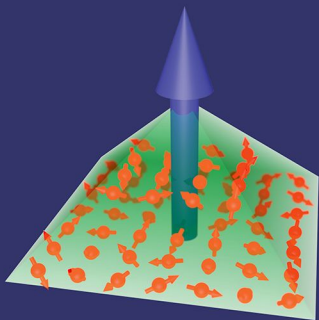


Electron & Nuclear Spin Dynamics in Semiconductor Nanostructures

M. M. Glazov



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M. M. Glazov

Ioffe Institute, 194021, St. Petersburg, Russia

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In memory of my father

Preface

In recent decades, the physics community has experienced a revival of interest in spin effects in solid state systems. This interest is two-fold: On one hand, the solid state systems, particularly semiconductors and semiconductor nanosystems, allow one to perform benchtop studies of the quantum and relativistic phenomena. The spin-orbit interaction effects on electron spin states and the hyperfine interaction of electron and nuclear spins are among the most prominent examples of such a class of effects. On the other hand, interest is supported by the prospects of realizing spin-based electronics where electron or nuclear spins may play the role of quantum or classical information carriers. This has resulted in the formation of a novel research field; spin electronics or, in short, spintronics.

The range of spin effects is vast. It extends from the effects of spin orientation and detection to the processes of spin transfer and manipulation. Moreover, a variety of spin systems in semiconductors have been studied including the spins of charge carriers, that is of electrons and holes, spins of nuclei, spins of impurities, and so on. Needless to mention, each semiconductor system brings about certain specific aspects of spin phenomena resulting in a multitude of spectacular and unique facets of spin physics in semiconductors. Many of these effects are observed experimentally and have firm theoretical grounds. As a result, spin physics in semiconductors has turned into a mature topic of research. To date, various aspects of spin effects in semiconductors have been concisely reviewed in collective books *Optical Orientation*, edited by F. Meier and B. P. Zakharchenya (North-Holland, 1984) and *Spin Physics in Semiconductors*, edited by M. I. Dyakonov (Springer, 2008), sometimes called “Old Testament” and “New Testament” of semiconductor spin physics, respectively. Each of the topics addressed in these references could become the subject of a separate monograph reflecting the state of the art in the corresponding area of spin physics.

This book aims at a rather detailed presentation of intriguing physics in the interacting systems of electron and nuclear spins in semiconductors, with particular emphasis on low-dimensional structures. The two spin systems naturally appear in practically all widespread semiconductor compounds. For instance, in most prominent GaAs, AlAs, or InAs materials that serve as building blocks for many semiconductor nanostructures, all host lattice nuclei have nonzero spins. The hyperfine interaction of charge carrier and nuclear spins gives rise to the spin exchange between these two systems. As a result, the understanding of the intertwined spin systems of electrons and nuclei is crucial for in-depth study and control of the spin phenomena in semiconductors. This is one of the reasons why the physics of electron and nuclear spins takes a special place in semiconductor spintronics. From the point of view of possible applications, the extremely long

lived (for up to days), nuclear spin polarization attracts a lot of interest in the community in issues related to the deliberate polarization of nuclei and detection of spin coherence. Interest in the physics of electron and nuclear spins is also strongly motivated by the beautiful and complex physics occurring in this many-body and nonlinear system.

Hopefully, the present book will be useful for postgraduates and researchers willing to enter the field of electron and nuclear spin dynamics in semiconductors, and comprehend the general physical picture of the interacting electrons and nuclei. Here, I have attempted to present the phenomenology and qualitative analysis together with the more rigorous models. The basics of experiments are outlined, however, the details on particular experimental methods and setups are not included because this goes far beyond my expertise. A certain amount of reference information and details on theoretical approaches included in this book might make it useful for specialists in the field as well.

It is a great pleasure to thank those who have helped me with writing the book. I am grateful to the students of Academic University in St. Petersburg (Russia) who have attended my lectures on spin effects in semiconductors. Their interest and stimulating questions have helped me to deeper understand things, which I thought were trivial. I am very grateful to my colleagues both at Ioffe Institute and all over the world for collaboration and support. I am indebted to my colleagues T. Amand, I. D. Avdeev, E. Chekhovich, M. V. Durnev, A. Greilich, I. V. Ignatiev, E. L. Ivchenko, K. V. Kavokin, V. L. Korenev, X. Marie, M. Yu. Petrov, A. V. Poshakinskiy, D. S. Smirnov, A. Tartakovskii, B. Urbaszek, D. R. Yakovlev, I. A. Yugova, and V. S. Zapasskii for valuable discussions and critical reading of the manuscript. I owe my wife, Marina Semina, and my mother, Nina Glazova, a debt of gratitude for their continuing support and help.

St. Petersburg, Russia
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Introduction

Creation, detection, and manipulation of the spin degrees of freedom of electrons and nuclei; phenomena of spin relaxation, decoherence, and dephasing; and the processes of spin transfer between different subsystems are among the most important problems studied in semiconductor spintronics. These effects taking place in semiconductor nanostructures with localized charge carriers, and the underlying interactions in spin systems are the focus of this book. The aim of this short introductory chapter is to provide an outline of the material contained in the book and explain the logic of its presentation.

Chapter 2 on “Spin Systems in Semiconductor Nanostructures” contains an introduction to a rich variety of phenomena taking place in the interacting system of electrons and nuclei in semiconductors. It includes the basics of the electronic properties of nanostructures and of spin physics, an overview of the fundamental interactions in the electron and nuclear spin systems, selection rules at optical transitions in semiconductors, and spin resonance phenomenon as well as optical orientation and dynamical nuclear polarization effects. The last section 2.6 contains an analysis of the particular features of spin dynamics arising in structures with localized electrons, such as quantum dots, which are studied later in the book. The aim of this chapter is to provide the basic minimum of information needed to read the remaining chapters. In the following chapters, we provide references to the basic notions and concepts introduced in Chapter 2 that might be helpful for readers interested in the contents of just a few particular chapters.

Chapter 3 on “Spin Resonance” is devoted to one of key phenomena in the field of spin physics; namely, the resonant absorption of an electromagnetic wave under conditions where the splitting between spin levels in a magnetic field is equal to the photon energy. This method is particularly relevant for identification of nuclear spin effects, because the resonance spectra provide “fingerprints” of different involved spin species and make it possible to distinguish different nuclear isotopes. As discussed in this chapter, nuclear magnetic resonance also provides access to local magnetic fields acting on nuclear spins and is caused by the magnetic interactions between the nuclei or by the quadrupole splittings of nuclear spin states caused by the environment. We also outline specific manifestations of spin resonance in the optical response of semiconductors; that is, optically detected

magnetic resonance, Section 3.3. This technique has become an indispensable tool due to its high sensitivity and relative simplicity of realization in semiconductors.

Chapter 4 on “Hyperfine Interaction of Electron and Nuclear Spins” discusses the interaction that underlies most of phenomena in the field of electron and nuclear spin dynamics. This interaction originates from the magnetic interaction between nuclear and electron spins. In many cases, it is reduced to the Fermi contact interaction whose strength is proportional to the probability of finding an electron at the nucleus. A more involved situation is realized for the valence band holes in III-V or II-VI materials where the hole Bloch functions vanish at the nuclei, see Section 4.2. The modification of the hyperfine coupling Hamiltonian in nanosystems is also analyzed. The chapter contains also an overview of experimental data aimed at determination of the hyperfine interaction parameters in semiconductors and semiconductor nanostructures.

One of the main manifestations of the hyperfine interaction is the transfer of spin between the electron and nuclear systems. This effect is studied in detail in Chapter 5 entitled “Dynamical Nuclear Polarization.” While thermal orientation of nuclei in a magnetic field is negligible due to their small magnetic moments, hyperfine coupling can effectively induce the spin flux from electrons to nuclei, or vice versa, provided that the electron spin system is driven out of equilibrium; for example, by means of optical orientation. In this chapter, we present the microscopic approach to the dynamical nuclear polarization effect based on the kinetic equation method followed by a more phenomenological but very powerful description of dynamical nuclear polarization in terms of the nuclear spin temperature concept. In this way, one can account for interactions between neighbouring nuclei without solving a complex many-body problem. The hyperfine interaction also induces the feedback of polarized nuclei on the electron spin system giving rise to a number of nonlinear effects such as bistability of nuclear spin polarization and the anomalous Hanle effect. These and other nonlinear effects in the interacting electron and nuclear system are presented in Sections 5.3–5.5 both from theoretical and experimental viewpoints.

Chapter 6 focuses on details of the optical manipulation of electron spin states. It also addresses manifestations of the electron and nuclear spin dynamics in the optical response of semiconductor nanostructures via spin-Faraday and spin-Kerr effects. Coupling of spins with light provides the most efficient way for nonmagnetic spin manipulation. The main aim of this chapter is to provide theoretical grounds for optical spin injection, ultrafast spin control, and readout of spin states by means of circularly and linearly polarized light pulses. The Faraday and Kerr effects induced by electron and nuclear spin polarization are analyzed both by means of a macroscopic, semi-phenomenological approach, and by using the microscopic quantum mechanical model. Theoretical analysis is supported by experimental data.

After establishing the main interactions in the electron and nuclear spin systems and describing the protocols for electron spin initialization, control, and detection, we turn to the discussion of the electron spin decoherence and relaxation

phenomena. These processes limit the conservation time of spin states as well as the response time of the spin system to external perturbations. In particular, Chapter 7 on “Electron Spin Decoherence By Nuclei” focuses on the hyperfine interaction induced decoherence of localized electron spins. After formulation of the central spin model where the spin of charge carrier interacts with the bath of nuclear spins, we present different methods to calculate the spin dynamics within this model. A simple but physically transparent semiclassical treatment where the nuclear spins are considered as largely static classical magnetic moments is followed by a more advanced quantum mechanical approach where the feedback of electron spin dynamics on the nuclei is taken into account. The chapter is concluded by Section 7.4 containing an overview of experimental data and its comparison with the models.

Although the hyperfine interaction serves as a major source of electron spin dephasing, it can be strongly suppressed by an external magnetic field. This is because a substantial difference between the electron and nuclei Zeeman splittings gives rise to the vanishing probabilities of electron-nuclei flip-flop processes making hyperfine coupling-induced spin relaxation inefficient. Hence, other interactions provide electron spin relaxation in strong fields. Electron spin relaxation in sufficiently strong magnetic fields is addressed in Chapter 8 on “Electron Spin Relaxation Beyond the Hyperfine Interaction.” Spin relaxation mechanisms related to the joint action of electron-phonon interaction and spin-orbit coupling are discussed in detail. Experimental manifestations of such processes and comparison between the experiment and theory are presented in Section 8.4. We also discuss in Section 8.5 a very specific situation where the spin-flip is caused by the magneto-dipole radiative transitions between the Zeeman-split levels.

Chapter 9 on “Electron Spin Precession Mode-Locking and Nuclei-Induced Frequency Focusing” is devoted to a rich variety of effects in spin dynamics arising from the conditions of pump-probe experiments. Here, we consider the situation where electron spin is injected by a periodic train of circularly polarized pump pulses and precesses between the pulses in an external magnetic field. Nontrivial effects such as resonant spin amplification and spin coherence mode-locking take place due to the commensurability of the repetition period of pump pulses and the charge carrier spin precession period. Theoretical approaches to describing the electron and nuclear spin coherence in such a case and experimental manifestations of these unusual regimes of spin dynamics are discussed in detail.

While the standard approach to studying electron and nuclear spin dynamics experimentally can be briefly summarized as excitation of spin system and monitoring its evolution (or detecting the time-integrated response), a lot can be learned from the equilibrium behavior of spins. In thermal equilibrium, both electron and nuclear spin systems are unpolarized on average, but characterized by nonzero fluctuations. The intensity and dynamics of these inevitable stochastic fluctuations of spins contain information on spin relaxation and decoherence times, spin precession period, and interactions in spin systems. The theory of spin fluctuations in semiconductor nanosystems as well as experimental advances in the

field of spin noise spectroscopy are reviewed in Chapter 10. Specific situations where the spin noise spectroscopy can be particularly useful for spin dynamics studies are discussed, the analysis of recent progress in the field of nonequilibrium spin fluctuations is presented as well.

The book is concluded by Chapter 11 on “Strong Coupling of Electron and Nuclear Spins: Outlook and Prospects” where some prospects in the field of electron and nuclear spin dynamics are outlined. Particular emphasis is put on a situation where the hyperfine interaction is so strong that it leads to a qualitative rearrangement of the energy spectrum resulting in the coherent excitation transfer between the electron and nucleus. The strong coupling between the spin of the charge carrier and of the nucleus is realized, for example, in the case of deep centers in semiconductors or in isotopically purified systems. We also discuss the effect of nuclear spin polaron (i.e., ordered state) formation at low enough temperatures of nuclear spins, where the orientation of the carrier spin results in alignment of the spins of nuclei interacting with the electron or hole.

Spin Systems in Semiconductor Nanostructures

2.1 Brief overview of spin systems in semiconductors

Spin is the *intrinsic magnetic moment* or *intrinsic angular momentum* of the particle. In contrast to the orbital angular momentum and the magnetic moment related to the orbital motion of charged particles, the presence of spin is not caused by any classical motion. The electrons, protons, and neutrons are fermions and possess a spin of $1/2$. Strictly speaking, the intrinsic angular momentum associated with the spin for these particles equals $\hbar/2$, but the reduced Planck constant $\hbar$ is often omitted, and hereafter we follow this convenient practice.

The protons and neutrons combine and form the nuclei that arrange in the crystalline lattices of semiconductors. In the prominent semiconductor material of GaAs, the relevant (stable) nuclei isotopes are ^{69}Ga , ^{71}Ga , and ^{75}As , all of which have nonzero spins of $3/2$. We recall that the superscript at the upper left of the chemical symbol denotes the mass number; that is, the total number of protons and neutrons constituting a nucleus. The most relevant isotopes of In are ^{113}In (stable, about 4% of natural abundance) and ^{115}In (weakly radiative with a lifetime of >400 trillion years, 95% abundance; i.e., stable from the view point of practical applications) have spins of $9/2$. Hence, in GaAs and InAs semiconductors and nanostructures all host lattice nuclei that have nonzero spin. This might not be the case for other important semiconductors, for example, the stable isotopes of Si: ^{28}Si , ^{29}Si , and ^{30}Si with abundances of about 92, 5, and 3%, respectively, have spins of 0 (^{28}Si , ^{30}Si) and $1/2$ (^{29}Si). Therefore, the majority of nuclei in silicon have zero spin. Some of the isotopes relevant to the semiconductors under study, together with their abundance and spins, are listed in Table 2.1.

Hence, in semiconductors we naturally have the spin system of charge carriers, electrons and holes, and of the host lattice nuclei. One of the specific features of semiconductors is the strong interaction of the charge carriers with light. The polarization state of the electromagnetic wave can be associated with the spin (*spin angular momentum*) of the photon. Unlike electrons, protons and neutrons, the photons are bosons with spin equal to 1. The right- and left-circularly

Table 2.1 Nuclear isotopes with nonzero spin in several group-IV, III–VI, and II–VI semiconductors. The asterisk denotes unstable but abundant isotopes. Magnetic moments, gyromagnetic ratios, and quadrupole moments (for $I > 1/2$ in barns = 10^{-28} m^2) are given. From (Fuller, 1976) and Evaluated Nuclear Structure Data File <http://www-nds.iaea.org/>.

Isotope	Abundance %	Spin I	Magnetic moment μ_I/μ_N	Gyromagnetic ratio γ_I (MHz/T)	Quadrupole moment Q (barn)
^{27}Al	100	5/2	+3.6415069	+11.103	+0.147
^{75}As	100	3/2	+1.439475	+7.315	+0.314
^{13}C	1.1	1/2	+0.7024118	+10.708	
^{111}Cd	12.8	1/2	−0.5948861	−9.069	
$^{113}\text{Cd}(\ast)$	12.2	1/2	−0.6223009	−9.487	
^{19}F	100	1/2	+2.6288	+40.077	
^{69}Ga	60.1	3/2	+2.01659	+10.248	+0.171
^{71}Ga	39.9	3/2	+2.562266	+13.021	+0.107
^{199}Hg	16.9	1/2	+0.5058855	+7.712	
^{201}Hg	13.2	3/2	−0.5602257	−2.846	+0.387
^{113}In	4.3	9/2	+5.5229	+9.355	+0.799
$^{115}\text{In}(\ast)$	95.7	9/2	+5.5408	+9.386	+0.86
^{95}Mo	15.9	5/2	−0.9142	−2.787	−0.022
^{97}Mo	9.6	5/2	−0.9335	−2.846	+0.255
^{14}N	99.6	1	+0.403761	+3.078	+0.0193
^{15}N	0.4	1/2	−0.28319	−4.317	
^{31}P	100	1/2	+1.130903	+17.24	
^{33}S	0.76	3/2	+0.6438	+3.272	−0.084
^{121}Sb	57.21	5/2	+3.3634	+10.255	−0.36
^{123}Sb	42.79	7/2	+2.5498	+5.553	−0.49
^{77}Se	7.6	1/2	+0.535	+8.156	
^{29}Si	4.7	1/2	−0.5553	−8.465	
^{123}Te	0.9	1/2	−0.7369478	−11.235	
^{125}Te	7	1/2	−0.8885051	−13.545	
^{67}Zn	4	5/2	+0.875479	+2.669	+0.150
^{183}W	14.3	1/2	+0.11778	+1.796	

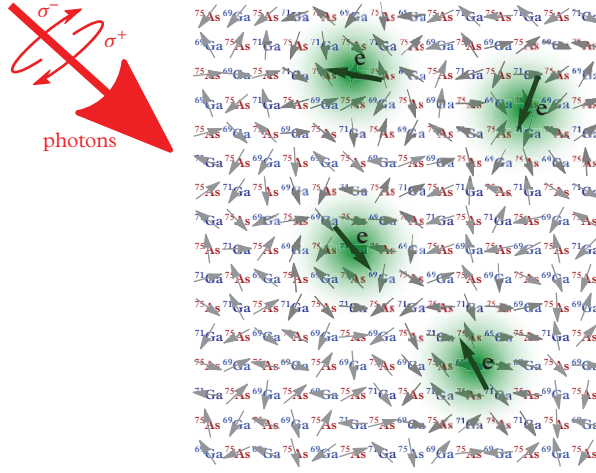


Figure 2.1 Schematics of the basic spin systems in a semiconductor, which are the focus of this book, illustrated with the example of GaAs. The large arrow denotes σ^+ and σ^- polarized photons carrying angular momentum. GaAs crystalline structure is sketched as a lattice of ^{69}Ga , ^{71}Ga , and ^{75}As isotopes, each carries a nonzero spin shown by the small gray arrows. The localized electron wavefunctions (e.g., bound to neutral donors) are illustrated by shaded circles, dark thick arrows show electron spins. The dimensions of the bound state wavefunctions and the lattice constant are not shown to scale.

polarized light corresponds to the photon spin components on the light propagation direction being $+1$ and -1 , respectively. The spin component of 0 is forbidden for photons because the electromagnetic field is transversal.

Figure 2.1 sketches these three spin systems, which are the most relevant to the following. The interactions in the spin systems outlined in Section 2.3 allow for spin transfer between photons, electrons, and nuclei. In many cases, the charge carriers act as intermediaries between the spins of photons and lattice nuclei: The electron spins can be efficiently polarized in the process of light absorption due to the *spin-orbit interaction* (this is the process known as *optical orientation*) and electrons can transfer spin to the lattice nuclei via the *hyperfine interaction*. In this chapter, we start with a concise introduction to the electronic properties of semiconductors and semiconductor nanostructures (Section 2.2), then discuss interactions in spin systems in Section 2.3, outline the main effects in spin systems in semiconductors in Sections 2.4 and 2.5, and conclude this chapter with the motivation for the study of spin dynamics in the interacting system of the lattice nuclei and localized electron spins (Section 2.6).

2.2 Basics of semiconductor nanostructure physics

The motion of electrons in bulk crystals resembles electron dynamics in free space. There are allowed energy bands with a continuous energy spectrum separated by the forbidden intervals known as band gaps. The formation of forbidden bands is because the electron potential energy is periodic in space. Due to the *translational symmetry* of bulk materials the electron states are characterized by the following quantum numbers: The continuous variable $\mathbf{k}$ being the quasi-wavevector or, in short, the wavevector of the electron, and discrete parameters n being the band label and $s = \pm 1/2$ being the electron spin index. Usually, the spin-orbit coupling mixes orbital and spin degrees of freedom, therefore, it is convenient to use a single subscript n that accounts both for the band and spin states. Accordingly, the wavefunction $\Psi_{\mathbf{k},n}(\mathbf{r})$ of the electron in semiconductor can be presented in the form of the Bloch function

$$\Psi_{\mathbf{k},n}(\mathbf{r}) = \frac{1}{\sqrt{v}} \exp(i\mathbf{k} \cdot \mathbf{r}) U_{\mathbf{k},n}(\mathbf{r}). \quad (2.1)$$

Here, v is the normalization volume, $\mathbf{r} = (x, y, z)$ is the electron position vector, and $U_{\mathbf{k},n}(\mathbf{r})$ is the periodic Bloch amplitude: At all translations that match the crystalline lattice with itself, the function $U_{\mathbf{k},n}(\mathbf{r})$ remains invariant. Equation (2.1) is the Bloch theorem that follows from the translational symmetry arguments. The values of the $\mathbf{k}$ are usually selected within the first *Brillouin zone* of the $\mathbf{k}$ -space, since the addition to $\mathbf{k}$ or subtraction from $\mathbf{k}$ of a reciprocal lattice vector can be accounted for by the change of the periodic amplitude in Eq. (2.1). The functions (2.1) satisfy the Schrödinger equation

$$\mathcal{H}_0 \Psi_{\mathbf{k},n} = E_n(\mathbf{k}) \Psi_{\mathbf{k},n}, \quad \mathcal{H}_0 = \frac{\hat{\mathbf{p}}^2}{2m_0} + V(\mathbf{r}) + \mathcal{H}_{so}. \quad (2.2)$$

Here $\hat{\mathbf{p}} = -i\hbar \nabla_{\mathbf{r}} = -i\hbar(\partial/\partial x, \partial/\partial y, \partial/\partial z)$ is the electron momentum operator, m_0 is the free electron mass, $V(\mathbf{r})$ is the *periodic* single-particle potential and $\mathcal{H}_{so}$ is the spin-orbit Hamiltonian, whose form is presented later in Eq. (2.11).

Equation (2.1) demonstrates that the electron state in a crystal is, like in free space, a plane wave, but, unlike in free space, modulated with a function $U_{\mathbf{k},n}(\mathbf{r})$ oscillating on the scale of the lattice constant. Just like in free space, the electron energy $E_n(\mathbf{k})$ is a continuous function of the wavevector $\mathbf{k}$ in the allowed bands, however, the $E_n(\mathbf{k})$ dependence can be both qualitatively and quantitatively different from that of the electron in vacuum. Figure 2.2 illustrates some of the relevant energy bands in the prototypical system of GaAs: the lowest empty band, that is, the *conduction band*, nearest excited conduction bands as well as the topmost occupied bands, the *valence bands*. The conduction and valence bands are separated by the forbidden band gap, E_g , for instance, in GaAs $E_g \approx 1.5$ eV. Of course, the band filling by the charge carriers depends on the temperature and

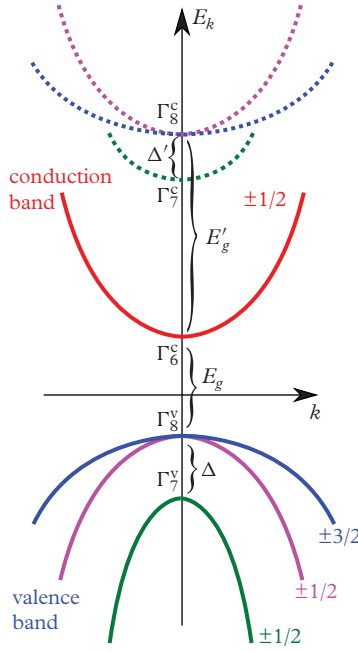


Figure 2.2 Sketch of the conduction and valence bands in a GaAs-like semiconductor. The remote conduction bands are shown by dotted lines. E_g and E'_g are the band gaps, Δ and Δ' are the spin-orbit splittings of bands, the bands are labelled by the irreducible representations Γ_6 , Γ_7 , and Γ_8 relevant at $\mathbf{k} = 0$, with the superscripts c and v denoting the conduction and valence bands respectively. Additionally, the bands, are labelled by the spin components, see Section 2.3 for details. The electron representation for labelling the valence band states is used.

presence of impurities; that is, doping. Hence, the classification of bands in terms of conduction and valence is performed for pure semiconductors in the limit of zero temperature.

Calculations of electron dispersion is a complex task where various atomistic methods such as density functional theory, pseudopotential method, tight-binding approximation are used. However, as a rule, one is interested in the electron states in the vicinity of some point $\mathbf{k} = \mathbf{K}_0$ in the Brillouin zone where in the extrema of the conduction band, $E_c(\mathbf{k})$, or the valence band, $E_v(\mathbf{k})$, energy dispersions are realized. GaAs is the direct band gap semiconductor, where the nearest in energy extrema of conduction and valence bands are formed at $\mathbf{K}_0 = 0$. This point is also denoted as the Γ -point, see Figure 2.2, and we focus on this situation hereafter. The Bloch amplitude $U_{\mathbf{k},n}$ is decomposed in the series of the periodic amplitudes at the Γ -point $U_{\mathbf{k},n} = \sum_{n'} C_{n'}^{(n)}(\mathbf{k}) U_{0,n'}$, where the subscript $\mathbf{0}$ denotes $\mathbf{k} = 0$. Making use of the $\mathbf{k} \cdot \mathbf{p}$ -perturbation theory (see Appendix B) one can show that the column vector $\hat{C}^{(n)}(\mathbf{k})$ formed of the decomposition coefficients $C_{n'}(\mathbf{k})$ satisfies the matrix equation

$$\mathcal{H}(\mathbf{k})\hat{C}^{(n)}(\mathbf{k}) = E_n(\mathbf{k})\hat{C}^{(n)}(\mathbf{k}). \quad (2.3)$$

Here, $\mathcal{H}(\mathbf{k})$ is the *effective Hamiltonian*. Within the second-order $\mathbf{k} \cdot \mathbf{p}$ -perturbation theory, the matrix elements $\mathcal{H}_{n;n'}(\mathbf{k})$ of the effective Hamiltonian $\mathcal{H}(\mathbf{k})$ are linear and quadratic in the components of the wavevector $\mathbf{k}$. In the simplest possible case where only one band is taken into consideration and the spin degree of freedom is disregarded, the effective Hamiltonian takes the form $\mathcal{H}(\mathbf{k}) = \hbar^2 k^2 / 2m$, where m is the *effective mass*.

Within the $\mathbf{k} \cdot \mathbf{p}$ approach one usually includes only the nearest bands in the Hamiltonian $\mathcal{H}(\mathbf{k})$, while contributions from the remote bands are accounted for by the quadratic in the wavevector, $\sim k^2$, and sometimes, higher order terms in the $\mathcal{H}(\mathbf{k})$. The form of the effective Hamiltonian in the bulk material can be established by means of symmetry analysis: The invariants method allows one to find a general form of $\mathcal{H}_{n;n'}$ while the parameters of the Hamiltonian are determined by comparison with atomistic calculations and experiments. For further details on the $\mathbf{k} \cdot \mathbf{p}$ -method we refer the reader to Appendix B and the books (Bir and Pikus, 1974; Ivchenko and Pikus, 1997; Voon and Willatzen, 2009) where this method is justified and presented in every detail.

In the presence of external electric and magnetic fields, impurities, lattice deformations, as well as in nanostructures, such as *quantum wells* (QWs), *quantum wires* (QWRs), and *quantum dots* (QDs), the electron potential energy is no longer periodic in space and the Bloch theorem does not apply anymore. For example, in quantum wells the electron motion is free in the well plane and restricted along the well normal; in quantum wires the electrons are free to move along one direction, the wire axis; and in quantum dots the motion of charge carriers is restricted in all three spatial directions. Generally, the atomistic methods should be used to calculate the wavefunctions and energies of size-quantized states. These methods have high accuracy but, as a rule, relatively high complexity, which does not allow one to trace the physics of the effects easily. Therefore, in many cases it is instructive to use approximate methods, namely the *effective mass method* or *effective Hamiltonian method*, which is valid if the charge carrier localization in nanostructures or in the impurity potential takes place on the length scale, which exceeds by far the lattice constant.

In the effective Hamiltonian method, the electron wavefunction is sought in each homogeneous part of the system as a linear combination of bulk solutions, see Eq. (2.1),

$$\Psi(\mathbf{r}) = \sum_n \varphi_n(\mathbf{r}) U_{0,n}(\mathbf{r}). \quad (2.4)$$

The exponential function $\exp(i\mathbf{k} \cdot \mathbf{r})$ describing a plane wave in Eq. (2.1) is replaced by an *envelope function* $\varphi_n(\mathbf{r})$ describing the size quantization effect, similar to the case of a free particle where the plane wave is replaced by the appropriate wavefunction in the given potential. The envelope functions are

smooth on the scale of the lattice constant; they obey the Schrödinger equation in bulk layers with the effective matrix Hamiltonian, Eq. (2.3), where $\mathbf{k} = -i\nabla$. The functions $\varphi_n(\mathbf{r})$ are matched at the heterointerfaces by the appropriate boundary conditions.

The amount included in the consideration bands depends on the phenomenon studied, required accuracy, and so on. If just one band is accounted for, as an example, the conduction band, and the spin is disregarded for a moment, the effective Hamiltonian takes the particularly simple form,

$$\mathcal{H}(\mathbf{k}) = \frac{\hbar^2 k^2}{2m} + V_{\text{eff}}, \quad (2.5)$$

where $V_{\text{eff}} = V_{\text{eff}}(\mathbf{r})$ is the effective potential energy. This model is referred to as the effective mass approximation. For the conduction band in GaAs $m \approx 0.067m_0$, where m_0 is the free electron mass. In heterostructures formed of homogeneous bulk semiconductors, the potential energy $V_{\text{eff}}(\mathbf{r})$ takes a constant value within each homogeneous region, the discontinuities of $V_{\text{eff}}(\mathbf{r})$ at the heterointerfaces equal to the band offsets between the materials. The effective potential can also include contributions due to external fields. The effective mass differs in different homogeneous regions as well. In nanostructures formed by a smooth variation of composition, both V_{eff} and m are smooth functions of coordinates, and the kinetic energy is usually represented as

$$-\frac{\hbar^2}{2} \nabla \frac{1}{m(\mathbf{r})} \nabla.$$

Similarly, for an electron interacting with the ionized donor in a bulk semiconductor, the potential $V_{\text{eff}}(\mathbf{r})$ is usually approximated by the attractive Coulomb potential, $V_{\text{eff}}(\mathbf{r}) = -e^2/(\kappa r)$, where $r = |\mathbf{r}|$ and κ is the static dielectric constant. Equation (2.5) is similar to the Hamiltonian of a free electron interacting with a potential field V_{eff} . Just as for an electron bound to an attractive potential, in semiconductors the attractive potential due to the electric field of the ionized donor or due to the band offsets results in the formation of localized states with a discrete energy spectrum. The engineering of the potential $V_{\text{eff}}(\mathbf{r})$ by means of advanced growth techniques opens up possibilities to design the wavefunctions of the size-quantized states in nanostructures and tune their energies on demand. The details on the size quantization effect in nanostructures can be found, for example, in (Bastard, 1988; Davies, 1998; Ivchenko and Pikus, 1997; Winkler, 2003; Ivchenko, 2005) where various semiconductor systems are considered.

2.3 Interactions in spin systems

Free electron spin states are described by the spinor wavefunctions denoted as $|\uparrow\rangle, |\downarrow\rangle$. These are eigenfunctions of the spin- z component operator, $\hat{s}_z$, with the

eigenvalues being $\pm 1/2$, respectively. In this basis, the electron spin operator is conveniently represented by the Pauli matrices:

$$\hat{\mathbf{s}} = \frac{1}{2}\boldsymbol{\sigma}, \quad \boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z), \quad (2.6)$$

where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (2.7)$$

The intrinsic magnetic moment $\hat{\boldsymbol{\mu}}$ of a free electron is related to its spin as $\hat{\boldsymbol{\mu}} = -2\mu_B\hat{\mathbf{s}}$, where $\mu_B = |e|\hbar/(2m_0c)$ is the Bohr magneton with c being the speed of light. As a result, for a free electron the Hamiltonian describing its spin interaction with the external magnetic field $\mathbf{B}$, that is, the *Zeeman effect*, has a particularly simple form

$$\mathcal{H}_B = -(\hat{\boldsymbol{\mu}} \cdot \mathbf{B}) = \frac{g_0}{2}\mu_B(\boldsymbol{\sigma} \cdot \mathbf{B}), \quad (2.8)$$

where $g_0 = 2$ is the free electron g -factor (also known as the *Landé factor*).

In crystals and nanostructures, the electron Zeeman effect is described by a similar Hamiltonian

$$\mathcal{H}_B = \frac{1}{2} \sum_{\alpha, \beta} g_{\alpha\beta} \sigma_\alpha B_\beta, \quad (2.9)$$

where we introduced the tensor of g -factors $g_{\alpha\beta}$. Here $\alpha, \beta = x, y, z$ are the Cartesian coordinates. Equation (2.9) can be recast in the form

$$\mathcal{H}_B = \frac{\hbar}{2} (\boldsymbol{\sigma} \cdot \boldsymbol{\Omega}_B), \quad (2.10)$$

where the pseudovector $\boldsymbol{\Omega}_B$ with the components

$$\Omega_{B,\alpha} = \frac{\mu_B}{\hbar} \sum_{\beta} g_{\alpha\beta} B_\beta,$$

has a transparent physical meaning: It is the frequency of the spin precession in the external field $\mathbf{B}$. The $\boldsymbol{\Omega}_B$ is the *Larmor frequency* of electron spin precession. The number of independent components of $g_{\alpha\beta}$ is determined by the symmetry of the system. For instance, in cubic semiconductors such as GaAs or InAs, the tensor $g_{\alpha\beta} = g\delta_{\alpha\beta}$ reduces to a scalar, g , and $\delta_{\alpha\beta}$ is the Kronecker δ -symbol. The difference between the free electron g -factor and the g -factor in many semiconductors is quite significant; for example, in GaAs for conduction band

electrons $g \approx -0.44$, while in InSb $g \approx -50$. The difference results from the *spin-orbit interaction*. The size quantization of electron states in nanosystems results both in renormalization of the g -factor with respect to the bulk values, and to the appearance of new independent components of the g -factor tensor (Ivchenko and Kiselev, 1992; Kalevich and Korenev, 1993; Ivchenko, 2005; Yugova *et al.*, 2007a).

The spin-orbit interaction or *spin-orbit coupling* provides the coupling between the spin states of the electron and its orbital motion. The spin-orbit interaction is a relativistic effect but, for the semiconductors under study, it is sufficient to account for the relativistic effects in the lowest order. The spin-orbit coupling is thus described by the Hamiltonian (Bir and Pikus, 1974; Landau and Lifshitz, 1977)

$$\mathcal{H}_{so} = -\frac{i\hbar^2}{4m_0^2c^2} (\boldsymbol{\sigma} \cdot [\nabla V(\mathbf{r}) \times \nabla]), \quad (2.11)$$

Here, $V(\mathbf{r})$ is the periodic potential in the crystal, Eq. (2.2), and the operator $\mathcal{H}_{so}$ acts in the basis of Bloch functions, $\Psi_{\mathbf{k},n}(\mathbf{r})$, arranged in the form of spinors,

$$\hat{\Psi}_{\mathbf{k},n}(\mathbf{r}) = \Psi_{\mathbf{k},n,+1/2}(\mathbf{r})|\uparrow\rangle + \Psi_{\mathbf{k},n,-1/2}(\mathbf{r})|\downarrow\rangle = \begin{pmatrix} \Psi_{\mathbf{k},n,+1/2}(\mathbf{r}) \\ \Psi_{\mathbf{k},n,-1/2}(\mathbf{r}) \end{pmatrix}.$$

The interaction in the form of Eq. (2.11) has a profound effect on the electron states in semiconductors and acts in many cases as a driving force for electron spin dynamics. Here we recall the most important manifestations of the spin-orbit coupling.

In semiconductors like GaAs, the conduction band in the vicinity of the Γ point is formed mainly of s -like atomic states (with the orbital momentum being 0), while the valence band is mainly formed of the p -like atomic states (with the orbital momentum being 1). Formally, the orbital part of the Bloch amplitude of the conduction band at $\mathbf{k} = 0$, $\mathcal{S}(\mathbf{r})$, is invariant under all operations in the point symmetry group T_d relevant to the zinc-blende lattice crystals. It corresponds to the irreducible representation A_1 ; see Appendix A for details. Accounting for the spin the pair of Bloch amplitudes $U_{0,n,+1/2} = \mathcal{S}(\mathbf{r})|\uparrow\rangle$, $U_{0,n,-1/2} = \mathcal{S}(\mathbf{r})|\downarrow\rangle$ transforms according to the irreducible spinor representation Γ_6 of the T_d point group. There are three orbital Bloch amplitudes, $\mathcal{X}(\mathbf{r})$, $\mathcal{Y}(\mathbf{r})$, $\mathcal{Z}(\mathbf{r})$, of the valence band at the Γ -point, which form the basis of the irreducible representation F_2 of the T_d point symmetry group. These three functions transform at all symmetry operations just like the components of the polar vector, such as the position vector $\mathbf{r}$, or the electric field $\mathbf{E}$. Accounting for the spin, we have six functions, for example, $\mathcal{X}(\mathbf{r})|\uparrow\rangle$, $\mathcal{Y}(\mathbf{r})|\downarrow\rangle$, which form a basis of the representation $F_2 \otimes \Gamma_6$. This representation is reducible, it can be decomposed in the sum of irreducible spinor representations $\Gamma_8 \oplus \Gamma_7$. The dimensions of these representations are 4 and 2, respectively. Hence, accounting for the spin-orbit interaction, one expects the splitting of the valence band top into the quadruplet (Γ_8 representation) and the doublet (Γ_7 representation), see Figure 2.2. The splitting between the Γ_8 and Γ_7

states is called the spin-orbit splitting of the valence band and denoted as Δ . It amounts to about 340 meV in GaAs, 390 meV in InAs, 750 meV in GaSb, and reaches almost 1 eV in InSb (Vurgaftman *et al.*, 2001; Adachi, 2005, 2009).

In fact, this fine structure of the valence band states is similar to the fine structure of the $2p$ -level of the hydrogen atom. Indeed, the periodic potential $V(\mathbf{r})$ in Eq. (2.11), is invariant under all transformations of the crystal point group, the derivatives $\partial V/\partial x$, $\partial V/\partial y$, $\partial V/\partial z$ transform as components of a vector $\mathbf{r} = (x, y, z)$. Hence, the selection rules for the matrix elements of the operator $\mathcal{H}_{so}$ are exactly the same as in the atomic case and, correspondingly, the matrix elements of the $\mathcal{H}_{so}$ operator in the basis of the valence band Bloch amplitudes $\mathcal{X}(\mathbf{r})|\uparrow\rangle, \mathcal{X}(\mathbf{r})|\downarrow\rangle, \dots$ differ from those in the atom by a common factor only. By analogy with the hydrogen atom, the valence band basic functions at the Γ point are usually labelled by the total angular momentum $\mathcal{J}$ and its component $\mathcal{J}_z$. The angular momentum $\mathcal{J}$ is the sum of the valence band electron spin (1/2) and orbital (1) momenta. It runs through the values from $-3/2$ to $+3/2$ for the Γ_8 states:

$$|\Gamma_8, +3/2\rangle = -\frac{\mathcal{X} + i\mathcal{Y}}{\sqrt{2}}|\uparrow\rangle, \quad (2.12a)$$

$$|\Gamma_8, +1/2\rangle = \sqrt{\frac{2}{3}}\mathcal{Z}|\uparrow\rangle - \frac{\mathcal{X} + i\mathcal{Y}}{\sqrt{6}}|\downarrow\rangle, \quad (2.12b)$$

$$|\Gamma_8, -1/2\rangle = \sqrt{\frac{2}{3}}\mathcal{Z}|\downarrow\rangle + \frac{\mathcal{X} - i\mathcal{Y}}{\sqrt{6}}|\uparrow\rangle, \quad (2.12c)$$

$$|\Gamma_8, -3/2\rangle = \frac{\mathcal{X} - i\mathcal{Y}}{\sqrt{2}}|\downarrow\rangle, \quad (2.12d)$$

and takes the values $\pm 1/2$ for the Γ_7 states:

$$|\Gamma_7, +1/2\rangle = -\frac{1}{\sqrt{3}}[\mathcal{Z}|\uparrow\rangle + (\mathcal{X} + i\mathcal{Y})|\downarrow\rangle], \quad (2.13a)$$

$$|\Gamma_7, -1/2\rangle = \frac{1}{\sqrt{3}}[\mathcal{Z}|\downarrow\rangle - (\mathcal{X} + i\mathcal{Y})|\uparrow\rangle]. \quad (2.13b)$$

Here, we omitted the argument of the orbital functions, $\mathbf{r}$, and used the canonical basis for the angular momentum basic functions, see; for example, (Edmonds, 1974; Varshalovich *et al.*, 1988).

It is noteworthy that at $\mathbf{k} \neq 0$ the Γ_8 quadruplet splits into two branches, usually denoted as the heavy-hole band (with a larger absolute value of effective mass) and the light-hole band (with a smaller magnitude of effective mass), see Figure 2.2. As illustrated in Figure 2.2, for $\mathbf{k}$ directed along one of the cubic axes $\langle 100 \rangle$, these bands correspond to the states with the projections of the total angular momentum onto the $\mathbf{k}$ being $\pm 3/2$ and $\pm 1/2$, respectively. Note that the situation is somewhat more complex for the arbitrary orientation of $\mathbf{k}$, moreover, in some semiconductors the signs of the heavy- and light-hole effective masses can be different. The details of the valence band fine structure can be found, for example, in (Bir and Pikus, 1974; Ivchenko and Pikus, 1997; Yu and Cardona, 2010).

Furthermore, the spin-orbit interaction affects the electron and hole dispersion in semiconductor and low-dimensional structures where the free motion is possible at least along one spatial direction. Indeed, in noncentrosymmetric systems, such as in zinc-blende lattice semiconductors and in many nanostructures, the energies of two spin states with a given wavevector $\mathbf{k}$, $E_{n\pm 1/2}(\mathbf{k})$, are not equal. The spin-dependent contribution in the conduction band effective Hamiltonian of a semiconductor with the T_d point symmetry is cubic in the wavevector (Dresselhaus, 1955)

$$\mathcal{H}_D = \gamma_c \left[\sigma_x k_x (k_y^2 - k_z^2) + \sigma_y k_y (k_z^2 - k_x^2) + \sigma_z k_z (k_x^2 - k_y^2) \right], \quad (2.14)$$

with γ_c being the coefficient, $\gamma_c \approx 20 \text{ eV}\text{\AA}^3$ for GaAs. The contribution of Eq. (2.14) to the effective Hamiltonian is termed the *Dresselhaus term*. Here, we use the set of cubic axes, $x \parallel [100]$, $y \parallel [010]$, $z \parallel [001]$. Such a form of spin splitting of the conduction band is readily understood from the symmetry arguments: The set of the Pauli matrices forms the basis of the F_1 representation of the T_d point symmetry group, the combinations $k_x(k_y^2 - k_z^2)$, $k_y(k_z^2 - k_x^2)$, $k_z(k_x^2 - k_y^2)$ also transform according to the same irreducible representation. For the valence band states, both $\mathbf{k}$ -linear and $\mathbf{k}$ -cubic terms are allowed (Pikus *et al.*, 1988; Ivchenko and Pikus, 1997). In wurtzite semiconductors and in nanostructures the symmetry is, as a rule, lower and allows for the coupling between the spin and the linear combinations of the wavevector components (Rashba and Sheka, 1959; Ohkawa and Uemura, 1974; Vas'ko, 1980; Bychkov and Rashba, 1984; Dyakonov and Kachorovskii, 1986). In any case, the spin-orbit contributions to the effective electron Hamiltonian can be written in a form similar to Eq. (2.10) (Winkler, 2003; Ivchenko, 2005; Dyakonov, 2017):

$$\mathcal{H}_{so} = \frac{\hbar}{2} (\boldsymbol{\sigma} \cdot \boldsymbol{\Omega}_{\mathbf{k}}). \quad (2.15)$$

Here, the pseudovector, $\boldsymbol{\Omega}_{\mathbf{k}}$, is an odd function of the electron wavevector, $\boldsymbol{\Omega}_{\mathbf{k}} = -\boldsymbol{\Omega}_{-\mathbf{k}}$. This is the result of the time reversal, $t \rightarrow -t$, symmetry: in nonmagnetic systems at the $t \rightarrow -t$ transformation both spin and wavevector change their signs ensuring the invariant form of the interaction. The spin-dependent combinations involving even powers of the wavevector components are thus forbidden. Equation (2.15) demonstrates that the electron moving in a noncentrosymmetric structure experiences an effective magnetic field, whose strength and direction depend on the electron wavevector. Hence, the electron orbital motion becomes coupled with the electron spin dynamics.

So far, we have discussed the single particle states and the single particle energy dispersion. Importantly, the interaction between electrons is spin-dependent as well. This is because of the Pauli principle, which imposes the antisymmetry of the total wavefunction of fermions with respect to the particle permutations. Even if the spin-orbit interaction is absent and, correspondingly, the spin part of the two-electron wavefunction can be separated from the orbital part, the symmetry

of the orbital part of the wavefunction is enforced by the symmetry of its spin part. According to the momentum summation rules, the total spin S of the electron pair can be either 0 or 1. The spin part of the function with $S = 0$, that is, *spin singlet*, reads

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2), \quad (2.16)$$

and it is antisymmetric with respect to the particle permutations (the subscripts 1 and 2 enumerate the electrons). Hence, the orbital function of the spin singlet is symmetric with respect to the electron permutations. By contrast, the spin parts of the $S = 1$ *triplet* have the form

$$|\uparrow\rangle_1|\uparrow\rangle_2, \quad \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2), \quad |\downarrow\rangle_1|\downarrow\rangle_2, \quad (2.17)$$

being symmetric with respect to the permutations, making the corresponding orbital wavefunction antisymmetric. Difference of the orbital wavefunctions results in the difference of the Coulomb interaction energies. Hence, the states with $S = 0$ and $S = 1$ are split due to the Coulomb interaction. In many cases it is convenient to avoid the antisymmetrization procedure and introduce the effective *exchange interaction* in the form of interaction between the spins as

$$\mathcal{H}_{exch} = -\mathcal{J}\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{s}}_2, \quad (2.18)$$

where $\hat{\mathbf{s}}_i$ is the spin operator of i th electron, $i = 1, 2$, and $\mathcal{J}$ is the exchange interaction constant. One can readily check that the eigenstates of the $\mathcal{H}_{exch}$ operator are indeed the singlet and the triplet states, see Eqs (2.16) and (2.17), with the energies $3\mathcal{J}/4$ and $-\mathcal{J}/4$, respectively. In the presence of the spin-orbit interaction, the scalar product of the spin operators in Eq. (2.18) is replaced by the combination (Kavokin, 2004; Bădescu *et al.*, 2005; Gangadharaiah *et al.*, 2008; Glazov and Kulakovskii, 2009)

$$-\sum_{\alpha\beta} \mathcal{J}_{\alpha\beta} \hat{s}_{1,\alpha} \hat{s}_{2,\beta},$$

where the components of the exchange constant tensor $\mathcal{J}_{\alpha\beta}$ are determined by the symmetry of the system, orbital wavefunctions of the electrons and the spin-orbit coupling. It is worth stressing that the exchange interaction is just a consequence of the permutation symmetry requirements and the Coulomb interaction. The exchange interaction is of major importance in diluted magnetic semiconductors, for example GaAs:Mn, where the paramagnetic ions with unpaired electron spins, such as manganese, are incorporated. It is also very important for *excitons*, Coulomb-correlated electron-hole pairs: The exchange interaction between an electron and a hole determines to a large extent the exciton

fine structure and serves as a driving force for the spin dynamics of excitons (Bir and Pikus, 1974; Ivchenko, 2005). It is noteworthy that, for electronic excitations in semiconductors, it is by far more important compared with the magnetic interaction between the charge carriers.

Magnetic interactions play a role where the spins of the lattice nuclei are involved. For a brief introduction to the physics of nuclei we refer the reader to the concise lecture course by Landau and Smorodinsky (2014). The basic interactions involving nuclear spins in solids are presented in (Meier and Zakharchenya, 1984; Abragam, 2002; Dyakonov, 2017). Consider a nucleus with the spin I . The nucleus can be treated as a point-like magnetic moment $\boldsymbol{\mu}$ since the size of the nucleus is in the range of $(1 \dots 10) \times 10^{-5} \text{ \AA}$, so it is far smaller than the lattice constant in semiconductors, as well as, naturally, the de Broglie wavelength of the electron. The magnetic moment operator $\hat{\boldsymbol{\mu}}$ is related to the nuclear spin operator $\hat{\mathbf{I}}$ as

$$\hat{\boldsymbol{\mu}} = \hbar \gamma_I \hat{\mathbf{I}} = \frac{\mu_I}{I} \hat{\mathbf{I}}, \quad (2.19)$$

where γ_I is the gyromagnetic ratio of the nucleus and μ_I is the nuclear magnetic moment. The interaction of nuclear spin with the magnetic field $\mathbf{B}$ is described by the Hamiltonian

$$\mathcal{H}_{B,N} = -(\hat{\boldsymbol{\mu}} \cdot \mathbf{B}). \quad (2.20)$$

Analogous to Eq. (2.9), one also introduces the nuclear g -factor, g_n , as $g_n = \hbar \gamma_I / \mu_N$, where $\mu_N = |e| \hbar / (2m_p c)$ is the nuclear magneton and m_p is the proton mass, recasting Eq. (2.20) in the form:

$$\mathcal{H}_{B,N} = -g_n \mu_N (\hat{\mathbf{I}} \cdot \mathbf{B}), \quad g_n = \frac{\mu_I}{I \mu_N}. \quad (2.21)$$

Note the difference in signs in Eqs (2.9) and (2.21): Formally, it comes from the fact the the nucleus is positively charged, such a sign convention is used in (Abragam and Bleaney, 1970; Kalevich *et al.*, 2017) and is quite convenient. Typically, the nuclear g -factors are in the order of unity, hence, in the same magnetic field the Zeeman splittings of nuclear spin sublevels are about three orders of magnitude smaller than those of the electron; see Table 2.1 where the magnetic moments and gyromagnetic ratios for some relevant isotopes are listed.

The nuclear magnetic moment $\boldsymbol{\mu}$ produces the magnetic field that acts on the electron spins and on the spins of the neighboring nuclei. The *hyperfine interaction* Hamiltonian of the conduction band electron and nuclear spin in III–V and II–VI semiconductors can be written in the form

$$\mathcal{H}_{hf} = A v_0 (\hat{\mathbf{I}} \cdot \hat{\mathbf{s}}) |\varphi_c(\mathbf{R})|^2, \quad (2.22)$$

where v_0 is the *primitive cell* volume and $\varphi_c(\mathbf{R})$ is the conduction band envelope function, Eq. (2.4), at the nucleus. Note that primitive cell of GaAs-type semiconductor contains two atoms. Sometimes the term unit cell is used in the literature. Here and in what follows we use primitive cell term to avoid confusion. The constant A depends on the material and nucleus type; it is, as a rule, in the order of $100 \mu\text{eV}$. For the conduction band electrons in GaAs-type semiconductors, the hyperfine interaction results from the contact Fermi interaction of the electron and nuclear spins because the orbital Bloch amplitude $\mathcal{S}(\mathbf{r})$ is nonzero at the host lattice nuclei. For the valence band holes, the orbital Bloch amplitudes $\mathcal{X}(\mathbf{r})$, $\mathcal{Y}(\mathbf{r})$ and $\mathcal{Z}(\mathbf{r})$ vanish at the nucleus and the hyperfine interaction of hole, and nuclear spins is provided by the dipole–dipole interaction. It is about an order of magnitude weaker compared with the electrons. The hyperfine interaction results in the transfer of spin between the electron and nuclear spin systems in semiconductors. Equation (2.22) demonstrates that the polarized nuclei exert the effective magnetic field, known as the *Overhauser field*, onto the electron spin. The polarized electrons also create an effective magnetic field acting on the nuclear spins known as the *Knight field*. The theory of the hyperfine interaction of the charge carrier spins with the host lattice nuclei is presented in Chapter 4.

The interaction between the lattice nuclei spins is simply the *dipole–dipole interaction* of the localized magnetic moments. The Hamiltonian of interaction between the nuclear spins $\mathbf{I}$ and $\mathbf{I}'$ has the form

$$\mathcal{H}_{dd} = \frac{\mu_I \mu_{I'}}{II'} \left(\frac{\hat{\mathbf{I}} \cdot \hat{\mathbf{I}}'}{r^3} - 3 \frac{(\hat{\mathbf{I}} \cdot \mathbf{r})(\hat{\mathbf{I}}' \cdot \mathbf{r})}{r^5} \right), \quad (2.23)$$

where $\mathbf{r}$ is the vector connecting the nuclei. A typical magnetic field (also known as the *local magnetic field*, B_L) created at a given nucleus by the other host lattice nuclei is in the order of $B_L \sim 0.1 \text{ mT}$. In contrast to electrons in semiconductors, an overlap of nuclei is negligible and the exchange interaction between the nuclei is irrelevant. Physically, the dipole–dipole interaction between the nuclear spins results in the redistribution of the spin polarization among the ensemble of nuclei, that is, in the nuclear spin diffusion and in the thermalization of the nuclear spin system. It follows from Eq. (2.23) that the dipole–dipole interaction does not conserve the total spin of the nuclei and, as a result, may serve as a source of nuclear spin relaxation.

Moreover, a nucleus with spin I higher than $1/2$ has a nonzero electric *quadrupole moment*. Qualitatively, its presence is caused by the non-spherical charge distribution in the nucleus, hence, the quadrupole moment serves as a measure of nuclei being prolate or oblate. As a result, the nuclear spin interacts with the electric field gradients:

$$\mathcal{H}_Q = \sum_{\alpha\beta} \frac{\partial^2 \mathcal{V}}{\partial r_\alpha \partial r_\beta} \frac{|e|Q}{6I(2I-1)} \left[\frac{3}{2} (\hat{I}_\alpha \hat{I}_\beta + \hat{I}_\beta \hat{I}_\alpha) - I(I+1) \right]. \quad (2.24)$$

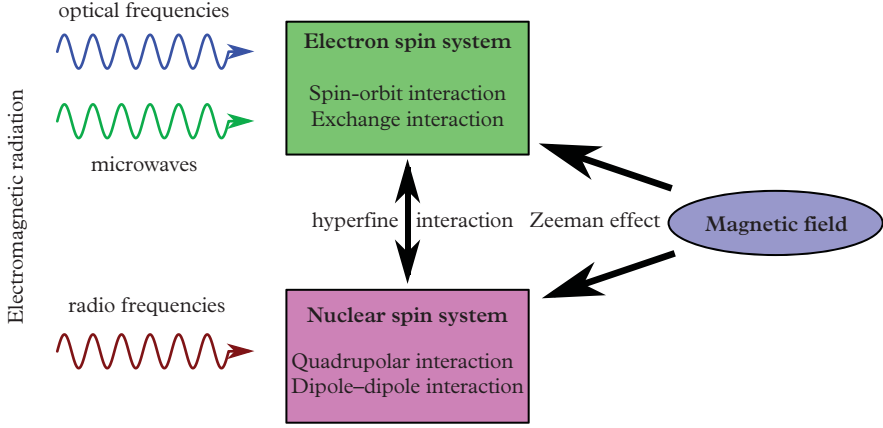


Figure 2.3 Chart of the important interactions in spin systems.

Here, $\mathcal{V}$ is the potential of the electric field, and Q is the constant, namely the quadrupole moment, which depends on the isotope, see Table 2.1. The form of the *quadrupole interaction* Hamiltonian, Eq. (2.24), can be easily understood from the symmetry arguments: The products of the pseudovector components $\hat{I}_\alpha \hat{I}_\beta + \hat{I}_\beta \hat{I}_\alpha$ transform exactly in the same way as the components of the rank-two tensor $\partial^2 \mathcal{V} / \partial r_\alpha \partial r_\beta$. It is noteworthy that for a spin 1/2, the symmetrized products $\hat{I}_\alpha \hat{I}_\beta + \hat{I}_\beta \hat{I}_\alpha$ either vanish or reduce to the unit 2×2 matrix, and the interaction in Eq. (2.24) vanishes. The quadrupole interaction defines the eigenstates of the nuclear spin system in an absence or in weak magnetic fields. It is especially important in strained systems, such as quantum dots or other heterostructures with lattice constant mismatch, where the elastic deformations induce the field gradients (Sundfors, 1969a; Bulutay, 2012). In solid solutions such as $\text{Ga}_x\text{Al}_{1-x}\text{As}$ some of the Ga nuclei are substituted with the Al ones, as a result, the arsenic nuclei experience the electric field gradients due to the asymmetric environment (Meier and Zakharchenya, 1984). Figure 2.3 illustrates the main interactions in the spin systems in semiconductors and impact of electromagnetic radiation with different frequencies on the electron and nuclear spins.

2.4 Optical orientation of electron spins and the Hanle effect

Nonmagnetic injection, detection, and manipulation of electron and nuclear spins are among the most essential tasks of semiconductor spintronics. The possibilities to control the electron spins by static or alternating electric field, including the electric field of the light wave, are enabled by the spin-orbit interaction. The *optical*