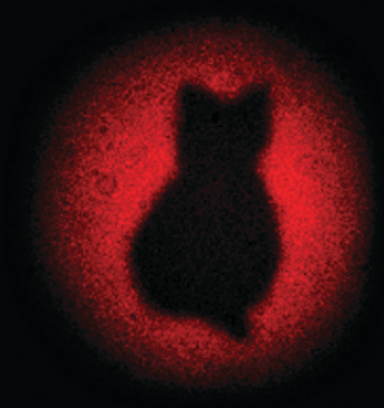
WILLIAM J. MULLIN

QUANTUM WEIRDNESS



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William J. Mullin

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For Sandra

Preface

Quantum mechanics is the science that is fundamental to understanding all nature. It explains the central workings of atoms, molecules and of the more complicated systems they form: solids, liquids, gases, plasmas, including biological systems. It has allowed us to penetrate the atom to see that protons and neutrons are made of quarks and gluons. We now know about the multitude of elementary particles, such as neutrinos and Higgs bosons.

What quantum mechanics tells us is that particles, for example, the electron or the helium atom, once thought to be small solid entities, often act more like waves than particles. In an opposite situation, light, once thought to be entirely explained by wave behavior, is found to have energy carried by particle like quanta called photons. There is a particle–wave duality in nature that was not fully realized before about 1920. Erwin Schrödinger developed his famous wave equation in 1925 and we have been using it ever since to explain the universe.

Waves do things that seem strange if you are expecting particle like behavior. If I drop a stone in a pond, a wave spreads out from the source and is soon all over the pond, while we would expect a particle to be more like the stone that caused the wave, centered in one place at a time following a well-defined path. There can be two or more waves on the pond at the same time, and they can interfere with each other. We will see that such a superposition means a particle can interfere with itself and basically be in many places at once. If you can identify a wave's wavelength, the distance between crests, then the wave must extend out some distance; if it is localized into only one crest, you are unable to identify a wavelength. In quantum mechanics, this is related to the famous Heisenberg uncertainty principle, in which simultaneous certainty about a particle's position and momentum is impossible. Waves can penetrate into places that are a bit unexpected. Quantum tunneling results in an electron more or less going right through the equivalent of a solid wall on occasion. On the other hand, particles often act, well, like particles: one often sees pictures of tracks from detectors in large accelerators and they seem to travel in well-defined lines there—very much as one would expect particles to behave. How does the wave interpretation apply there?

The predictions of the Schrödinger equation agree remarkably well with experiment, but the way we seemingly must interpret what it is telling us often seems very bizarre. The oddness is often described as "quantum weirdness." As another example, in one experiment two particles separate until they are very far apart. We make a measurement on particle 1 at a point A and get a result. Instantly, we are able to predict with certainty what the result will be on a similar experiment on particle 2 at point B. In classical mechanics, this is possible only if the particles possess built-in properties that are correlated before the measurement, and determine their results. But quantum mechanics seems to say that it was the measurement process that creates the results! Einstein called this "spooky action at a distance" and thought that a good theory should not include such nonlocality. But later experiments confirmed that this remarkable nonlocality seems a necessary component of the theory. We have been trying to puzzle out the weirdness ever since.

Quantum effects in nature can seem weird to us because we grow up seeing the world in the light of Newtonian mechanics, even if we have not studied physics at all. Balls follow smooth arcs, more or less like the particle tracks. We can certainly specify a baseball's position and speed simultaneously. Balls bounce off walls rather than tunneling through. So we become accustomed to this "classical" Newtonian way of viewing things; when atoms or electrons behave very differently, that is, quantum mechanically, we find surprises.

A deep understanding of quantum mechanics often requires some complicated advanced mathematics. One can say the words that interpret the equations, but without the math, one sees only half the picture. On the other hand, a lot of the basic ideas are accessible with just a minimum of algebra. Algebra alone does not give the whole picture, but it does yield considerably more depth of understanding than just words without *any* math. So the reader of this book should know basic algebra (and some trigonometry). An introductory classical (Newtonian) physics background is useful too, but we provide an appendix of physics terms to help those who have not had any physics or who have forgotten much of it. The algebra background is probably more important than the physics. There are four sections of the book in which the math manipulations, while still algebra, are somewhat more difficult than average. These are Secs. 12.1, 13.3, 14.1, and 15.4. If necessary, skip the math there, and get the basic idea from the words.

This book is not a text in quantum mechanics. It does not solve the Schrödinger equation for any situation. While it discusses the nature of the wave functions in standard cases like the harmonic oscillator, the particle in a box, and the hydrogen atom, it does not tell how these are derived. Moreover its sampling of topics is more likely to be among those that satisfy the criteria of either illustrating the weirdness of quantum mechanics or some fundamental aspect of it.

The bibliography has a mixture of references to elementary treatments and full research books and papers in order to give credit to authors whose works were useful to me in writing this book and for readers who are at a more advanced level and want to pursue further information.

My thanks to Christopher Caron, a high-school-age friend whose interest in learning about quantum mechanics stimulated the writing of much of the material in this book. My long-time research collaborator Franck Laloë of École Normale Supérieure in Paris has for years helped spark my interest in quantum questions, and his advanced treatise *Do We Really Understand Quantum Mechanics* has been one inspiration for the present, much more elementary book. I thank Franck and UMass colleagues Robert Krotkov and Guy Blaylock for critical readings of the manuscript. However, errors and poor explanations are entirely my responsibility. Many thanks to copyeditor Elizabeth Farrell for making the text read more smoothly.

WJM
Amherst, MA

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1

Waves

> Just take your time—wave comes. Let the other guys go, catch another one.
>
> Duke Kahanamoku

1.1 Some history

Quantum mechanics is the basic theory of matter. It was originally developed in the early part of the twentieth century to explain the properties of the atom, but it has gone much deeper and broader than that. It ranges from systematically explaining the properties of the elementary particles (quarks, electrons, protons, etc.) to giving a deep understanding of solids, liquids, and gases. It is the most successful scientific theory ever developed, allowing us to calculate details to many decimal places of accuracy.

The only known limitation of quantum mechanics is in joining it to the theory of gravity, that is, Einstein's general relativity theory; we still do not have a quantum theory of gravity. Nevertheless, quantum mechanics has still been remarkably successful in understanding many features of the cosmos, such as how the elements evolved in stars, and how black holes can decay. It is possible that a more fundamental approach, such as "string theory," will allow a synthesis of quantum mechanics and gravity. We will see.

In a first course in physics, we learn Newton's laws, $F = ma$, etc. These laws (and those of Maxwell, for radiation) tell us how to understand the motion of baseballs, satellites, and many properties of light and radiation. Such "classical mechanics" explains how large objects behave. But it does not seem to explain the properties of small objects, like atoms. Classical mechanics gives us the basic concepts of mass, momentum, energy, etc., that we need to interpret everyday life. We also carry those concepts over into quantum mechanics, but they are interpreted somewhat differently there. When we do an experiment on an atom, we must necessarily use large objects as our measurement devices (we have to have knobs we can turn with our hands), and there is a meeting of the large and the small, that is, of classical mechanics and quantum mechanics. But the assumption here is that quantum mechanics is the more fundamental theory and that it can, in principle, be applied to the large objects, so that we can derive Newton's laws from quantum mechanics. Moreover, there are instances where large-scale behavior (as, say, in the "superflow" of liquid helium at low temperature) defies Newton's laws, and only quantum mechanics can explain the experiment. Modern electronic equipment (based on the transistor) is based on our knowledge of solid-state physics from quantum mechanics. Nevertheless, we tend to look at nature from a classical mechanical point of view. When we throw a baseball, we have an intuitive understanding of where it will go. If it suddenly changed course by 90°, we would think that was weird. If the baseball seemed to be in two places at once, we might believe we were being tricked by a magician. But, in quantum mechanics, having a particle in two places at once is exactly what we are led by experiment to believe is true. Such weirdness is what we have to deal with.

As successful as quantum mechanics has been as a calculational device, there is a fundamental problem: interpreting just what it means about the nature of matter. The basic object in quantum mechanics is the wave function; we know it is to be used in a probabilistic way, but that still leaves open whether it is a "real" object or a kind of bookkeeping system existing just in our minds. We are led by nature into describing a kind of instantaneous wave function communication or correlation at arbitrary distances that would seem to violate Einstein's relativity principle that things cannot travel faster than the speed of light, and yet, well... no, it sneakily manages *not* violate it. The basic effects are just downright weird when we try to look at them from the point of view of classical mechanics. What quantum mechanics actually means has been debated for 90 years, and the debate continues even more energetically today. Most physicists in the past had not worried about the philosophical issues of interpretation; they left that to take place in esoteric journals like *Foundations of Physics* and took the attitude of "shut up and compute," since they could explain their experiments independently of the profound implications, which were left to the philosophers and a few physicists.

However, in recent years, experimenters in laser and atomic physics have come to be able to manipulate individual atoms, and the weird behavior has led to the possibility of quantum computing and similar applications that may mean the weirdness has practical applications! This has meant that the funny business in quantum mechanics has escaped from the esoteric journals and now is in the frontline physics journals, with even more people involved. Weirdness is now relevant, and even philosophical interpretation has become a hot subject.

In classical mechanics, a baseball, the moon, an atom, and an electron are all particles, while light is a wave. And yet, in quantum mechanics, an electron sometimes behaves like a wave and sometimes like a particle. The terms "particle" and "wave" are said to be complementary; the concept that applies depends on the experiment one is doing. The particle of light is the photon, which also has this dual behavior. The fact that an object can morph between wave and particle behavior adds to the weirdness when we try to view experiments from a classical mechanics point of view. *The amazing thing is that the mathematics of quantum mechanics can accommodate both behaviors and explain either kind of experiment; it is the interpretation we try to place on the mathematics that gets us into debates.*

The basic law of quantum mechanics is the Schrödinger equation, which, for a single particle in one dimension, looks like this (don't be alarmed!):

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\psi(x,t). \tag{1.1}$$

This beast is certainly one of the most fundamental equations of physics. Solving it is an exercise in advanced calculus, which is not the path these notes will follow. However, we *will* center our attention on the wave function $\psi(x,t)$ and how it is used and interpreted in quantum mechanics. We will avoid all of the advanced math and just use basic algebraic manipulations "at worst." The Schrödinger equation is what is called a wave equation; it has solutions that describe waves. So, we need to understand the basic properties of waves. But we also need particle concepts. So, we will spend some space going over the basic particle and wave concepts here.

1.2 Classical wave dynamics

We will use a few standard physics ideas, like momentum and energy, in our discussion. These originally arose in classical Newtonian mechanics. We have listed and briefly explained a number of these terms in the appendix at the end of the book. Just a qualitative understanding of these terms is probably all that is required for reading this book. However, in quantum mechanics, we solve a wave equation, and the wave function indeed acts like a wave, so we need a bit more detailed understanding of the physics of waves. That subject is usually treated in discussions of water, sound, or electromagnetism. Such ideas will be immediately used to understand matter waves. As a supplement to the rapid introduction to waves given in this chapter, I suggest the tutorial and animations given in the website associated with my book on sound: http://billmullin.com/sound/, which we reference occasionally below.

Common waves travel in an elastic medium, a material that has the ability to rebound from a displacement. Suppose we have a number of masses connected by springs. If you displace the first mass and let go, the nearest connected spring is stretched, which displaces the second mass, which stretches the second spring, etc. The potential and kinetic energies are carried down the chain in the form of a wave. To see an animation of this, go to this website associated with my sound textbook: http://billmullin.com/sound/AnimationPages/FigX-6.html. Similarly, if you drop a stone in water, the water is displaced downward, which pulls the neighboring water down, and a wave progresses outward from where the stone entered. Waves occur in many places: in the air, as sound waves; on guitar strings that have been plucked; and in the vacuum of space, as electromagnetic radiation such as, for example, light waves or x-rays. Here, we will concentrate on the waves on a stretched string (say, a bungee cord). I shake one end of the elastic cord and that causes a wave to travel down it. As in a water wave, the medium (a particular part of the string) moves up and down, but the wave itself (the elastic energy) travels along the string. So, we distinguish between *medium* motion and *wave* motion. If the wave motion is horizontal, the medium motion is up and down, that is, perpendicular to the wave motion; we call this a transverse wave. A useful animation, from the website that shows the distinction between wave motion and medium motion, is given at http://billmullin.com/sound/AnimationPages/FigsI-1&2.html.

There are several types of transverse traveling waves; we distinguish between impulsive and oscillatory waves. The former is a single or a series of bursts. (The corresponding sound wave would be formed by clapping one's hands.) It moves along as a localized bump on the string. An oscillatory wave has regular repeating pulses.

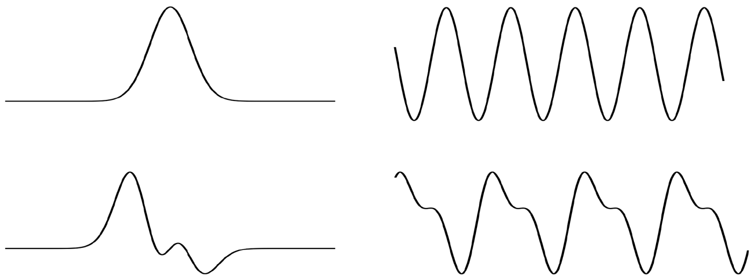

Figure 1.1 Four wave types. The two on the left are pulse waves, while the two on the right are oscillatory (imagine the form continues to repeat beyond the figure). The top oscillatory wave is sinusoidal, but the lower one is non sinusoidal.

An important case of this is the sinusoidal wave, which has a special mathematical shape. Figure 1.1 illustrates an instantaneous picture of these waves. You have to imagine them moving along, say, to the right. The animation shows a sinusoidal wave in motion. A pulse wave in motion and reflecting from a wall can be seen at http://billmullin.com/sound/AnimationPages/FigsI-16&17.html.

It is possible to put two distinct waves on the same medium, but they will add to form a composite wave; this adding is known as *superposition*. (The second oscillatory wave in Fig. 1.1 is the superposition of two sinusoidal waves of different wavelengths [discussed later on in this section]). A very important property of waves is interference. Note that the transverse displacement of a wave can be positive or negative (above or below normal). If two waves pass through each other, where the two positive portions, or two negative portions, of the waves overlap, they will add, forming a larger displacement; but where a positive overlaps a negative portion, they will tend to cancel out. The reinforcement is called constructive interference, and the canceling is destructive interference. Take a look at the animation examples at http://billmullin.com/sound/AnimationPages/FigsI-19&20.html.

Waves reflect off boundaries; for example, sound wave reflecting from canyon walls result in an echo. In a concert hall, sounds "reverberate" by reflection, adding to the quality of the music. On a string (and elsewhere), the reflected wave can interfere with the original wave, producing what are called standing waves, which we talk about below.

Sinusoidal waves have a specific terminology; in the sinusoidal wave pictured in Fig. 1.1, the distance between the peaks (or the distance between the troughs) is the wavelength, denoted by the Greek letter lambda (λ); the time for any point of the medium to undergo a complete cycle (a round trip of some point on the string from the maximum through a minimum and back up) is called the period (T; measured in seconds). The frequency f is the number of cycles per second (this used to be indicated as cps, but now the units are hertz [abbreviated as Hz]). Since the seconds per cycle is indicated by T, we have the relation

$$T = \frac{1}{f}. \tag{1.2}$$

If the period is 0.5 s, the frequency is 2 Hz. The amplitude of the wave is the maximum distance any point is displaced from the normal string position. Thus, the distance from the minimum displacement to the maximum is twice the amplitude.

The distance a sinusoidal wave travels in a period T is the wavelength λ. Thus, the wave velocity (distance per time) is given by

$$v = \frac{\lambda}{T} = f\lambda, \tag{1.3}$$

a fundamental wave formula. The velocity of the wave v is a property of the medium (depending on its elasticity, density, etc.) and is unchanging from wave to wave on the same medium. Thus, the shorter the wavelength of the wave, the higher its frequency, so the product is a constant.

As examples, consider sound and light: middle C in music has a frequency of 261.6 Hz, a wavelength of 1.32 m, and sound wave velocity of 345 m/s. Red light has a frequency of about 4.5×10^{14} Hz, and a wavelength of 6.7×10^{-7} m, with a wave velocity of 3.0×10^{8} m/s.

Suppose we have a guitar string stretched tautly between two posts. Any wave on the string will be reflected from the ends, and the reflections will interfere with the original wave. The result is a wave that does not seem to move to the right or left but just up and down—a standing wave. An animation showing how two traveling sinusoidal waves form a standing wave is shown at http://billmullin.com/sound/AnimationPages/FigsII-2.html. Figure 1.2 shows a standing wave and its motion through one period. There are points at which there is always complete destructive interference; these are the nodes denoted by N in the figure. Positions at which maximum constructive interference occurs are the antinodes A in the figure.

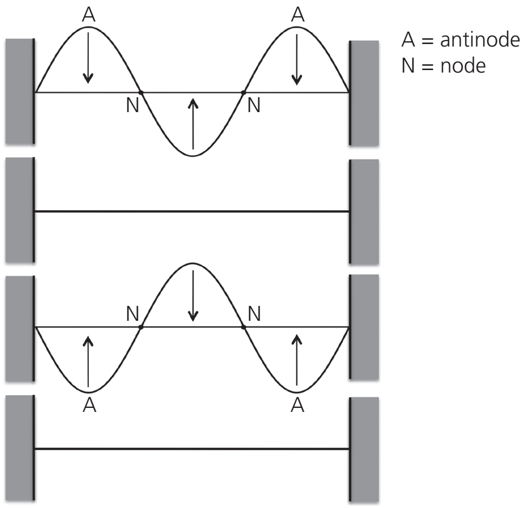

Figure 1.2 A standing wave at times 0, $T/4$, $T/2$, $3T/4$, and T, where T is the period of the wave.

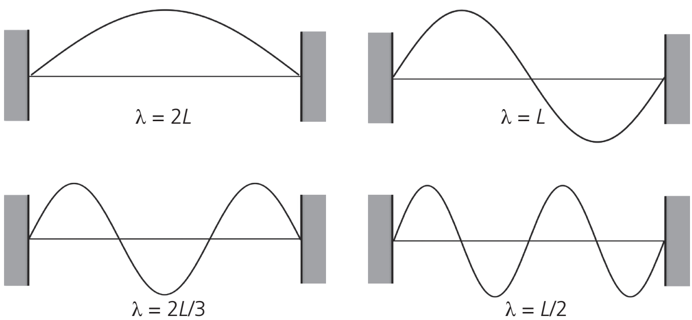

Figure 1.3 Standing waves with wavelengths $2L$, $2L/3$, L, and $L/2$, where L is the distance between the endpoints.

Standing waves can have only the wavelengths that "fit" into the distance between the walls, that is, have zero displacements at the ends. The four standing waves with the longest wavelengths are shown in Fig. 1.3. The wave with the wavelength $2L$ is called the fundamental wave. It has the frequency $f_1 = v/\lambda = v/2L$. The next longest wavelength is L, and that wave has the frequency $f_2 = v/L = 2f_1$. The next one has the frequency $f_3 = 3f_1$, and this continues. This sequence of frequencies is called the harmonic series. The frequency f_1 is the fundamental, or first, harmonic, f_2 is the second harmonic, etc. Not every frequency can occur on the string, only those that are multiples of the fundamental frequency. The spectrum of frequencies is discrete. (You might say it is *quantized*.)

When I strum or pluck a note on a guitar string, I rarely get a resulting wave that is purely only one of these harmonics; rather, I get a complicated wave that is a superposition of many harmonics. The sound resulting is pleasant because all the frequencies are in tone with one another; they are all multiples of the fundamental. Any arbitrary complex wave shape can be considered as a superposition of these selected harmonic standing waves. The nonsinusoidal oscillatory wave in Fig. 1.1 is made up of first and second harmonics. The sound excited by the string also has this property: it is composed of sinusoidal waves of these same frequency components. Figure 1.4 shows some complex wave forms of musical instruments. Each can be considered as being made up of a discrete set of harmonic sinusoidal waves; they differ in the relative amplitudes of the various components. When we analyze a wave into its harmonic sinusoidal components, we are doing what is called Fourier analysis. Any traveling repeating (oscillatory) wave can be constructed from a discrete harmonic sinusoidal series.

We can also do a Fourier analysis of an arbitrary wave shape, even if it is not repeating. For example, the pulse wave in Fig. 1.1 can be Fourier analyzed. However, the sinusoidal waves needed will not be a discrete

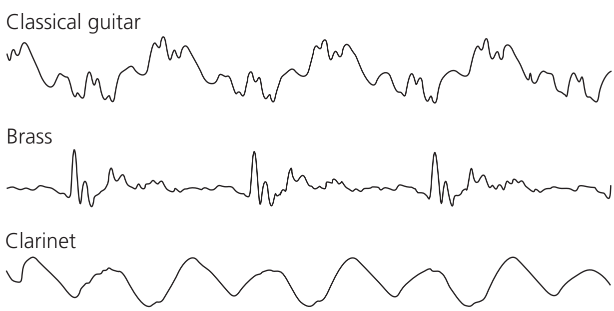

Figure 1.4 Wave forms from selected musical instruments. What is plotted is pressure amplitude versus time. Our guitar string would produce a similar complex wave form. Each can be considered a superposition of sinusoidal harmonic waves.