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# Necessary Existence



Alexander R. Pruss &  
Joshua L. Rasmussen

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Great Clarendon Street, Oxford, OX2 6DP,  
United Kingdom

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First Edition published in 2018

Impression: 1

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Published in the United States of America by Oxford University Press  
198 Madison Avenue, New York, NY 10016, United States of America

British Library Cataloguing in Publication Data

Data available

Library of Congress Control Number: 2017954208

ISBN 978-0-19-874689-8

Printed and bound by  
CPI Group (UK) Ltd, Croydon, CR0 4YY

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# Acknowledgments

Special thanks to Nathan Ballantyne, Luke Barnes, Todd Buras, Sean Carroll, Trent Dougherty, Felipe Leon, Rachel Rasmussen, Daniel Rubio, Christopher Tomaszewski, Luke van Horn, Peter van Inwagen, and Craig Warmke for their valuable feedback on earlier drafts. We are also grateful to Christopher Tomaszewski for his work on the index, and to our families for their patience with us.



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# 1

## Introduction

People have at all times been talking of an *absolutely necessary* Being.

—Immanuel Kant<sup>1</sup>.

### 1.1 The Question of Necessary Existence

Virtually everything we encounter in ordinary experience can, apparently, fail to exist. Cars, iPads, telephone poles, towers, flowers, mittens, kittens, bricks, sticks, planets, stars, dust: none of these things *have to* exist, it seems. We can easily imagine a universe without such things; and there was a time before anything of this sort existed. It seems the things with which we are familiar are *contingent*—i.e., possibly absent.<sup>2</sup>

Is everything contingent? Or, might there also be one or more things that exist *of necessity*? A necessary thing, as we are thinking of it, would be something that exists no matter what possible situation obtains. Its non-existence at any time would be impossible in the strongest sense. So, for example, a necessary thing cannot be assembled or disassembled. It cannot snap into being or snap out of being. It cannot *not* exist—no matter what. Is there anything like that?

<sup>1</sup> Kant and Müller 1907.

<sup>2</sup> There is Williamson's proposal (Williamson 2001) that *everything* exists of necessity. But even Williamson allows for contingency in the world. He says it is contingent, for instance, whether a given physical thing is physical (pp. 12–13). And in general, where one might ordinarily say that objects pass in and out of existence, Williamson will say they pass in and out of a certain basic category of being (a category he calls 'concrete,' which is to be distinguished from the category of causally-capable things, which we call 'concrete'). Someone in Williamson's shoes could wonder, therefore, whether anything is necessarily on the side of this basic category which they are currently on. Many of the questions and arguments we raise have parallels in the Williamson setting, and we leave working out these parallels to the interested reader.

In this book, we are primarily interested in the prospect of a necessary *concrete* thing, which we take to be anything capable of causation. We intend to use the term ‘cause’ in a minimal sense to designate anything that acts as an antecedent condition (or entity) that is at least partially causally responsible for some event. Causes need not be sufficient for their effects: for instance, we could say that *Adam’s smoking* caused *Adam’s lung cancer*, even though his smoking didn’t *have to* cause lung cancer. Also, we allow substances to be causes, even if ‘substance causation’ is to be analyzed in terms of ‘event causation.’<sup>3</sup> In general, we take no sides on what sorts of things can be causes; we leave it open, for instance, whether numbers, properties, propositions, sets and other so-called ‘abstract’ entities may have causal powers and so also count as ‘concrete’ in our stipulated sense. Our question, then, is this: is there anything that (i) possibly causes something (is *concrete* in our sense) and (ii) exists no matter what? In keeping with tradition, we will call anything that satisfies both (i) and (ii) a ‘necessary being.’ We inquire: are there any necessary beings?

We will set out a case for an affirmative answer. We will lay the groundwork in Chapter 2, where we motivate a standard logic of the *necessary* and the *possible*. Then, over the course of six chapters, we will present six arguments for the existence of a necessary being. The first argument is an up-to-date defense of a traditional explanation-based argument from contingency. The next five arguments are new possibility-based arguments which make use of twentieth-century advances in the logic of necessity. We aim to present the arguments as possible pathways to an intriguing and far-reaching conclusion. In the final chapter, we will address what we take to be the most challenging objections to the existence of necessary beings. Finally, in an appendix, we will offer a number of additional arguments for a necessary being, without detailed discussion, in the hope of inspiring further inquiry.

## 1.2 Why Necessary Existence Matters

The question of necessary existence is relevant to several fields of inquiry, including cosmology, ontology, and theology. Start with cosmology. Many

<sup>3</sup> So, for example, one might analyze ‘John caused the fight’ as ‘John’s rude comments caused the fight.’

physicists and cosmologists are extremely interested in questions about ultimate explanations. Stephen Hawking states that his goal as a physicist is “a complete understanding of the universe, why it is as it is and why it exists at all” (Boslough, 1989, p. 77). Echoing a similar sentiment, cosmologist Sean Carroll writes, “We are looking for a complete, coherent, and simple understanding of reality” (Carroll, 2005, p. 634). Brian Greene, a theoretical physicist, asserts that an ultimate explanation of the universe “would provide the firmest foundation on which to build our understanding of the world” (Greene 1999). The search for an ultimate explanation invites a question: what kind of an explanation can be ultimate? Can contingent reality alone constitute an ultimate explanation?

Physicist and cosmologist Lawrence Krauss proposes that the ultimate foundation of contingent reality is, in a certain sense, *nothing* (Krauss 2012). His proposal is provocative. And it inspires curiosity: his state of ‘nothing’ includes laws and conditions, and one may like to know what could explain their existence. Might other laws and conditions have obtained instead? If so, then what accounts for the existence of these particular starting conditions? Why did *they* obtain? One theory is that there is no answer—no deeper explanation. But consider the alternative: a *necessary* concrete reality lies at the ultimate explanatory foundation of our universe. That’s an importantly different kind of answer relevant to cosmogony.

Suppose, for a moment, that there actually are one or more necessary beings. Call it, or them, “*N*.”<sup>4</sup> Now *N* either is describable in the language of physics, or it is not. Suppose, first, that *N* has a physical description. Then the most complete cosmogonic theory would make reference to *N*. In other words, our most accurate scientific theory of the world would make reference to one or more necessary beings. That would be highly interesting to know, if it were true. Alternatively, suppose *N* cannot be described in the language of physics. In that case, even the most *complete* physical theory would fail to describe all the basic components of reality. In other words, there would be more to concrete reality than science would be capable of telling us about—even in principle. That, too, would

<sup>4</sup> To handle plurality, we may either: (i) suppose ‘*N*’ refers to a mereological sum of all necessary beings, (ii) suppose ‘*N*’ refers to the totality of necessary reality in the way we might speak of ‘heaps’ or ‘holes’ without prejudging the question of whether our ontology should include these, or (iii) treat ‘*N*’ as a plural referring device.

be interesting to know, if it were true. So, if there is a necessary being, then either cosmology is incapable of revealing an ultimate explanation of reality, or cosmological theories that make no reference to a necessary being are incomplete. Either result would be of great interest.

But now suppose instead that there is no necessary being. Then perhaps Krauss' theory is right: everything came from 'nothing' (in some sense). In this case, reality has no necessary foundation: reality is contingent all the way down. Krauss' theory may be true, but *only if* things don't bottom out in necessary beings. For if necessary beings lie at the explanatory foundation of our cosmos, then contingent reality does not—and cannot—come from *nothing* (unless a necessary being could count as 'nothing'). The question of necessary existence is relevant, then, to discerning which cosmogonic theories are metaphysically possible.

Necessary existence is also relevant to ontology. The ontologist endeavors to identify the most fundamental categories of reality. She asks: "What basic kinds of things are there?" One's answer to this question provides a framework for dealing with a wide range of philosophical questions. The traditional view is that we may rightly divide reality into *concrete* things (such as substances and events) and *abstract* things (such as numbers, properties, relations, and sets). Furthermore, some philosophers think that the divide between concrete and abstract things coincides with the divide between *contingent* and *necessary* things. We may wonder, however, whether the category of concrete things could overlap the category of necessary things. Is concreteness compatible with necessary existence? The answer to that question depends upon the answer to the question of this book: are there any necessary concrete things?

The question of necessary existence is also directly relevant to the current debate over *metaphysical nihilism*—the thesis that it is metaphysically possible for there to not have been anything at all. If there is a necessary being, then metaphysical nihilism is false: an empty world is impossible.<sup>5</sup> But if there isn't a necessary being, then the door to metaphysical nihilism is open.

Why care about metaphysical nihilism? One reason is that it bears on one of the deepest and longest standing questions: why is there anything at all? Metaphysical nihilism, if true, precludes what may be the simplest and

<sup>5</sup> See, for example, Baldwin 1996, Lowe 2002, Paseau 2002, Rodriguez-Pereyra 2002, Cameron 2007, Efrid 2009, and Hoffmann 2011. Note that some of these subtraction arguments have the more modest conclusion that there could be no spatial things.

most straightforward answer: there is something because the alternative is impossible (see, e.g., Rundle 2004). Metaphysical nihilism rules out this answer because it implies that there actually *could* have been nothing. On the other hand, if metaphysical nihilism is false, then the simplest ultimate explanation of existence is metaphysically possible. We consider this possibility to be enormously interesting and worth investigating.

Let us consider, finally, the relevance of necessary existence to theology. Many people are quick to associate the term ‘necessary being’ with God. There is an obvious historical reason for this: arguments for a necessary being have for centuries been a backbone of natural theology. Of course, arguments for a necessary being aren’t by themselves arguments for theism. The usual theistic arguments are multi-stage arguments, where an argument for a necessary being is just an initial stage. Nontheists may resist the subsequent stages; and so they may conceive of necessary concreta in naturalistic-friendly terms. Even so, arguments for a necessary being are a venerable and foundational part of natural theology, since God has classically been conceived as a supreme and unique necessary being. So the question of necessary existence is relevant to the question of God’s existence.

To be clear, ‘necessary being’ is by no means synonymous with ‘supreme being’ or ‘God.’ Richard Swinburne, for instance, thinks God is a contingent being. And, as we have already suggested, one might conceive of a necessary being in nontheistic terms. Smith (2001), for example, has proposed that the necessary being is a timeless *point* that acts as a causal condition for our cosmos. We should distinguish, therefore, between a necessary being and a supreme being. Arguments for or against the one are not automatically arguments for or against the other.

Nevertheless, sound arguments for or against a necessary being would significantly impact theology. Suppose Swinburne, for example, were persuaded that there is a necessary being. Would he continue to think God is contingent? Probably not. After all, God is supposed to be the ultimate source of all other concrete entities in every world where God exists. So, if God exists and there is a necessary being (that is, a necessary concrete thing), then in every world where God exists, either God is identical with that necessary being or God is that necessary being’s cause. But if God is identical with that necessary being, then God is a necessary being, after all. And surely if God is the *cause* of a necessary being, then God is also a necessary being. It seems, then, that a sound argument for a necessary being would provide a reason to think that *if* there is

a supreme being, it is a necessary being. Moreover, a sound argument *against* the existence of a necessary being would provide a reason to think that the classical concept of God as necessarily existent has no application to reality. These are significant theological results.

### 1.3 The “Necessary Being” Survey Results

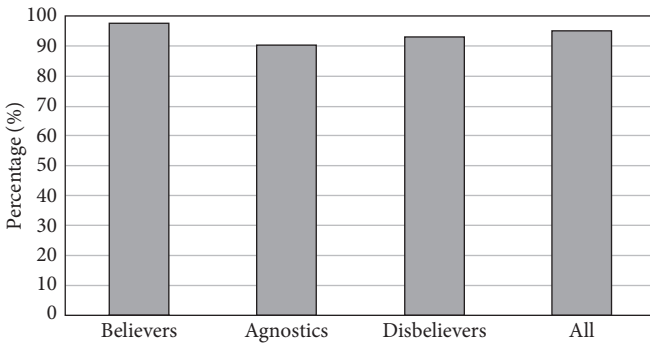
Can arguments for a necessary being actually persuade people? It is commonly thought that philosophical arguments, especially those concerning *ultimate* causes, do little more than reinforce the beliefs of those who already have convictions on the matter. But could there be arguments for a necessary being whose premises are plausible to those who don’t *already* think there is a necessary being? If not, what value is there in this inquiry? Is it possible to make genuine progress on a topic like this?

In view of such questions, we conducted an informal experiment to estimate the intuitive appeal of various arguments for a necessary being. The experiment consists of an online, interactive survey located here: [www.necessarybeing.com](http://www.necessarybeing.com).<sup>6</sup> The survey begins by supplying links to key definitions, including a definition of the crucial term, ‘necessary being.’ The survey follows with a series of questions about premises in several arguments for a necessary being. The first question is: “Is there a Necessary Being?” The choices for each question are “it seems so,” “it seems not,” and “I can’t say.” The survey is interactive in that subsequent questions depend upon answers.

We implemented a program that analyzes the participants’ answers. Some sets of answers correspond to premises in a deductively valid argument for a necessary being. As soon as a participant reports answers that belong to a set of such premises, then that participant is taken to a ‘proof’ page that shows how to deduce the existence of a necessary being from the reported answers. If a participant never reports such answers, they are taken to a more mundane ‘thank you’ page. In either case, participants are then invited to report whether they are a philosopher and/or a professor of philosophy.<sup>7</sup>

<sup>6</sup> It was previously located at [www.necessarybeing.net](http://www.necessarybeing.net).

<sup>7</sup> We posted the link to our survey on *Prosblogion* and *Matters of Substance* blogs. We didn’t screen anyone who took the quiz. We simply recorded the results from anyone who completed the quiz within a prespecified time frame.



**Figure 1** Percentage of participants whose responses entail the existence of a necessary being.

Our goal was to investigate whether anyone who doesn't already believe in a necessary being might accept premises that jointly entail that there is a necessary being. If there are such people, we also want to know whether they are few and far between.

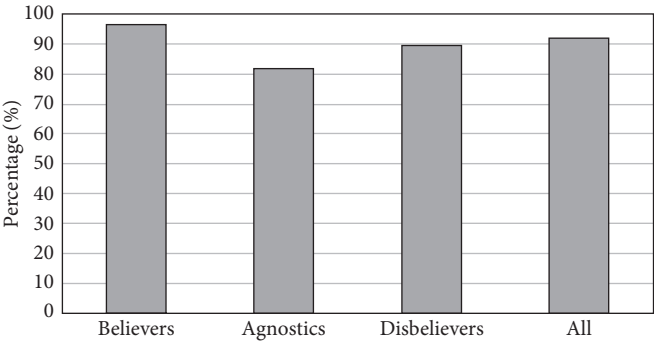
We will now give the results of our survey. The results are based upon the answers of 2,322 participants. These participants comprise all those participants who took the quiz for the first time between August 15, 2012 and April 15, 2013. About half of them (49.9%) responded "it seems so" to the question "Is there a Necessary Being?" Call these 'believers.' 21.8% reported "it seems not" (disbelievers), and 28.3% reported "I can't say" (agnostics).

So what percentage of participants reported answers that comprise the premises of an argument for a necessary being? The overall answer (including believers and non-believers alike) is 94.8%. Among *disbelievers*, the percentage is not much lower: 93.1%. Figure 1 is a breakdown of the percentages of believers, agnostics, and disbelievers who reached a 'proof' page.<sup>8</sup>

We can see that believers were the most likely to reach a 'proof' page. No surprise there. Interestingly, the disbelievers were slightly more likely to reach a 'proof' page than the agnostics. Perhaps this is because agnostics tend to be more reserved in their epistemological inquiries,

<sup>8</sup> The results were fairly stable over time. We calculated a standard deviation of 0.88% for month-to-month variation.





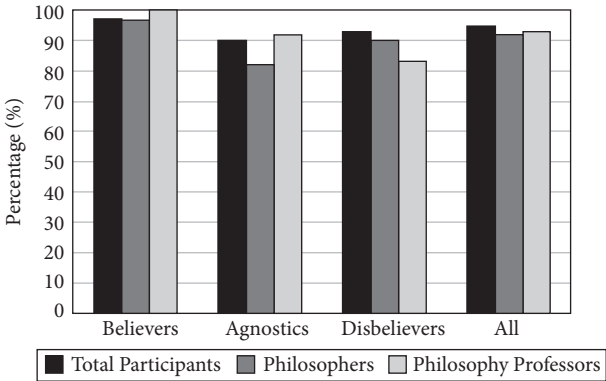
**Figure 2** Percentage of philosophers whose responses entail the existence of a necessary being.

leading them to report “I can’t say” for many of the responses that would otherwise lead to a ‘proof’ page for a necessary being.

We also tabulated results for the 446 participants each of whom reported to be a *philosopher*. Those results are shown in Figure 2.

We see that philosophers were slightly more skeptical than the broader population, but not by much.

The chart in Figure 3 summarizes the results for the foregoing categories of participants, plus the 46 philosophers who reported that they were professors. We see that the professors were the most likely



**Figure 3** Response summary.

to give reports that are consistent with their belief or disbelief in a necessary being.

These results support the hypothesis that there are skeptics of a necessary being who are inclined to accept, on initial review at least, premises that jointly entail that there is at least one necessary concrete thing. Moreover, we have confirming anecdotal evidence: a few participants e-mailed us to report that they were persuaded by the ‘proof’ page to think that there is a necessary being. This result is especially interesting because the survey does not offer independent support or motivation for any of the premises (as we will do in the case of the premises in the arguments to come, aside from those in the appendix).

We should highlight a few disclaimers. First, the survey does not test intuitions regarding any argument *against* a necessary being. Someone who finds plausible premises in an argument for a necessary being may *also* find plausible premises in an argument against a necessary being. Thus, the quiz does not test the participants’ rationality: disbelievers may be rationally inclined to consider certain premises as *prima facie* plausible, even if those premises jointly entail that there is a necessary being. In fact, the survey results could help some participants better understand why they rationally ought not accept a premise they had found *prima facie* plausible.

Second, the presentation of the proofs is semi-formal. We worked hard to clearly define all the key terms so that there would be no questions about whether the proofs are logically valid. Moreover, several professional philosophers critically reviewed the proofs before we conducted the survey. Nevertheless, we can’t promise that no one may interpret a line in a way that leads to a question about the arguments’ validity. It is worth noting that during an initial trial period (before August 15, 2012) a few participants reported ambiguities in certain proofs. In light of those reports, we made adjustments to eliminate ambiguities. Fortunately, no ambiguities affecting validity were reported during the experimental period. Although we take each of the proofs to be deductively valid, we do not claim that there cannot be any debate over how to interpret the formal features of a particular proof.

Third, the survey contains *multiple* arguments for a necessary being. We haven’t tested how likely it is that participants will accept the premises of any one particular argument. Rather, we have tested the likelihood that

they will accept the premises of at least one from a number of interrelated arguments.

We understand that the results of the survey are open to various interpretations. One thing is clear, however: there are unexplored and underexplored arguments for a necessary being. Therefore, we offer our book as a resource for studying and investigating new arguments for a necessary being.

## 2

# Metaphysical Possibility and Necessity

### 2.1 Introduction

It is metaphysically possible for there to be golden mountains, dog-headed humanoids, and violations of mass-energy conservation. It is metaphysically necessary that there be infinitely many primes and that every cat have DNA. And it is metaphysically impossible for there to be square circles, married bachelors or self-caused beings. Moreover, it is either metaphysically necessary or metaphysically impossible for the  $10^{100}$ th digit of  $\pi$  to be even, though no one has any idea which it is.

The last example emphasizes that the modality here is not epistemic. To say that a proposition is metaphysically necessary is not to say that our evidence requires us to believe it, and to say that it is metaphysically possible is not to say that our evidence allows for it.

In this chapter, we will distinguish metaphysical modality from narrowly logical modality and then argue that metaphysical modality is governed by the axioms of the system S5. We will then briefly discuss how two-dimensionalists' notions of conceptual necessity and possibility would also suffice for our purposes.

We shall use  $\Box$  and  $\Diamond$  for metaphysical necessity and possibility, respectively. In this chapter we will have some occasion to discuss impossible worlds. In other chapters, the word 'world' will be used only for *possible* worlds.

### 2.2 What is Metaphysical Modality?

Famously, Findlay (1948) argued that for any being, it is logically possible that that being not exist, and hence a necessary being is a

self-contradiction.<sup>1</sup> If Findlay is right, then the arguments of this book are doomed to failure. Findlay's line of thought was dependent on seeing necessity as something like what one might call 'narrowly logical necessity' (though even so his arguments may not have been successful).

Instead, the modality we are talking about in this book is what is sometimes called "broadly logical" necessity and possibility (see, e.g., Plantinga 1974). It needs to be distinguished from "narrowly logical" notions. Narrowly logical notions of modality arise naturally from the thought that impossibility is self-contradiction. Formally, given a logical system  $L$ , we can get a notion of  $L$ -necessity as provability in  $L$  and  $L$ -possibility as non-disprovability (or "non-self-contradiction") in  $L$ . For this notion to be useful, it seems we need a set of axioms for  $L$  that is finite or at least recursively specifiable.<sup>2</sup>

However, we will get important modal facts wrong if we reduce modality to this narrow-logical notion. Here are three reasons to think so. First, we need additional *non-formal* axioms about natural kinds (including, perhaps, natural kinds that are not exemplified in our world) in order to capture such necessities as that cats contain DNA, that water is material, or that acute thinkers aren't acute angles. Second, it is plausible that true arithmetical claims are necessary even while Gödel's First Incompleteness Theorem entails that they are not all  $L$ -necessary (Pruss 2011, Section I.2). Third, and perhaps most convincingly, Gödel's Second Incompleteness Theorem, together with the highly plausible premise that self-contradictions can't be *necessary*, implies that necessity isn't reducible to provability.

We will elaborate on this third reason for those interested in the technical details. Those not interested in these technical details are invited

<sup>1</sup> Cf. Swinburne 2012.

<sup>2</sup> Consider two alternatives. First, suppose we leave unspecified what the axioms are. Then the theory is compatible with every other serious theory of modality. That's because the proponent of some theory  $T$  of modality could just specify  $L$ 's axioms to be all the propositions that are necessarily true according to  $T$ , and every serious theory of modality agrees that everything that logically follows from necessities is necessary. Alternatively, suppose we instead say that axiomatizability is a primitive notion. Then the theory has no advantage over the theory that necessity is a primitive notion: it replaces a mysterious modal notion of necessity with what is on its face an even more mysterious modal notion, that of being objectively an axiom. Perhaps, though, there is some useful way of specifying axioms that is not recursive in nature. If so, then that theory would need to be examined separately, and we leave that option for future study.

to skip ahead. The Second Incompleteness Theorem tells us that if  $L$  is a consistent system with recursively specifiable axioms and sufficiently many uncontroversial axioms of arithmetic, then one cannot prove  $L$ 's consistency in  $L$ . Suppose  $L$  is such a consistent system. In the interest of readability, our argument for the inadequacy of  $L$ -modality will be very quickly sketched, without careful distinctions between formulas and their Gödel numbers. Here it goes. Let  $\Box_L$  and  $\Diamond_L$  be  $L$ -necessity and  $L$ -possibility, respectively. Fix *any* sentence  $p$ . It follows from Gödel's Second Incompleteness Theorem (Boolos and Jeffrey 1995, p. 188) that there is no  $L$ -proof of  $L$ 's consistency. Now, one can  $L$ -prove that if  $L$  is inconsistent (i.e., if a self-contradiction can be  $L$ -proved), then one can  $L$ -prove  $p$ , just by running in  $L$  one of the standard proofs that everything follows from a self-contradiction. Hence, one can  $L$ -prove that if  $L$  is inconsistent, then  $\Box_L p$  (i.e.,  $p$  has an  $L$ -proof). Hence, using contraposition, if one could  $L$ -prove  $\sim \Box_L p$ , one could  $L$ -prove that  $L$  is consistent. By Second Incompleteness, however, one cannot  $L$ -prove that  $L$  is consistent, and so one cannot  $L$ -prove  $\sim \Box_L p$ . Thus,  $\sim \Box_L \sim \Box_L p$ , and it follows that  $\Diamond_L \Box_L p$ .

We have just shown that  $L$ -modality makes  $\Diamond_L \Box_L p$  true for every sentence  $p$ —i.e., every sentence is  $L$ -possibly  $L$ -necessary. But we do not want this to be the case for the correct notions of necessity and possibility. Suppose for instance that  $p$  is some self-contradiction, say  $0 = 1$ . Then, it is  $L$ -possible that  $p$  is  $L$ -necessary. But we certainly do not want to say that it is possible that it is necessary that  $0 = 1$ . The universal truth of  $\Diamond_L \Box_L p$  shows that narrowly logical modality is untenable as an across-the-board account of modality. At best, it is an account of modality applicable to sentences without modal operators—and even then the First Incompleteness Theorem causes a problem, as we noted.

We might say that Gödel's First Incompleteness Theorem tells us that there are truths (even arithmetical ones) that are not provable, while the Second tells us that there are necessities that are not provable. Thus, narrowly logical modality understood in terms of provability gives us the wrong answers across the board: it not only leaves out modal facts about natural kinds and Gödelian arithmetical claims, but it makes  $0 = 1$  be possibly necessary.

Of course, modulo some set-theoretic worries, we *can* make  $L$ -modality coextensive with metaphysical modality, if we do not require the axioms to be recursively specifiable. After all, we could just take

all metaphysically necessary truths to be axioms. But then we trivialize *L*-modality.

Metaphysical necessity and possibility are not defined in terms of provability in a formal system. What are they, then? There are many accounts on the table, and our arguments are meant to be neutral in regard to all the best accounts. For instance, one of the authors of this book favors a Platonic account, on which some propositions have a fundamental modal property like necessity or possibility (see, e.g., Plantinga 1974), while the other favors a causal-powers account, on which possibilities are grounded in the powers of existing objects (see Pruss 2011 for a discussion of both this account and a number of others). (One might even combine the two stories: perhaps the possession of a fundamental modal property could be grounded in facts about the powers of objects.) One need not figure out the grounds or nature of modality in order to acquire concepts of metaphysical modality. We acquire the concepts of necessity and possibility by ostension, as we think about examples of modal propositions, like the ones presented at the beginning of the chapter. We may then separate the relevant modal concepts from neighboring ones (like the narrowly logical ones) with which they can be confused. Thus, no definition is required for comprehension.

## 2.3 Modal Logic

### 2.3.1 Overview

The modal system S5 is the main modal system of this book. We will shortly discuss the requisite modal axioms and rules of inference. Our arguments are best formulated in a free logic, i.e., a logic where terms need not refer (Nolt 2010). In such a logic, there is an existence predicate *E!* such that *E!t* holds if and only if *t* exists, i.e.,  $\exists x(x = t)$ . This allows a simple formulation of the claim that *x* exists necessarily:  $\Box E!x$ . And in the sense in which we use the term ‘necessary being’ in this book, we may characterize a necessary being as follows:

NECBEING. Entity *x* is a necessary being if and only if  $\Box E!x$  &  $\Diamond \exists y \text{ Causes}(x, y)$ .

One also needs to avoid confusing *E!x* with the “exists uniquely” quantifier  $\exists!x$ .

### 2.3.2 *Uncontroversial Aspects*

We assume four uncontroversial rules of modal logic. First, the De Morgan equivalences between necessity and possibility:  $\Diamond \sim p \Leftrightarrow \sim \Box p$  and  $\sim \Diamond p \Leftrightarrow \Box \sim p$ . In other words, a proposition is possible if and only if its negation is not necessary; and it is impossible if and only if its negation is necessary.

Second, one thing that the narrowly logical account of modality got right is this: if  $p$  is a theorem—i.e., has a proof (from the axioms)—then it is necessary. This leads to the Rule of Necessitation: given a proof of  $p$  from axioms alone, one may infer  $\Box p$ .

Third, any plausible account of metaphysical necessity will yield the Distribution Axiom: if  $\Box(p \rightarrow q)$ , then  $\Box p \rightarrow \Box q$  (where  $p \rightarrow q$  is material implication). The axiom basically states that necessity transfers across entailment: so, if  $p$  is necessary, and if  $p$  entails  $q$ , then  $q$  is necessary. (We will consistently use ' $p$  entails  $q$ ' to mean that  $\Box(p \rightarrow q)$ .)

Fourth, we accept Axiom T, which states that we can move from necessity to actuality:  $\Box p \rightarrow p$ . By the De Morgan equivalences and contraposition, this is equivalent to the axiom that what is actual is possible:  $p \rightarrow \Diamond p$ . While this axiom is uncontroversial for metaphysical modality, it is certainly controversial for other kinds of modality. For instance, it is false for epistemic modality: something can be epistemically necessary (say, in the sense of being entailed by what one justifiably believes, or in the sense of having unit epistemic probability) and yet false.

### 2.3.3 *S4 Axiom*

#### 2.3.3.1 VIOLATIONS

The S4 Axiom says that if  $p$  is possibly possible, then  $p$  is possible. Equivalently, if  $p$  is necessary, it is necessarily necessary.

There certainly are modalities that violate S4. We'll give two examples. First, let us say that something is practically possible for you provided that you could bring it about at a cost not exceeding  $c$  dollars. Suppose  $p$  would cost  $2c$  dollars. Then it might well be the case that at a cost of  $c$ , you could make  $p$  be practically possible—for you might be only another  $c$  away from getting to  $p$ . Thus,  $p$  is practically impossible, but it is practically possible for  $p$  to be practically possible.

Second, and more seriously, say that  $p$  is nomically possible provided that  $p$  is compatible with the laws of nature. Now one of the laws of