters, one dealing with the history of modal logic from Kant until the late twentieth century and the other discussing logic and semantics in the twentieth century. Separate chapters are reserved for the philosophy of alternative logics, for the

philosophical aspects of inductive logic, for the relations between logic and linguistics in the twentieth century, and for the relations between logic and artificial intelligence. Eastern logic is not covered, but the main schools of Indian logic are presented in the last chapter of the volume. While the former part of the volume is chronologically divided, the chapters of the latter part follow a thematic division.

Note

I have used extracts from my article "Peirce and the Logic of Logical Discovery," originally published in Edward C. Moore (ed.), *Charles Peirce and the Philosophy of Science* (University of Alabama Press, Tuscaloosa, 1993), 105–118, with the kind permission of University of Alabama Press. The chapter also contains passages from my review article "Perspectives on Peirce's Logic," published in *Semiotica* 133 (2001), 157–167, which appear here with the kind permission of Mouton de Gruyter.

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2

Late Medieval Logic

Tuomo Aho and Mikko Yrjönsuuri

1. The Intellectual Role and Context of Logic

Our aim is to deal with medieval logic from the time when it first had full resources for systematic creative contributions onward. Even before that stage there had been logical research and important logicians. The most original of them, Abelard, achieved highly significant results despite having only a very fragmentary knowledge of ancient logic. However, we shall concentrate on the era when the ancient heritage was available and medieval logic was able to add something substantial to it, even to surpass it in some respects. A characterization such as this cannot be adequately expressed with years or by conventional period denominations; we hope though that the grounds for drawing boundaries will become clearer during the course of our story.

1.1. Studies

It was characteristic of later medieval logic that it was pursued as an academic discipline, as a major component in an organized whole of studies. Indeed, after the Middle Ages, logic has never been allotted so large a share in the activities of the universities. Moreover, logic was connected to certain classical texts that were seen as natural foundations of this science. Thus, it is reasonable first to say something about the system of studies in general and about the nature of these works in particular.

Ever since Rome, school teaching had always centered on the *trivium* of grammar, rhetoric, and dialectic. When schools developed and the most prominent clusters of schools began to turn into universities, these disciplines

found their place in the faculty of arts (*artes*). Dialectic, the art of arguing and reasoning, was largely concerned with logical issues, and was often taken to be the most important art of the *trivium*. Thus, the outcome was that every student had to take extensive courses in logic. Perhaps the dialectical background can throw some light upon the linguistic and semantic tone of medieval logical thought.

The faculty of arts was always much bigger than the higher faculties (theology, law, medicine). If there was a theological faculty in a university, it was associated with advanced studies and required a preliminary education in arts. But philosophical and logical research was pursued by theologians even after proceeding to the higher faculty; in fact, the most competent scholars often preferred the privileged higher faculty. Thus the history of logic must take into account the production of both faculties. Many commentaries on Peter Lombard's theological *Sentences* contain important passages on logic, and topics related to logic are often dealt with in the so-called *quodlibetal disputations*, to mention just two examples.

We cannot pay much attention to the history of universities, though we can say that the process of university education started in Italy in the twelfth century, Bologna being the oldest university. Paris, however, was undoubtedly the most important university for philosophy, and it received its official statutes in 1215. Paris was a permanent international center for current philosophical and theological discussion. Another place where logical research was often especially popular was Oxford. These were the two capitals of medieval logic, although the center of gravity shifted to Italy in the less innovative period toward the end of the fourteenth century. During the fourteenth century, universities spread to the east and to the north. There were 15 universities in 1300, 30 in 1400, and about 60 in 1500, naturally of very different size and quality, though one component of studies was standard everywhere, and that was logic.

1.2. The Growth of Logic

Medieval philosophers normally made use of an array of authoritative classical texts, which were taken to be trustworthy, though not infallible. The curriculum was organized around these texts, and very often the problems discussed were put forward as questions of interpretation and explication of the texts. Hence the general breakthrough of Aristotelianism in the thirteenth century represented a great change, establishing Aristotle as the main source of academic studies. But in *logic* Aristotle had even before that been regarded as the greatest of authors, and anti-Aristotelian reactions did not seriously extend to logic. Rather than being rejected in the Middle Ages, Aristotle's own work in logic was built upon and developed ever further toward the end of the period.

The famous standard translation for most of Aristotle's texts was that by William Moerbeke. With the logical works the case was different: Though

Moerbeke translated some of them in the 1260s, the authority of the old Roman translation by Boethius (c. 480–524) remained unquestioned. The *Organon* that late medieval logicians used was the Latin text of Boethius. (For *Posterior Analytics*, no translation by Boethius is known; its standard rendering was made by James of Venice before 1150.) These translations are actually quite accurate, although written in a very formal and literal idiom.

Three concise basic works belonged to the kernel of logic throughout the Middle Ages. These were Aristotle's own short *Categories* and *De interpretatione*, and Porphyry's introduction, *Isagoge*. In addition to these, the so-called old logic (*logica vetus*) used Boethius's logical works and a few minor ancient texts (by, e.g., Apuleius and Augustine). The shape of logic changed considerably when Aristotle's complete works of logic became known in the middle of the twelfth century. That opened the way for the new logic, *logica nova*, and in a relatively short time the corpus of *logica vetus* was practically replaced by new works. Even Boethius's treatises on syllogisms fell into disuse. Except for Aristotle and Porphyry, the only work that retained its place was *Liber sex principiorum*, a treatise explaining the categories that Aristotle himself does not dwell upon.

The period of *logica nova* used as its authoritative corpus all the six works in Aristotle's *Organon: Categories*, *De interpretatione*, *Prior Analytics*, *Posterior Analytics*, *Topics*, and *Sophistici elenchi*. At first, dialecticians were especially fascinated by fallacies and sophisms (*Soph. el.*), but gradually the investigation turned more toward the formal theory of syllogism (*Pr. Anal.*). During the thirteenth century, they encountered problems that could not be answered by straightforward Aristotelian principles, and were thus drawn to new fields of logic. After the introduction of such new subjects, logic came to be called *logica moderna*, in contrast to *logica nova*, now called *logica antiqua*. This way of speaking, however, did not imply any break with the earlier Aristotelian tradition, only an expansion of investigation.

The first complete handbooks of *logica moderna* date from the second quarter of the thirteenth century. The earliest known overview is *Introductiones in logicam* by William of Sherwood from the 1230s, but the greatest success of all was the *Tractatus*, also called *Summulae logicales*, by Peter of Spain (probably from the 1240s). This comprehensive work maintained its status as a famous standard textbook throughout the later Middle Ages and the Renaissance, even in the time of printed books. It also served as the source for numerous shorter courses. Similar ambitious textbooks were written by Roger Bacon (*Summulae dialectices*, 1250s) and Lambert of Auxerre (*Logica*, 1250s). In a way, these works can be seen as a synthesis of the founding period of *logica moderna*: On the one hand, they were the first systematic presentations of whole logic, on the other hand, they completed the new so-called terminist logic. Simultaneously a more profound philosophical discussion was started by the influential Robert Kilwardby, who wrote one of the first commentaries on *Prior Analytics* (1240s).

As to the logical content in the overall presentations of logic, significant advance comes only much later, in the generation of Walter Burley (c. 1275–1344) and William Ockham (c. 1285–1347). Ockham's *Summa logicae* is from the early 1320s, Burley's *De puritate artis logicae* from the late 1320s. These works manifest a turn in logical literature toward new problems and to a more theoretical way of thinking. The greatest representative of the next period is John Buridan (c. 1300–1361?); a comprehensive picture of his teaching in Paris is given in his *Summulae de Dialectica*. In the latter half of the fourteenth century, logic was already highly technical. In particular, a series of Englishmen distinguished themselves, among them William Heytesbury (d. 1372?), Ralph Strode (d. 1387?), and Richard Lavenham (d. 1399). A kind of summary of this stage is the enormous *Logica magna* (c. 1400) by the Italian Paul of Venice (c. 1369–1429).

1.3. Non-Latin Traditions

Our account will be only about the Latin West. The significance of Arabic philosophy must be emphasized, and yet we shall not discuss the Arabic logic per se, as it had its creative phase long before the time of Western late medieval philosophy. Aristotle's *Organon* was translated into Arabic in the ninth century in Baghdad, and a commentary tradition started soon after that. Logic was honored as a kind of grammar of reasoning, and for example, al-Farabi (c. 870–950) underscored its importance as the "forecourt of all philosophy." Avicenna (Ibn Sīnā, 980–1037), on the other hand, was already a brilliant, independent exception: During his time, logical research was already declining, and commentaries were replaced by handbooks. His work had a profound influence on Western theories of meaning. In the twelfth century, the Spanish Arabic school revived commentaries, and the last commentator, Averroes (Ibn Rushd, 1126–1198), was also the greatest. The works of Averroes, "the Commentator," were soon translated and became highly appreciated in Europe. In logic he was not as dominant as in metaphysics or in natural philosophy, but undoubtedly his works belong to the background that was always present. Averroes's thought survived mainly in the West. In the Islamic world, logic was integrated into studies of theology and law, and even handbooks were gradually replaced by more or less elementary textbooks. During the period we describe, from the thirteenth century on, Arabic logic no longer produced anything but new versions and editions of established textbooks.

On the other hand, a rich tradition of Jewish philosophy was alive in Europe through the late Middle Ages. Logic was not its favorite field, but some Jewish authors paid considerable attention to logical questions. However, these studies had little interaction with Latin logic, and thus had to rely solely on Aristotle as commented by Averroes and al-Farabi. Still, there were some innovations, the most interesting figure being Gersonides (1288–1344). Writing in a rigorous manner, he made a number of criticisms of traditional doctrines; among other

things, he rejected the old Averroistic construction of modal syllogistics in *Prior Analytics* (thus paralleling contemporaneous Latin developments).

Even today, very little is known about Byzantine logic. Apparently an uninterrupted interest in logic, "the instrument of philosophy," existed among Byzantine scholars. It produced mainly Aristotelian commentaries, often in the neo-Platonist spirit. Its independent progress was severely hindered by a conservative, philological approach to Greek sources, and occasionally also by religious scruples against the pagan heritage.

1.4. Texts

Aristotle was the essential basis of later medieval logicians, but other classical ideas also played their part. First, Greek Aristotelian commentators had discussed various problems in Aristotle's logic and its correct systematization, and their work became partly known (either directly or through Arabic sources). Second, the Stoics had argued that Aristotelian predicate logic was insufficient and required some background from the propositional logic that they studied as the real logic. No complete Stoic works were preserved, but these Stoic themes were transmitted, for example, by Augustine's *Dialectica* and by Boethius. We shall meet similar problems in the medieval theory of demonstration in topics and consequences. Because of the Stoic influence, medieval logicians were always in a different position from the ancient Peripateticians in that they were aware of the necessity of essentially nonsyllogistic inference. Furthermore, logic was obviously influenced by classical grammar, which provided it with categories like nouns, verbs, and other parts of speech, as well as central syntactical notions, the main authority being Priscian's grammar of Latin. Finally, some logical material had found its way into the work of famous ancient authors, among them Cicero, and the Christian fathers.

From the middle of the thirteenth century, there was a rapid increase both in logical studies and in Aristotelian studies in general. Soon the obligatory logical curriculum included the whole of the *Organon*. Aristotle's text is so concise and difficult that it was always accompanied by commentaries and explanatory texts. It was required that students mastered this material thoroughly, and practical logical exercises became very popular as a supplement to lectures. A major and growing part of studies was dedicated to logic. If we understand logic in the widest sense, it appears that more than half of the program of an arts faculty could be about logic. At least we may note that logic had an undisputed place in medieval learning, and that it was not a specialist subject since almost all leading philosophers wrote about logic.

Most logical works were closely connected to university teaching. The usual teaching method in medieval universities was that a text was lectured on and explained in detail. The intention was to build a consistent interpretation of the text, to eliminate ambiguities and to resolve the problems and conflicts the text gave rise to, and a typical medieval method of study included disputations where some theses were argued for and against. The character of university

teaching goes some way to explaining the literary types that became widespread. In addition to simple lecture drafts, there were all kinds of commentaries, ranging from elementary glosses to large systematic books. There were treatises (*tractatus*), that is, manuals or more advanced surveys of some field, which gradually became more independent of the underlying texts. The liking for argumentation and disputation produced *quaestiones*, analytic works where some specific question is resolved or a thesis defended. (Later, systematic studies were organized in the form of a series of questions even though they were often referred to as commentaries.) We must also remember that logical subjects are often encountered as digressions in other works, for example, in the extensive sentence commentaries of theologians.

There has been quite a decisive improvement in the accessibility of medieval logic over the last three decades, when numerous texts have been published. However, a large amount of material still remains unpublished and even completely unstudied. In fact, it is quite possible that our whole view of the outlines of medieval logic will undergo a change; indeed, such changes have occurred before, and systematic historical research of this logic is still a very young enterprise.

1.5. Interdisciplinary Relations

Obviously, logic had a well-established place in the system of disciplines in the Middle Ages. But what kind of interaction did logic have with the other sciences? Unfortunately, it is not easy to say anything definite about this. First of all, formal philosophy of science was studied by logicians in connection with the *Posterior Analytics*, which discusses the correct form and nature of deductive theories. In this way, the methodology and philosophy of science were a part of medieval logic. Also, the occasional attempts to create calculative scientific speculations used heavy logic, but in general there was little concrete connection of logic to particular natural sciences, which took care of their own subjects. On the other hand, metaphysics—universally considered a real science—was always relevant for logic. Thus, semantic theory, so prominent in medieval logic, is immediately bound to metaphysical questions. Just as early supposition theory employs a metaphysical basis, so in the late-medieval nominalist trend it is impossible to separate logical from ontological thought.

The role of theological matters is less transparent. Obviously theology needed systematic thought and conceptual analysis, and was hence favorable to logic. The conceptual examples and difficulties that logicians examined were very often drawn from theology. Generally, the significance of theology for logic must have been positive. Their union was made problematic, however, when many philosophers began to think that some mysteries of faith, such as the Trinity, were not only inscrutable but literally beyond logic—even if their exact formulation could be a task for logic. Thus Ockham and Buridan thought that certain theological notions had to be explicitly declared unsuitable for use as

substitution instances in ordinary logical principles, and a few authors were even more radical on this point. On the other hand, there was—of course—some religious hostility which regarded logical reasoning as an unhealthy method in theological matters.

2. Language as the Subject Matter of Logic

Thinkers in the Middle Ages were anxious to discuss the correct system and classification of sciences. Since their philosophy of science was realist, they believed that the classification should be based on the order of nature. Logic, however, clearly has special features that make its place in this scheme problematic. Is it a science that has as its subject matter some part or aspect of reality? Or is it merely the art of using linguistic idioms? Or is its function something else altogether?

2.1. A Science "of Words" or "of Reason"

The first known medieval textbook of logic, Garlandus Compotista's *Dialectica* from the late eleventh century, already sets the discussion of this topic on a track that was to have crucial influence on the kinds of innovations that were to be achieved in medieval logic. Throughout the Middle Ages, logical theories had a very intimate relation to actual language use. According to Garlandus, logic is concerned with actual utterances (*voces*). After Garlandus, Abelard, for example, continues on the same track, but refines the position: As he sees it, statements are not built from mere spoken sounds but from words that have a signification (*sermones*). Thus, they also constitute the subject matter of logic. Logic is "a science of words" (*scientia sermonicalis*).

It seems that well into the thirteenth century the idea that logic studies actual language use remained basically unchallenged. Teachers and students of logic considered that their studies helped in the acquisition of argumentative skills for actual scientific disputations. Given the status of Latin as the language of all medieval learning, it was natural to make the appropriate logical distinctions from the viewpoint of spoken Latin. This gave an important status to essentially linguistic structures even in the later developments of medieval logic.

In approaching many particular features of medieval logic, it is crucial to remember this pragmatic way of looking at the subject matter of logic. In the Middle Ages, the art of logic was not taken to be concerned with abstract structures in the way modern logic and modern mathematics are, but with actual linguistic practices of reasoning. It was generally accepted that logic is, at least in some sense, a practical science giving advice on how to understand and make assertive statements and how to argue and reason in an inferential manner—though opinions varied whether this practical characterization of logic was accurate in any deeper sense.

In the Arab world logic was thought of in a different manner, and thus toward the thirteenth century under Arabic influences the Latin world became aware of a different way of looking at the character of logic as a field of research. According to al-Farabi, the *logos* (in Arabic, *al-nutq*) discussed in logic occurs on two levels, one inscribed in the mind, and the other existing externally in spoken sounds. Thus, we may even separate different senses of the Greek word *logos* in accordance with the level of discourse at issue. Avicenna was also influenced by al-Farabi's discussion, and gave even further impetus to the idea that logic is concerned with intellectual structures rather than with what we do in spoken discourse. Thus, logic should be called "a science of reason" (*scientia rationis*), as the Latin world translated the idea.

In the thirteenth-century Latin tradition, both the idea of logic as "a science of words" and as "a science of reason" had a foothold. In his major classification of all the university disciplines, *De ortu scientiarum*, Robert Kilwardby (c. 1215–1279) gave a definition of the nature of logic that combined the two views. It is worth taking a closer look at his definition, because it also clarifies the medieval way of locating branches of logic in terms of Aristotle's logical works in the *Organon*.

According to Kilwardby, logic is "a science of words" (*scientia sermonicalis*) in the sense that "it includes grammar, rhetoric and logic properly so-called." But as Kilwardly immediately points out, "in the other sense, it is a science of reason," and in this sense it is "distinguished from grammar and rhetoric." It may seem that here Kilwardby would be demarcating two different disciplines both ambiguously called "logic." But this is not really his intention, as he hastens to explain: Logic properly so-called must in his opinion be listed as one of the "sciences of words"; it is the science of words that attends to their rational content. As he sees it, logic does not study arguments as mere words nor as mere rational structures, but as rational structures presented in linguistic discourse. The grammatical and rhetorical features of these arguments, for example, do not pertain to the art of logic. Logic studies the rational structures expressed and understood in linguistic discourse—neither rational structures as such, nor linguistic structures as such.

The core of logic can in Kilwardby's view be found in Aristotle's *Prior Analytics*. This is because at its core, logic is concerned with reasoning, and this is the main topic of *Prior Analytics* and its system of syllogistic reasoning. It is of some interest to note that Kilwardby is very Aristotelian in claiming that all forms of valid reasoning can be reduced to the categorical syllogism discussed in *Prior Analytics*. This was not the received view at the time, and Kilwardby's position did not win unconditional approval. Abelard had discussed the theory of conditional inference and clearly would not have accepted such a principle. Indeed, conditional inferences were throughout the Middle Ages a standard part of logical curriculum. Soon after Kilwardby, toward the end of the thirteenth century, the theory of consequences (*consequentiae*) grew into a self-conscious general theory of inference that had no specific reference to the syllogistic system; syllogism was increasingly presented as a special case of inference.

Kilwardby pushes aside one aspect of Aristotle's discussion in the *Prior Analytics*. According to Kilwardby, only dialectical and demonstrative syllogisms are relevant to logic, while the rhetorical syllogisms discussed by Aristotle fall out of the scope of logic because they take "the form that is suited to the singular, sensible things considered by the orators." Logic as a science is concerned with universal rational structures as captured in discursive reasoning.

In Kilwardby's presentation of the structure of logic, the system developed in the *Prior Analytics* is put to further use in the *Posterior Analytics* and the *Topics*. As Kilwardby sees it, the division into different works is based on the matter to which the syllogistic structures are applied. The *Posterior Analytics* discusses the way in which the syllogistic form is applied to "specific matter" and yields scientific demonstrations. For its part, *Topics* is concerned with "common matter" and shows how we can construct good inferences relying on generic considerations. Aristotle's *Sophistici Elenchi*, for its part, plays in Kilwardby's view the role of considering what can go wrong in constructing an inference.

As Kilwardby shows, the role of *De interpretatione* and *Categoriae* can also be considered in terms of the syllogism. *De interpretatione* considers the propositional structures that are essential for constructing syllogisms. A syllogism must be construed so that it has a middle term, and for this purpose it is necessary to see how assertive statements usable as premises can be built to consist of two terms conjoined affirmatively or disjoined negatively. *Categoriae* goes into the structure even more deeply, considering the terms and their signification in reality.

From the mid-thirteenth century onward, Avicenna's conception of logic as a science of reason gained increasing currency in philosophical discussions on the subject matter of logic. To some extent this happened at the expense of the earlier view of logic as a science of words. As we have already seen, Kilwardby restricts the meaning in which logic is a science of words so that it no longer carries much weight. Albert the Great's general position concerning the nature of logic is similar, but in the beginning of his commentary on Aristotle's *Categories* he takes the explicit position that logic is strictly speaking not a "science of words" at all. Rather, logic is concerned with argumentation, and argumentation should be referred to reason rather than to words.

Albert's student Thomas Aquinas (1224?–1274) followed him in this matter. In his more elaborate system, the subject matter of logic consists of three conceptual operations of the mind, namely, formation of concepts, of judgments, and of inferences. This systematization can be traced back to Plotinus and the neo-Platonic commentators of Aristotle's logic in a more explicit way than Kilwardby's system. The first two operations are discussed, respectively, by Aristotle's *Categories* and *De interpretatione*, and the third by the other four works included in the *Organon*. As Aquinas saw it, making a judgment—and, in fact, anything that logic is concerned with—requires an intellectual act of understanding. Thus, making a judgment is not primarily to be understood as a speech act but as a mental act. According to Aquinas, externally

expressed linguistic structures should be seen as results and representations of intellectual acts, and only in this intermediate way does logic come to be concerned with linguistic structures. In the first place, logic is concerned with the intellectual operations by which the universal features of material reality are understood.

The detailed structure of Aquinas's presentation of the subject matter of logic has the crucial feature that it relies heavily on the Aristotelian idea that all inferences can be presented as syllogisms. As Aquinas saw it, all of logic can be understood in terms of syllogistic structures. Since he thought that logic deals with the three basic operations of the intellect, any inference would have to be based on them. However, there are understandably quite stringent limitations on the extent to which logic can be derived from these basic operations. For example, with the claim that all assertions are made by the composition of a predicate with a subject, Aquinas was almost forced to reject conditionals as assertions. However, hypothetical propositions had a long tradition deriving from Stoic logic and had been dealt with already by Boethius, and thus Aquinas was compelled to comment on them. As he put it in the first section of his commentary on *De interpretatione*, hypothetical propositions "do not contain absolute truth, the understanding of which is needed in demonstration ... but they signify something to be true on condition." According to Aquinas's logic, conditionals could not be used as premises in scientific demonstrations.

Neither Albert nor Aquinas worked much with the actual details of logical systems, and their discussion has more of the character of the philosophy of logic. However, the distinctive flavor of medieval logic showed itself in its close connections to actual language use, and it incorporated analysis of a much wider variety of linguistic structures than the simple predications included in the syllogistic presented in the *Prior Analytics*. Moreover, Abelard's work had already made medieval logicians acutely aware of a concept of inferential validity that was essentially unconnected to the syllogistic structure. While Kilwardby, Albert the Great, and Aquinas defended the strong Aristotelian program of reducing all inferential validity and thereby all logic to an analysis of the syllogistic system, actual work in logic was taking another course.

In the subsequent development, Aquinas's three operations of the mind were often referred to, but usually understood in a loose and suggestive manner. It became standard to treat logic with the organizational principle that *Categories* studies concepts and *De interpretatione* propositions, while *Prior Analytics* and the three subsequent Aristotelian works concentrate on inferences. It was, furthermore, commonly accepted that there are many traditional logical genres inherited from the twelfth century that do not fit into this basic scheme. For example, there was an abundance of literature on the so-called syncategorematic terms, analyzing the logical properties of words such as "except" (*praeter*), "begins" (*incipit*), "whole" (*totum*) and "twice" (*bis*). Such problems had little connection to the development of syllogistic systems. Furthermore, it remained

a problem to explain how such logical genres could fit into the description of logic as a science of reason, because many of them were quite clearly motivated by the analysis of linguistic structures.

2.2. Mental Language

An interesting alternative way of characterizing logic as concerned with mental concepts rather than Latin words was being developed at the time Aquinas was working, and it gained momentum among logicians in the latter half of the thirteenth century. It was based on quite a different understanding of the workings of the human mind from that of Aquinas's Aristotelian outlook. Roger Bacon (c. 1214–c. 1293) rejected the idea that the human understanding works only with real universals existing intentionally in the mind. Rather, the mind should be understood in terms of a discourse consisting of singular acts of intellection whereby different singular things are understood in different ways. According to Bacon, logic is not concerned with an external discourse but with the internal discourse of the mind, with "mental expressions and terms" (*dicciones et termini mentales*). In other words, Bacon posits a mental language to serve the role of the subject matter of logic. As we shall see, this approach was to play a major role in later developments.

First, however, we must take a closer look at the content of Bacon's suggestion. One of the central classical texts that Bacon refers to was Boethius's distinction between three levels of discourse (*oratio*): intellectual, spoken, and written. In making the distinction, Boethius was commenting on Aristotle's *De interpretatione* 16a10, and Boethius's way of reading the passage was well known in the late Middle Ages, but it remained a debated issue how one should understand the intellectual level of discourse and how one should relate logic as a discipline to these levels.

It seems clear, though, that Bacon understood the intellectual discourse in a way that can with good reason be called linguistic. He even takes pains to show how word order functions in this discourse. Without going into details, it is sufficient here to point out that he looked at the structure of mental sentences in terms of Aristotelian predication: The subject comes first, then the predicate, both with their "essential determinations." They are then followed with the various "accidental parts" of the composition. Especially his way of dealing with these "accidental parts" shows how looking at thought as a linguistic phenomenon gives Bacon a clear advantage in comparison to Aquinas from the logician's point of view. Through his theory of mental language, Bacon is able to attribute considerably more logically relevant linguistic structure to the intellectual level. One of the aims of this enterprise was—as is evident to any logician—to show how to solve ambiguities of scope arising in Latin through the relatively loose rules concerning word order. In this way, Bacon worked toward a theory of an ideal language to serve logical functions as early as the 1240s, if the current scholarly opinion of the date of his *Summa de sophismatibus et distinctionibus* is correct.

The mental discourse that Bacon was after is abstracted from spoken languages like Latin and overcomes their arbitrariness. However, there is also the other side of the coin: He specifically wanted to find the basis for certain logically relevant Latin structures from mental discourse. Although he thought that grammatical gender has no correlate in mental discourse, the subject-predicate structure and many syncategorematic expressions have. Indeed, Bacon seems to find from the mental discourse even more than a logician would need. In many issues, it becomes apparent that he was working more as a linguist than as a logician. His aim was a universal grammar rather than a universal language suitable for logic.

Commentators have, accordingly, connected Bacon to the movement of speculative grammar emerging in the latter half of the thirteenth century. The approach to linguistic analysis employed by this school is often called "modist." The label reflects the specific use of a threefold series of concepts: "Modes of being" (*modi essendi*) in reality were paralleled in language by "modes of signification" (*modi significandi*) and in the mind by "modes of understanding" (*modi intelligendi*). The movement was more closely connected to language theory than logical theory, and accordingly we will only discuss it briefly here.

The main idea of modist theory was to approach Latin expressions as generated from a universal grammatical structure accurately reflecting the structure of reality. That is, they thought that grammar is (in the words of Bacon) "substantially one and the same in all languages, although varied in its accidents." Other central figures of this movement include Boethius of Dacia, Martin of Dacia, and Radulphus Brito. At the beginning of the fourteenth century, the program lost ground, although much of the terminological innovations, including the term "mode of signifying," survived until the Renaissance in the standard vocabulary of logicians.

According to the modists, all words have two levels of meaning. Words have in addition to their own specific meanings certain more general meanings, or so-called modes of signifying. To be more exact, a phonological construction gains a special meaning when it is connected to a referent that it "is imposed" (*imponitur*) to mean (in the so-called first imposition). Furthermore, the word is also "imposed" (in the second imposition) to mean its referent in a certain grammatical category with certain modes of signifying. For example, pain can be referred to by a variety of Latin words in different grammatical categories: *dolor* refers to it as a noun, *doleo* as a verb, *dolens* as a participle, *dolenter* as an adverb, and *heu* as an interjection. In all these words the special signification is the same, but the modes of signifying are different.

The modists found no theoretical use for the most central logical term of the terminists, "supposition" (*suppositio*; it will be described with more detail in the next section). In their view, the varieties of ways in which words are used in sentential contexts are based on modes of signifying contained in the words, and thus they were not willing to admit that the sentential context as such would have an effect on how the term functions—which is one of the leading principles

of the supposition theorists. Rather, their approach was generative in the sense that the sentences were to be generated from words that have their signification independently. This approach made it unnatural to distinguish the sentential function of a term from its signification. It may, however, be noted that the term "consignification," meaning the function of syncategorematic terms in the terminist approach, was used by modists to express the way in which phonological elements of actually used words mean modes of signification: For example, the Latin ending *-us* "consignifies" nominative case, singular number, and masculine gender. The thirteenth-century grammarians recognized the congeniality of syncategorematic terms and modes of signifying: Both are understood as the elements of discourse that show how the things talked about are talked about and what in fact is said about them.

From the viewpoint of the history of logic, it is important to recognize that from the twentieth-century viewpoint, the modist conception of grammar can be characterized as making the subject a "formal science." The criteria of congruence were taken to depend solely on the grammatical structure, or the consignifications of the elements of the sentence, regardless of the special significations of the terms used in the sentence. Modists thought of the generation of language as putting semantically significant elements into grammatical structures. It seems that at least the Parisian master Boethius of Dacia wanted to develop also logic into this direction and wanted to make a distinction somewhat like the twentieth-century distinction between logical form and semantic content. Nevertheless, it was only some decades later at the time of John Buridan that the substance of logic was thoroughly reconsidered from this viewpoint.

2.3. The Universality of Logic

From the viewpoint of practicing logicians, the debate concerning the subject matter of logic at the end of the thirteenth century probably seemed like a search for a credible account of the universal basis of the invariable features of argumentation found in the logical analysis of actual use of language. That is, what is the universal basis on which the validity of an inference formulated in a particular language is grounded?

It was accepted as relatively clear that logic is about actually or potentially formulated tokens of terms, propositions, and arguments that are linguistic in some sense of the word. It was clear that such discursive arguments existed in such external media as spoken or written Latin expressions. However, logic aimed at, and appeared to have found, some kind of universality, and such universality apparently could not be achieved if logic was tied to a particular spoken language. Instead, thirteenth-century discussions converged in finding the universality of logic in intellectual operations. But what are these intellectual operations? Can we speak of a mental language serving as the domain of logic? In particular, is a mental proposition linguistic in any relevant sense? And because it was assumed that an affirmative predication is

based on or performs a composition, one had to ask what exactly does this composition put together.

At the turn of the fourteenth century, we find different logicians giving different answers to these specific questions. At that time, the most common answer was the one inspired by Bacon. It was based on looking at the mental discourse from the viewpoint of "imagined spoken words," and accepting it as the privileged medium of logical arguments. This kind of explanation is straightforward and relatively acceptable from the metaphysical viewpoint, but is, of course, less satisfactory in explaining the kind of universality achieved in logic. If mental language is nothing but imagined Latin words, there seems to be little reason for assuming it to have any more universal status than Latin has. Yet that appears to be what Bacon wished to propose.

The realist Walter Burley seems to have approached the problem from the viewpoint of the universality of logic. Given his realist metaphysics, it is understandable that he contributed the concept of "real proposition" (*propositio in re*). He aimed at explaining mental propositions as consisting of real external things, which are conceived and propositionally combined in the mind. This model of the metaphysical basis of mental language of course works only if conceptual essences are understood in a realist way without separating them from the things themselves. Also, such "real propositions" are not very language-like.

The nominalist William Ockham formulated the most innovative and by far the most influential theory of mental language. He ridiculed the position of Burley by asking how it could be that the subject of a proposition formulated in Oxford is in Paris while the predicate is in Rome. A suitable example of such a proposition would be "Paris is not Rome." Ockham seems to have gone back to Bacon's theory, but with the awareness of some of its shortcomings.

With his nominalist metaphysical outlook, he strongly held the view that all the metaphysically real things involved in mental propositions are particular mental acts or states. But the substantial logical strength of his theory of mental language was really that it was formulated in a way that was sufficiently neutral from the metaphysical point of view. Indeed, Ockham himself started with the idea that mental language consists of *ficta* (that is, of intellectually imagined objects of thought that do not have any kind of existence outside the mind but are simply "made up" by the mind) but ended with the view that mental language is better understood as consisting of intellectual acts intentionally directed at real or possible things. At one stage of his career he was working on the theory of mental language without being able to make up his mind which of these two rather different views would provide the appropriate metaphysical foundations.

In the first chapters of *Summa Logicae*, William Ockham addresses the Boethian idea of three levels of language. In opposition to Aquinas's treatment of the same topic, Ockham claims that written language is subordinated to spoken language rather than signifies it. Similarly, spoken language is subordinated to mental language rather than signifies it. That is, according

to Ockham, all three languages similarly signify things in the external world. They are, furthermore, all equally languages. Written language is inscribed on external material things, and spoken language exists as a continuum of sounds. Similarly, mental language consists of real qualities of the thinking mind. Furthermore, in Aquinas's picture intellectual acts were the significations of linguistic expressions and by their nature could not serve as a medium of communication. For Ockham, mental language could by its nature serve equally as a medium of communication if only there were beings who could perceive its expressions apart from the "speaker" him- or herself. In fact, Ockham thought that we have every reason to suppose that the angels described in the Christian doctrine communicate in the same language in which we think.

The main difference between mental language and the two other kinds of language is the naturalness of mental language. Unlike spoken ordinary languages, which we nowadays call natural, Ockham's mental language is natural in the sense of not being conventional. The expressions of mental language have their significations naturally, without explicit or implicit consent or any other kind of conventionality involved. A mental word is capable of signifying only the things it really signifies, and it signifies exactly those things to all competent users of the word. (It may be noted that Ockham admits that in angelic communication some mental expression may be unfamiliar to the perceiver and thus unintelligible to him.) In principle, there are no ambiguous terms in the mental language. This is one of the central features that make Ockham's mental language an ideal language, which is then suitable for the purposes of a discipline like logic.

There are also two other senses in which Ockham aims at description of an ideal universal language. On the one hand, he tries to describe in general terms what must be required of any language that is used for thinking, and assumes that mental language has only such necessary features without any accidental "ornaments of speech." Since these features are necessary requirements of thought, all thought must comply with them. Thus, Ockham constructs a theory of a language that is universal in the sense of being used by all intellects that think discursively.

On the other hand, according to Ockham, mental language is directly related to the constitution of the world. It reflects accurately mind-independent similarities between real things. Thus, a fully developed mental language would be universal in its expressive power: There cannot be any feature of the world that could be conceived by an intellectual being but not expressed in mental language. Everything that can be thought can also be cast in terms of mental language. From this principle it also follows that all linguistic differences between expressions of spoken languages that result in different truth values (which are not "ornaments of speech") have their correspondents on the level of mental language.

From the logical point of view, perhaps the most interesting ideal feature of Ockham's mental language is its compositionality, which makes it a recursive system. Complex expressions get their meaning from their constituent parts

in a systematic way. In this respect, mental language shows similarities to twentieth-century formal calculi, although it is much more complex.

According to Ockham, the expressions of mental language consist of categorematic and syncategorematic parts with specific linguistic roles (we will return to this distinction more fully in the next section). A categorematic term (e.g., "animal") signifies real individuals and refers to them as the other elements of the propositional context determine. A syncategorematic term (e.g., "every") does not signify any external things but rather, as Ockham puts it, "performs a function with regard to the relevant categorematic term." Typically, syncategorematic terms affect the way in which the significations of the categorematic terms result in reference (or *suppositio*) in the sentential context. We may say that the categorematic terms of a sentence determine which things are talked about, whereas the syncategorematic terms determine how they are talked about and what is actually said about them. The number of basic categorematic terms of the ideal mental language accords to the variety of things that could exist in the world; they express the natural kinds of possible things. Ockham's view of the number and selection of syncategorematic terms is more difficult to determine. On the one hand, it is clear that he is thinking of a much wider variety of such logical constants than twentieth-century logic used. On the other hand, it is equally clear that most of his logical rules concern the effects of syncategorematic terms on logical relations between sentences.

Because the compositional characteristics of mental language depend on the distinction between categorematic and syncategorematic terms, Ockham's mental language seems to conform to the twentieth-century ideal principle of logical formalism, namely, the idea that all sentences directly reveal their logical form. This seems to be one of the features of the mental language that Ockham is most interested in, and much of his logic is devoted to systems elaborating on the functions of syncategorematic expressions. However, Ockham's theory has interesting details that reflect a conscious decision not to accept logical form (as we nowadays understand it) as the guiding universal principle in determining the logical validity of an inference.

The theory of mental language was also discussed and developed after Ockham, but without major revisions. The most important innovator was John Buridan, who altered much of the terminology used in defining language and gave a rather different account of how the simple terms of language are learned, but these revisions resulted in few changes that would be relevant to our purposes here. After Buridan, some minor topics like the role of proper names and individual terms, and the nature of word order as explanatory of issues of scope were discussed. These can hardly be called revolutionary with regard to the nature and purposes of logic.

At the peak of its success, medieval logic had thus found a definition of its subject matter that provided a relatively reasonable explanation both of its universality and of its dependency on discursive linguistic structures. For the logicians of the second quarter of the fourteenth century, logic was the art

of constructing and using mental propositions. It studied the basic syntactic features of mental language, the ways and forms of assertions that can be produced in it, and the ways these assertions can be organized in inferential relations. Because mental language was understood as capable of expressing all possible universal structures of discursive thought, logic studied the universal art of reasoning.

3. Terms

3.1. The Notion of a Term

Textbooks of "traditional logic" used to divide their material into three sections: the doctrines of terms, propositions, and inferences. This practice is based on ancient grounds, of course, but Aristotle nowhere says that all logic should be so divided, and medieval logic did not at first do so. In thirteenth-century logic books, often the chapters are still relatively independent, or at least not organized according to such a general plan. But then, at the turn of the century, this idea soon became dominant. We find it, for example, in both Burley and Ockham, in spite of their sharp disagreement. We are going to follow this familiar order, starting with *terms*.

Everybody agreed that terms were the ultimate units of discourse. In a way this is obvious, but the emphasis on this fact in logical contexts also has a nontrivial sense which shows the Aristotelian character of medieval logic. For the logic that Aristotle had developed was term logic, unlike that of the Stoics. But Aristotle gave two different explanations of terms. In *Categories* he speaks about noncomposite expressions ("such as 'man', 'ox', 'runs', or 'wins' "). In *Prior Analytics* he says (24b16–18): "I call that a term into which a proposition is resolved, i.e., the predicate or that of which it is predicated, when it is asserted or denied that something is or is not the case." These explanations lead to very different uses of the word "term."

In the first sense, a term is simply any word. Many medieval logicians mentioned even meaningless words, like "ba," "bu," but only to concentrate on ordinary words. In this sense, which is that of grammarians, it is only required that a term is a noncomposite significant element of the language. Or it can be a composite expression signifying one thing.

In the second sense, which is more exciting for the logicians, a term is something that can stand as a subject or a predicate of a proposition. This excludes wide classes of words from the status of terms. According to the strictest definition, a term is only that type of nominal expression that can figure as S or P in a categorical proposition "S is P."

This leads to a question concerning the structure of terms because S and P can be complex expressions. Buridan, for instance, took a strictly propositional view and argued that a simple proposition has exactly two terms. In this usage a term is identified with an extreme (*extremum*) of a proposition. But

since the various words occurring within such terms can be extremes in other propositions, authors kept on saying that propositions can have complex terms that are composed of simple terms.

The theory of terms is obviously connected to grammar, and Priscian's classifications had a strong influence on earlier writers. But logically it was important to eliminate Latin contingencies and consider as general cases as possible. However, that is a problematic requirement regarding terms: What could be those language-independent terms? Different ways to tackle this question systematically were offered first by so-called speculative grammar, and then by the mentalistic interpretation of language, which was finally victorious, but both approaches emphasized the universality of language. For late medieval logicians, the terms were in the first place mental terms that occurred in mental propositions.

3.2. Categorematicity

A distinction that is especially important for logic was made between *categorematic* and *syncategorematic* terms. This distinction was well known to all logicians, and they usually introduced it immediately after the definitions of terms. The source of these notions was in grammar, but logicians gave them a new function, following a hint from Boethius. Priscian had written about "syncategorematic, i.e., consignificant, parts of speech": Most words are grammatically categorematic since they can occur as subjects or predicates, but for instance, conjunctions, prepositions, adverbs, and auxiliary verbs cannot. They are syncategorematic and signify only together with other words. Logicians proceeded from this picture to distinguish two ways of meaning and to describe the logical behavior of philosophically interesting syncategorematic words.

Syncategorematic words were first studied in special treatises. This genre of *Syncategoremata* was popular from the last quarter of the twelfth century to near the end of the thirteenth century. Well-known treatises of this kind were written by Peter of Spain, William of Sherwood, Nicholas of Paris, and even the famous metaphysician Henry of Ghent. Later, the subject was incorporated into general textbooks of logic. The distinction itself had its systematic place at the outset of the exposition of the theory of terms, since it was utilized in many questions; particular syncategoremata were then discussed in their due places.

Even in the fourteenth century most authors apparently based their definitions of syncategoremata on different ways of signifying. According to Ockham, "categorematic terms have a definite and determinate signification. . . . Examples of syncategorematic terms are 'every', 'no', 'some', 'all', 'except', 'so much', and 'insofar as'. None of these expressions has a definite and determinate signification." Buridan states: "Syncategorematic terms are not significative *per se*, as it were, but only significative with another." Paul of Venice still defended this view against "a common definition" that a syncategorematic term cannot be the subject or the predicate or a part of either. Such a purely syntactical criterion had been supported by Albert of Saxony (1316–1390).

Syncategorematic expressions were usually counted as terms. A theoretical reason for this was a slogan that was in use at least from Peter of Ailly onward: A term is a sign that in a proposition represents something or *somehow*. Syncategoremata, indeed, signify "somehow" (*aliqualiter*), for thirteenth-century treatises had already pointed out that syncategoremata serve to show how the categorematic terms ought to be understood. It is thus essential for their signifying that they are joined with other terms to elaborate their meanings.

Present-day readers will easily associate syncategorematic terms with logical constants. This is partly correct but must not be taken too literally. For one thing, the class of syncategoremata of language is much wider than the small sets of logical constants nowadays. However, the medievals ignored most syncategoremata and studied only those which seemed to be philosophically interesting. These were just words with special logical peculiarities, and hence, for these terms, the comparison with logical constants may be justified. The lists of different logicians varied greatly, but several dozens of words were thus discussed. Among them belonged sentential connectives; words like "only" and "except"; quantifiers; modal operators; words like "whole" and "infinite"; some verbs like *incipit* and *desinit* ("begins" and "ends"); and the copula *est*, that is, the copulative use of the verb "to be," *esse*. General textbooks listed them but did not usually go into details of particular syncategoremata. In the fourteenth century, such closer study often took place by means of sophismata: In this literature it was typical to analyze sentences that were problematic or ambiguous because of syncategorematic words (see section 6).

Buridan expressly said that the matter of a proposition consists of purely categorematic terms while syncategoremata belong to its *form*. From this point of view, it is interesting to notice that the notion of syncategorematicity proved difficult because it did not determine a precise class. Thus Buridan had trouble with attitude operators: Verbs like "to know" and "to promise" clearly have a formal function and yet they are independently meaningful. The two criteria, the semantical and the grammatical, did not always coincide, and Peter of Ailly suggested that they should be wholly separated. A term could therefore be syncategorematic either "by signification," or "by function," or in both ways.

3.3. Predicables

In a proposition something is said of something, as Aristotle taught. It is therefore logically important to have some idea of the various types of things that can be thus predicated, the *predicables* (*praedicabilia*). Medieval logicians based their classification here on Porphyry and Boethius. Obviously, a predicable is something that can be said (predicated) of something else, but in a stricter sense, it is only a universal term that can be predicated of many things. This distinction was made already in thirteenth-century textbooks, and it is easy to see that predicables have a close connection to the most famous medieval metaphysical problem, the problem of universals.

Explaining Aristotle's *Categories*, Porphyry mentioned five types of universal terms: *species*, *genus*, *differentia*, *proprium*, and *accidens*. These were the "five universals" (*quinque voces*) that recur in medieval discourses. They reveal various relations of the predicate to the subject: What kind of information does the predicate give us about the subject? When it is said that S is P, the predicate P may express a species to which S belongs, or a genus to which every S belongs, or a characteristic essential feature of them (*differentia*), or a nonessential but necessary property of every S and only them (*proprium*), or their accidental feature (*accidens*). (The P of species is a somewhat obscure case here because it can be predicated of individuals, too, unlike the others.) Added to a genus, a specific difference (*differentia specifica*) defines a species, which in turn can be a genus for lower subspecies. In this way, the famous "Porphyrian tree" is generated, ranging from uppermost genera down to individuals.

The doctrine of predicables was a standard part in medieval logic texts, and it was a relatively unproblematic part: The difficulty, of course, is metaphysical and concerns the essential, necessary, and accidental qualities. Logicians, however, used the five universals as metalinguistic tools to classify predicates. A more ontological question is that of categories, or *praedicamenta*, as logicians preferred to call them. The first category is substance; the other categories are ways in which something can belong to a substance. Aristotle studied quality, quantity, and relation in his *Categories*, and more briefly he discussed even place, time, position, habit (having), passion, and action. With some variants, medieval *praedicamenta* treatises give the same list of 10 members.

As Buridan says, "this treatise is found in many summulae, but in many it is not." Indeed, it is not obvious why logicians need to discuss a question that seems purely metaphysical. But there was a motive for those who included this treatise in their summulae—an assumption of the parallelism between predication and being. Except substance, all categories both "are said of things" and "are in things." Thus, a classification of ways of being in a substance also produces a classification of questions and answers that can be made concerning an entity, and this is a logically relevant achievement. Later, nominalists give up the assumed parallelism and analyze categories simply metalinguistically, as classes of terms. Ockham, for instance, has a long discussion in which he wishes to show how terms of other categories are secondary to substance and quality.

We may note in passing that predicables and categories have a very different role among the speculative grammarians of the late thirteenth and early fourteenth centuries. For them, terms are intelligible because they manifest the same characters and structures as the entities of the world; the "modes of signifying" belonging to grammatical features of lexical meaning and inflection are functions that reflect categorial features of objects. Such an approach leads to a special view of metalinguistic issues. Hence it is also natural that these authors, the modists, concentrated on rather abstract lexical contents and were not very interested in the semantic properties of concrete occurrences of terms in particular sentences.

3.4. Significance

The main body of the theory of terms consisted of *proprietates terminorum*. The tireless analysis of these "properties of terms" displays the intense interest in *philosophical semantics* that was characteristic of later medieval philosophy. This is a field that seems to be a medieval invention. In Aristotle and other ancient sources, there were only scattered remarks on semantic questions, and it can hardly be said that they attempted to establish any self-conscious theory of semantics. On the other hand, after scholastic philosophy these problems were often considered futile, and explicit philosophical semantics was largely rejected. But the medieval theory has had a striking revival in the latter half of the twentieth century, when philosophical semantics has again grown into a complex discipline, often struggling with questions that bear an obvious resemblance to medieval themes.

Undoubtedly the two most important properties of terms are *signification* and *supposition*. They have often been compared to present-day "meaning" and "reference," but this comparison must not be taken literally. For one thing, the emphasis was on the words and signs: Unlike many accounts of meaning and reference, the medieval doctrine viewed signification and supposition mainly as something that the words do or as something that is done by means of words.

Let us start with *signification*. Logicians were aware of the ambiguity of this word. Usually, instead of interpreting signification as a signified entity of some sort, they started from "acts of signifying" and assumed that terms had a property of being significant. (In this respect, terms differed from other words which had no signification by themselves.) A word signifies, or has signification, because of its "institution," or according to another common account, because of its "use" in language. In short, signification is the role of the term in language. The same idea acquires a new slant with the introduction of mental terms. It then becomes standard to claim that spoken words have their significations because of linguistic conventions, whereas the mental terms are *natural signs* that have their significations necessarily, without any stipulation. Signification is generally connected to mental acts of understanding: A linguistic term signifies that of which it makes a person (a speaker or a hearer) think, a mental term is itself an act of thinking of something, a representation. (To quote John Aurifaber: "signifying is an accident of the intellect, but a word is the thing by means of which the intellect signifies.") The thing thus signified has "intentional being."

Even before the mentalistic turn, it was usual to find the essence of words in signification. Thus Thomas Aquinas said that "signification is like the form of a word"—the matter was the phonological shape, the form was its signifying capacity. Later, it was said that mental concepts have their significations "formally," and spoken and written words essentially function as instruments of this signification.

Signification is the defining property of all terms; thus it is natural that it can be defined no further. Late medieval philosophers seem to agree that

signification is a basic notion that can only be explained by illustration. For them, it was obvious that a term signifies something, but there was a great, partly metaphysical controversy about what this something is.

Boethius had already said that words signify concepts, that is, corresponding mental entities. This gave an impulse for the view that words signify concepts immediately and objects indirectly. (Such a "semiotic triangle" had been discussed earlier by Greek Aristotelian commentators.) This opinion became prevalent among the Thomists. Aquinas himself had pointed out that a term signifies a general nature that is abstracted from individual entities. The later Thomists emphasized that the concepts were signs, too: Thus the words do signify objects "principally" (most important), although they do it only "indirectly" (through the concepts).

A contrary position was championed by Bacon, and it won general support at the end of the thirteenth century. It started from the obvious fact that terms are used in propositions, and the propositions are about objects and not concepts. Thus all terms must signify objects. However, nonexistent objects cause problems which compel logicians to make reservations concerning that general principle. What is signified is, for instance, the object "regardless of its being or not being" (Kilwardby), or the object "*secundum quod* the intellect perceives it by itself" (Duns Scotus). Moreover, spoken words and mental terms signify the same objects. According to Ockham, it is a basic fact that words are "subordinated" to the corresponding mental terms in such a way that they signify the same things. He apparently did not think that this use of language could be further explained. Buridan was not satisfied with this kind of answer and again interpreted the subordination as a type of signification: Words also signify concepts, in some sense. Later discussion became rather complex when different positions were combined and refined.

Admitting then that terms signify something extramental, it is still not clear what this *significatum* is for general terms. The question is inevitably connected to the theory of universals. The realist answer is that the term signifies something general; "man" signifies a universal, a species, a property, or a common nature of "man in general." The nominalist answer is that the term signifies all relevant individuals; "man" signifies each man. Both answers cause trouble, which shows the uneasy union of signification and denotation. For it was assumed, after all, that a term signifies what it is true of, and this characterization would better suit denotation.

Syncategorematic words have no signification in the strict sense. However, most logicians were not as rigorous as Ockham, who said that they do not signify at all. Even Buridan was willing to admit that they did not signify things but ways of thinking. And both realists and nominalists agreed that syncategorematic words could "consignify," that is, participate in forming significant wholes.

There is even another sense of the word "consignification." In addition to its basic signification, a word can have some consignification that further determines its content. Especially thirteenth-century authors often use this

approach to explain the role of features like case endings and—the most discussed example—tenses. The idea is that the actual occurrences of words get richer contents than bare lexical words.

3.5. Supposition

Denotation was first discussed by means of *appellation*, a notion borrowed from the "appellative nouns" of grammar. Appellation is the relation between a general term and the things actually belonging under it at the moment of utterance. Often this notion was applied only to the predicates of propositions, but at least from William of Sherwood onward it had unrestricted use.

The "property of terms" that caused the most extensive study was *supposition*. The word derives from grammatical contexts. According to Priscian, a word has a supposition when it is placed as the subject of a proposition. This meaning was usual in the twelfth century. On the other hand, grammar had also formed the idea that a word supposits because it refers to an individual. Gradually this became the central aspect, and the supposition of terms was their way to denote individuals. As the supposition theory expanded, logicians had to seek for suppositions even for other terms than the subjects of propositions—for predicates and parts of complex terms. The question of supposition began to concern the denotation of terms quite generally, and at the same time appellation lost much of its importance, turning into a special case of supposition.

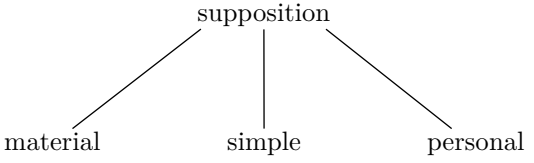
Supposition theory was a challenging subject especially because the supposition of terms depends on their position in a proposition. Each word that is not equivocal always has the same signification, but its supposition varies in different propositions. As Ockham said, "supposition is a property of a term, but only when it is in a proposition." This compelled the logicians to develop classifications for the several kinds of supposition. As many scholars have pointed out, precisely this propositional approach was characteristic of the theory of supposition. It must, however, be noted that Peter of Spain admitted even a "natural supposition" (*suppositio naturalis*) that belongs to a term immediately because of its own signification, and this idea was preserved by many Parisian logicians.

Thirteenth-century terminist textbooks already include a detailed and clearly developed doctrine of supposition. In Paris during the second half of the century, this tradition had to give way to the modistic influence, but it survived largely undisputed in Oxford. Subsequently, the ideology of mental terms made it again generally accepted in the beginning of the fourteenth century. After this it became part of the permanent apparatus of late medieval logic.

"To supposit" is obviously a technical term; it means something like "to stand for," and this indeed was an alternative expression. Early terminists like Sherwood thought that *supposition* belongs only to substantives that are posited as subjects (i.e., subposited under predicates), whereas the denotative

function of verbs and adjectives is *copulation*. Soon, however, it became the rule to merge these cases and connect supposition to every categorematic term. The definition of this general supposition is not evident. Perhaps it is clearest simply to quote concise definitions from two authors: "When a term stands for something in a proposition in such a way that we use the term for the thing and the term (or its nominative case, if it is in an oblique case) is truly predicated of the thing (or a pronoun referring to the thing), the term supposits for that thing" (Ockham). "All and only those terms supposit which, when something is pointed out by the pronoun 'this' or several things by the pronoun 'these', can truly be affirmed of that pronoun" (Buridan).

We shall try to sketch an overview of the divisions of supposition. First of all, in some cases the supposition is "improper" because the word is used in a nonliteral or metaphoric way; let us concentrate on "proper supposition" only. The definition of its various types displays both semantic and syntactic factors. It seems that the *suppositum* of a word can be of three fundamentally different semantic kinds, and the supposition is accordingly called either *material*, *simple*, or *personal*.



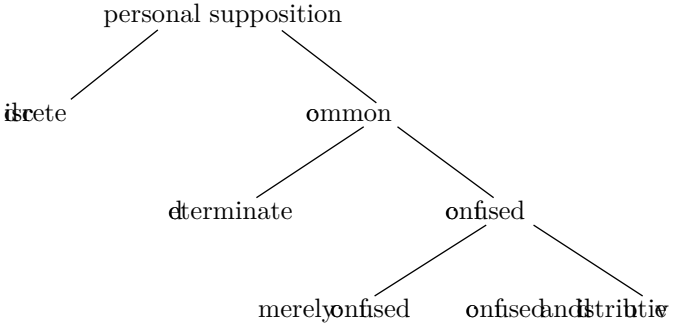
A term has *material* supposition (*suppositio materialis*) when it stands for itself. It must be kept in mind that people in the Middle Ages did not use quotation marks, and material supposition is an alternative way to cope with some problems of use and mention. Sherwood notes that material supposition can be of two types: The word supposits itself either as a sheer utterance or as something significant. His examples are "man" in "Man is monosyllabic" and "Man is a noun."

The supposition is *simple* (*simplex*) when a word stands for a concept. The classical elementary example is "Man is a species." To realists, the *suppositum* then should be equated with some extramental conceptual *significatum*. "If 'man is a species' is true, the term 'man' supposits its *significatum*. . . . The word 'man' does not primarily signify anything singular; thus it signifies primarily something general, and this is a species" (Walter Burley). According to nominalists, the simple *suppositum* is a mental entity, such as an intention.

In the most common case, the word supposits some things that it signifies. For historical reasons, this was called by the surprising name of *personal* supposition (*suppositio personalis*). Because both simple and personal supposition are related to the meaning, unlike material supposition, they were often together called *formal* supposition. On the other hand, nominalists liked to reduce concepts to mental words, so in a sense Buridan and Peter of Ailly are

more straightforward than Ockham when they do not admit simple supposition as an independent class, counting it as material.

The main task is the classification of personal supposition, and here syntactic matters interfere. Let us start by providing the next diagram, representing the early state of the classification, and then proceed to explanations of its titles.



This scheme was in fact given by William of Sherwood, except that he makes the difference between common and discrete supposition in another context.

Discrete supposition (*suppositio discreta*) belongs to discrete terms: that is, to proper names and demonstrative expressions, like "this man" or "this." Then the *suppositum* is the unique object that is signified. All other terms have *suppositio personalis communis*.

This *common* supposition is further divided into determinate and confused kinds. The supposition is *determinate* (*determinata*) when it allows instantiation, as we might say. But medieval logicians had no such notion. Early authors thought that a determinately suppositing term stands for one determinate object. Ockham improved on this, saying instead that determinate supposition supports *descent* to singulars, that is, to sentences that are got by substituting singular terms in place of a general term. Thus, in "A man is running" the term "man" supposits determinately because we can legitimately infer that "This man is running or that man is running or...," and each member of this disjunction in its turn allows *ascent* back to the original sentence.

The supposition is *merely confused* (*confusa tantum*) if the proposition does not allow instantiation but is instead implied by its particular instances. As Ockham puts it, "in the proposition 'Every man is an animal', the word 'animal' has merely confused supposition; for one cannot descend to the particulars under 'animal' by way of a disjunctive proposition. The following is not a good inference: every man is an animal, therefore, every man is this animal or every man is that animal or every man is..." But it is also worth noticing that "Every man is this animal or that animal or that..." does indeed follow.

Finally, the word has *confused and distributive* supposition (*confusa distributiva*) if it allows descent to all singulars but does not support any ascent. Here the reference concerns "distributively" each and every one of the individuals. For example, in "Every man is an animal" the term "man" has confused

distributive supposition. It is correct to infer "This man is an animal and this man is an animal and..."; or as we write nowadays, "The man a is an animal and the man b is an animal and..." On the other hand, none of these singular propositions implies the original sentence.

For confused supposition—and especially for the descent to singulars—it is important to decide what the adequate class of individuals ought to be. There was some debate on this point about the correct formulation until it was agreed that the terms had to be duly *ampliated*, in other words, extended from the basic case of all individuals presently belonging under the concept, such as all actual men, to include all past and future men as well, and in later logic even all possible instances (all possible men). So terms could be examined either with their actual supposition or with an extended supposition.

It is not obvious what the motive behind supposition theory really was. Early authors possibly just wanted to capture various kinds of referring. But when Ockham and his followers started to build a more complex theory, with rules of descent and ascent, they probably did pursue something else. Thus, the supposition theory has been compared to the modern framework of quantification theory, and clearly it has something to do with the problems of multiple quantification and scope—problems that had no explicit place in Aristotle's logic. Also, it can be seen as an attempt to warrant certain inference types, like those of descent and ascent. The interpretation here is still a matter of controversy.

4. Proposition

The core of the medieval theory of judgment centers around the standard definition of proposition (*propositio*), deriving from the late ancient period through Boethius. The definition runs as follows:

> A proposition is an expression that signifies something true or false.
> *Propositio est oratio verum falsumve significans.*

This definition accords with the classical theory of definition. It consists of the generic part (expression) and the distinguishing characteristic (signifying something true or false). For our purposes, however, it seems more useful to divide it into three parts and look at the concepts of truth and falsity separately from the problem of what it is that the proposition exactly signifies.

4.1. Propositions Are Expressions

As we have already seen, medieval authors understood logic as a discipline whose subject matter is linguistic discourse. It is well in line with this general approach that they also thought of the propositions studied in logic as sentences actually uttered in some language, typically either spoken or written. As we saw in section 2, a central issue in the determination of the subject matter

of logic was whether (and in what sense) we could distinguish a special class of mental propositions. Thus, thoughts can be propositional only in so far as they have linguistic structure. A proposition, as the medievals thought of it, is something that is put forward as a sentence, and thus it has actual existence in time and typically also in space. As we will soon see in more detail, it was not the case that medieval logicians would have failed to make a distinction between the actual utterance and that which it expresses. Rather, they simply thought of propositional truth as an issue that comes up in connection with claims actually put forward, not as a property of abstract entities.

From the viewpoint of twentieth-century logicians, this feature of medieval conceptual practice has some implications which are worth pointing out, although they are ultimately superficial. According to medieval parlance, a proposition has to exist (i.e., has to be actually put forward in some language) to have a truth value, and it has its truth value in respect to some specific instant and context. Thus, a proposition like "there are no negative propositions" cannot be true, since it falsifies itself, though it is clear that the case could be as it claims. Also, the same proposition can have different truth values in different situations. The truth value of "Socrates is seated" varies when Socrates either stands up or sits down. Furthermore, the truth of "this is a donkey" varies depending on what the demonstrative pronoun refers to. Indeed, all logical properties that a proposition has presuppose that it exists; thus medieval logicians often pointed out that their study applies to propositions, not eternally, but on all occasions in which they are put forward.

4.2. Propositions Carry Truth Values

Not all significative expressions are propositions. Boethius's textbook distinguishes between "perfect" and "imperfect" expressions with the idea that an imperfect expression does not make complete sense but the hearer expects something more. More important, Boethius continues by listing questions, imperatives, requests, and addresses in addition to indicative sentences that make an assertion and count as propositions. This listing of the kinds of expressions is based on grammatical categories, and similar strategies were also followed in subsequent discussions. It may be of some interest to note that Buridan, for example, takes it to be worth an argument to reject Peter of Spain's claim that sentences in the subjunctive mood (like "if you were to come to me, I would give you a horse") do not count as propositions.

It seems that medieval logicians disagreed on whether a proposition that is just mentioned without being asserted carries a truth value. The distinction between apprehensive and judicative uses of a propositional complex was rather standard. Ockham, for example, argues that it concerns propositions so that even an apprehended proposition has a truth value, although no stance is taken to it in the apprehension. Judgments, as he sees it, take stances on truth values, but propositions have them by themselves. Buridan, for his part, seems to rely on similar considerations to show that the sentential complex at issue

is not a proposition. He seems to have thought that sentences are able to carry truth values through being asserted. We will return to this issue in connection with molecular propositions.

For the most part, medieval logicians accepted the laws of noncontradiction and the excluded middle. Thus, every proposition has one and only one truth value. But neither of the two principles remained unchallenged. Aristotle's famous sea battle in *De interpretatione* chapter 9 was widely discussed and within that debate it was also suggested that contingent propositions about the future perhaps do not yet have a truth value. This did not become the standard view. Similarly, in the widespread discussions concerning limit-decision problems and particularly the instant of change, it was suggested that perhaps contradictories are both true at the instant of change. Instead of accepting this, the standard line was to provide elaborate analyses of limit decision relying on mathematical considerations concerning infinitesimal magnitudes.

As is well known, in the more philosophical discussions concerning the nature of truth medieval logicians often put forward the principle of correspondence: Truth is *adaequatio rei et intellectus*. This definition was not, however, much used in the specific context of logic. There the term "truth" was mostly used with the more limited meaning of propositional truth, and it proved difficult to exemplify from the real world anything that corresponded to a propositional complex. Thus, truth could hardly be explained as a relation between a real thing and a proposition. In the Aristotelian approach, things are referred to by using simple terms, and no simple expression—a mere term—can have a truth value. Truth rather arises from "composition" or "division" of terms in a predication, and depends on how this composition or division accords with how things really are. In his *Syncategoreumata*, Peter of Spain gives an elaborate suggestion that there is some kind of real composition, typically explicable with reference to the way in which everything in the world is composed of matter and form. According to Peter's suggestion, the truth of a sentence depends on whether this "real composition" is expressed adequately.

The standard Aristotelian *dictum*, "it is because the actual thing exists or does not that the statement is called true or false" (*Cat.* 12; 14b21–22), was not always understood in this manner. A typical way of explicating the claim that a proposition is true was to say that it "signifies as it is" (*significat sicut est*) or something to the same effect. By such formulas logicians tried to avoid committing themselves to positing any real entity with which the true proposition would have direct correspondence. Instead, the expression often worked in a way analogous to what has lately been called "disquotational": allowing transformation of the claim "p is true" into the simple claim "p."

Ockham's *Summa logicae* (I, 43) contains an interesting discussion of in what sense truth is predicated of a proposition. In his opinion, it is not a real quality of the proposition. This can be proved by the fact that a proposition may change from truth to falsity by fully external change. For example, when something ceases to move, the truth value of the proposition "this thing

is moving" changes without the proposition itself changing. According to Ockham's explanation, "true" is a connotative term signifying that things are as the proposition signifies. However, this remark leaves open the issue of what it is for things to be as the proposition signifies.

4.3. Are There Any Propositional Significates?

Stoic logic, and in its wake important early medieval authors like Boethius and Peter Abelard, made a distinction between a declarative sentence and its *dictum*, or that which "is said." Thus, the *dictum* expresses, or it simply *is*, the content of the proposition without being itself a proposition. For example, the proposition "Socrates is seated" (*Socrates sedet*) says or puts forward the *dictum* "that Socrates is seated," which in Latin is typically expressed as an accusative plus infinitive construction (*Socratem sedere*). Over the centuries, many logicians discussed the status of the *dictum*. Also, the related distinction between a proposition (as an expression) and its total significate (in distinction from the separate significates of its constituents) became a topic of an interesting dispute toward the second quarter of the fourteenth century.

In his early work, *Commentary on the Sentences*, Ockham puts forward a theory according to which belief always concerns a proposition formulated in mental language. That is, when a person assents to something, he has to formulate a mental proposition expressing that which he assents to. He then reflexively apprehends the proposition as a whole and assents to it. It seems that Ockham's motivation for this theory was the view that there is no way to grasp propositional content apart from formulating a proposition in mental language. Thus, if objects of beliefs are true or false, they must be formulated in mental language.

Several contemporaries of Ockham did not straightforwardly accept the idea that the object of belief must always be an actually formulated proposition. Even Ockham himself shows some hesitation toward this theory in his *Quodlibetal questions*, which he composed later. It seemed to many authors that when one believes, for example, that God exists or that a man is running, the object of belief is somehow out in the world and not merely a proposition in the mind. The idea is that people do not always believe in sentences, but at least sometimes it should rather be said that they believe things to exist in a certain way. This consideration made medieval logicians search for something like propositional content outside the mind and a number of different theories of how it could be found emerged.

In Walter Chatton's theory, the object of the assent has to be some extramental thing. If you believe that a man is running, the object of your belief is the man at issue. Thus, the significate of the proposition "a man is running" is the man. Chatton recognized that his theory has the problematic consequence that the simple term "a man" and the propositional complex "a man is running" signify the same thing. As Chatton saw it, the difference in these two expressions is not in what they signify but in how they signify it.

The terminology he used in this connection refers back to modist grammatical theories.

It seems that both Ockham and a younger contemporary, Adam Wodeham, reacted against Chatton's theory. In his *Quodlibetal disputations*, Ockham makes a further distinction concerning propositional assent, in effect allowing it to be the case that you give assent without reflexively considering a mental proposition. In such a case, you simply form the proposition and give your assent in an unreflective way as connected to rather than directed at the mental proposition. As Ockham curiously points out, this kind of assent is not at issue in scientific knowledge, only in beliefs of ordinary life. According to Ockham's obscure remarks, nothing is the object of this kind of assent.

Wodeham seems to have continued from this basis in his theorizing. He wanted, though, to allow that even the nonreflective kind of assent is about something, and the significate of the proposition appeared to be a suitable candidate for an object. However, its metaphysical status seemed quite unclear to the medieval mind. According to Wodeham, the significates of propositions need to be categorically different from the significates of the terms. As he put it, propositions do, of course, signify the things signified by their terms, but no thing or combination of things is the adequate total significate of the propositional complex. The adequate significates of propositions are such that they can only be signified by propositions; even further, they do not belong to any of the Aristotelian categories nor can they be called things.

Wodeham's theory became known as a theory endorsing "complex signifiables" (*complexa significabilia*). Such entities were rejected by most subsequent logicians, including major figures like John Buridan, but accepted by some, most famously by Gregory of Rimini—in subsequent discussion, the theory became known as his theory. In the third quarter of the fourteenth century, discussion of what propositions signify was abundant. Is it something like a mode of being? Or just a mental act of composition? Do propositions in fact signify anything more than just the things denoted by the terms, or perhaps even just the thing denoted by the subject?

The fourteenth-century discussion concerning complex signifiables seems to have made it clear to late medieval logicians that their logic was based on a metaphysical view of the world as consisting of things and not of states of affairs. The constituents of the world could be referred to by terms, but to make claims about the world, a different kind of mental act was needed. Paradigmatically, one had to construct a complex expression asserting a composition of multiple entities.

4.4. Predication

In Aristotelian logic, the ground for all judgments is laid by the predicative structure, where two terms are either joined or disjoined as the subject and the predicate. After Boethius, it remained customary in the Middle Ages to treat affirmative predication and negative predication as two different kinds

of statement, and also to take negation simply as "destroying the force of the affirmation." Thus, it is not necessary here to treat negative predication distinctly from the basic affirmative case.

The affirmative predication consists, as already Boethius recognized, not only of the two terms but also of the copula. Thus, when Aristotle remarks that a predication can be constructed either with a verb (e.g., "a man runs," *homo currit*) or with a participle (e.g., "a man is running," *homo currens est*), this was normally interpreted as meaning that the latter form is to be taken as primary. In the latter, the copula "is" was said to be added as a third part (*tertium adiacens*). In Latin, the copula was of course the standard verb "to be" (*esse*), which was also used in the simple existential claim "a man exists" (*homo est*). This use of *est* as *secundum adiacens* had to be explained since it appeared to lack either the copula or the predicate. As Boethius saw it, the verb serves here a double role. This solution was accepted in the Middle Ages, and thus there was no need to see it as an altogether different kind of statement. Buridan even argued against ordinary linguistic practice that logically one should prefer the formulation "a man is a being" (*homo est ens*).

Given that the copula joins the two terms into a predicative proposition and gives the sentence its assertive character, it still remains unclear exactly how it joins the terms together. It seems that this was one of the most fundamental points of disagreement among medieval logicians. For modern scholars it has proved rather difficult to find a satisfactory description of how the simple predication was understood in the Middle Ages.

One crucial nontrivial issue seemed clear, though. Throughout the Middle Ages, it was commonly assumed that in the absence of specific contrary reasons, the verb "to be" even as the copula retains its signification of being. Thus, all affirmative predications carry some kind of existential force, while negative predications do not. In an affirmation, something is affirmed to exist; a negation contains no such affirmation of existence. But beyond this simple issue, interpretations of the nature of predication seem to diverge widely.

Most of the twentieth century discussions of the exact content of the different medieval theories of predication have been based on the Fregean distinction between the different senses of "to be." Scholars have distinguished between inherence theories and identity theories of predication, despite the evident threat of anachronism in such a strategy. For want of a better strategy, we also have to rely on that distinction here. But instead of trying to classify authors into these two classes, let us simply look at the motivations behind these two rather different ways of accounting for what happens in a predication.

The idea of the inherence theory is that the subject and the predicate have crucially different functions in the predication. While the function of the subject is to signify or pick out that which is spoken of, the function of the predicate is to express what is being claimed of that thing. The idea is, then, that the Aristotelian form signified by the predicate inheres in the thing signified by the subject. Peter of Spain seems to defend this kind of theory

of predication when he tries to show that the copula signifies that relation of inherence obtaining between matter and form, or between a subject and its accident. Aquinas seems to follow this account.

Scholars have disagreed about Abelard's theory, and it indeed seems that his rich discussion of the topic provided grounds for several kinds of different subsequent theories. On the one hand, he seems to lay the basis for the inherence theory. On the other, he defends the idea that to look at the exact truth conditions of a predication like "a man is white" (*homo albus est*), it should be analyzed into a fuller form "that which is a man is that which is white" (*idem quod est homo est id quod album est*; *Logica ingredientibus* 60.13). With such a formulation he seems to have in mind the idea that for the affirmative predication to be true, the subject and the predicate must refer to the same things. This is commonly called the identity theory of predication.

Abelard's "that which is" (*quod est*) formulation remained part of the actual practice of logical writing for several centuries. It can be found commonly from logical texts throughout the Middle Ages, although it was not always offered as an explanation of the truth conditions of predication in general. The formulation has the special feature that it appears to give the subject and the predicate of a predication a similar reading. Both are to be understood as referring to some thing, and then the assertion put forward in the proposition would be the identity of these two things. This seems to amount to the identity theory of predication in Fregean terms.

In the fourteenth century, both Ockham and Buridan seem to have quite straightforwardly defended the idea that the Aristotelian syllogistic is based on identity predications. As they put it, the simple predication "A is B" is true if and only if A and B supposit for the same thing. For the most part, truth conditions of different kinds of propositions can be derived from this principle.

Somewhat interestingly, Ockham nevertheless recognizes the need of basic propositions expressing relations of inherence. For Ockham, the predicate "white" is a so-called connotative term, and therefore a somewhat special case. According to his analysis, the predication "Socrates is white" (*Sortes est albus*) should be analyzed into "Socrates exists and whiteness is in Socrates" (*Sortes est et Sorti inest albedo*). In his metaphysical picture, Ockham allows both substances and qualities to be real things, and if one is allowed to use only so-called absolute terms that supposit in a sentence only those things which they signify, the relation of inherence (*inesse*) is not expressible with an identity predication. Qualities inhere in substances, but they are not identical with substances. The whiteness at issue in the claim "Socrates is white" is not Socrates, it is a quality inhering in Socrates. Socrates is not whiteness even if he is white.

In his *Summa logicae*, Ockham has some special chapters on propositions involving terms in oblique cases (in cases other than the nominative). The just-mentioned proposition "whiteness is in Socrates" is a paradigm case of

such a proposition (in Latin, the subject has to be in the dative case *Sorti*; in English, the effect of the case is represented with the preposition "in"). Furthermore, all propositions involving the terms that Ockham calls "connotative" require in their logical analysis that oblique cases are used. The main claim of the short chapters of *Summa logicae* addressing propositions containing such terms is that their truth conditions cannot be given by the simple rule of thumb that the subject and the predicate must supposit for the same thing in an affirmative sentence. Consequently, the rules for syllogisms formulated with such propositions are also abnormal. In effect, Ockham excludes propositions with oblique terms from the ordinary syllogistic system, thus leaving a surprising gap in his logical system.

In his logic, Buridan proceeds differently. For the purposes of the syllogistic system, he requires that all propositions should be analyzed into a form where truth conditions can be given through variations of the rule that in affirmative sentences the subject and the predicate supposit for the same thing. This allows him to apply the standard syllogistic system to all propositions. The solution is at the price of greater semantic complexity. Buridan has to allow so-called connotative terms (including, e.g., many quality terms like "white") as logically simple terms despite their semantic complexity.

Both Ockham and Buridan apparently thought that identity predication is the logically privileged kind of predication. Nevertheless, they also both accepted the Aristotelian substance-accident ontology to such an extent that they had to find ways of expressing the special relation of inherence. While Ockham allowed exceptions to the syllogistic through irreducible propositions expressing inherence, Buridan opted for a syllogistic system with obviously complex terms expressing inherential structures.

4.5. Negations

As the medieval logicians saw it, the simple predication "A is B" contains altogether four different places where a negation can be posited:

1. It is not the case that A is B.
2. A is not B.
3. Not-A is B.
4. A is not-B.

It is of course clear that 1 is closest to the negation used in twentieth-century logic. In it, the negation is taken to deny the whole proposition. According to Boethius's commonly accepted formulation, the force (*vis*) of the predication is in the copula, and hence denying the copula denies the whole proposition. Thus, the negation in 2 has the same effect as in 1. (As Buridan notes, for quantifiers and other modifiers, the location of the negation may still make a difference.)

2 is the standard negation of medieval logic. It is the direct contradictory of the corresponding affirmative predication. In particular, it is noteworthy that this negation does not carry any existential presuppositions. Thus, "a chimera is not an animal" is true simply because no chimeras exist.

3 and 4 are affirmative statements containing an infinite term, as terms of the type "not-A" were called. In these cases, the negation is connected directly to a term and not to a proposition. An infinite term was taken to refer to those things to which the term itself does not refer. Thus, not-man refers to anything that is not a man. Because these negations do not make the proposition negative, 3 and 4 carry existential content: Some B must exist for 3 to be true, and some A for 4 to be true.

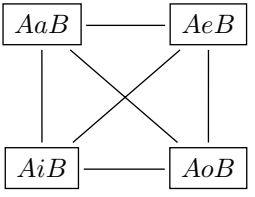
Although the syntactic idea of attaching a negation to a term was universally accepted in the Middle Ages, logicians seem to have disagreed about whether the term-negation should be taken to be essentially the same negation as the propositional one but in a different use. In his *Syncategoreumata*, Peter of Spain seems to reject this idea. He presents the two negations as genuinely different in themselves. His discussion is connected to a theory where even simple names and verbs signify in a composite sense. Thus, the idea is that "man," for example, means a composition of a substance with a quality, a substance having the quality of being human. Thus, the infinite term "not-man" signifies a substance that has not entered into a composition with the quality of being human. Ockham, for his part, preferred to reduce negating a term to ordinary propositional negation, claiming that the meaning of "not-man" can be explained as "something which is not a man." Buridan allows infinite terms a significant role in his syllogistic system, and thus seems to go back to thinking that the negation involved in them is fundamentally distinct from that which he calls "negating negation"—that is, the propositional negation that has power over the copula.

Given that negations can be put in many places even in a simple predication, medieval logicians gained skill in handling combinations of different negations. The idea that two negations cancel each other (provided that they are of the same type and scope) was also well known.

4.6. Quantifiers

Aristotelian predications typically have so-called quantity. Medieval logic commonly distinguished between universal ("every A is B"), particular ("some A is B") and indefinite propositions ("A is B"). As Boethius already pointed out, the indefinite predication that lacks any quantifier is equivalent to the particular one. Some logicians did specify certain uses that violate this rule of thumb, but such exceptions are rare. In addition to quantified and indefinite predications, singular predications were also discussed (e.g., "Socrates is running"). They had a subject term that was a proper name or some suitable demonstrative pronoun.

Basic quantified predications were given vowel symbols as mnemonic labels from the first two vowels of the Latin verbs "affirm" (*affirmo*) and "deny" (*nego*). Thus, the universal affirmative was shortened as AaB, where A is the subject and B the predicate. Similarly, the particular affirmative was AiB, the universal negative AeB and the particular negative AoB. These four predications were further organized into the so-called square of opposition to show their interrelations.



The upper two, the universal affirmative and the universal negative, were called *contraries*; they cannot be true simultaneously, but could both turn out to be false. Similarly, the particular affirmative and particular negative were *subcontraries*; they cannot be false simultaneously, but both could turn out true. The relation between the universal and the particular was called *subalternation* on both sides; the particular follows from the universal but not vice versa. The propositions in the opposite corners were called *contradictories*, since one of them had to be true and the other false.

In the Middle Ages, a substantial amount of ink was used discussing whether a universal affirmative could be true when only one thing of the relevant kind exists. The paradigm example was "every phoenix exists," and many logicians rejected it with the requirement that there must be at least three individuals to justify the use of "every." Toward the end of the thirteenth century this discussion seems to disappear, apparently in favor of the view that one referent is enough; the existential presupposition was never dropped, however.

Another issue of detail that was also widely discussed later was the case of universal predications of natural sciences, which capture some invariable that does not appear to be dependent on the actual existence of the individuals at issue. A suitable example is "every eclipse of the moon is caused by the shadow of the Earth." According to a strict interpretation of the existential presupposition, such predications prove false most of the time—which seems somewhat inconvenient. Two fundamentally different suggestions for a better reading of the predication were put forward. Ockham seems to favor the idea that what really is at issue here is the conditional proposition "if the moon is eclipsed, the eclipse is caused by the shadow of the Earth." This solution draws on the traditionally recognized idea that the conditional is implied by the universal affirmation. However, Buridan opted for another solution. As he reads the universal affirmation at issue, its verb should be read in a nontemporal sense. In such a reading, past and future eclipses also provide instances satisfying the diluted existential presupposition.

4.7. Complex Terms

Medieval logicians allowed that not all terms of a predication are simple. A predication can, of course, always be divided into the subject and the predicate together with the copula and the appropriate quantitative, qualitative, and modal modifiers. But the two terms may possibly be further analyzable (e.g., "a just man is talking," where the subject "just man" consists of two parts), and indeed this was a topic that attracted much attention during the Middle Ages. Toward the middle of the fourteenth century, discussion on this topic resulted in a detailed theory on the interaction between different kinds of combinations of categorematic and syncategorematic elements that can be found in a predication. To tackle with issues of scope an elaborate system of word order rules was introduced for the technical Latin used by logicians.

It seems that thirteenth-century logicians did not take it to be a serious problem that complex predications do not behave in ways that suit the needs of syllogistics. Following Aristotle's remark (*Analytica priora* I, 36; 48b41–49a5), syllogisms with oblique terms in the various cases were usually discussed separately, and thus it seems that the thirteenth-century logicians probably thought that more complicated predications do not necessarily fit into the ordinary syllogism. As we already noted, Ockham makes this slight inconvenience clear in his *Summa logicae*. It seems that Ockham fully understood that the traditional syllogistic logic does not always work if actually used linguistic structures are given full logical analyses. Also, he explicitly allows that there is no general way of giving the truth conditions of the various kinds of complex predications; in particular, he points out that even as simple a construction as the genitive case makes the standard truth conditions of identity predications inapplicable. "The donkey is Socrates's" is an affirmative predication. However, its truth requires, but it is not sufficient for it, that the subject and predicate supposit for different things ("donkey" for a domestic animal owned by a person, and "Socrates" for the owner of the animal). More generally, Ockham thought that mere identity predications are not sufficient to explain the expressive power of the actually used language. A richer variety of propositions had to be accounted for, but in fact they found no place in syllogistic logic. Thus, syllogistic logic was not a complete system covering all valid inferences.

After Ockham, Buridan took another approach. As he saw it, all categorical propositions can be reformulated as straightforward Aristotelian predications fitting the needs of the ordinary syllogism and having the rule of identity or nonidentity of supposition as the criterion of truth. For this purpose, he had to modify the traditional systems of combining different categorematic and syncategorematic elements so that they appear as geared toward building up terms whose suppositions can be decided. Perhaps most important, he saw that he could not assume that standard Aristotelian predications would be found as the end results of logicolinguistic analysis. Rather, he understood the building blocks of the syllogistic system—identity predications—to be more

or less artificial constructions built from complex terms. For example, for the purposes of syllogistic logic the sentence "the donkey is Socrates's" must be read as "the donkey is Socrates's thing," although the predicate "Socrates's thing" clearly is not a simple term of the ideal mental language.

Buridan did not assume that all mental or spoken propositions would be identity predications. Rather, he assumed that for the purposes of syllogistic logic, any proposition could be transformed into an equivalent identity predication. By such means, syllogistic logic could serve as a complete system containing all inferences.

Buridan's strategy involved, therefore, a massive expansion of the syllogistic system toward incorporating increasingly complex terms. Whereas logicians up to Ockham had accepted that a wide variety of propositions are nonstandard from the viewpoint of syllogistic logic, Buridan builds rules on how the content of these nonstandard propositions can be expressed by standard structures involving very complex terms. Buridan provides elaborate rules concerning complex terms. The idea is to show how nouns and verbs interact with different syncategorematic expressions and produce terms that fit into standard Aristotelian predications. In Buridan's view, all propositions can be transformed so that the truth conditions can be expressed through the criteria of an identity predication. In affirmative sentences, the terms must supposit for the same thing, while in negative sentences, they must not supposit for the same thing.

To see the full strength of Buridan's new system, let us consider a somewhat more complicated example. Buridan analyzes "Each man's donkey runs" (*cuiuslibet hominis asinus currit*) in a new way. Traditionally, this Aristotelian sentence was understood as a universal affirmation consisting of the subject "man" in the genitive case, and a complex predicate. This analysis makes the subject supposit for men, and the predicate for running donkeys so that the assertion cannot be read as an identity predication. Thus, standard syllogistics are not applicable to a proposition like this. Most logicians up to and including Ockham seem to have been satisfied with the implied limitations of the syllogistic system. Buridan, however, analyzes the proposition as an indefinite affirmation. It has a complex subject "each man's donkey," which includes two simple categorematic terms ("man," "donkey"), a marker for the genitive case (the genitive ending "'s"), and a quantifier (the universal sign "each"). The quantifier does not make the proposition universal, because it has only a part of the subject in its scope and must therefore be understood as internal to the subject term. As a whole, the subject supposits for sets of donkeys such that each man owns at least one of the donkeys in the set. The predicate of the proposition is a simple term, "running." It supposits for sets of running things. Construed in this way, the predication can be evaluated with the standard criteria of truth, and standard syllogistics can be applied to it.

It seems clear that Buridan took seriously the programmatic idea that the Aristotelian syllogistic system should provide a universal logical tool which did not allow major exceptions to behave in nonstandard ways. But instead of analyzing complex propositions into combinations of predications with simple

terms, Buridan provides elaborate rules concerning the ways in which complex terms are built.

4.8. Hypothetical Propositions

In the Middle Ages, not only conditionals but also conjunctions and disjunctions were called hypothetical (*hypothetica*) propositions. Otherwise the treatment of conditionals and disjunctions causes no surprises to a modern reader familiar with basic propositional logic. Walter Burley, for example, gives the following account. Conjunctions are propositions consisting of two further propositions that are joined with the conjunction "and" or something equivalent. Their truth conditions require that the propositions thus joined are true. Negating a conjunction makes reference to another type of hypothetical, namely disjunction, because denial of a conjunction requires only that one or the other of the conjuncts is denied. Disjunction, for its part, is defined in the inclusive manner: Its truth conditions require that one of the parts is or both of them are true. Denial of a disjunction produces a conjunction, and as Burley notes, denial of a disjunction of contradictories (e.g., "Socrates runs or Socrates does not run") produces a conjunction which includes contradictories.

Certain interesting issues are raised in more detailed discussions of conjunctive and disjunctive propositions. One such is the nature and exact content of conjunctive and disjunctive terms used in propositions like "every man runs or walks." Are they reducible to conjunctive and disjunctive propositions and why not exactly? How ought they be accounted for in inferential connections? Another, more philosophical issue was the question of whether the parts of conjunctions and disjunctions are strictly speaking propositions. As Buridan notes, the "force of the proposition" (*vis propositionis*) in a disjunction is in the connective, and thus not in either of the disjuncts. Hence, it is only the whole and not the parts that carry truth value in the composition. When someone utters a disjunctive proposition consisting of contradictories, he does not, according to Buridan, say anything false, although one of the parts would be false if uttered as a proposition. Thus, hypothetical propositions do not, strictly speaking, consist of categorical propositions but of linguistic structures exactly like categorical propositions.

It seems that medieval logicians treated conjunctions and disjunctions in a straightforwardly truth-functional manner. It seems equally clear that their treatment of conditionals differs from the twentieth-century theory of material implication. Indeed, in the Middle Ages theory of conditionals was mainly developed in connection with a general theory of inference, under the label "consequences." Conditionals were taken to express claims concerning relationships of inferential type.

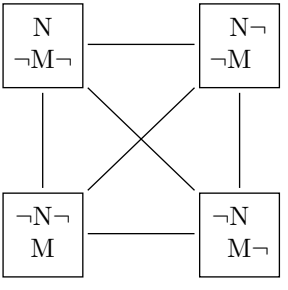
Medieval logicians also distinguished further types of hypothetical propositions. In Buridan's discussion (1.7), altogether six kinds of hypothetical propositions are accounted for, including conditional, conjunctive, disjunctive,

causal ("because the sun shines above the Earth, it is daytime"), temporal ("Socrates runs when Plato disputes"), and local ones ("Socrates runs to where Plato disputes"). Buridan even vaguely suggests that perhaps other Aristotelian categories may also give rise to hypothetical propositions in a way similar to temporal and local hypotheticals. It is clear that this approach to hypothetical propositions relies on other ways of combining propositions than just the truth-functional ones. The connective may express something more than just a truth function.

4.9. Modal Operators

Logical issues connected to possibility and necessity, which in the twentieth century have been studied as alethic modal logic, were a central research topic in late medieval logic. These modal terms were usually discussed together with other modifiers operating in similar syntactic roles. For example, the twentieth-century fields of study known as deontic logic (dealing with permissibility and obligation) and epistemic logic (dealing with concepts of knowledge and belief) have their counterparts in the Middle Ages, where these issues were discussed together with possibility and necessity.

Most medieval logicians discussed altogether four modal operators crucial to modern alethic modal logic: possibility, impossibility, contingency, and necessity. These were defined in relation to each other so that the necessary was usually taken to be possible but not contingent, whereas the impossible was taken to be neither possible nor contingent. Like the square of opposition of simple predications, modal predications were often organized into a square of modal opposition following Aristotle's presentation in *De interpretatione* (ch. 13). It is particularly noteworthy that following Aristotle's model, the square of modal opposition typically contained just the modal operators, not complete sentences. In a somewhat schematized way, the basic square can be illustrated as follows:



In this square, the relations of contrariety, subcontrariety, subalternation, and contradiction were said to behave as they would in the basic square of simple predications.

Following Aristotle, medieval logicians made a distinction between two ways of understanding a modal predication to make sense of examples like the

possibility that someone sitting walks. Understood *de dicto*, there is no such possibility. The sentence "someone sitting walks" is impossible. But understood *de re*, there is such a possibility, since the person who is sitting may be able to walk. Thus, if the modal predicate "can walk" is understood *de re*, or as concerning the person who is actually sitting now, the sentence might be true. All subsequent major logicians discussed this distinction in some form or other.

In the sections concerning modal propositions in Ockham's *Summa logicae*, it is clear that the *de dicto* reading is given logical priority. Using another traditional terminology, Ockham prefers to call it the composite sense (*sensu composito*) and does not oppose it to a *de re* reading but to the roughly similar divided sense (*sensu diviso*). Ockham apparently thinks that modality is a property of propositions rather than terms, and aims at reducing readings *sensu diviso* to *sensu composito* through analyzing modal propositions in *sensu diviso* into propositions *sensu composito*.

There are three main models by which modern scholars have been able to account for the way in which medieval logicians understood what it means to say that something is possible: the statistical model, the potency model, and the consistency model. During the medieval period, the modal concepts used by particular logicians typically ought not to be explained through reference to a single model. Rather, these three different strands of thought have influenced to varying degrees the modal thinking of different medieval logicians.

The basic intuition explained by the statistical model is that all and only those things seem to be possible which sometimes occur. If something never happens, it means that it can't happen. The potency model, for its part, explains the intuition that whether something is possible depends on whether it can be done. For something to be possible it is required that some agent has the potency to realize it, though it is not required that the thing is actually realized. However, because normally there are no generic potencies that remain eternally unrealized (why should we say that humans can laugh if no one ever did?), this model becomes clearly distinct from the statistical model only when God's omnipotence is understood to reach wider than just actual reality. God could have created things or even kinds of things which he never did, and these things remain therefore eternally unrealized possibilities.

It seems that throughout the Middle Ages, God's omnipotence was thought to be limited only by the law of noncontradiction. Contradictions are not real things, and therefore God's power is not limited, although we can say that he cannot realize contradictions. This consideration seems to have been one of the reasons why logicians in the thirteenth century increasingly used the criterion of consistency to judge claims about possibility. But it seems that the development of syntactic logical techniques also made it natural to demarcate a class of propositions that are impossible in the traditional sense but nevertheless seem to involve no contradiction (e.g., that man is irrational, or that man is not an animal). Especially the traditional technique of laying down a false or even impossible thesis for an obligational disputation (see the

following) seems to have encouraged consideration of consistent propositions or sets of propositions that are in some sense impossible. Some authors, like Boethius of Dacia, even use the special expression *compossibilitas* to refer to this kind of concept of consistency as distinct from possibility. As is well known, from Duns Scotus onward, several logicians made this kind of concept of consistency crucial for possibility in general.

The medieval discussion can be characterized as aiming at finding a way to account for these rather distinct intuitions of what it means to say that something is possible. A certain shift in emphasis is visible. Whereas earlier authors pay more attention to the statistical idea at the expense of consistency, later authors tend to neglect or argue against intuitions captured in the statistical model while emphasizing consistency as the criterion of possibility.

5. Classical Forms of Inference

5.1. Syllogisms

We must next turn to the "theory of inference." Ignoring probable inferences for now, we can say that this part of logic tries to describe how some propositions necessarily follow from others, from their premises. The propositions of a certain sequence have such properties that the last one must follow necessarily from its predecessors. An important type of inference is the *syllogism*—the inference on which Aristotle concentrated in his *Prior Analytics*.

The syllogism was the best-known and paradigmatic type of inference throughout the Middle Ages. In the thirteenth century, when logicians studied demonstrative inference, they were almost exclusively concerned with syllogistics; but afterward, when a more general inference theory developed, the policies of various authors differed widely. Thus Ockham still devotes the main part of his inference theory to a detailed analysis of syllogistics, and so does Buridan, whereas Burley regards it as a well-known special case of the more interesting subject of inference in general. The syllogism is probably the most famous item of "traditional" logic, but actually it has a not very dominant place in the works of medieval logicians. (For instance, in the *Logica magna* of Paul of Venice it is the subject of only one of 38 treatises.) However, it is systematically and historically so important that we must discuss it in relatively more detail.

All authors start by presenting or elaborating the highly condensed definitions in the beginning of the *Prior Analytics*. The often-quoted characterization in *An. Pr.* 24b19–20 says: "A syllogism is a discourse (*oratio*) in which, certain things having been supposed, something different results of necessity because these things are so." In a broad sense, any formally valid inference could be called a syllogism. But in the stricter sense, a syllogism has precisely two "things supposed," two premises. There has been quite a lot of discussion on

whether Aristotelian syllogisms are better understood as conditionals ("if p and q then r") or as deductive inferences ("p, q; therefore r"). The latter interpretation is perhaps more popular nowadays, and apparently it is also the correct way to see medieval syllogistics, at least in its classical stage. (Obviously the two things have a systematic correspondence, the relation that we nowadays call the deduction theorem, and many medieval authors were fully aware of it.) This means that syllogisms are like natural deductions of present-day logic.

Though medieval syllogistics followed Aristotle closely, there were some formal differences. Thus Aristotle—for special reasons—had formulated his syllogistic propositions as "Y belongs to X," "mortal belongs to man." This manner was never adopted in Latin; the medieval logicians wrote just "X is Y," "man is mortal."

Aristotle himself had brought the theory of nonmodal syllogistics to such perfection that there was little room left for initiative or disagreement. However, medieval texts produced a more systematic form for the theory, obviously aiming at didactic clarity.

A syllogism consists of two premises and one conclusion; the first premise is called major, and the second premise minor. Each proposition has two terms, a subject and a predicate, connected by a copula. But the two premises have a term in common, the so-called medium, and the terms of the conclusion are identical with the two other terms of the premises. Syllogisms can then be classified according to their configuration Subj–Pred into four different *figures* as follows:

	I	II	III	IV
major	M–B	B–M	M–B	B–M
minor	A–M	A–M	M–A	M–A

Further, each syllogistic proposition belongs to its type a, e, i, or o because of its quality and quantity: They are affirmative or negative, universal or particular. *If* we proceed by defining that the conclusion must always have the structure A–B, then it is obvious that each figure includes $4^3 = 64$ alternative combinations, and the total number is 256. But this is not exactly the classical method, so let us have an overview of the syllogism as it was usually presented.

Medieval logicians have a full and standard apparatus for syllogistics as early as the first terminist phase. They list the same valid syllogisms, usually in the same order, and also call them by the same names. The textbooks of William of Sherwood and Peter of Spain supply these names, which stem from some unknown earlier source and even the famous mnemonic verse composed on them. The names have three syllables, one for each sentence, containing the logical vowels a, e, i, and o. (In the following list of syllogisms, we mention these names that have recurred in all later logic.) The valid syllogisms were known as *moods*.

The *first* figure includes the four famous syllogisms from which Aristotle starts:

every M is B, every A is M, therefore every A is B (*Barbara*)
no M is B, every A is M, therefore no A is B (*Celarent*)
every M is B, some A is M, therefore some A is B (*Darii*)
no M is B, some A is M, therefore some A is not B (*Ferio*)

The *second* figure has four moods:

no B is M, every A is M, therefore no A is B (*Cesare*)
every B is M, no A is M, therefore no A is B (*Camestres*)
no B is M, some A is M, therefore some A is not B (*Festino*)
every B is M, some A is not M, therefore some A is not B (*Baroco*)

Furthermore, the *third* figure contains six moods:

every M is B, every M is A, therefore some A is B (*Darapti*)
no M is B, every M is A, therefore some A is not B (*Felapton*)
some M is B, every M is A, therefore some A is B (*Disamis*)
every M is B, some M is A, therefore some A is B (*Datisi*)
some M is not B, every M is A, therefore some A is not B (*Bocardo*)
no M is B, some M is A, therefore some A is not B (*Ferison*)

After the Renaissance, logicians continue by giving the five moods of the fourth figure: *Bramantip*, *Camenes*, *Dimaris*, *Fesapo*, and *Fresison*. That, however, is not the orthodox Aristotelian way. Aristotle knew inferences like these but did not include a fourth figure in his theory. Instead, he wanted to place these syllogisms into the first figure. Following his remarks, Theophrastus developed a clear account of the matter, and it was well known in the Middle Ages through Boethius. In Theophrastus's account, the major term need not be the predicate in the conclusion, which can also have the inverted order B–A. This gives us the five so-called indirect moods of the first figure:

every M is B, every A is M, therefore some B is A (*Baralipton*)
no M is B, every A is M, therefore no B is A (*Celantes*)
every M is B, some A is M, therefore some B is A (*Dabitis*)
every M is B, no A is M, therefore some B is not A (*Fapesmo*)
some M is B, no A is M, therefore some B is not A (*Frisesomorum*)

This method can replace the fourth figure, though it does introduce a certain unsatisfactory asymmetry.

The problem of the missing figure has caused much scholarly debate that we cannot enter into here. Medieval logicians were quite aware of the problem since they had seen at least Averroes's comments on the fourth figure. Arguments were often given to refute "objections" questioning the sufficiency of three figures. Apparently medieval authors were unanimous in thinking that the fourth figure could be eliminated with the indirect moods of the first figure. They either said that there were only three figures, or more precisely, like Albert of Saxony, that the fourth is superfluous. It is noteworthy that they did not regard the order of premises as essential.

Thus there are 19 valid syllogistic moods. A small addition was obtained by allowing the five "subaltern" moods, which yield a particular conclusion though a universal one would be valid too. For example, *Barbari* instead of *Barbara* leads to "some A is B." This step would not be accepted in modern logic where universal implications have no existential import, and it indicates clearly that medieval syllogistics assumed that every term really had existential reference.

Aristotle had only implicit allusions to singular propositions in syllogisms, and it was a good achievement that medieval logicians constructed a full and systematic theory of singular syllogisms. Ockham was the most active worker here. He emphasizes that the singularity of terms makes no difference for the validity of inference. This amounts to a considerable reinterpretation of the whole notion of a propositional term. Moreover, he gives explicit cases of singular syllogisms in each figure, for example, the third figure "expository syllogisms" like "x is B, x is A, therefore some A is B." (For nominalists like him, the question had special epistemological relevance because of the basic status of truths about individuals.) Some later Ockhamists even drew a dichotomy across the whole syllogistics between expository syllogisms and those with general mediums.

5.2. Theory of Syllogistics

Syllogistics, undoubtedly, is just a small portion of logical inferences, but systematically it is extremely important. The unique thing in classical syllogistics is that it was a formal theory. Its results are not separate truths achieved by trial and error; instead, they are derived in a deductive manner. This had largely been achieved already in the *Prior Analytics* and continued by ancient commentators. Medieval logicians were very interested in this project.

The most important tool here is *conversion*. It is a completely general method that pertains to all propositions of the S–P form, but it finds good use in syllogistic theory. Briefly, in a conversion the subject and the predicate change places, and conversion rules tell when such a transposition is legitimate. The following set of (nonmodal) conversion rules was universally accepted. First, in simple conversion AeB converts with BeA, and AiB converts with BiA. In other words,

some A is B if and only if some B is A, and

no A is B if and only if no B is A.

Second, in conversion *per accidens* AaB implies BiA, and AeB implies BoA (this negative one is the only rule that was not in Aristotle):

if every A is B then some B is A, and

if no A is B, then some B is not A.

These are only *per accidens*, because they change the quantity and do not hold in the opposite direction. Third, ever since Boethius even contraposition was taken as a type of conversion. It preserves the quality and quantity but "changes the finite terms into infinite ones." For example, "if every A is B then every non-B is non-A." Fourteenth-century logicians noticed that contraposition need not be valid when any of the terms is empty—an existential assumption is required.

With conversions, some syllogistic moods can be derived from others. The idea is that if certain syllogisms are selected as basic, others can be derived from them by a clever use of fixed methods. Aristotelians called this process "reduction," present-day logicians would call it *proof*. Conversion was the most important method of reduction. The other method was *reductio ad impossibile*: A mood is valid because the negation of the conclusion leads to the negation of a premise. With these methods, all syllogistic moods could be reduced to the direct moods of the first figure—in fact even further, to *Barbara* and *Celarent*. This was basic stuff in all textbooks, and the consonants in the names of moods refer to the methods of reduction. (S: convert simply; P: convert *per accidens*; M: transpose the premises; C: reduce *ad impossibile*.)

These privileged syllogisms are cases of *dici de omni et nullo*, in which the conclusions can be seen as immediate corollaries of simple affirmation or negation. As Buridan explains, "*dici de omni* applies when nothing is taken under the subject of which the predicate is not predicated, as in 'Every man runs'. *Dici de nullo* applies when nothing is taken under the subject of which the predicate is not denied." So direct first figure syllogisms are immediately self-evident, and medieval logicians, like Aristotle, called them "perfect." Others are imperfect in the sense that their validity needs to be shown.

The growth of syllogistic theory naturally leads to the philosophical question of its foundations. Such a problem can arise from two perspectives: One may wonder about the status of syllogistics in the totality of logic, or one may ask if particular syllogistic inferences depend on some other principles.

a. The question about the general status of syllogistics became current when the theory of consequence developed in the beginning of the fourteenth century (see section 6). Aristotle had started from syllogisms and proceeded to a brief discussion of other inferences; now logicians took the opposite direction. In the thirteenth century, some logicians' attitude seems to be that all strict demonstrative logic is syllogistical, but the more people were concerned with logical research, the clearer it became that other inferences are valid, too; and

this was then explicated by means of the concept of consequence. However, the relation between syllogistics and consequences is not very clear. Syllogistics is a part of consequence theory, in the sense that one particular type of consequences are "syllogistic consequences." (This is especially clearly said by Buridan, whereas Ockham prefers to keep the titles unconnected.) And syllogisms hold because they are good or solid consequences, in our words, logically *valid* ones. But does syllogistics depend somehow on other parts of the theory? It seems that medieval logicians did not think so. They were aware of the importance of propositional logic—after all, the Stoic heritage had survived—but they did not work in the present-day fashion and build first a propositional calculus, then a predicate logic on it. Burley is an interesting case here: He really starts from the simple consequences of propositional logic. But he had no followers in this respect, and contrary to what has been suggested, even he does not apparently aim at any stratification of logics here.

b. More concretely, one might ask if the validity of a particular syllogistic mood is based on some principles, or if a syllogism involves the use of other logical laws. This problem does not appear in terminist manuals, but it is discussed in the 1240s by Robert Kilwardby. He insists that the necessity of *dici de omni et nullo* is of such a self-evident nature that it cannot be regarded as a genuine inference step. Many logicians agreed with him. Kilwardby even asks if syllogisms presuppose separate inferences of conversion, and argues that it is not so. Suppose that no B is A; just add "every A is A" as the second premise, and you get the converted sentence, "no A is B," by *Cesare*. Similarly in other cases, we see that conversion reduces to syllogism. This idea was not generally accepted, but conversion was occasionally considered so immediate a transformation that it could not be called an inference at all.

Soon, however, an alternative view was articulated. About 1270, Peter of Auvergne refers to *loci*, the governed steps of argumentation theory, and says that "every syllogism holds because of a *locus* from a more extensive whole to its part." Simon of Faversham, Radulphus Brito, and others then developed this thought that a syllogism must involve a "principle of consequence." The conclusion is somehow included in the premises. But the remarks are brief and obscure. In any case, they anticipate the fourteenth-century view of logically necessary consequence relation that is not peculiar to syllogisms.

5.3. Modal Syllogisms

Aristotle devotes a large part of his *Prior Analytics* to modal syllogisms. But unlike nonmodal syllogistics, this area remains in a very unsatisfactory state. The modalities he there studies are necessity, impossibility, and contingency. He wishes to produce a complete set of syllogisms in which some propositions have such modalities; further, he tries to systematize these syllogisms like the nonmodal ones. Here he needs conversion, *reductio*, and a third method, *ekthesis*, based on defining new predicates. Medieval logicians replaced *ekthesis* with a more elegant method of expository syllogisms.

The main problem is that Aristotle's theory looks incoherent. His set of accepted syllogisms might be the outcome if all modal propositions were read *de re* only, as concerning the modal properties of individuals. But then the conversion rules do not hold: Obviously "every A is something necessarily B" does not convert to "some B is something necessarily A." Moreover, his choice of valid syllogisms contains some oddities.

Ancient commentators struggled with these puzzles, and medieval Aristotelians could not avoid them. Peter of Spain's *Summulae* does not really discuss modal logic, but Kilwardby, Lambert, and Albert the Great try to save Aristotle's doctrine. They resort to a very strong interpretation of necessity, proposed by Averroes, which concerns only necessities which hold per se because of essences. Even this technique demands some arbitrary decisions, and in any case it amounts to a severe restriction of modal syllogistics.

A similar approach seems to have continued through the thirteenth century. The first known work that introduces new methods is the commentary by Richard of Campsall, written about 1308. Campsall's own theory is conservative, since he wants to maintain the Aristotelian syllogisms and conversions by means of a strict and somewhat confused *de re* reading. But the novelty is that he makes a systematic distinction between divided and composite readings. It is connected to the idea, initiated by Duns Scotus, of simultaneous alternative states of affairs.

This new semantics of modal notions made possible a new and different approach to modal logic. From this point of view, modal logic was seen to be much wider than the part that Aristotle had developed, and the relations of modes could be systematized in a new way. The basic notions were now necessity and possibility, which could be understood as realization in all and some alternatives respectively. The first exact presentation of the resulting syllogistics was the very thorough account in Ockham's *Summa logicae*. In Paris, the orthodox Aristotelian model survived much longer, but Buridan's *Tractatus de consequentiis* (1335) provides a modern theory, which is almost as full as Ockham's. A third and more concise classical text is in Pseudo-Scotus's commentary on *Prior Analytics* (c. 1340).

The new modal logic gave plenty of room for the notion of contingency, and it caused some disagreement, but for simplicity we bypass this and concentrate on the syllogistics of possibility and necessity. The composite and divided readings of them were strictly distinguished. The composite readings are easier, and accordingly they were less discussed. They were indeed *de dicto* in the sense that strictly speaking they only make a singular nonmodal claim about a *dictum*; for example, "it is necessary that some A is B" is interpreted as "the *dictum* 'some A is B' is necessary." The syllogistic for such propositions follows from the general consequence theory. Ockham and Buridan agree that in every mood, if both premises are prefixed with necessity N, the conclusion is necessary too. On the other hand, a syllogism MMM, with all the three propositions modalized as possible, does not hold because the premises need not be compatible. Ockham also remarks on NMM and MNM.

Much more problematic were divided premises, that is, propositions with genuinely modalized copulas. The main device for dealing with them was ampliation (see section 3.5), which extends the subject term to refer to supposita that occur in alternative nonactual states of affairs; thus "every A is possibly B" will be read "everything which is or possibly is A is possibly B." But ampliation may be blocked by adding *quod est A*, "what (actually) is" A. Now it is striking that ampliation was understood in two different ways. Ockham assumed that ampliation is good for possibilities (and contingencies)—but he did not accept it for necessities. In other words, only actual things could be said to have necessary properties. The reasons for this are not clear; perhaps he thought that necessities always involve some existence postulate. Buridan, in his turn, said clearly that all modalities amplify the subject in the same way, and this became the common view, that is, if the subject of a modality is not explicitly restricted to what is, it is freely amplified. (We must therefore be cautious if we wish to use present-day possible world apparatus here.) Buridan drew an octagonal diagram of the propositions "Every/Some A is necessarily/possibly B/not B" and analyzed all the 56 logical relations between them. This made the map of modalities much clearer.

Combinations of syllogistic moods, modalities, and restrictions produce a huge number of cases, and logicians could not mention every case explicitly, although they did pursue a full theory of them. They also comment on cases where some propositions are nonmodal. We can only sketch some outlines now. In the direct first figure, everybody accepted MMM syllogisms as valid. Buridan and Pseudo-Scotus accept NNN, MNM, and NMN. The seemingly surprising NMN here shows the effect of ampliation. (For instance, every M is necessarily B, some A is possibly M, therefore some A is necessarily B.) Ockham accepts NNN only when restricted to actuals; for Buridan's school this is another valid syllogism, like several other moods resulting from a restriction of subjects of N or M. Buridan also accepts, for example, _NM with an assertoric major. In the second figure, Buridan mentions NNN, NMN, and MNM (and Pseudo-Scotus mistakenly adds MMM). These again have restricted versions (in the style of: if every actual B is necessarily M and every actual A is possibly not M, then no A is B). But Ockham allows no valid syllogisms here. In the third figure, all accept MMM. Buridan and Pseudo-Scotus accept NNN, NMN, and MNM, while Ockham accepts only restricted versions of these. Some of them, not precisely the same ones, are in Buridan.

Ockham's theory looks somewhat unfinished: His view of ampliation causes trouble, and he derives a great number of results by discussing individual examples one by one. Buridan, on the other hand, uses a very elegant deductive method with, for example, cleverly formulated conversion rules. His theory is the summit of medieval modal logic. His pupils Albert of Saxony and Marsilius of Inghen continued to give comprehensive accounts of modal syllogistics, with some usually unsuccessful innovations, but after them modal syllogisms seem to have fallen out of fashion.

5.4. Topics and Methodology

An important part of medieval logic was *topics*. The dialectics of the old *trivium* mostly belonged to it. The ultimate source was Aristotle's *Topics*, but a second and simpler authority that replaced it for a long time was Boethius's *De differentiis topicis*. The main subject in this inquiry was *loci*, *locus* being Latin for Aristotle's *topos* (literally "place," here something like "consideration"). Aristotle does not define his *topos*, whereas Boethius gives two meanings for *locus*. It can be a "maxim," a self-evident sentence that needs no further proof, but it can also be a logically relevant feature that distinguishes two sides. Confusingly, the distinction can be between sentences, like affirmative and negative, antecedent and consequent, or between concepts, like genus and species, part and whole. For example, the distinction between genus and species supports the maxim: What belongs to the genus belongs to the species.

Boethius's double notion of *loci* long guided medieval topics. On the other hand, Aristotle emphasized an aspect which was not so prominent in Boethius: Topics concerns dialectical argumentation, the finding, testing, and examining of plausible theses. Hence it is not restricted to methods of demonstrative scientific proof of necessary results.

Treatises as early as the eleventh century discuss topics, and this interest culminates with the Aristotelian revival of the thirteenth century. Thus, Peter of Spain gives a detailed list of various *loci* which follows Boethius closely. An important idea in such lists is that *loci* are supposed to guarantee the validity of an inference or argument that was not immediately valid because of its form. For instance, Peter's inference "The housebuilder is good, therefore the house is good" is surely not formally valid—and not even quantified—but it is "confirmed" by the *locus* of cause and effect: "That whose efficient cause is good, is itself also good."

We see that the result is still not conclusively proved, but the addition connects the argument to syllogistics. This need of support is characteristic of "enthymematic" arguments, demonstrative or not. Nowadays we are accustomed to think that they are valid because of some suppressed deductive premises, but medieval authors did not always see the matter so. Often they thought that the support came from a rule and not from an implicit premise. The terminists were inclined to think that all valid arguments are reducible to syllogisms; topics gives metalogical directions for finding suitable middle terms for the reduction.

After the early terminists, topics was still constantly discussed. After all, the *Topics* was a big book in the *Organon* and belonged to the obligatory courses, at least in part. But the heyday of topics was over when *logica moderna* was developed. It was no longer a really inspiring field, although it undoubtedly had some importance: Topics apparently influenced the growth of consequence theory (see section 6.1), and the doctrine of *loci* was also relevant in discussions concerning the foundation of syllogisms.

When the *consequentia* theory developed, both syllogisms and nonsyllogistic inferences could be seen as cases of the same general patterns. As a result, topics lost an important function. The arguments that were formerly studied in topics were, in the fourteenth century, normally included in consequences. Also, it is significant that topics was no longer connected to enthymemes but to dialectical arguments, that is, its special character was seen as epistemic. Usually, the leading logicians no longer treated topics as a separate subject at all—Ockham, for instance, studied topical arguments only as a relatively uninteresting special case. On the other hand, Buridan still painstakingly devoted a whole treatise to topics. Later the interest in topics diminished even more; Paul of Venice did not speak of it. However, commentaries on Aristotle's *Topics* were written throughout the fourteenth and fifteenth centuries, but no new ideas were presented.

The Aristotelian theory of science was highly abstract; while it had little contact to concrete problems, it did have a close connection to logic. The basic source for medieval discussion was *Posterior Analytics*, though direct commentaries on this difficult work were not very common. In the Aristotelian picture, developed for example by Aquinas, an ideal science consists of a system of demonstrative syllogisms. Their premises must be true, necessary, and certain. Premises can be derived by other syllogisms, but ultimately they rest on evident necessities. As Kilwardby says, "the demonstrator considers his middle term as necessary and essential, and as not possibly otherwise than it is; and so he acquires knowledge, which is certain cognition that cannot change." Science is thus a system of syllogisms about causes and essences; it can use logical principles, but logic itself is obviously not a science. Much of this grandiose view later had to be given up, when first Scotus problematized the notion of necessity and then Ockham problematized the notion of evidence.

6. New Approaches to Inferences

During the thirteenth century, four new domains of logical research broadly falling into the scope of propositional logic emerged: *consequences*, *obligations*, *insolubles*, and *sophisms*. In overall treatments of logic like Ockham's *Summa logicae* and Buridan's *Summulae dialectica*, these new branches of logic were discussed in the place traditionally occupied by treatments of dialectical topics in the sense in which they referred to what Aristotle discusses in his *Topics*. This is not to say that the traditional theory of dialectical topics, for which Cicero and Boethius had provided the classical texts, had disappeared altogether. Nor can we say that these new areas of logic had replaced the tradition of dialectical topics. Rather, the purposes aimed at by research in these new areas were seen to be approximately similar to those traditionally aimed at by the theory of dialectical topics, and consequently the new fields were taken to complement traditional discussions. In modern terms, we can say that the point of gravity was moving from the theory of argumentation toward formal logic.

Let us start with consequences, considering four different issues pertaining to this crucial area of logic. Late medieval discussion of consequences aimed at giving clear and specific determinations of (1) what is a consequence, (2) the definition of the validity of a consequence, (3) how they should be classified, and (4) rules concerning valid consequences.

6.1. What Is a Consequence?

In general, late medieval treatments of consequences understood them as inferences. That is, they were not called "true" (*vera*) or "false" (*falsa*), but rather were said to be "good" (*bona*), or simply "to be valid" (*valeo*) or "to hold" (*teneo*), or in the opposite case "to fail" (*fallo*). Despite an acknowledged close connection to conditional propositions, consequences were usually discussed separately as belonging to a different place in the overall structure of logic. Ockham, for example, discusses conditionals within his theory of propositions, and turns to consequences as a theory of nonsyllogistic inference in the beginning of III, 3 in *Summa logicae*: "After treating syllogism in general and demonstrative syllogism, we now have to turn to the arguments and consequences that do not apply the syllogistic form."

The genre of logical writings on theory of consequences seems to have arisen in the thirteenth century from recognition of the fact that a general theory of inferential validity can be formulated in addition to, or as an extension of, the traditional syllogistic system. As such, medieval logicians had been aware of the idea at least since Abelard's work, and Boethius had already composed a special treatise on what he called "Hypothetical syllogisms," that is, on propositional logic. Nevertheless, Walter Burley's *De puritate logicae* seems to have been the first overall presentation of logic to discuss the theory of inference systematically starting from general issues of consequences and moving toward more particular issues after that, allowing syllogistic only the minor position of a special case.

That most medieval logicians saw consequences as inferences and not as propositions is reflected in the fact that they aimed at formulating general rules (*regulae*) of valid inferences; traditional dialectical topics were also seen to belong to this set in addition to a number of more formal ones. The outstanding exception in this picture is John Buridan and his *Tractatus de consequentiis*. He explicitly defines consequences as hypothetical propositions consisting of two parts, the antecedent and the consequent, joined by a connective like "therefore" (*ergo*). Thus, Buridan's consequences amount to conditional propositions with specific content. He treats consequences as pieces of discourse that assert the validity of an inference from the antecedent to the consequent: "One follows from the other" (*una sequatur ad aliam*). Accordingly, Buridan does not discuss or lay down metalinguistic rules (*regulae*) of consequences in this treatise, but instead asserts "conclusions" (*conclusiones*) concerning what can be truly said about the kinds of sentences following from each other.

It seems clear that all prominent medieval logicians saw the distinction between the acceptability of performing an inferential step and the assertion that a valid inferential relation obtains. Whereas most logicians thought that consequences should be understood as inferences, Buridan made the opposite decision. For him, a consequence was a proposition, a conditional claim concerning an inferential relation between the antecedent and the consequent. He seemed to have had no followers in this opinion, but because of his prominent position in late medieval logic, his surprising stand has caused a number of misunderstandings concerning the issue both for medieval authors and for modern commentators.

6.2. Criteria of Validity

The simplest way to formulate the definition of inferential validity was to ground it on the idea that it is impossible for the antecedent to be true and the consequent false. Indeed, it seems that all late medieval definitions of validity can be seen as variously qualified or modified versions of this principle. In the first known treatise directly dedicated to consequences, Burley's *De consequentiis*, we find the definition that a consequence is valid if "the opposite of the consequent is repugnant to the antecedent." The problem with this definition is that it seems unclear in which sense we are to take the word "repugnant," since it is often used in a way that already contains reference to inferential connections. Indeed, Burley elsewhere opts for alternative definitions closer to the modal criterion.

In Buridan, we find the following list of three alternative descriptions concerning when some proposition "is an antecedent to another" or, in other words, a consequence is valid:

(a) "that is antecedent to something else which cannot be true while the other is not true" "*illa alia non existente vera*";

(b) "that proposition is antecedent to another proposition which cannot be true while the other is not true when they are formed simultaneously" "*illa alia non existente vera simul formatis*";

(c) "that proposition is antecedent to another which relates to the other so that it is impossible that howsoever it signifies, so is the case, unless howsoever the other signifies, so is the case, when they are formed simultaneously" "*sic habet ad illam quod impossibile est qualiterqumque ipsa significat sic esse quin qualiterqumque illa alia significat sic sit ipsis simul propositis*."

Buridan finds each of these three descriptions problematic, but accepts the last, if it is understood in a suitably loose manner. The problem with the first definition is related to the standard medieval requirement that a proposition must be actually formulated to be true. This makes it clear that almost no consequence would be valid according to the first criterion, since the consequent need not be

formulated when the antecedent is. The second aims at correcting this problem through the simple addition "when they are formed simultaneously," but falls prey to it as well. A consequence like "no proposition is negative, therefore no donkey runs" should be invalid, but turns out to be valid on criterion (b) as well as on (a), because the antecedent is never true when it is actually put forward. Thus, it cannot be true without the consequent being true even if they were simultaneously formed. With criterion (c), Buridan takes another strategy. He recognizes that the consequential relation should not be seen to obtain with the sentences themselves, not even between potentially formulated ones, but rather between their contents. However, Buridan did not believe that such sentential contents would exist (see the section about propositional significates, *complexa significabilia*), and therefore the formulation of the criterion (c) makes problematic ontological commitments. Apparently he could not find a formulation that would avoid them, and thus we are left without a satisfactory description of inferential validity.

It seems, nevertheless, that Buridan's strategy of transporting criteria of validity from the actual sentences to their significations or contents became a generally accepted one. In some interesting sense, which still puzzles modern scholars, Buridan's further discussions on the topic take a "mentalistic" turn in the conception of logical validity. He considered that logical validity depended on the mind in a more crucial sense than many of his predecessors. Some formulations by his followers made this mentalistic turn even more obvious in ways that we shall see in the next section.

6.3. Classifications of Valid Consequences

The most traditional medieval distinction among kinds of valid consequences was the distinction between those valid "as of now" (*ut nunc*) and those valid "simply" (*simpliciter*). Validity *ut nunc* was taken to mean something like validity given the way things now are: From "every animal is running," it follows *ut nunc* that Socrates is running, at the time in which Socrates exists as an animal. After his death, the consequence ceases to be valid. Simple validity, on the other hand, meant validity in all circumstances. In this sense, from "every animal is running," it follows that "every human is running." It is noteworthy that validity *ut nunc* also contains some kind of necessity, and thus it cannot be compared to twentieth-century material implication.

Late medieval logicians put their main interest in two other, philosophically more interesting distinctions. Somewhat confusingly the concepts "form" and "matter" were used in both distinctions, so that when we come to Paul of Venice, a consequence may be, for example, "formally formal" or "materially formal," since he combines the two distinctions into one systematic presentation. In both distinctions the issue was to separate a class of consequences that were valid in a privileged manner: not only valid, but "formally valid."

In one sense, formal validity meant a substitutional kind of validity, where a consequence is formally valid, if it "is valid for all terms" (*tenet in omnibus*

terminis), and only materially valid if its validity is based on the special content of some of the terms used in the inference. In this sense, the paradigm examples of formally valid inferences were syllogisms in the Aristotelian figures, but also examples like *modus ponens* could be put forward. In the other sense, an inference was called formally valid only if the consequent was "formally included" (*includit formaliter*) in the antecedent or in the "understanding" (*intellectus*) of the antecedent; this kind of formal validity was often called "natural" or "essential" validity. It seems that the roots of both distinctions can be traced back to the early Middle Ages. At least Kilwardby gives ground for both distinctions. Nevertheless, the two distinctions seem to have had a somewhat different history. Furthermore, the concept of material validity remained in most treatments rather obscure. It seems, however, that especially as related to the latter definition of formal validity based on inclusion, material validity was often understood as having to do with certain properties of the propositions used. The paradigm cases of materially valid inferences followed the rules "from the impossible anything follows" and "the necessary follows from anything."

Let us first look at the latter kind of formal validity, the one based on the idea that the antecedent must "formally include" the consequent. The concept of "formal inclusion" seems to have been developed by late thirteenth-century theologians, such as Henry of Ghent, Godfrey of Fontaines, and Duns Scotus. In many texts the topic comes up in a discussion concerning the role of the third person in the divine Trinity, employing the special technique of obligations (see following). These discussions resulted in elaborated theories of what it means to say that a concept is included in another concept, or that an assertion conceptually includes and thus entails another claim. The primary examples studied by medieval logicians included inferences like "a human exists, therefore, an animal exists," and the explanation of their "formal" validity was based on the necessary conceptual or essential relation between the species "human" and the genus "animal." The concept "human" was said to "formally include" the concept "animal," and thus the inferences based on this relation were said to be "formally valid." In twentieth-century terms, we would rather describe them as analytically valid inferences.

William Ockham was aware of this discussion and aimed at bringing the results into the systematic context of logical theory. In the classification of *Summa logicae*, a consequence is formally valid if it is valid by general rules of a specific kind. They must concern the syntactic features of the propositions (*forma propositionis*) involved in the consequence. Also, the rules must be self-evident (*per se nota*). This part of the definition is in effect identical with, or at least comes very close to, the substitutional type of definition of formal validity. But on the same page Ockham also admits as formally valid consequences that are valid by something he calls an "intrinsic middle." His example is "Socrates does not run, therefore a man does not run," which is valid by the "intrinsic middle," "Socrates is a man." It seems that Ockham wanted to present this type of formal validity to allow also inferences based

on something like conceptual inclusion within this group, the inclusion being expressible as an intrinsic middle.

Some 10 years later, Ockham's student Adam Wodeham explicitly distinguished between two different ways of understanding the concept of formal validity. One of them uses only the substitutional criterion, while the other accepts as formally valid all consequences based on truths known in themselves (*per se nota*). Insofar as Wodeham's *per se nota* refers to all analytic truths and not only conceptual inclusion, the definition is wider than that derived from the traditional slogan "formally includes," but it is clearly on the same track.

Material validity is defined by Ockham with reference to something he calls "general conditions of the propositions," and he gives the *ex impossibile quodlibet* rule as an example. In Ockham's case this is strange, since he clearly knew that from a contradiction it is possible to derive anything with rules which he allows to be formal. Do we, thus, have inferences that are both material and formal?

In his definition of formal validity, Buridan presents only the substitutional principle, without mentioning the idea of conceptual inclusion. His examples of inferences which are valid but not formally so, however, show that he was aware of the criterion but did not want to use it. He straightforwardly claims that those inferences, which are valid so that all substitutions of the categorematic terms with other terms are also valid, are formally valid. Among formally valid inferences, Buridan explicitly counts inferences from contradictions, though of course not from weaker impossibilities like from "a man is not an animal." These he classifies as material.

It seems that in the latter half of the fourteenth century, Buridan had few followers in his classification principles. Only Albert of Saxony seems to have accepted the substitution principle as the sole criterion of formal validity. The majority of logicians seem to have wanted to develop an idea which is closer to what was later in the twentieth century called analytic validity. The criterion of formal validity was, therefore, formal or conceptual inclusion of the conclusion in the premises. As an interesting special case, Paul of Venice presents a system that uses both concepts of formal validity, thus producing a very elaborate system.

6.4. Rules of Consequences

Usually medieval discussions of the theory of consequences also included a selection of rules warranting valid inferences. Instead of anything close to a complete listing of such rules, we must here satisfy ourselves with a look at the types of such rules presented in the medieval discussions.

We have already encountered two such rules: "from the impossible anything follows" and "the necessary follows from anything." These rules were practically never completely rejected in the later Middle Ages. However, their applicability in specific contexts was often limited, and as they were typically classified as

materially valid, they were understood as belonging to a somehow inferior kind.

Medieval authors knew the main rules of so-called classical propositional logic. For example, detachment is put by Burley as concisely as possible with a term variable: "If A is, B is; but A is; therefore B is." Transitivity rule is presented by Burley as the consequence "from start to finish" (*a primo ad ultimum*). He also discusses other examples of basic propositional logic of the kind, but when we turn to the later fourteenth century, the selection of rules of this type leaves nothing to be hoped for.

One type of rules of consequences that seems to have interested medieval authors quite widely is based on epistemic operators. These were often discussed by direct comparison with modal rules; if something is necessary, it is the case, and if something is known, it is the case. More interesting (and more disputable) examples of relevant inference schemes are more complicated. The rule "if the antecedent is known, the consequent is also known" was often held to be valid only on the further condition that the consequence itself is known.

The first rule of consequences in Ockham's *Summa logicae* is that "there is a legitimate consequence from the superior distributed term to the inferior distributed term. For example, 'Every animal is running; hence every man is running'." All medieval logicians accepted this example as valid, though they often formulated the rule differently, and the explanation of the kind of validity varied. Buridan, who relied on the substitution principle, thought that the consequence is valid in a standard syllogistic mood with the help of a suppressed premise. But almost all other logicians thought that something like the rule given by Ockham suffices for showing the validity. The reference to the relation between a superior and an inferior term given in the rule was understood in terms of the criterion on conceptual containment. It seems that in twentieth-century terms, the rules of this type could be characterized as regulating analytic validity.

These types of rules already bring us close to Aristotle's program in the dialectical topics presented in the *Topics*. This work was indeed much used in compilations of the listing of the rules for consequences. Also, in many works the lists contain rules that have more of the character of the theory of argumentation than of formal logic. The rules for consequences are indeed one of the places where the differences between modern and medieval conceptions of logic are most clearly visible.

6.5. Obligations

The genre of late medieval logical literature that has perhaps been the most surprising for modern commentators carries the title obligations (*obligationes*). The duties or obligations at issue in the treatises carrying this title were of a rather special kind. The basic idea was based on the Socratic question/answer game as described and regulated by Aristotle in the *Topics*. In the specific medieval variant of the game the opponent put forward propositions that had

to be granted, denied, doubted, or distinguished by the respondent. In giving his answers, the respondent was expected to pay recognition to the truth, but especially to some special obligation given to him in the beginning of the exchange by the opponent. This duty was understood to override the general duty of following the truth, but not the general logical duty of respecting arguments and avoiding contradictions.

Here we cannot go into details concerning the different variants of the system, although a number of interesting logical issues arose through the study of the particular kinds of possible duties. The main type of an obligational disputation, as medieval authors knew it, was based on a *positum*, a sentence put forward by the opponent in the beginning as something that the respondent has to grant. This sentence was typically false, and often even impossible in some way not directly implying a contradiction (conceptually impossible, naturally impossible, etc.). Then the opponent put forward further propositions, and in answering them the respondent had to pay attention to inferential relations between the *positum* and these later *proposita*. Altogether four main alternative sets of exact rules of how the inferential connections ought to be recognized were developed in the Middle Ages.

According to one late thirteenth-century system, described by the Parisian logician Boethius of Dacia in his commentary on Aristotle's *Topics*, the respondent must grant everything that the opponent puts forward after the *positum*, with the sole exception of propositions that are inconsistent (*incompossibile*) with the *positum* or the set of *posita*, if there are several. Boethius divides propositions into "relevant" and "irrelevant" ones with the criterion of an inferential connection to the *positum*. Those inconsistent with the *positum* are called repugnant (*repugnans*), and those following from it are called sequent (*sequens*). The repugnant ones must be denied and the sequent ones must be granted. Others are irrelevant, and Boethius claims that the respondent must grant them, since this implies nothing for the *positum*.

In his discussion, Boethius relied on an already traditional terminology, but not all of the earlier authors would have agreed with his rules. The early fourteenth-century discussion took place mainly in England, and there a different set of rules came to be accepted as the traditional system. According to these rules, the respondent should of course grant the *positum* and anything following from it. Similarly, he should deny repugnant propositions. But he should grant true irrelevant propositions and deny false ones. After having granted or denied such propositions, he should take them into account in the reasoning. He should grant anything that follows from the *positum* together with propositions that have been granted earlier or whose negations have been denied. Thus, the respondent must keep the whole set of his answers consistent, but otherwise follow the truth.

Duns Scotus claimed that in an obligational disputation based on a false *positum* one need not deny the present instant, but one can understand the counterfactual possibility at issue in respect to the present instant. (Unlike many of his predecessors, Scotus denied the principle "what is, is necessary,

when it is.") After Scotus, it became customary to think of the set of answers after the disputation as a description of some consistently describable situation. This brought obligational disputations close to counterfactual reasoning and thought experiments.

Richard Kilvington suggested in his sophismata an interesting revision of the rules apparently based on the idea that the disputation ought to describe the situation that would obtain if the false *positum* were true. He claimed that this principle ought to be taken as the rule guiding answers, giving the respondent a duty to grant what would be true and deny what would be false if the *positum* was true.

Kilvington's suggestion did not gain many followers. Most authors kept to the traditional rules, probably because Kilvington's rules seemed too vague. Formally valid inferential connections were taken to provide a better foundation for obligational disputations. But another revision was also suggested, and for some time it gained more followers. Roger Swineshed suggested that all answers ought to be decidable solely on the basis of the *positum* without recognition of any subsequent exchange. Swineshed's suggestion was that the respondent ought to grant the *positum* and anything following from it, and deny anything repugnant with it. Other propositions were to be taken as irrelevant, and they were not to be respected in the reasoning. This had the implication that irrelevant propositions would have to be kept separate from the mainline of the disputation, as a kind of second column in the bookkeeping. As Swineshed explicitly recognized, contradictions between the two columns could arise so that, for example, a conjunction may be denied when one of its conjuncts is granted as the *positum* and the other is granted as true and irrelevant.

The main logical topic studied in obligational disputations was logical coherence. The disputations were in essence structures allowing propositions to be collected together into a set, with evaluation of the coherence of the set as the crucial issue at each step. The different rules formulated the alternative exact structures for such a procedure.

6.6. Insolubles

Early treatises on obligations are often connected with treatises carrying the title "insolubles" (*insolubilia*). In these treatises something is laid down in a way similar to how the obligational *positum* is laid down, but the crux of the discussion is that the given propositions appear to describe a possible situation and yet they entail a contradiction. The case is thus paradoxical. As a common example from the obligational treatises themselves, we may mention the rule that the respondent ought not accept "the *positum* is false" as his *positum*. The case is, of course, closely analogous to what is nowadays known as the Liar Paradox.

It is not clear that obligational disputations were the original context of the genre of logic that came to be called *insolubilia*, since the first treatments of such paradoxes in their own right seem to be equally early and have other

sources, too. But the way medieval logicians formulated their versions of the Liar Paradox comes with an obligational terminology and context.

If we turn to the mature treatises of the early fourteenth century, the paradigmatic insoluble is the proposition "Socrates is saying what is false," and the assumed situation is that Socrates utters this and only this proposition. Then it is shown that if the sentence is true it is false (because if it is true, what it signifies is the case), and if it is false it is true (because it signifies that it is false, and that was assumed to be the case). Because these results cannot stand together—every proposition is true or false but not both—a contradiction seems to follow from what is clearly possible, for the only assumption seems to be that Socrates makes a simple understandable claim.

Medieval logicians discussed a wide variety of carefully formulated analogous paradoxes, and it seems that some of them were formed to counter specific purported answers to the paradox. For example, if the paradox is claimed to result from direct self-reference, we may be asked to consider other examples. For example, medieval logicians considered cases where two or more people make assertions about the truth or falsity of each other's claims and thus produce a paradoxical circle. A paradox reminiscent of the Liar Paradox can be produced without any proposition referring to itself—the paradox is not dependent on direct self-reference. It is also interesting to note that some practical analogs of the paradox were considered. Assume, for instance, like Buridan, that Plato is guarding a bridge when Socrates wants to cross it. Then Plato says, "If you utter something false I will throw you into the river, and if you utter something true I will let you go." Socrates replies, "You will throw me into the river." Now, what should Plato do? Cervantes makes Sancho Panza face a similar problem when he is the fake governor of an island, and indeed, Cervantes probably got the paradox from some medieval treatment of logic.

The variety and the history of the different solutions of the insolubles is too wide and complicated to be even summarized here. Some main alternative solutions presented in the medieval discussion must suffice for now.

In the early discussions, the so-called nullifiers (*cassantes*) claimed that the one who utters a paradoxical sentence "says nothing." If Socrates says only the sentence "Socrates says what is false," he has not really uttered a proposition at all, and thus no truth value is needed. The problem, of course, is to explain precisely why the utterance fails to be a proposition. Some authors gave the reason that a part, like "false," cannot refer to its whole; but this thesis is too generalized.

In his *Sophistical Refutations*, Aristotle mentions the case where somebody says something that is simultaneously both true and false. This remark occurs in connection to the fallacy of confusing truth in a certain respect and absolute truth (*secundum quid* and *simpliciter*). Thus applications of this fallacy were often tried in solving insolubles, but understandably the results were not very convincing. One related suggestion was that insolubles were to be treated not as cases of genuine self-reference but instead as cases where a certain shift of reference (*transcasus*) takes place. When Socrates says that he is lying, he

simply cannot mean that very utterance itself, and therefore we must look for some other utterance in the immediate vicinity. In the assumed case, this approach makes the insoluble false simply because there is nothing else that Socrates says.

Fourteenth-century logicians found all these suggestions too simple-minded. In the early 1320s, Thomas Bradwardine used symbolic letters for propositions and assumed that every proposition a signifies, in addition to its ordinary signification, even "a is true." (Strictly speaking, this was formulated as a general doctrine only later.) Substituting $a =$ "a is false," we get "a is false and a is true," a contradiction that shows that a is false. A similar strategy is further refined by William Heytesbury (1335). He puts the issue within the framework of obligation theory, discussing cases where insolubles are pressed on the respondent. All insolubles turn out to be false, but he admits that there is no general solution; what is needed is a careful study of what exactly is extraordinary in the signification of each relevant sentence.

Some authors, like Swineshed round 1330, argued that an insoluble proposition "falsifies itself." This requires a new opinion about truth: For the truth of a proposition, it does not suffice that it signifies what is the case, but it also must not falsify itself. This fundamental novelty may have been one reason why the theory was not generally accepted—and, moreover, its applications soon lead to obscurities.

Later, Gregory of Rimini and Peter of Ailly tried to utilize the doctrine of mental language in this context. The complex theory that Peter developed (in the 1370s) argues that spoken insolubles correspond to two conflicting mental propositions, whereas a mental proposition cannot ever be insoluble. This idea became well known but did not gain general acceptance.

To sum up, we may say that the common view was that certain propositions were called insoluble not because of logical puzzles that could not be solved but because providing a solution "is difficult," as many authors remark. It was generally agreed that insolubles were false. Only a few authors took seriously the possibility that the paradox might be a genuine one, one that did not allow any satisfactory solution. But even they did not think of insolubles as a threat to the system of logic as a whole. Insolubles were not considered to undermine the foundations of logic but simply to be one interesting branch of logical studies. One might surmise that this can derive from the idea of looking at logic as an art dealing with the rational structures embedded in the mental basis of ordinary language, rather than as a calculating system based on special foundations.

6.7. Sophismata

Buridan's *Summulae de Dialectica* concludes with an almost 200-page section containing sophisms (*sophismata*), which are examples construed in a rather distinct way so that they make the need of logical distinction clearly visible. Buridan's work is no exception; different kinds of collections of such sophisms

are commonly found in medieval logic manuscripts. It seems that they were used in medieval logic teaching as exercises to show how general logical systems could be applied in practical contexts. But often they also contain interesting material that is not discussed in systematic treatises.

Separate collections of sophisms circulated throughout the Middle Ages. Perhaps the most famous of the early examples of such aids of teaching was known as the *magister abstractionum*. Little is known of the person, and he may not have been a single person. It is possible that we simply have a collection of examples which circulated among teachers of logic, who would each add their own examples and drop out others. Later, many authors of logical textbooks compiled their own collections of sophisms. This is what we find for example in the case of Buridan.

In early fourteenth-century Oxford, such a textual genre gained new significance by assuming a relatively specific independent role not only in the university curriculum (where undergraduate students in their first years of university were called "sophists" [*sophistae*]) but also in logical study. The collections of sophisms composed by Heytesbury, Kilvington and some other members of the so-called group of Oxford calculators were an important locus of logicolinguistic and mathematical study providing important results that were later used by pioneers of early modern science.

A sophism in this sense of the word consists of (1) the *sophisma sentence*; (2) a *casus*, or a description of an assumed situation against which the sophisma sentence is evaluated; (3) a *proof* and a *disproof* of the sophisma sentence based on the casus; and (4) a resolution of the sophism telling how the sophisma sentence ought to be evaluated and how the arguments to the contrary should be countered.

In the discussion of sophisma 47 in his collection, Kilvington assumes that the procedure in solving a sophisma must abide with the rules of obligational disputations. That is, the casus is to be understood as having been posited in the obligational sense, and thus anything following it would have to be granted and anything repugnant to it would have to be denied. From this viewpoint, the proof and disproof can be articulated as obligational disputations. Although an explicit commitment to using obligational rules such as Kilvington's is rare in the collections of sophisms in general, obligational terminology is omnipresent.

In many sophisms, the problematic issue was to show how the sophisma sentence was to be exactly understood. For this reason, sophismata became an especially suitable place for determining exact rules of scope and the interpretation of words serving important logical roles. Indeed, this is the context where late medieval logicians developed the exactitude in regulating logical Latin that was ridiculed by such Renaissance humanists as Juan Luis Vives.

Heytesbury's *Rules for Solving Sophisms* (1335) is a good representative of the genre and can thus be used as an example here. It consists of six chapters. The first is on a topic we have already mentioned, so-called insolubles. The second discusses problems of epistemic logic with sophisms based on the words

"to know" and "to doubt." The third tackles problems connected to the use of pronouns and their reference. In the remaining three parts, Heytesbury turns to problems that may be better characterized as natural philosophy rather than logical analysis of language. The fourth part considers a traditional topic, the verbs "to begin" and "to cease," and thereby issues connected to limit decision problems and temporal instants. The fifth part, on maxima and minima, continues on the same tract from a different viewpoint. The sixth and final part is dedicated to "three categories," referring to the Aristotelian categories of place, quantity, and quality. Especially this last part and its discussions of speed and acceleration proved very fruitful in the early development of modern science despite the fact that all the cases studied in it are purely imagined and lack any sense of experiment. For example, instead of real bodies in motion, medieval logicians considered imagined bodies in motion. In fact, this chapter and others of its kind show how the medieval *secundum imaginationem* method, relying only on logicolinguistic analysis, was able to provide results that have often been misguidedly attributed to experimental scientists working centuries later.

One of the specific techniques used in solving sophisms deserves treatment of its own in a history of logic. In early thirteenth-century texts, a sentence like "Socrates begins to be pale" was analyzed as something like "Socrates was not pale and Socrates will be pale." The analysis was accompanied with a discussion on which of the two conjuncts in the particular kind of change at issue should be given in the present tense, and how one should formulate the continuity requirement that Socrates, say, will be pale immediately after the present instant, even before any given determinate future instant. Such an analysis became a standard technique used in a large variety of cases and was called "exposition" (*expositio*). Without going into the particulars of the specific verb "to begin," it is worth pointing out here that the idea in such an analysis is to break down the sentence containing the problematic "exponible term" into a conjunction or a disjunction that is equivalent in its truth conditions. For fourteenth-century logicians, it was a commonly accepted doctrine that there is a large number of terms that admit, or in the contexts of a sophism, demand such an analysis. Furthermore, this kind of analysis was taken to be necessary for practically all philosophically central terms if there was a need to treat them in a logically exact manner.

7. The End of the Middle Ages

7.1. Later University Logic

Undoubtedly, the main plot of medieval Aristotelian logic lies in the development that began from the early terminists and led to the stage of Burley and Ockham, and then had its academic culmination in the systematic work of Buridan. But the discipline of logic survived after that, and some new special features appeared and new developments took place in the late Middle Ages.

It is probably true to say that logicians were no longer very original during this time. But here it is necessary to emphasize that logic was a widespread and multiform discipline; the volume of material is very great, and much of it is still unexamined.

A gradual change happened in philosophy in general during the fourteenth century, a change whose background is hard to explain. It has been pointed out that the whole cultural climate was no longer the same: The fourteenth century included great political upheavals; the Church had difficulties that led to the great schism; various protest movements appeared, and so on. All this contributed to the loss of the previous unity. It is customary to start the "autumn of the Middle Ages" from 1350, but this demarcation is largely symbolic; the only concrete thing that can support it is the Black Death, which killed many philosophers in 1349. After 1350, philosophy was still practiced in the old style, and logic has hardly ever been as prominent a part in philosophy as in the latter half of the fourteenth century. However, the overall authority of philosophy and logic started to diminish.

Let us try to sketch an overview of the historical development. Ockham, a political dissident, had never made an uncontested breakthrough—in fact, he was considered an extremist even among nominalists. In logic, however, his thought had a wide influence. Buridan, then, had more indisputable prestige, and as regards logic, his influence became dominant in Parisian philosophy during the 1340s. In this field he had two extremely competent pupils, Albert of Saxony (d. 1390) and Marsilius of Inghen (d. 1396). After the generation of Buridan's students, the position of Paris weakened, although it was still the most famous university.

England underwent a quite distinctive process. In the beginning of the fourteenth century the best logicians were English, and even after them there were original figures in Oxford, like Bradwardine, Heytesbury, and Billingham. Then, after 1350, logic turned to great technical sophistication but little essentially original appeared in the works of logicians such as Hopton, Lavenham, Strode, Feribrigge, and Huntman. Soon after 1400, a complete collapse took place in England, and only some elementary texts were produced during the fifteenth century.

But English logic was, however, very influential in the late Middle Ages on the Continent. English works of the fourteenth century were studied and commented on in Italy. Particularly Ralph Strode's logic achieved great fame. Paul of Venice had studied at Oxford, and he transmitted the comprehensive English tradition to the Italian logicians of the fifteenth century: Paul of Pergula, Gaetano of Thiene, and others.

Moreover, the fifteenth century is the era of the triumph of the university, which also involved a geographical expansion of philosophical studies. Hence we meet a number of new active centers of logic emerging in Central Europe, in universities like Prague, Cracow, and Erfurt.

A typical feature in fifteenth-century philosophy is a conscious turn toward old masters. Thus, philosophy formed into competing schools with their own

clear-cut doctrines; this process was promoted by the commitment of religious orders to their official authorities and by the allotment of chairs in philosophy. These Thomist, neo-Albertist, Scotist, and nominalist currents were not very innovative in logic, though some of their leaders were first-rate logicians (like the Scotist Tartaretus).

The form of logical works changed gradually. Instead of voluminous commentaries, two other types of work became popular: shorter discussions of individual subjects, and more general *summulae* expositions. A far-reaching step was the innovation of printing, which led to the promotion of textbooks in particular. (The first printed logical book was the *Logica parva* by Paul of Venice, in 1472.) On the whole, we can say that logic was no longer very creative; there were few original results, and perhaps they were not even actively pursued. We can feel some signs of the later sentiment that the science of logic had already been completed. In statements like this, we must remember, though, that there has been particularly little historical research on fifteenth-century logic.

Attention was often concentrated on earlier results; thus there was much interest in all kinds of special cases and counterexamples, which we cannot discuss here. Generalizing crudely, we might say that the exponibilia, the sophismata, and the insolubilia became especially popular themes, whereas the fundamental questions of terms, propositions, and inferences were less debated. Modal logic seems to disappear, though it has a surprising revival at the end of the fifteenth century (with Erfurtians like Trutvetter). At the same time there is also a revival of philosophical logic in Paris (e.g., Scotsmen around John Mair).

The strictly formal part of older logic, such as syllogistics, was still taught everywhere, and occasionally even cultivated in so far as there was an opportunity to develop it. A famous example is the innovation of the so-called *pons asinorum*. Fifteenth-century authors formulated clearly this virtually mechanical method for finding a suitable minor premise by means of which a given conclusion can be syllogistically inferred from a given major.

In less formal matters, we encounter an interesting line by examining the widely read *Speculum puerorum* (1350s) by Richard Billingham. He discusses the *probatio*, literally "proof" but also meaning "trial," of propositions and concludes that it is only possible by a further *probatio* of its terms. "Immediate" terms are simple, but others can be submitted to some of the three forms of such a treatment. First, "exponible" terms can be replaced by several occurrences of simpler terms in a conjunction of simpler propositions which is equivalent to the original one. Thus "only a man" is exponible in the proposition "only a man runs," and its exposition leads to the equivalent "a man runs and nothing but a man runs." "Resoluble" terms are replaceable by simple terms, leading (not to equivalents but) to truth grounds; thus "a man" is resoluble to "this" since "a man runs" has a truth ground "this runs and this is a man." And for "official" terms, it can be shown that they hold an office together with *dicta*, like the modal and attitudinal operators do. By means of the *probatio* of terms,

the proposition ought to acquire a logically elementary form, and problems arising from difficult constructions can then be handled. Finally Billingham gives some grammatical rules for advancing without error in the *probatio*.

Similar ideas can be found in a number of later English and Italian authors who discuss such basically non-Aristotelian themes. The attention turned to *logical grammar*. Logic courses often started from *logica vetus*, continued with material from the Aristotelian *Prior* and *Posterior Analytics*, and then concentrated on the new themes. This feature can be seen as a mark of a shift from logic in the strict sense toward *conceptual analysis* of logically difficult items: problematic concepts, ambiguous linguistic constructions, and so on. Accordingly, much attention was awarded to questions of grammatical deep structure and its accurate expression by means of variants in lexical forms and word order. The serious nature of these problems can now be appreciated again, in the light of present-day grammatical theory, but it is of course true that fifteenth-century authors did not have a sufficient technical apparatus for mastering their Latin sentences. It is also easy to understand that these undertakings seemed useless and annoying to many critics.

7.2. Reactions

It is common to speak about "medieval logic," and one easily thinks of it as a monolithic totality. Perhaps we have managed to say that the truth is much more complex. But all the authors we have discussed so far had a solid Aristotelian background. There were, however, even other tendencies, which started to grow during the fifteenth century. We might prepare the way for the novelties by mentioning earlier dissidents.

The Aristotelian methodology in science was rather restrictive, and for a long time repeated attempts had been made to find a place for something more innovative. Bacon is perhaps the most famous among these authors: He showed great curiosity in matters of empirical science and made initiatives in the philosophy of science. But his logic seems to follow well-known Aristotelian lines. A much more perplexing case is Raimundus Lullus (Ramón Llull, c. 1235–1316). Having no academic training, he did not care about *logica moderna*; instead, he sought to create an original way of argumentation that would undeniably prove Christian dogmas to infidels. This so-called *Ars magna*, to which he gave several formulations, uses various basically neo-Platonic sources. As basic concepts, he chooses some central divine attributes and cross-tabulates them with certain logicometaphysical aspects. This ought to produce, in the way of multiplication tables, a scheme of interesting manifestations. Lullus also suggested that concepts should be written on concentric circles and arguments performed by rotation of the circles. In fact, Lullus never achieved any logical results, and his program rests heavily on theological premises. But he introduced the idea of purely combinatorial procedure (with symbolic letters), and this was something that fascinated many later authors. "Lullists" reappeared during the fifteenth century, and even Leibniz was interested in Lullus.

Late medieval university logic acquired a respected opponent when Italian humanists began to propagate their new ideals. In the middle of the fourteenth century, Petrarch had violently attacked scholasticism and particularly logic, making it clear that professional logic was a corrupt and useless discipline that could not benefit a literary civilization. His leading followers, such as Bruni and Bracciolini, were more detailed in their criticisms. According to them, what is sensible in logic is delivered through the studies of language and dialectics, whereas university logic is mostly incomprehensible sophistry. They also pointed out, correctly, that medieval logic consisted of additions made by barbarians to the classical heritage.

The early humanists mainly expressed nothing but their discontent, but a more substantial alternative logic was developed by the famous philologist Lorenzo Valla (1407–1457) in his *Dialecticae disputationes*. He argued that a lot of the scholastic problems were actually illusory and resulted from obscure and abstract misinterpretation of questions that were essentially linguistic. Valla admitted that a small kernel of elementary logic was needed, as ancient Romans had already admitted, but for him formal validity was not as interesting as the informal convincing power of arguments. Thus he focused on the dialectical theory of reasoning and discussion, emphasizing matters of grammar and style. His work anticipates the revival of topics in a new form.

A similar nonscholastic development was continued by many other authors. Gradually the humanist influence extended outside Italy to the whole of Europe, and there grew a conscious effort to form a simple logic free of tradition. In this process, the new logic also found a place in the academic environment and much logical literature turned to dialectical issues, new ancient sources became known, and logic definitely entered the era of printed books. All this amounts to a basic transformation, and the next part can well start with it.

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3

Logic and Philosophy of Logic from Humanism to Kant

Mirella Capozzi and Gino Roncaglia

1. Humanist Criticisms of Scholastic Logic

The first impression of a reader who "crosses the border" between medieval and Renaissance logic may be that of leaving an explored and organized field for a relatively unexplored and much less ordered one. This impression is emphasized by the fact that while in the medieval period we can assume, despite relevant theoretical differences, some consensus about the nature and purpose of logic, such an assumption cannot be made with reference to the postmedieval and Renaissance period: The many "logics" coexisting and challenging each other were often characterized by deeply divergent assumptions, articulations, and purposes. As far as logic is concerned, we could almost be tempted to use this "explosion of entropy" as the very marker of the shift between the medieval and the Renaissance period.

The development of humanism, with its criticism of the late medieval logical tradition, is not the only factor contributing to this situation, but surely is a relevant one. Excessive and artificial subtlety, lack of practical utility, barbarous use of Latin: These are the main charges that humanist dialecticians made against scholastic logic. Such charges do not simply point out formal deficiencies that could be eliminated within a common logical framework, but call for a change of the logical paradigm itself. The effort to address such charges had a deep influence on the evolution of logic and resulted in a variety of solutions, many of which were based on contaminations between selected but traditional logical theories, on the one hand, and mainly rhetorical or

Though we decided on the general structure of this chapter together, sections 1–4 and 8 are by Gino Roncaglia, while sections 5–7 and 9–11 are by Mirella Capozzi.

pedagogical doctrines on the other. But the charges themselves were initially made outside the field of logic: One of the very first invectives against scholastic logic came from Francesco Petrarca (1304–1374), hardly to be considered a logician (Petrarca 1933–42, I, 7).

The central point at issue is the role of language. The late medieval scholastic tradition used language as a logical tool for argumentation, and favored the development of what J. Murdoch (1974) aptly called "analytical languages": highly specialized collections of terms and rules which—once applied to specific and definite sets of problems—should help guarantee the formal precision of reasoning. In this tradition, the use of a simplified and partly artificial Latin could help the construction of sophisticated formal arguments. The humanists, on the contrary, privileged the mastery of classical Latin. For them, language—together with a few simple and "natural" arguments taken from ancient rhetoric—was a tool for an effective and well-organized social and pedagogical communication.

Besides the different theoretical standpoints, there is a social and cultural gap between two different intellectual figures. Scholastic-oriented teachers are usually university professors who tend to consider logic, philosophy, and theology as specialized fields. For them, knowledge is reached through a self-absorbing (and largely self-sufficient) intellectual activity, whose formal correctness is regulated by logic. Many humanist dialecticians, on the contrary, do not belong to and do not address themselves to the academic world: They consider logic a tool to be used whenever language is used with rhetorical or practical purposes, and regard a broad "classical" culture more important than a specialized and abstract one (see Jardine 1982, 1988).

One should be careful, however, in assessing the reasons for the privilege humanists accorded to rhetoric. For the humanists, logic—or rather dialectic, to use the term that, already present in the Ciceronian tradition and in the Middle Ages (see Maierù 1993), was preferred by most humanist and Renaissance authors—*has* to do with the use of arguments. But to be practically effective, such arguments have to be natural, aptly chosen, easily stated and grasped, expressed in good, classical Latin. And they don't need to be demonstrative arguments: Probable arguments are also included within the scope of dialectic.

One should also be careful in considering humanism as a monolithic movement aimed at banishing all reminiscence of medieval logic. Humanism is not chronologically subsequent to scholasticism, and many humanists knew late scholastic logical texts fairly well, such as those by Paul of Venice. Some even praised them (Vasoli 1968, 20–23; Perreiah 1982, 3–22; Mack 1993, 14–15). Nevertheless, formally correct and truth-preserving arguments were considered as only some of the tools available to a good dialectician. The latter's aim is to master the art of using language (*ars bene disserendi*), the Ciceronian *disserendi diligens ratio*, and this requires not only demonstrative skills but also the ability to persuade, to construct probable arguments, to obtain consensus.

The definition of dialectic provided by Rudolph Agricola (1444–1485)—one of the many Renaissance variations on Cicero's own—is representative of this

point of view. According to it, dialectic is the "ars probabiliter de qualibet re proposita disserendi" (art of speaking in a probable way about any proposed subject). The explication of "probabiliter" clarifies the broad scope of the term (see Mack 1993, 169–173, where "probabiliter" is translated as "convincingly"): "probable (*probabile*) in speaking is not only what is actually probable, that is, as Aristotle states, what is accepted by all, or by the most part, or by the learned. For us, probable is what can be said about the proposed subject in an apt and adequate way" (Agricola 1967, 192). This meaning of the concept is broad enough to include good old-fashioned demonstrative arguments in the field of dialectic (Risse 1964–70, I, 17–18), but they are no longer the only kind of arguments a dialectician should take into account.

A first introduction to sources, principles, and precepts of humanist-oriented logic is provided by the works of the prominent humanist dialectician Lorenzo Valla (1407–1457), who, significantly, received his cultural training mostly within the humanist circles of the papal *curia*. While some of the earlier humanists were content with a dismissal of scholastic logic—Petrarca's and Bruni's invectives against the *barbari britanni* being the most often quoted testimony of this attitude (Garin 1960, 181–195; Vasoli 1974, 142–154)—in his *Repastinatio dialecticae et philosophiae* (Valla 1982), Valla added to heavy criticism of traditional logical doctrines a complete and systematic reassessment of the nature and purpose of dialectic from a humanistic point of view.

According to Valla, dialectic deals with demonstrative arguments, while rhetoric deals with every kind of argument—demonstrative as well as plausible ones. Therefore dialectic is to be considered as a part of rhetoric, and rhetoric has to provide the widest spectrum of argumentative tools to all branches of learning. Moreover, dialectic should be simple and disregard all the questions that, though discussed by logicians with technical logical tools, actually pertain to Latin grammar. During the Middle Ages the relation between logic and grammar had been closely investigated by the so-called modist logicians. They worked at a sophisticated *speculative grammar*, based on an ontologically grounded correspondence between ways of being, ways of thinking, and ways of signifying. Valla's grammar, on the contrary, is based on the Latin of classical authors, and therefore on a historically determined *consuetudo* in the use of language. Valla thus carries out what has been described as a "deontologization" of language (Camporeale 1986; Waswo 1999).

Valla devotes the first of the three books of his *Repastinatio* to the foundations of dialectic and to a discussion of the Aristotelian doctrine of the categories. Here, too, Valla applies his general rule: simplification through reference to concrete uses of Latin, rather than to an abstract metaphysical system. The 10 Aristotelian categories are thus reduced to 3—substance, quality, and action—and examples are given to show how the remaining categories can be reduced to quality and action. Similarly, the transcendental terms, which according to the medieval tradition "transcend" the division among the 10 categories and are reciprocally convertible, are reduced to the only term "res." The reason why Valla prefers the term "res" to the traditional

"ens" is that "ens" in classical Latin is not a noun but a participle that can be exposed as "that thing (*res*) which is." Therefore the term "res" is the true fundamental one. This example shows how Valla explains problematic terms or sentences by offering a reformulation considered more precise and easier to analyze. The practice of explanation through reformulation was familiar to medieval logicians under the name of *expositio*, but Valla uses *expositio* to reach linguistic, rather than logical clarification.

Valla's second book is devoted to proposition and addresses the question whether all propositions should be reduced to the basic tripartite form: subject–copula–predicate ("*A* est *B*"). This question was the object of a long debate, continued during the whole period we are dealing with (Roncaglia 1996), and had usually been investigated under the assumption that it was the *logical* structure of the proposition at issue. Valla, on the contrary, perceives the problem as related to the *grammatical* structure of the proposition, and accordingly offers a negative answer, since in the use of Latin the construction "est + participle" (*Plato est legens*) is not equivalent to the use of an indicative form of the verb (*Plato legit*). The Spanish humanist Juan Luis Vives (1492–1540) will share the same attitude (see Ashworth 1982, 70).

To support his contention, Valla considers propositions like *Luna illuminatur*, which—in Latin—can be transformed into a tripartite form only through a shift in meaning. A further argument is drawn from the idea that the participle form of the verb may be seen as somehow derivative with respect to the indicative form. Therefore—if something is to be reduced at all—it should be a participle like *legens*, to be reduced to *qui legit* (Valla 1982, 180). Logicians should not superimpose their logical analysis to the "good" use of language, but should rather learn from it. Language should be studied, described, and taught, rather than "corrected" from an external point of view.

Valla did not consider the study of modal propositions as pertaining to logic (hence his complete refusal of modal syllogistic). This refusal—common to most humanistic-influenced Renaissance philosophy—is once again defended on linguistic rather than purely logical grounds. Why should we attribute to terms like "possible" and "necessary" a different status from that of grammatically similar terms like "easy," "certain," "usual," "useful," and so on? (Valla 1982, 238; see Mack 1993, 90; Roncaglia 1996, 191–192.)

Valla's third book, devoted to argumentation, preserves the basic features of Aristotelian syllogistic, but dismisses the third figure and, as already noted, modal syllogisms. Owing to his desire to acknowledge not only demonstrative but also persuasive arguments, Valla pays great attention to hypothetical and imperfect syllogisms and to such nonsyllogistic forms of argument as *exemplum* and enthymemes.

The final section of Valla's work is devoted to sophistic argumentations. Medieval discussions of sophisms allowed logicians to construct interesting, complex, and borderline situations to test the applicability and the effectiveness of their logical and conceptual tools. Valla is fascinated by the persuasive and literary strength of "classical" problematic arguments, such as the *sorites* (a

speech proceeding through small and apparently unavoidable steps from what seems an obvious truth to a problematic conclusion) or the *dilemma*, in which all the alternatives in a given situation are considered, only to show that each of them is problematic. Valla does distinguish "good" and "bad" uses of these kinds of "arguments," but his criterion is basically that of practical usefulness in persuasive rhetoric.

Valla's *Repastinatio* is also a typical example of the importance humanists assigned to the "invention" (*inventio*) of arguments, connected with topics. Renaissance dialecticians considered Aristotle's *Topica* as a systematic treatment of practical reasoning, and complemented it with Cicero's *Topica* and with the treatment of topics included in Quintilian's *Institutio oratoria*, which—rediscovered in 1416—had become one of the most popular textbooks on rhetoric by the end of the century, while Boethius's *De differentiis topicis*, widely used in the Middle Ages (Green-Pedersen 1984), had only few Renaissance editions (Mack 1993, 135). Both Cicero's and Quintilian's treatment of topics helped shift the focus from "formal" disputations to rhetorical and persuasive ones.

The most complete and influential Renaissance study of topics is contained in Agricola's *De inventione dialectica* (Agricola 1967, 1992). Agricola grounds his conception of topics on his realist conception of universals (Braakhuis 1988). In his opinion, things are connected by relations of agreement and disagreement, and topics are orderly collections of common marks, which help us organize and label relations, and find out what can or cannot be said about a given thing in an appropriate way. While being systematically arranged, topics, according to Agricola, are not a closed system: The very possibility of viewing things from different angles and perspectives, of relating them in new ways, not only enables us to draw or invent arguments but also allows us to find new common marks.

We have already considered Agricola's definition of dialectic. In his opinion, topics are the method of dialectical invention, while the discourse (*oratio*) is its context. There are, however, two different kinds of dialectical discourse: exposition (*expositio*) and argumentation (*argumentatio*). The former explains and clarifies, and is used when the audience doesn't need to be convinced, but only enabled to understand what it is said. The latter aims at "winning" assent, that is, at persuading. Although argumentation is connected with disputation, necessary arguments are not the only way to win a disputation: Plausible and even emotionally moving arguments should be considered as well. Agricola's concept of argumentation is thus connected with rhetoric, a connection strengthened by the fact that both use natural language. This explains why Agricola has no use for the kind of highly formalized, analytical language used by medieval and late medieval logic.

However negative Valla's and Agricola's attitude toward the logical tradition, it was never as negative as that of Petrus Ramus (Pierre de La Ramée, 1515–1572). According to his biographer Freigius, Ramus's doctoral dissertation (1536) defended the thesis: "everything that Aristotle said is misleading

(*commentitium*)." This does not imply—as many assumed—that Ramus considers all Aristotelian theories to be false: In his opinion, Aristotle is guilty of having artificially complicated and corrupted the simple and "natural" logic which Aristotle's predecessors—notably Plato—had devised before him (Risse 1964–70, I, 123–124). Scholastic logic is obviously seen by Ramus as a further step in the wrong direction.

Various versions of Ramus's logic (including the 1555 *Dialectique*, in French: Ramus 1996; for a survey of the different editions of his works and of the stages marking the complex development of Ramus's dialectic, see Bruyère 1984) were published between 1543 and 1573. After his conversion to Protestantism in 1561, his library was burned, and he had to flee from Paris. Ramus died on August 26, 1572, killed on the third day of the St. Bartholomew's massacre. His being one of the Huguenot martyrs undoubtedly boosted the fortune of his already popular works in Calvinist circles.

Ramus's concept of dialectic is based on three main principles: Dialectic should be *natural* (its foundations being the "eternal characters" which constitute, by God's decree, the very essence of our reasoning), it should be *simple* (it deals with the correct way of reasoning, but disregards metaphysical, semantic, and grammatical problems as well as unnecessary subtleties), and it should be *systematically organized*, mainly by means of dichotomic divisions. Therefore, Ramus's books extensively used diagrams, usually in the form of binary trees: A feature that may be connected—as argued by Ong (1958)—with the new graphical possibilities offered by printed books, and that will influence a huge number of sixteenth- and seventeenth-century logic textbooks, not only within the strict Ramist tradition.

The first and foremost division adopted by Ramus is Cicero's division between invention (*inventio*) and judgment (*iudicium* or *dispositio*). They are the first two sections of logic. A third section, devoted to the practical and pedagogical exercise of dialectic (*exercitatio*), is present in the first editions of Ramus's logical works but disappears after 1555.

The *inventio* deals with the ways arguments are to be found. Because arguments are to be found and classified by means of topics, according to Ramus, the treatment of topics should precede, rather than follow (like in Aristotle), that of judgment. Ramus's table of topics, organized by means of subsequent dichotomic divisions, is strongly influenced by Agricola and by Johannes Sturm (1507–1589), who taught dialectic and rhetoric in Paris between 1529 and 1537 and greatly contributed to the popularity of Agricola in France.

Ramus's treatment of judgment is also unconventional. While in traditional logic this section presupposes an extensive treatment of proposition, Ramus deals with this subject in a sketchy way and adds an independent (albeit short) section on the nature and structure of proposition only in the 1555 and successive editions of his work. In the last edition Ramus follows Cicero in using the term *axioma* to refer to a categorical proposition (having used earlier the term *enuntiatio* or *enuntiatum*), while he always gives the more specific meaning of major premise of a syllogism to the term *propositio*.

Syllogism and its various forms (including induction, example, and enthymeme) constitute the core of the "first judgment": the first of the three sections in which Ramus divides his treatment of judgment in the earlier editions of his dialectic. Ramus's explicit effort is that of simplifying Aristotelian syllogistic, but during the years between the 1543 edition of the *Dialecticae institutiones* and his death, his syllogistic underwent so many changes that it is impossible to give a faithful account of it in a few pages. Typical of Ramus's syllogistic is his use of the terms *propositio*, *assumptio*, and *complexio* to refer to the major premise, minor premise, and conclusion of a syllogism, and his tendency to favor a classification of syllogisms according to the quantity of the premises, considering as primary moods those with two universal premises. In the earlier editions of his dialectic, Ramus held that all moods with particular premises should be reduced to universal moods. He admitted some of them later on, but banned the *reductio ad impossibile* used to reduce second and third figure moods to the first figure. But Ramus's better known innovation in the field of syllogistic is the so-called Ramist moods: syllogisms in which both premises are singular, accepted on the ground that individuals could be seen as (lowest) species. The discussion about Ramist moods will keep logicians busy for most of the subsequent century.

The second section of Ramus's treatment of judgment (called "second judgment" in the earlier editions of his work) deals with the ways to connect and order arguments by means of general principles. Ramus attributes great importance to this "theory of method," which he further develops in the later editions of his logical works, and which in his opinion shapes the whole system of science (also offering the conceptual foundation for an extensive use of dichotomies). According to Ramus, the dialectical method (*methodus doctrinae*) goes from what is most general to what is most particular. This is done by means of divisions that, in turn, are drawn on the base of definitions expressing the essence of the concepts involved. Division and definition are thus the two main tools of method. The opposite route, going from particular instances to more general concepts (*methodus prudentiae*), might be used when either the lack of a more general conceptual framework or reasons of practical convenience force us to dwell on single or partial pieces of information. However, it cannot guarantee certainty; and is therefore mainly used in rhetorical discourse aiming at persuasion, rather than in demonstrative reasoning. In Ramus's opinion, however, the distinction between *methodus doctrinae* and *methodus prudentiae* does not imply that we have two methods: We have only one method—based on an ideal "knowledge space" organized by means of definitions and divisions—that, in given and concrete situations, also allows for tentative and partial bottom-up routes.

Thus conceived, the dialectical method is governed by three laws, which constitute the Ramist counterpart of the Aristotelian-Scholastic *de omni*, *per se* and *universaliter primum* principles. Ramus calls them the laws of *truth*, *justice* and *wisdom*: in the field of science every statement (i) should be valid in all its instances; (ii) should express a necessary (essential) connection of

the concepts involved; (iii) should be based on subject and predicate that are proper and proportionate (allowing for simple conversion).

Ramus's logic was very influential in the second half of the sixteenth and in the first half of the seventeenth century (Feingold, Freedman, and Rother 2001). However, "pure" Ramist scholars—mostly active in the Calvinist areas of Germany, in Switzerland, in Holland, and in England—were to face an almost immediate opposition not only in Catholic but also in Lutheran universities, and saw their influence decrease after the beginning of the seventeenth century. Much more influential (and more interesting) were the many "eclectic" logicians who either tried to reconcile Ramus's and Melanchthon's logical views (Philippo-Ramists) or introduced some Ramist themes within more traditional (and even Aristotelian) contexts.

2. The Evolution of the Scholastic Tradition and the Influence of Renaissance Aristotelianism

Despite humanist criticisms, the tradition of scholastic logic not only survived during the sixteenth and seventeenth centuries but evolved in ways that are much more interesting and articulated than most modern scholars suspected until a few decades ago. Our knowledge of this evolution is still somehow fragmentary, but the scholarly work completed in recent years allows some definite conclusions. We can now say that in this evolution of the late scholastic logical tradition, six factors were particularly relevant: (i) the work of a group of Spaniards who studied in Paris at the end of the fifteenth and at the beginning of the sixteenth century and later taught in Spanish universities, influencing the development of logic in the Iberian peninsula; (ii) a renewed attention toward metaphysics, present in the Iberian second scholasticism and most notably in the works of Francisco Suárez (1548–1617), whose *Disputationes Metaphysicae* (Suárez 1965) influenced many authors all across Europe; (iii) the crucial role of the newly formed (1540) Society of Jesus, whose curriculum of studies (*Ratio Studiorum*) was to shape institutional teaching in all of Catholic Europe; (iv) the complex relations with humanism, and the influence of logicians like Agricola, whose doctrines, while taking as their starting point a humanist conception of logic, were nevertheless susceptible of somehow being absorbed or integrated within a more traditional framework; (v) the "new Aristotelianism" of authors like Jacopo Zabarella (1533–1589) and Bartholomaeus Keckermann (1572?–1609); and (vi) the renewed interest in scholastic logic, discernible in reformed Europe (and most notably in Germany) as a consequence of the doctrinal and theological conflicts with the catholic field and within the reformed field itself. In the following pages, we provide some details on this complex development.

At the end of the fifteenth century and in the first decades of the sixteenth, the Paris college of Montaigu became a center of logical research in which the late medieval logical (especially nominalist) tradition survived and to

some extent flourished. A group of Spanish and Scottish logicians, lead by the Spaniard Jeronimo Pardo (d. 1505) and by the Scottish John Mair (1467/9–1550), debated themes such as the nature of supposition and signification, the distinction between categorematic and syncategorematic terms, the role of beings of reason (*entia rationis*), the nature of proposition (further developing the late medieval discussions on mental propositions), modality, and the theory of consequences. Somehow connected to this Paris group, or active there at the beginning of the sixteenth century, were the Spaniards Antonio Núñez Coronel (d. 1521), Fernando de Encinas (d. 1523), Luis Núñez Coronel (d. 1531), Juan de Celaya (1490–1558), Gaspar Lax (1487–1560), Juan Dolz (fl. 1510), the Frenchman Thomas Bricot (d. 1516), the Belgian Pierre Crockaert (Pierre of Brussels, d. 1514), and the Scot George Lokert (d. 1547).

Particularly interesting is their discussion about the nature of *complexe significabile* (propositional complex), a subject already debated by medieval logicians. The medieval defenders of this theory, associated with the name of Gregory of Rimini (c. 1300–1358), held that the object of science is not the proposition itself but what is signified by it (and determines its truth or falsity); such total and adequate meaning of the proposition is neither a physical nor a purely mental being and is not reducible to the meaning of its parts. It is rather similar to a state of affairs, which can be signified only by means of a complex (the proposition) and is therefore called *complexe significabile*. The discussion on the nature (and usefulness) of the *complexe significabile* was connected to the discussion on the role of the copula, since the copula was usually considered as the "formal" component of the proposition, "keeping together" subject and predicate. The copula was thus considered as a syncategorematic term: a term that does not possess an autonomous meaning but helps determine the meaning of the proposition as a whole. The defenders of a "strict" *complexe significabile* theory did not need a separate discussion of the mental copula, because in their opinion the *complexe significabile* is a unity and cannot be analyzed in terms of its parts. But many authors—among them John Buridan (c. 1295–1356)—assigned a much more relevant role to the copula, seen as the (syncategorematic) mental act that, in connecting subject and predicate, establishes the proposition. It is this very theory that was discussed by many of the above-mentioned late fifteenth- and early sixteenth-century Paris-based logicians (see Ashworth 1978, 1982; Muñoz Delgado 1970; Nuchelmans 1980; Pérez-Ilzarbe 1999). Pardo's position in this discussion was the most original. In his opinion, the copula is not purely syncategorematic: It is subordinate to a conceptual schema that represents something (i.e., the subject) as related in a certain way to something else (i.e., the predicate) or to itself (Nuchelmans 1980, 49). In this way the copula, while retaining its formal function, also signifies something (*aliquid*), that is, the subject, as considered in a given way (*aliqualiter*), namely as modified by the relation with its predicate. The idea of the copula signifying *aliquid aliqualiter*, and not simply *aliqualiter*, and the special relevance attributed to the subject in determining the meaning of

the copula and of the proposition as a whole, were discussed, and generally criticized, by Pardo's successors. They especially investigated the role of impossible propositions, as well as propositions with a negative, privative, or impossible subject, and the problem of whether a quasi-syncategorematic nature could be attributed to the proposition as a whole.

The Iberian Peninsula was one of the strongholds of Catholicism. Moreover, as we have seen, it inherited many features (as well as textbooks and Paris-trained professors) from Parisian late scholasticism. This made the influence of the humanist movement—albeit discernible—less radical than elsewhere. Therefore, the Iberian Peninsula was the ideal context in which Catholic logicians—dwelling on the scholastic (chiefly Thomist) logical and philosophical tradition—could pursue the work of doctrinal and pedagogical systematization that was required by the struggle against the reformed field.

The Carmelite universities of Salamanca (*Salamanticenses*) and Alcalà (*Complutenses*) and the Jesuit university of Coimbra (*Conimbricenses*) each produced a complete philosophical course, including specific volumes devoted to logic. Of these the most influential was probably the Coimbra Logic, compiled by Sebastian Couto (1567–1639) but partially dependent on Pedro da Fonseca (1528–1599), who had been teacher at that university. Fonseca, the "Portuguese Aristotle," published the *Institutionum Dialecticarum Libri VIII* (Fonseca 1964) in 1564, a logical treatise built on the model of Peter of Spain and widely read throughout Europe. Fonseca's logic interprets the traditional emphasis on terms by giving a theoretical priority to the conceptual moment over the judicative one (truth and falsity are in concepts rather than in judgment) and among concepts, to singulars over abstracts and universals. To reconcile God's foreknowledge and human free will, and to handle the problem of future contingents—a theme of special interest for all Iberian philosophers—Fonseca developed, independently from Luis de Molina (1535–1600), a theory of the *scientia media*, or, as he says, of "conditioned futures," by which God foreknows all the consequences of any possible free decision.

Placing Fonseca's theories within a wider and more systematic treatment, the Coimbra logic offers a translation and a detailed commentary of Aristotle's *Organon*, which, in the form of questions, includes a discussion of most of the topics debated by sixteenth- and seventeenth-century logicians. The *Conimbricenses* reject the idea that beings of reason are the object of logic (in the scholastic tradition logical concepts such as "genus" and "species" were considered to be *entia rationis*, and the Thomist tradition considered them as the formal object of logic): Dwelling on the idea of logic as *ars disserendi*, they prefer to characterize it as a "practical science" dealing with the construction of correct arguments. Argumentation is, therefore, the first and main object of logical enquiry. Particularly interesting is the long section devoted to the nature of signs at the opening of the commentary on Aristotle's *De Interpretatione* (see Doyle 2001). The concept of sign is here taken in a broad meaning, as to include not only spoken, written, and mental "words," but also iconic

languages and arithmetical signs. It is to be remarked that the influence of Coimbra logic was not limited to Europe: Jesuit missionaries used it in Latin America and even in China.

If the teaching of logic in Coimbra is connected to Fonseca, another important figure of Iberian logic and philosophy, Domingo de Soto (1494/5–1560), is connected to Alcalà and Salamanca, where he taught. Soto made important contributions to a plurality of fields, so much so that it was said *qui scit Sotum, scit totum* (who knows Soto, knows everything). Despite his endorsement of Thomism—testified by his defense of the theory that the object of logic are beings of reason—Soto was open to Scotist, nominalist, and even humanist influences, and his commentary on Aristotle's logic (Soto 1543) criticizes the "abstract sophistries" of the late scholastic logical tradition. This, however, did not prevent him from discussing and adopting many late scholastic logical theories, including large sections of medieval theories of terms. His *Summulae* (Soto 1980) are a commentary on one of the key works of medieval logic, Peter of Spain's *Tractatus* (best known as *Summulae Logicales*; see previous chapter), and include an ample discussion of signification, supposition, and consequences (see d'Ors 1981; Ashworth 1990; Di Liso 2000). Soto adopts an apparently Ciceronian definition of dialectic, considered as the art of discussing *probabiliter*. As remarked by Risse (1964–70, I, 330), however, this should not be considered a rhetorical attempt to establish apparent plausibility, but rather as an attempt to establish rational assertibility. Among the interesting points of the *Summulae* are the treatment of *induction* in terms of *ascensus* (the passage from a conjunction of singular propositions—or from a proposition with a copulative term as subject or predicate—to a universal proposition, or to a proposition with a general term as subject or predicate) and a complex square of modalities, which takes into account the quantity of the subject. Soto's discussion of second intentions offers what has been interpreted as a sophisticated theory of higher-level predicates (Hickman 1980).

One of Soto's students in Salamanca was Franciscus Toletus (1533–1596), who later taught both in Zaragoza and Rome, in the Jesuit Collegium Romanum, and was the first Jesuit to be appointed cardinal. Toletus wrote both an *Introduction* and a *Commentary* on Aristotle's logic (Toletus 1985). Like Soto, Toletus adopts some humanist theories—he takes the definition of logic as *ratio disserendi* from Boethius and divides it into invention and judgment—but his logic is actually a synthesis of Aristotelianism and Thomism, deeply influenced by the late medieval logical tradition. He considers beings of reason as formal objects of logic—thus partly endorsing the Thomist position—but maintains that logic's material object is constituted by our concepts of things and, ultimately, by things themselves, for logical beings of reason are only second intentions, based on first-order concepts—thus partly endorsing the position of Arab commentators of Aristotle (Ashworth 1985b, xli). Of special interest is his extensive use of physical and geometrical examples within the discussion of categories, and his long discussion of contingent futures within the commentary on *De Interpretatione*.

The most important Jesuit philosopher working in Spain at the end of the sixteenth century was Francisco Suárez (1548–1617). His *Disputationes Metaphysicae*, first published in 1597 (Suárez 1965), constituted a reference text and a model for further works both in the Catholic and in the Reformed fields. According to Suárez, metaphysics offers a general and unified theory of real being (*ens reale*) and of its divisions, whereas logic deals with the way of knowing and explaining such divisions. Though the *Disputationes Metaphysicae* is not a logic textbook, it discusses many issues relevant to the philosophy of logic. Suárez pays great attention to relations, subdivided into real relations (only conceptually and not really distinct from the things on which they are grounded, but nevertheless to be considered as a category of beings) and conceptual relations, which are only a product of the mind and as such do not have any ontological status. Suárez's detailed discussion of both kinds of relations helps to explain the special interest that many scholastic-oriented logicians devoted to this topic in the seventeenth century.

The last of the *Disputationes*—disputation LIV—is devoted to a subtle discussion about beings of reason (*entia rationis*) and relations of reason. According to Suárez, beings of reason are not "real" (actual or possible) beings and do not share a common concept with real beings; their only reality is that of being object of the understanding (they only have objective existence in the intellect). Therefore, they are not to be included within the proper and direct object of metaphysics. They can nevertheless be dealt with within the context of metaphysical research, given their nature of "shadows of being" (Suárez 1996, 57) and given their usefulness in many disciplines, especially logic and natural philosophy. Suarez's opinion on *entia rationis* is thus different both from that of those—like the Scotist Francis of Mayronnes (1280?–1327?)—who simply denied their existence, and from that of those—like many Thomists, including Cardinal Cajetanus (Tommaso de Vio, 1469–1534)—who thought that there is a concept common to them and to real beings. Suárez included impossible objects in the range of *entia rationis*: His discussion is thus especially relevant to the history of the logical and ontological status of impossible entities (Doyle 1987–88, 1995). The discussion on the nature of *entia rationis* was a lively one in sixteenth-century Spain and was bound to continue in Catholic Europe during most of the seventeenth century. An interesting example is that of the Polish Jesuit Martinus Smiglecius (1564–1618). In his opinion, the opposition between *ens reale* and *ens rationis* is not grounded on the fact that the *ens rationis* is not a form of being, but on the fact that it is by definition a being which is not, and cannot possibly be, an *ens reale*. A being of reason is thus, according to Smiglecius, one whose essence implies the impossibility of its real existence. The fact that *entia rationis* cannot have real existence is, according to Smiglecius, a logical and not just a physical impossibility. They, however, can have conceptual (and hence intentional) existence.

In the later Middle Ages, English logicians had been famous for their subtleties: The logical, physical, and epistemic sophisms discussed by the so-called *calculatores*, working at Merton College in Oxford, deeply influenced late

fourteenth- and early fifteenth-century logic both in Paris and in Italy, and were exactly the kind of logical subtleties rejected by humanist logicians. During the fifteenth and in the first decades of the sixteenth century, however, the English logical tradition declined (see Giard 1985). This did not prevent a slow penetration of humanist ideas, testified by the 1535 statutes or the university of Cambridge, recommending the reading of Agricola and Melanchthon as substitutes for late medieval scholastic texts, and by the *Dialectica* published in 1545 by the Catholic John Seton (c. 1498–1567). The latter offers a drastically simplified treatment of traditional topics such as signification, supposition, categories, syllogism, but liberally uses nonformal arguments and literary examples, divides dialectic into invention and judgment, adopts Agricola's definition of dialectic as well as his classification of topics, and quotes, beside Cicero and Quintilian, modern humanists like Erasmus and Vives.

In the last decades of the sixteenth century, the debate on Ramism was to shake both English and continental universities. In England, Ramus found in William Temple (1555–1627) a learned defender and commentator, who, despite the strong opposition of his fellow Cambridge teacher and former master Everard Digby (1550–1592), managed to make of Cambridge, albeit for a short time, a stronghold of Ramism. The penetration of Ramism in Oxford was less substantial, and by the beginning of the new century the anti-Ramist positions were predominant in both universities. The defeat of Ramism was accompanied by the propagation of Aristotelianism—tempered by humanist-oriented attention toward classical literary examples rather than purely logical ones and toward rhetorical practices such as the *declamatio*—and by the circulation of the leading logic books published in the continent (among them Zabarella and Keckermann). The *Logicae Artis Compendium* by the Oxford professor Robert Sanderson (1587–1663; Sanderson 1985) is a good example of this new situation. Sanderson abandons the division of logic into invention and judgment, favoring a threefold division according to the three acts of the mind: The first, dealing with simple concepts, is associated with the treatment of simple terms; the second, dealing with composition and division, is associated with propositions; and the third, dealing with discourse, is associated with argumentation and method. Though this threefold division is present in the medieval and late medieval tradition and is discussed by the Conimbricenses, Zabarella, and Keckermann, Sanderson and other Oxford logicians seem to have been among the first to use it as the main division for logic textbooks (Ashworth 1985b, xli). In his logic, Sanderson includes medieval topics such as the theory of supposition and consequences, but their presentation is straightforward and not very elaborated. His discussion of method is more articulate and gives a foremost role to pedagogical concerns.

We have already mentioned the Padua professor Jacopo Zabarella, who, advocating a renewed, "pure," and philologically accurate Aristotelianism, absorbs both some humanist instances—visible in the pedagogical organization of his works and in the inclusion of Aristotle's *Rhetoric* and *Poetic* within a broad treatment of logic, on the ground of their dealing with probable