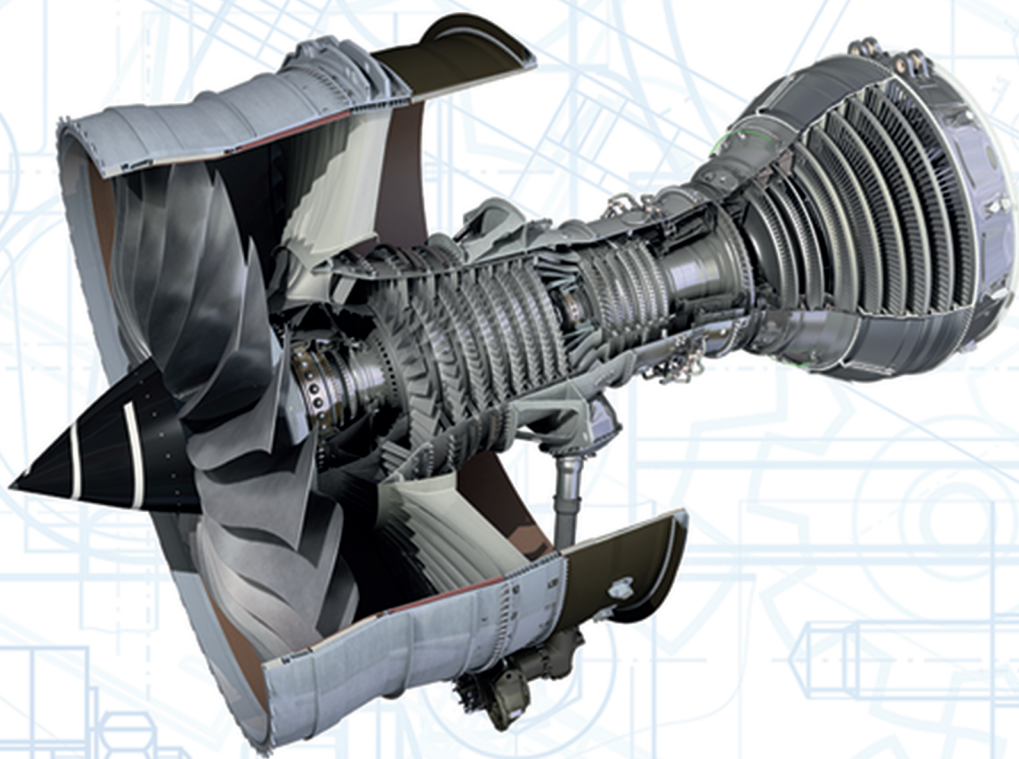


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INTRODUCTION TO
**ENGINEERING
FLUID MECHANICS**



MARCEL ESCUDIER

Introduction to Engineering Fluid Mechanics

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Marcel Escudier

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*To my wife Agnes, our son Stephen, and the memory of my Mother
and Grandmother*

Preface

A fluid is a material substance in the form of a liquid, a gas, or a vapour. The most common examples, to be found in both everyday life and in engineering applications, are water, air, and steam, the latter being the vapour form of water. The flow (i.e. motion) of fluids is essential to the functioning of a wide range of machinery, including the internal-combustion engine, the gas turbine (which includes the turbojet, turbofan, turboshaft, and turboprop engines), wind and hydraulic turbines, pumps, compressors, rapidly rotating discs (as in computer drives), aircraft, spacecraft, road vehicles, and marine craft. This book is concerned primarily with Newtonian fluids, such as water and air, for which the viscosity is independent of the flow. The quantitative understanding of fluid flow, termed fluid dynamics, is based upon the application of Newton's laws of motion together with the law of mass conservation. To analyse the flow of a gas or a vapour, for which the density changes in response to pressure changes (known as compressible fluids), it is also necessary to take into account the laws of thermodynamics, particularly the first law in the form of the steady-flow energy equation. The subject of fluid mechanics encompasses both fluid statics and fluid dynamics. Fluid statics concerns the variation of pressure in a fluid at rest (as will be seen in Chapter 4, this limitation needs to be stated more precisely), and is the basis for a simple model of the earth's atmosphere.

This text is aimed primarily at students studying for a degree in mechanical engineering or any other branch of engineering where fluid mechanics is a core subject. Aeronautical (or aerospace), chemical, and civil engineering are all disciplines where fluid mechanics plays an essential rôle. That is not to say that fluid flow is of no significance in other areas, such as biomedical engineering. The human body involves the flow of several different fluids, some quite ordinary such as air in the respiratory system and water-like urine in the renal system. Other fluids, like blood in the circulatory system, and synovial fluid, which lubricates the joints, have complex non-Newtonian properties, as do many synthetic liquids such as paint, slurries, and pastes. A brief introduction to the rheology and flow characteristics of non-Newtonian liquids is given in Chapters 2, 15, and 16.

As indicated in the title, this text is intended to introduce the student to the subject of fluid mechanics. It covers those topics normally encountered in a three-year mechanical-engineering-degree course or the first and second years of a four-year mechanical-engineering-degree course, as well as some topics covered in greater detail in the final years. The first ten chapters cover material suitable for a first-year course or module in fluid mechanics. Compressible flow, flow through axial-flow turbomachinery blading, internal viscous fluid flow, laminar boundary layers, and turbulent flow are covered in the remaining eight chapters. There are many other textbooks which cover a similar range of material as this text but often from a much more mathematical point of view. Mathematics is essential to the analysis of fluid flow but can be kept to a level within the capability of the majority of students, as is the intention here where the emphasis is on understanding the basic physics. The analysis of many

flow situations rests upon a small number of basic equations which encapsulate the underlying physics. Between these fundamental equations and the final results, which can be applied directly to the solution of engineering problems, can be quite extensive mathematical manipulation and it is all too easy to lose sight of the final aim. A basic understanding of vectors is required but not of vector analysis. Tensor notation and analysis is also not required and the use of calculus is kept to a minimum.

The approach to certain topics may be unfamiliar to some lecturers. A prime example is dimensional analysis, which we suggest is approached using the mathematically simple method of sequential elimination of dimensions (Ipsen's method). The author believes that this technique has clear pedagogical advantages over the more widely used Rayleigh's exponent method, which can easily leave the student with the mistaken (and potentially dangerous) idea that any physical process can be represented by a simple power-law formula. The importance of dimensions and dimensional analysis is stressed throughout the book. The author has also found that the development of the linear momentum equation described in Chapter 9 is more straightforward to present to students than it is via Reynolds transport theorem. The approach adopted here shows very clearly the relationship with the familiar $F = ma$ form of Newton's second law of motion and avoids the need to introduce an entirely new concept which is ultimately only a stepping stone to the end result. The treatment of compressible flow is also subtly different from most texts in that, for the most part, equations are developed in integral rather than differential form. The analysis of turbomachinery is limited to flow through the blading of axial-flow machines and relies heavily on Chapters 3, 10, and 11.

'Why do we need a fluid mechanics textbook containing lots of equations and algebra, given that computer software packages, such as FLUENT and PHOENICS, are now available which can perform very accurate calculations for a wide range of flow situations?' To answer this question we need first to consider what is meant by accurate in this context. The description of any physical process or situation has to be in terms of equations. In the case of fluid mechanics, the full set of governing equations is extremely complex (non-linear, partial differential equations called the Navier-Stokes equations) and to solve practical problems we deal either with simplified, or approximate, equations. Typical assumptions are that all fluid properties remain constant, that viscosity (the essential property which identifies any material as being a fluid) plays no role, that the flow is steady (i.e. there are no changes with time at any given location within the fluid), or that fluid and flow properties vary only in the direction of flow (so-called one-dimensional flow). The derivation of the Navier-Stokes equations, and the accompanying continuity equation, is the subject of Chapter 15. Exact analytical solution of these equations is possible only for a handful of highly simplified, idealised situations, often far removed from the real world of engineering. Although these solutions are certainly mathematically accurate, due to the simplifications on which the equations are based they cannot be said to be an accurate representation of physical reality. Even numerical solutions, however numerically accurate, are often based upon simplified versions of the Navier-Stokes equations. In the case of turbulent flow, the topic of Chapter 18, calculations of practical interest are based upon approximate equations which attempt to model the correlations which arise when the Navier-Stokes equations are time averaged. It is remarkable that valuable information about practical engineering problems can be obtained from considerations of simplified equations, such as the

one-dimensional equations, at minimal cost in terms of both time and money. What is essential, however, is a good physical understanding of basic fluid mechanics and a knowledge of what any computer software should be based upon. It is the aim of this text to provide just that.

Already in this brief **Preface** the names Navier, Newton, Rayleigh, Reynolds, and Stokes have appeared. In Appendix 1 we provide basic biographical information about each of the scientists and engineers whose names appear in this book and indicate their contributions to fluid mechanics.

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The author gratefully acknowledges the influence of several outstanding teachers, both as a student at Imperial College London and subsequently as a Research Associate at the Massachusetts Institute of Technology. My interest in, and enjoyment of, fluid mechanics was sparked when I was an undergraduate by the inspiring teaching of Robert Taylor. Brian Spalding, my PhD supervisor, and Brian Launder are not only internationally recognised for their research contributions but were also excellent communicators and teachers from whom I benefitted as a postgraduate student. As a research associate at MIT I attended lectures and seminars by Ascher H. Shapiro, James A. Fay, Ronald F. Probststein, and Erik Mollo-Christensen, all inspiring teachers. Finally, my friend Fernando Tavares de Pinho has given freely of his time to answer with insight many questions which have arisen in the course of writing this book.

Marcel Escudier
Cheshire, August 2016

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Notation

Each Roman, Greek, and mathematical symbol is followed by its meaning, its SI unit, and its dimension(s).

Lower-case Roman symbols

a	acceleration	m/s^2	L/T^2
c	blade chord length	m	L
c	concentration	kg/m^3	M/L^3
c	speed of sound	m/s	L/T
c	wetted perimeter	m	L
c_f	skin-friction coefficient	–	–
c_0	speed of light in vacuum	m/s	L/T
d	diameter	m	L
e	energy	J	ML^2/T^2
$\dot{e}_{xx}$	extensional strain rate in x -direction	$1/\text{s}$	$1/\text{T}$
f	non-dimensional velocity	–	–
f_x	body force per unit mass acting in the x -direction	m/s^2	L/T^2
f_D	Darcy friction factor	–	–
f_F	Fanning friction factor	–	–
$\bar{f}_F$	average Fanning friction factor	–	–
g	acceleration due to gravity	m/s^2	L/T^2
g_0	acceleration due to gravity at sea level ($z = z' = 0$)	m/s^2	L/T^2
h	height	m	L
h	spacing of parallel plates	m	L
h	specific enthalpy	kJ/kg	L^2/T^2
h_0	specific stagnation enthalpy	kJ/kg	L^2/T^2
$h_{0,REL}$	relative stagnation enthalpy	kJ/kg	L^2/T^2
i	angle of incidence	$^\circ$ or rad	–
j	number of independent dimensions	–	–
k	number of non-dimensional groups	–	–
k	radius of gyration	m	L
k	specific turbulent kinetic energy	m^2/s^2	L^2/T^2
$\bar{k}$	time-averaged specific turbulent kinetic energy	m^2/s^2	L^2/T^2
k_B	Boltzmann constant	J/K	$\text{ML}^2/\text{T}^2\text{K}$
l	length	m	L
l_K	Kolmogorov length scale	m	L
l_M	mixing length	m	L

m	mass	kg	M
m	wedge-flow exponent	–	–
m_A	added mass	kg	M
$\dot{m}$	mass flowrate	kg/s	M/T
n	amount of substance	kmol	M
n	number of physical quantities	–	–
n	power-law exponent in power-law viscosity model	–	–
p	static pressure	Pa	M/LT ²
p_G	gauge pressure	Pa	M/LT ²
p_H	hydrostatic pressure	Pa	M/LT ²
p_{REF}	reference pressure	Pa	M/LT ²
p_T	total pressure	Pa	M/LT ²
p_V	vapour pressure	Pa	M/LT ²
p_0	stagnation pressure	Pa	M/LT ²
$p_{0,REL}$	relative stagnation pressure	Pa	M/LT ²
$\bar{p}$	average static pressure	Pa	M/LT ²
p'	fluctuating component of static pressure	Pa	M/LT ²
p'	intermediate static pressure	Pa	M/LT ²
p^*	non-dimensional static pressure	–	–
$\dot{q}$	heat transfer rate	W	ML ² /T ³
$\dot{q}'$	heat transfer rate per unit length	W/L	ML/T ³
r	radial distance	m	L
s	arc length	m	L
s	cascade-blade spacing (or pitch)	m	L
s	distance along a streamline	m	L
s	specific entropy	m ² /s ² · K	L ² /T ² θ
s_0	specific stagnation entropy	m ² /s ² · K	L ² /T ² θ
t	elapsed time	s	T
t	temperature	°C	θ
$\tilde{t}$	non-dimensional time	–	–
t^*	non-dimensional time	–	–
u	specific internal energy	kJ/kg	L ² /T ²
u	velocity component in x -direction	m/s	L/T
$\bar{u}$	time-averaged value of velocity component u	m/s	L/T
u'	fluctuating component of velocity component u	m/s	L/T
u^*	non-dimensional value of velocity component u	–	–
u^+	velocity component u normalised by u_τ	–	–
u_P	velocity of plastic plug	m/s	L/T
u_0	centreline velocity	m/s	L/T
u_τ	friction velocity	m/s	L/T
v	specific volume	m ³ /kg	L ³ /M
v	velocity component in y - or r -direction	m/s	L/T
$\bar{v}$	time-averaged value of velocity component v	m/s	L/T

v'	fluctuating component of velocity component v	m/s	L/T
v^+	velocity component v normalised by u_τ	–	–
v_K	Kolmogorov velocity scale	m/s	L/T
w	specific weight	N/m ³	M/L ² T ²
w	velocity component in z - or θ -direction	m/s	L/T
$\bar{w}$	time-averaged value of velocity component w	m/s	L/T
w'	fluctuating component of velocity component w	m/s	L/T
w^+	velocity component w normalised by u_τ	–	–
x	distance along or parallel to a surface/streamwise distance	m	L
X	length	m	L
y	distance normal to a surface	m	L
y^+	distance y normalised by u_τ and ν	–	–
z	blade height (or length)	m	L
z	depth (i.e. distance measured vertically downwards)	m	L
z'	height (i.e. distance measured vertically upwards)	m	L
z'	geometric altitude	m	L
z'_G	geopotential altitude	m	L
z_C	depth of centroid	m	L
z_P	depth of centre of pressure	m	L

Upper-case Roman symbols

A	cross-sectional area	m ²	L ²
A	surface area	m ²	L ²
A^*	choking (or sonic) area	m ²	L ²
A_E	nozzle exit area	m ²	L ²
A_T	nozzle throat area	m ²	L ²
B	barometric (or atmospheric) pressure or external pressure	bar	M/LT ²
B	log-law constant	–	–
Bi	Bingham number	–	–
C_D	coefficient of discharge	–	–
C_D	drag coefficient	–	–
C_F	average friction factor	–	–
C_L	lift coefficient	–	–
C_P	pressure coefficient	–	–
C_P	specific heat at constant pressure	m ² /s ² · K	L ² /T ² θ
C_V	specific heat at constant volume	m ² /s ² · K	L ² /T ² θ
D	diameter	m	L
D	drag (or drag force)	N	ML/T ²
$\bar{D}$	mean diameter	m	L
D_H	hydraulic diameter	m	L
D_T	nozzle throat diameter	m	L
D'	drag force per unit length of surface	N/m	M/T ²

E	energy released	J	ML^2/T^2
E	Young's modulus	Pa	M/LT^2
Eu	Euler number	–	–
F	force	N	ML/T^2
F	non-dimensional stream function	–	–
F_B	buoyancy force	N	ML/T^2
F_θ	function in Thwaites' method	–	–
Fr	Froude number	–	–
G	mass velocity	$\text{kg}/\text{m}^2 \cdot \text{s}$	$\text{M}/\text{L}^2\text{T}$
G	shear modulus (fluid)	Pa	M/LT^2
G	modulus of rigidity (solid)	Pa	M/LT^2
H	height or depth	m	L
H	horizontal component of force	N	ML/T^2
H	boundary-layer shape factor	–	–
He	Hedstrom number	–	–
H_{12}	boundary-layer shape factor	–	–
I	second moment of area	m^4	L^4
I_C	second moment of area about an axis through the area's centroid	m^4	L^4
I_{xy}	product of inertia	m^4	L^4
K	bulk modulus of elasticity	Pa	M/LT^2
K	consistency index in power-law viscosity model	$\text{Pa} \cdot \text{s}^n$	M/LT^{2-n}
K	loss coefficient	–	–
K	turbomachine stagnation-pressure loss coefficient	–	–
Kn	Knudsen number	–	–
$1/K$	compressibility	$1/\text{Pa}$	LT^2/M
L	length	m	L
L	lift (or lift force)	N	ML/T^2
L^*	choking length	m	L
M	Mach number	–	–
M	molar mass	kg/kmol	–
M	momentum	$\text{kg} \cdot \text{m}/\text{s}$	ML/T
$\mathcal{M}$	molecular weight	kg/kmol	–
M_{REL}	relative Mach number	–	–
$\dot{M}$	momentum flowrate	$\text{kg} \cdot \text{m}/\text{s}^2$	ML/T^2
$\dot{M}'$	momentum flowrate per unit width of duct	kg/s^2	M/T^2
MG	metacentric height	m	L
N	molecular number density	$1/\text{m}^3$	$1/\text{L}^3$
N	number of molecules	–	–
N	rotational speed	rps	$1/\text{T}$
N_A	Avogadro number	$1/\text{kmol}$	$1/\text{M}$
N_P	turbomachine power-specific speed	–	–
N_S	turbomachine specific speed	–	–
P	piezometric pressure	Pa	M/LT^2

P	power	W	ML^2/T^3
Po	Poiseuille number	–	–
Pr	Prandtl number	–	–
$\dot{Q}$	volumetric flowrate	m^3/s	L^3/T
$\dot{Q}'$	volumetric flowrate per unit width	m^2/s	L^2/T
R	radius	m	L
R	reaction force	N	ML/T^2
R	resultant force	N	ML/T^2
R	specific gas constant	$m^2/s^2 \cdot K$	$L^2/T^2\theta$
R_E	mean radius of the earth	m	L
R_H	hydraulic radius	m	L
R_I	inner radius of annulus	m	L
R_O	outer radius of annulus	m	L
$\mathcal{R}$	molar gas constant (universal gas constant)	$kJ/kmol \cdot K$	$L^2/T^2\theta$
Re	Reynolds number	–	–
Re_x	Reynolds number based upon length x	–	–
Re_δ	Reynolds number based upon length δ	–	–
Re_C	critical Reynolds number	–	–
Re_D	Reynolds number based upon pipe diameter	–	–
Re_H	Reynolds number based upon hydraulic diameter	–	–
Rep	Reynolds number based upon plastic viscosity	–	–
S	fluid-structure interaction force	N	ML/T^2
St	Strouhal number	–	–
T	absolute temperature	K	θ
T	skin-friction coefficient = $\theta\tau_s/\mu U_\infty$	–	–
T	surface-tension force	N	ML/T^2
T	thrust (or thrust force)	N	ML/T^2
T	time interval	s	T
T	torque	$N \cdot m$	ML^2/T^2
T_0	stagnation (or total) temperature	K	θ
$T_{0,REL}$	relative stagnation temperature	K	θ
Ta	Taylor number	–	–
U	free-stream velocity	m/s	L/T
U_0	scaling velocity	m/s	L/T
U_∞	free-stream velocity	m/s	L/T
V	velocity	m/s	L/T
V	vertical component of force	N	ML/T^2
V_B	buoyancy force	N	ML/T^2
V_D	vertically downwards force	N	ML/T^2
V_U	vertically upwards force	N	ML/T^2
V_∞	terminal velocity	m/s	L/T
$\bar{V}$	average (bulk-mean) velocity	m/s	L/T
$\tilde{V}$	non-dimensional velocity	–	–
$\bar{V}^+$	average velocity $\bar{V}$ normalised by u_τ	–	–

$\mathcal{V}$	volume	m^3	L^3
$\mathcal{V}_C$	critical volume for validity of continuum hypothesis	m^3	L^3
$\mathcal{V}_D$	displaced volume	m^3	L^3
$\mathcal{V}_S$	submerged volume	m^3	L^3
V_∞	y -direction velocity at edge of boundary layer	m/s	L/T
W	relative velocity	m/s	L/T
W	weight	N	ML/T^2
W	width	m	L
W	work	J	ML^2/T^2
$\dot{W}$	rate of work input (power input)	W	ML^2/T^3
We	Weber number	–	–
X	length	m	L
Y	boundary-layer thickness	m	L
Y	surface tension	N/m	M/T^2
Z	depth of liquid	m	L

Lower-case Greek symbols (English word in parentheses)

α (alpha)	angle of attack	$^\circ$ or rad	–
α	absolute flow angle	$^\circ$ or rad	–
α	conical gap angle	$^\circ$ or rad	–
α	non-dimensional constant in Blasius' equation	–	–
α'	constant in shock-structure analysis	m^2/s	L^2/T
β (beta)	oblique shock angle	$^\circ$ or rad	–
β	relative-flow angle	$^\circ$ or rad	–
β	wedge angle	$^\circ$ or rad	–
γ (gamma)	ratio of specific heats	–	–
$\dot{\gamma}$	shear rate	$1/\text{s}$	$1/\text{T}$
$\dot{\gamma}_{xy}$	shear rate corresponding to τ_{xy}	$1/\text{s}$	$1/\text{T}$
δ (delta)	angle of deflection or deviation	$^\circ$ or rad	–
δ	boundary-layer thickness	m	L
δ	radial gap width	m	L
δA	element of area	m^2	L^2
δF	element of force	N	ML/T^2
δh	infinitesimal height difference	m	L
δH	element of horizontal force	N	ML/T^2
δm	element of mass	kg	M
δp	infinitesimal change or difference in pressure	Pa	M/LT^2
δs	infinitesimal change of distance	m	L
δt	infinitesimal change in time	s	T
δV	element of vertical force	N	ML/T^2
$\delta \mathcal{V}$	element of volume	m^3	L^3

δW	element of weight	N	ML/T ²
δx	element of streamwise or x -direction distance	m	L
δy	element of distance normal to a surface or y -direction distance	m	L
δz	element of depth or z -direction distance	m	L
$\delta z'$	element of height	m	L
δ^*	boundary-layer displacement thickness	m	L
δ_{SUB}	thickness of viscous sublayer	m	L
δ_1	boundary-layer displacement thickness	m	L
δ_2	boundary-layer momentum-deficit thickness	m	L
ϵ (epsilon)	turbulent kinetic energy dissipation rate	m ² /s ³	L ² /T ³
ϵ	upwash or downwash angle	° or rad	–
ε (epsilon)	eccentricity	m	L
ε	non-dimensional annular gap with	–	–
ε	surface-roughness height	m	L
ε^+	surface-roughness height normalised by u_τ and ν	–	–
η (eta)	dynamic viscosity	Pa · s	M/LT
η	boundary-layer similarity variable	–	–
θ (theta)	angle	° or rad	–
θ	boundary-layer momentum-deficit thickness	m	L
θ	contact angle	°	–
θ	turning angle	°	–
$\dot{\theta}$	angular velocity	rad/s	1/T
$\ddot{\theta}$	angular acceleration	rad/s ²	1/T ²
κ (kappa)	lapse rate	K/m	θ/L
κ	von Kármán's constant	–	–
κ	wavenumber	1/m	1/L
λ (lamda)	time constant	s	T
λ	pressure-gradient parameter	–	–
λ	Pohlhausen's pressure-gradient parameter	–	–
λ	wavelength of turbulence	m	L
λ_P	Poiseuille-flow pressure-gradient parameter	–	–
λ_θ	boundary-layer pressure-gradient parameter	–	–
μ (mu)	dynamic viscosity	Pa · s	M/LT
μ	Mach angle	° or rad	–
μ_{EFF}	effective viscosity	Pa · s	M/LT
μ_P	viscosity of plastic plug	Pa · s	M/LT
μ_T	eddy viscosity	Pa · s	M/LT
μ_∞	infinite-shear-rate viscosity	Pa · s	M/LT
ν (nu)	kinematic viscosity	m ² /s	L ² /T
ν	Prandtl-Meyer function	°	–
ν_T	kinematic eddy viscosity	m ² /s	L ² /T

ξ (xi)	blade stagger angle	$^{\circ}$ or rad	–
ξ	non-dimensional distance	–	–
ξ	turbomachine enthalpy-loss coefficient	–	–
ξ_P	non-dimensional radius of plastic plug	–	–
ρ (rho)	density	kg/m ³	M/L ³
σ (sigma)	density ratio	–	–
σ	relative density	–	–
σ	surface tension	N/m	M/T ²
σ_{xx}	normal stress in x -direction	Pa	M/LT ²
τ (tau)	characteristic time	s	T
τ	shear stress	Pa	M/LT ²
τ_K	Kolmogorov time scale	s	T
τ_S	surface shear stress	Pa	M/LT ²
$\overline{\tau_S}$	average surface shear stress	Pa	M/LT ²
τ_Y	yield stress	Pa	M/LT ²
τ_{xy}	shear stress acting in y -direction	Pa	M/LT ²
ϕ (phi)	angle	$^{\circ}$ or rad	–
ϕ	blade camber angle	$^{\circ}$ or rad	–
ϕ	turbomachine flow coefficient	–	–
χ (chi)	blade angle	$^{\circ}$ or rad	–
χ	boundary-layer scale factor	–	–
ψ (psi)	stream function	1/s	1/T
ψ	hydraulic machine pressure-change coefficient	–	–
ω (omega)	angular velocity	rad/s	1/T

Upper-case Greek symbols

Γ (gamma)	circulation	m ² /s	L ² /T
Γ	lapse rate	$^{\circ}$ C/km	θ /L
Γ_{AD}	adiabatic lapse rate	$^{\circ}$ C/km	θ /L
Δ (delta)	finite change or difference	–	–
Δ	scaling length	m	L
Δ_S	shock thickness	m	L
Δp	finite pressure difference	Pa	M/LT ²
Δp_0	reduction in stagnation pressure	Pa	M/LT ²
ΔZ	finite depth difference	m	L
$\Delta \rho$	density difference	kg/m ³	M/L ³
Θ (theta)	dilation	1/s	1/T
$\tilde{\Theta}$	ratio θ/δ , where θ = boundary-layer momentum-deficit thickness	–	–
Λ (lamda)	degree of reaction	–	–
Λ	molecular mean free path	m	L
Π (pi)	non-dimensional group	–	–
Π	shock strength	–	–

Π	wake parameter	–	–
Σ (sigma)	summation	–	–
$\tilde{\Phi}$ (phi)	ratio δ^*/δ	–	–
Ω (omega)	angular velocity	rad/s	1/T

Mathematical symbols

div	vector operator of divergence	1/m	1/L
∇	del (or gradient) operator	1/m	1/T
∇^2	Laplacian operator	1/m ²	1/L ²

Lower-case Roman subscripts

f	friction
r	radial direction
t	throat
x	x-direction
y	y-direction
z	z-direction

Upper-case Roman subscripts

A	actual
B	back (pressure)
C	centroid or critical
E	exhaust
F	fluid or fuel or full scale
G	centre of gravity or gas
H	based on hydraulic diameter
H_2O	water
I	inlet or inner surface
L	laminar or liquid or lower surface
M	manometer
M	model
O	outer surface
P	centre of pressure
REF	reference condition
S	isentropic or solid or submerged or surface
T	total or turbulent
TH	theoretical
U	upper surface

Lower-case Greek subscript

θ	θ -direction
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Numerical subscripts

0	stagnation or reference conditions
1	conditions upstream of a shockwave
2	conditions downstream of a shockwave

Superscripts

T	isothermal
*	choking (or critical or sonic) condition

Why do students of many branches of engineering need to study fluid mechanics? First and foremost, the answer is ‘design’. It can be argued that the principal purpose of engineering is **engineering design**, and it is frequently the case that considerations of fluid flow are crucial to the engineering-design process. It would be inappropriate here to discuss in detail what is meant by engineering design. Suffice to say, design is sometimes confused with **styling**, which refers primarily to the external appearance of a device or machine, whereas engineering design is concerned with its functioning and invariably involves calculations based upon the laws of physics. In this introductory chapter we indicate the wide and diverse range of practical situations where fluid mechanics plays a central role, often together with such related subjects as heat transfer, thermodynamics, and combustion. Although the emphasis in this book is on applications of fluid mechanics in mechanical, aeronautical, and civil engineering, other examples could be taken from biomedical, building, chemical, and environmental engineering. Within this book we also mention many of the natural phenomena for which fluids, and the way they flow, play a fundamental role. Although the origins of fluid mechanics can be traced to ancient Greek (Archimedes) and Roman (Frontinus) times, and important contributions were made in the 15th (da Vinci), 16th, 17th (Newton, Pascal), 18th (Bernoulli and Euler), and 20th centuries (Prandtl, Taylor), most of the major developments in the subject were made by engineers, mathematicians, and physicists in the 19th century (including Kelvin, Mach, Navier, Rankine, Rayleigh, Reynolds, and Stokes). Many effects, functions, equations, non-dimensional parameters (see Chapter 3), etc., are named after these pioneers and other major contributors to fluid mechanics: brief biographies are included in Appendix A.

A thorough understanding of the contents of this book should enable the student to

- use the results of **dimensional analysis** (Chapter 3) to scale up the results of wind-tunnel model tests¹. A typical example is in the analysis of wind-tunnel data for the aerodynamic behaviour of a **Formula 1 racing car**, as shown in Figure 1.1 (to illustrate the point, we could just as well have chosen, e.g. a fighter aircraft or a bridge).
- specify the characteristics of a **centrifugal pump**, as illustrated in Figure 1.2, required to handle large quantities of oil, based upon small-scale tests with water, again guided by dimensional analysis
- calculate the flow speed in a wind tunnel using a **Pitot-static tube** and a **U-tube manometer**, as shown in Figure 1.3 (the size of the manometer relative to the Pitot tube is

¹ Where the aerodynamic characteristics of an aircraft, a car, a locomotive, or any other vehicle are to be investigated in a wind or water tunnel, it is usual for the vehicle to be fixed in position with the fluid flowing around it. This change is known formally as a **Galilean transformation**. In a wind tunnel used to investigate vehicles in contact with a road, the surface in contact with the vehicle usually moves at the same speed (and direction) as the working fluid. Such an arrangement is referred to as a **rolling road**. Note too that the flow direction in all figures in this book is from left to right, a convention adopted in the majority of fluid mechanics textbooks.

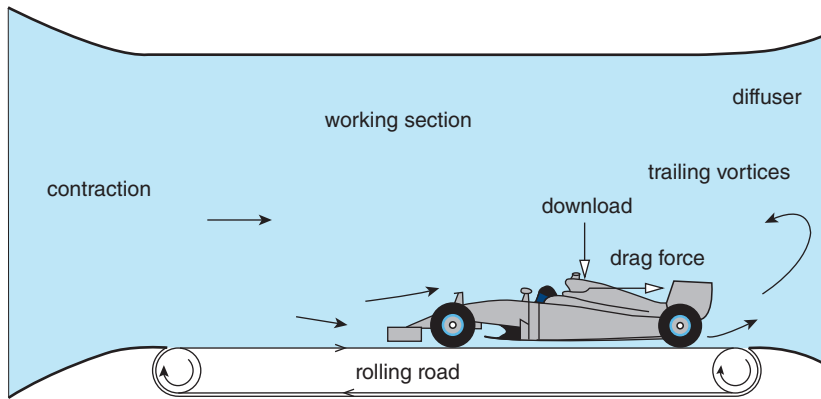


Figure 1.1 Wind-tunnel test of a racing car

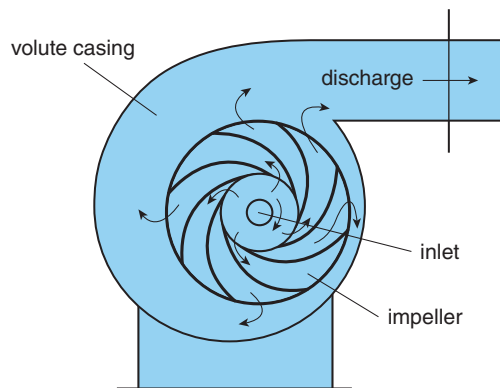


Figure 1.2 Centrifugal-pump testing

much reduced in the diagram). This calculation involves both **hydrostatics** (Chapter 4) and **Bernoulli's equation** (Section 8.9).

- using the principles of hydrostatics (Section 8.5), calculate the resultant force exerted by the water in a reservoir on the face of a dam, as shown by R in Figure 1.4²
- use the principles of hydrostatics to design a **floating boom** to contain an **oil slick**, as shown in Figure 1.5
- use Bernoulli's equation (see Chapter 8) to calculate the **lift force** resulting from the airflow over the surfaces of an aerofoil, as shown in Figure 1.6. A qualitative discussion of the underlying physical phenomena which explain lift is given in Section 17.7.
- use Bernoulli's equation to determine the flowrate at which **internal boiling** occurs at room temperature as a consequence of reduced pressure (so-called **cavitation**, discussed in Section 8.11) in the flow of a liquid through a constriction, such as a valve or, as illustrated in Figure 1.7, a **convergent-divergent nozzle**

² The inverted triangle is used to identify a free surface.

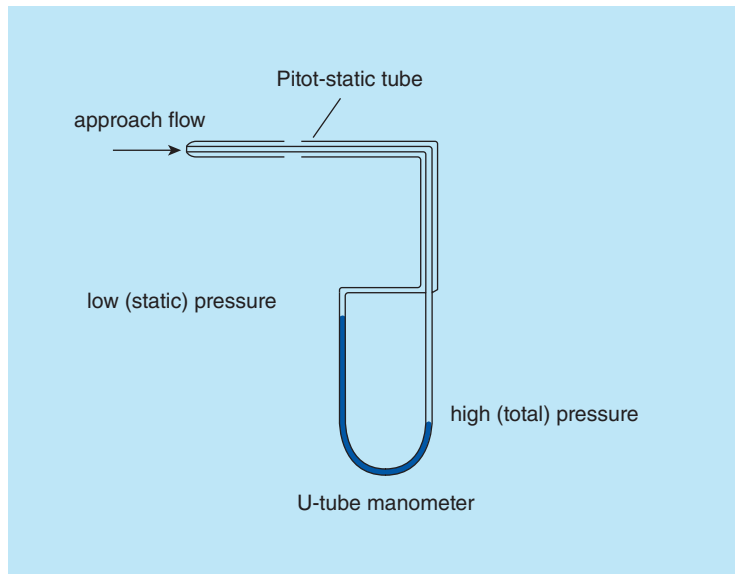


Figure 1.3 Pitot-static tube and U-tube manometer

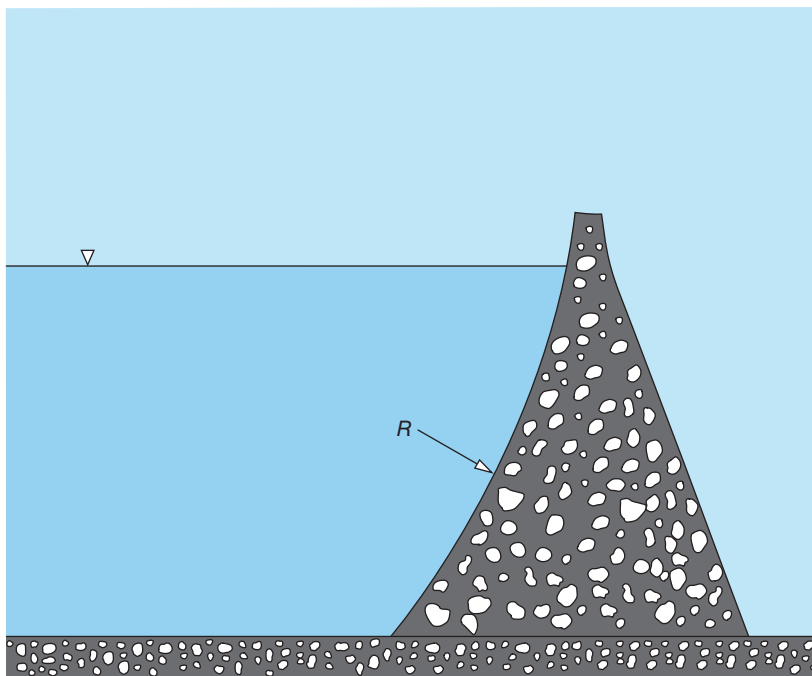


Figure 1.4 Hydrostatic force on the face of a dam

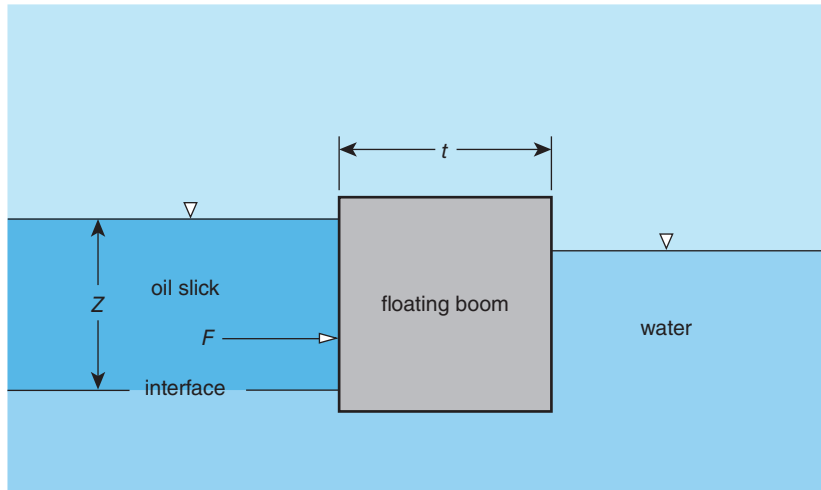


Figure 1.5 Floating boom designed to contain an oil slick

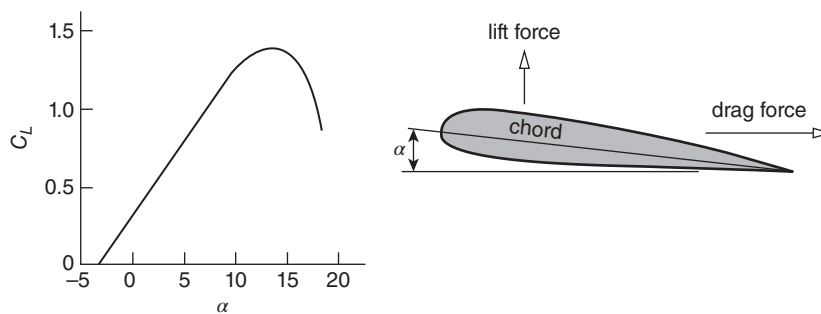


Figure 1.6 Aerodynamic lift generated by an aerofoil: Lift coefficient C_L versus angle of attack α

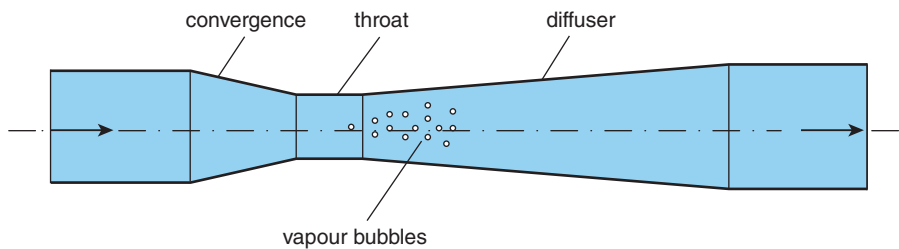


Figure 1.7 Cavitation in water flow through a convergent-divergent nozzle

- use the **mass-conservation** (Section 6.8) and **momentum-conservation** (Chapter 9 and Section 10.4) **equations** to calculate the thrust developed by a turbofan engine, such as that shown schematically in Figure 1.8, which is a simplified version of Figure 14.1
- use the continuity and momentum equations, together with Bernoulli's equation, to calculate the power output of a **Pelton hydraulic turbine** (Section 10.11), as shown in Figure 1.9

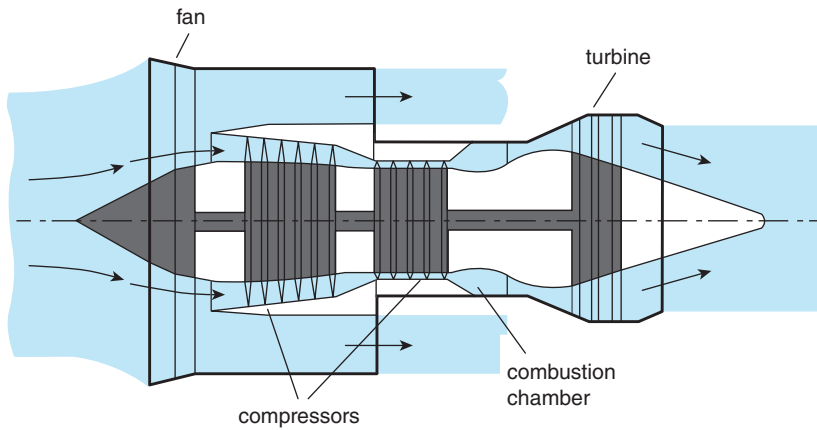


Figure 1.8 Schematic cross section of a turbofan engine

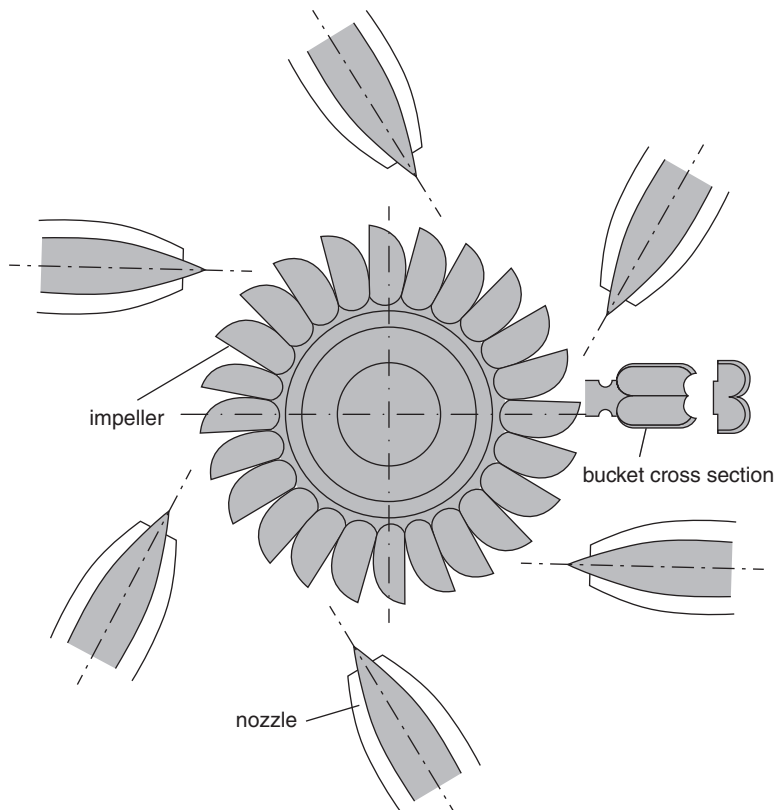


Figure 1.9 Schematic cross section of a Pelton turbine

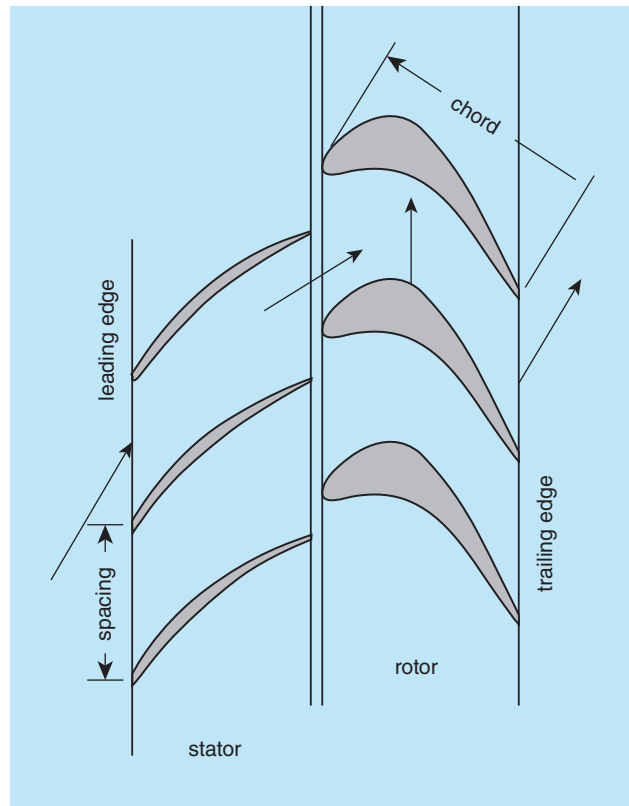


Figure 1.10 Turbine stage

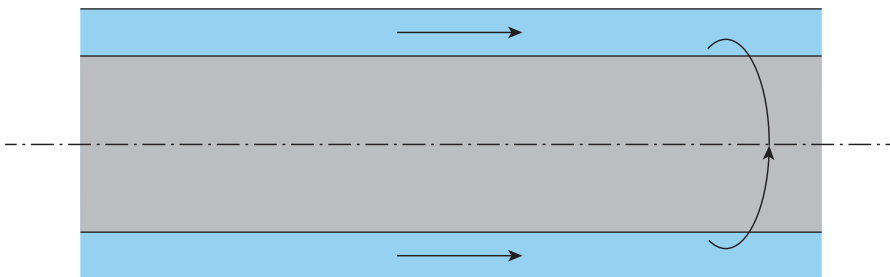


Figure 1.11 Viscous flow through a concentric annulus with centrefbody rotation

- use the mass- and momentum-conservation equations, the **steady-flow energy equation**, and the **perfect-gas law** to calculate the power output of a **turbine** (or **compressor**) stage (Section 14.8), as shown in Figure 1.10
- use the mass- and momentum-conservation equations, together with **Newton's law of viscosity**, to calculate the flow of a viscous fluid through a **concentric annulus** with **centrefbody rotation** as shown in Figure 1.11 (Section 16.5)

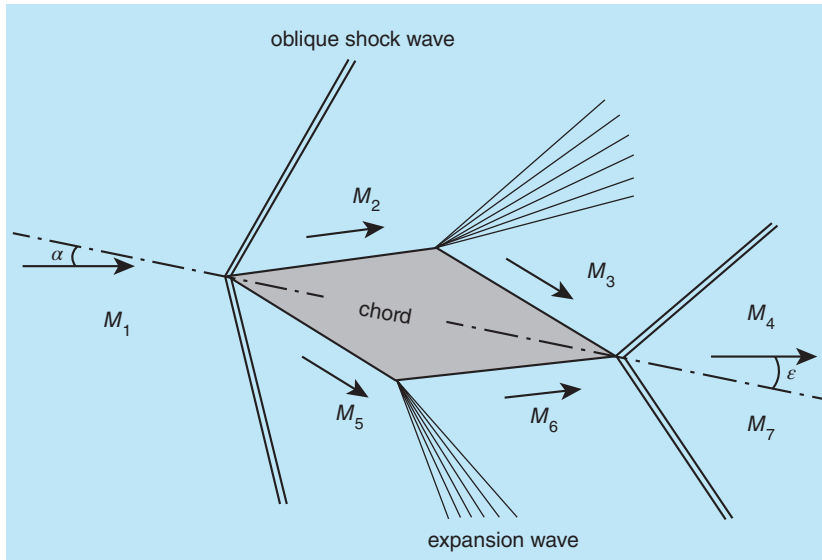


Figure 1.12 Supersonic flow with shock and expansion waves over a diamond-shaped aerofoil

- use the tabulated solution of the **Blasius equation** for a laminar boundary layer (Section 17.3) to calculate the drag force on a thin flat plate immersed in a viscous fluid flow
- use the Virginia Tech **Compressible Aerodynamics Calculator** (see Section 11.3) to calculate the **shock** and **expansion waves**, the **Mach number** and **pressure distributions**, and lift force (Section 12.3) of supersonic perfect-gas flow over a **diamond-shaped aerofoil**

The **thickness** of the **shock waves** in Figure 1.12 is greatly exaggerated (see Section 11.8). The subscripted M 's indicate the Mach numbers in each region of the flow.

The foregoing is just a selection of the engineering applications of fluid mechanics considered in this textbook. As we emphasise in the remainder of this chapter, there are few areas of life, whether man-made or natural, in which fluids and fluid mechanics do not play a vital role.

1.1 What are fluids and what is fluid mechanics?

Without salt-free water to drink, we die within about ten days, and become brain dead within about four minutes without the oxygen which makes up about 21% by volume of the air we breathe (the rest is mainly nitrogen, 78%). Water is a **liquid**, air is a **gas**, and both are what we call **fluids**. The total mass of air in the **atmosphere** which surrounds the earth (see Section 4.13) is estimated to be about 5.3×10^{18} kg (or 5.3 petatonnes³), and the total mass of

³ Peta- and exa- are two of the 20 approved prefixes of The International System of Units (SI) presented in Section 3.2.

water in all the oceans, lakes, rivers, etc., the so-called **hydrosphere**, is about 1.4×10^{21} kg (or 1.4 exatonnes). Given their abundance, and their importance to our very existence, it is hardly surprising that water and air are the two fluids encountered most commonly in fluid mechanics. There are, of course, many other familiar ‘everyday’ fluids: methane, ethane, hydrogen, helium, oxygen, and nitrogen are all gases which behave much like air; similarly, natural (as opposed to synthetic) fluids such as oil, petrol, mercury, honey, glycerine, and alcohol are all relatively simple liquids much like water, but with different densities, viscosities, and other properties (Chapter 2 is concerned with fluid properties and what makes fluids different from solids). We call these simple fluids, with viscosities independent of their motion (though not their temperature), **Newtonian**. Blood, synovial fluid (which lubricates our joints), custard, mayonnaise, salad cream, ketchup, hair gel, toothpaste, drilling fluid, fracking (or fracturing) fluids, freshly mixed cement slurry, and paint are all liquids but with viscous properties and flow behaviour very different from those of water. These differences arise primarily because such liquids have either a complex molecular structure or consist of a mixture of a simple liquid (such as water) and many tiny (often in the micron range) suspended particles. Because of the complexity of their viscosities, these liquids are termed **non-Newtonian**. The study of the viscous properties of non-Newtonian liquids is a subject in itself, called **rheology**. There is a brief account of non-Newtonian liquids in Section 2.10. Simple models for such liquids and their flow are discussed in Sections 15.5 and 16.6.

We know from everyday experience that liquids flow. Water flows from the mains supply when we open the tap. Water flows from the sink or bath into the drainage system. Tea flows from a teapot. Beer flows into our digestive system from a glass, bottle, or can, and then, usually after a biological/chemical transformation and temporary storage, flows out again from our urinary system. Blood flows through our arteries and veins, pumped by a natural or artificial heart. Air flows into our lungs, and carbon dioxide flows out into the atmosphere. Liquid or gaseous fuel flows into the engines of passenger vehicles, trains, aircraft, and ships, while exhaust gases flow out, again into the atmosphere. Town gas, a mixture consisting primarily of hydrogen, methane, and carbon dioxide, flows to our cookers and boilers, and products of combustion flow out. Air flows around us as we walk, run, or ride our bicycles. It flows over the bodywork of our cars, over the wings and fuselages of the aircraft in which we fly, and through the blades of wind turbines, causing them to rotate and generate electrical power. Oil, gas, brine, and drilling fluid flow from deep in the earth to the surface when we drill for oil or gas. Water flows from rivers into reservoirs, lakes, and the sea and from reservoirs through **hydraulic turbines** again to generate electrical power. It also flows into the boilers of power-generating steam turbines where it is converted into steam, a vapour. It flows around the hull of a ship or submarine. Lava, a non-Newtonian liquid, flows from an active volcano.

We should also be aware that some substances can exist in more than one state (or **phase**). Water, for example, can exist as ice (a solid), water (a liquid), or steam. The latter exhibits some of the characteristics of a gas, particularly at very high temperatures, and is termed a **vapour**. Many gases, including air, can be **liquefied** by subjecting them to very high pressure and/or low temperature.

Engineering fluid mechanics is concerned with analysing fluid flows, such as those mentioned above, in order to calculate the rates at which they flow, the changes in pressure as they flow, and the stresses and forces they exert on the machines and surfaces through and over

which they flow. The **law of conservation of mass**, **Newton's laws of motion**, and the **laws of thermodynamics** (principally the **first law** in the form of the **steady-flow energy equation**), together with appropriate representation of fluid properties, form the basis of the analysis. Before we go into further detail, it is useful to expand the catalogue of situations where fluid mechanics plays an essential role.

1.2 Fluid mechanics in nature

The height of the **atmosphere**, that is the altitude beyond which we are in the vacuum of outer space, is usually taken to be about 80 km. For many purposes, the atmosphere can be taken as a series of stationary spherical layers of air with the temperature variation shown in Figure 1.13.

We consider this **hydrostatic** model of the atmosphere in some detail in Section 4.13. We know, of course, that the atmosphere, especially the part of it we inhabit, is very often far from static; **meteorology** is the branch of fluid mechanics devoted to the study of its motion. Anyone who has seen time-lapse images of clouds knows that, in addition to being swept along by winds, they are in constant motion due to **thermals** (finite packets of warm air moving upwards which allows gliders to rise to altitudes up to about 15 km), **evaporation**, **condensation**,

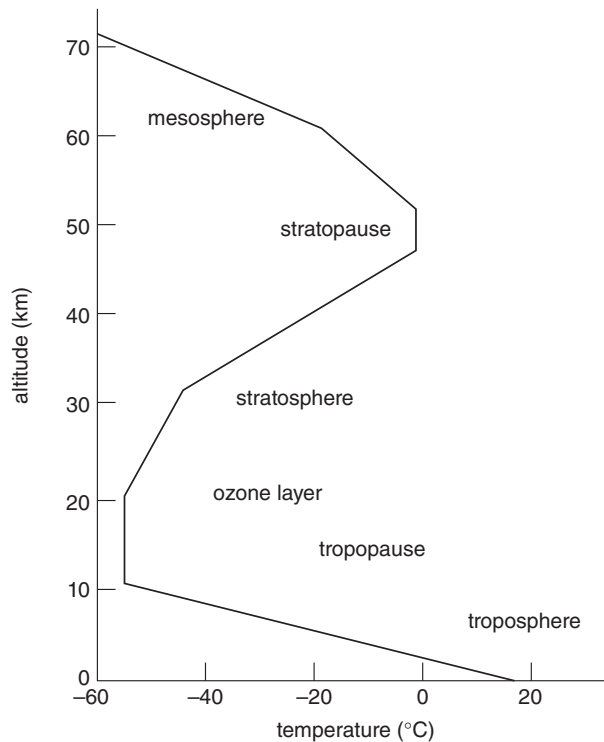


Figure 1.13 Structure of the atmosphere

and **shearing** (which gives rise to the **clear-air turbulence** often experienced by passenger aircraft). We should also mention dust devils, tornadoes, and hurricanes which are examples of the intense, often violent, **swirling motion** which can arise in the lower atmosphere due to combined thermal and shearing effects.

While it is essential that the earth is surrounded by a layer of air, it is just as important for humans (at least in our current state of evolution) that all the water in the hydrosphere is not distributed uniformly over the planet's surface. Were that the case, the water layer would be about 2.7 km deep. Instead, this water actually covers about 71% of the earth's surface, with regions of the deepest ocean being about 10 km deep, almost equal in magnitude to the height of Mount Everest. As with the atmosphere, much can be learned about the oceans, reservoirs, lakes, etc., by considering them to be at rest. Chapters 4 and 5 are devoted to **hydrostatics**—the study of fluids at rest—with a considerable fraction concerned with the forces exerted on surfaces, such as the face of a dam, as shown in Figure 1.4. **Oceanography** is the branch of fluid mechanics which deals with tides, currents, waves, **stratification** (water-density variations due to salinity and temperature changes with depth), and other phenomena associated with water motion in the oceans. Related topics involving fluid mechanics are erosion, sedimentation, whirlpools, river flows, and also the flow in canals and sewers, although the latter are man-made rather than natural systems. In principle, we could also include here the fluid mechanics associated with the wave-like body motion which fish, eels, aquatic mammals, and sperm use to swim.

Undeniably natural are the flows of lava from an active volcano and of hot water and steam from a geyser. The flow of formation fluids (oil, methane, hydrogen sulphide, brine, etc.), as well as drilling mud, from an oil well represents a mixture of man-made and natural phenomena. There would be no flow were it not for the man-made well, but the flow of formation fluids through porous rock involves natural fluids flowing through naturally occurring channels in a natural medium. Here again, however, in **hydraulic fracturing** (commonly referred to as '**fracking**') we are dealing with a combination of man-made and natural processes.

The study of flow in the circulatory, respiratory, urinary, and other biological systems is termed **biofluid mechanics**. As with all natural systems, an additional difficulty is that the geometry of the flow channels is not well defined and often not fixed. For example, arteries and veins are flexible and so change in cross section as blood pressure increases and decreases with every beat of the heart. To further complicate matters, blood is not a homogeneous liquid but consists mainly of red corpuscles, which are thin discs about $8\ \mu\text{m}$ in diameter with a thick rim, suspended in plasma. As a consequence of this composition, the effective viscosity (see Section 2.10) of blood decreases with shearing (relative tangential movement) and is slightly elastic (**viscoelastic**) in character, i.e. blood is a **non-Newtonian fluid**. At rest, blood has an effective viscosity about 100 times that of water, although this factor decreases to about five in the arteries. In any event, the saying 'blood is thicker than water' is entirely accurate. Although synovia, the fluid which lubricates our joints, is a homogeneous liquid, it is again a non-Newtonian fluid with **shear-thinning**, **viscoelastic** properties, in this instance because it has a polymer-like molecular structure.

1.3 External flows

As engineers, we are concerned primarily with fluids which flow either through or around man-made devices, which we term **internal** and **external flows**, respectively. In either case, **viscosity** (or to be more precise, **dynamic viscosity**) is the key fluid property which determines the details of the flow. Wherever velocity gradients occur in a flowing fluid, the fluid property viscosity leads to shear stresses and forces. A fundamental concept in fluid mechanics is the **no-slip condition** according to which, in the immediate vicinity of a solid surface, a consequence of viscosity is that the fluid is brought to rest (or, more generally, if the surface is itself moving, to the same velocity as the surface so that the relative velocity is zero). In essence, the fluid adheres to the surface. For an external flow, the major effects of viscosity are confined to a relatively thin region close to the surface called the **boundary layer**, the subject of Chapter 17.

In the case of buildings, smoke stacks (or chimneys), bridges, wind turbines, windmills, off-shore structures such as drilling platforms, etc., the external flow (there may be quite separate internal flows, such as exhaust gases) is provided by nature. The damage which sometimes occurs to these and other structures when high windspeeds arise tells us that the wind can impose massive forces on their surfaces. In certain circumstances, even at relatively low speed, a steady wind can excite vibrations (**flow-induced vibrations**) which can be of sufficient amplitude to cause structural damage. Huge plate-glass windows have been known to pop out of their frames due to wind-induced torsional oscillations of skyscrapers, as happened to the 241 m high **John Hancock Tower** opened in Boston in 1976. The best known example of wind-induced vibration was the complete destruction in 1940 of the **Tacoma Narrows Bridge** in Washington State, USA. Remarkably, in both instances, the vibration was initiated at windspeeds no greater than about 70 kph. In order to design structures which are safe, we need to calculate both the steady and periodic forces due to the wind, either from fundamental theory or, more likely, from a combination of theory and experimental data obtained from tests carried out in a wind or water tunnel. The use of experimental data, generalised using **dimensional analysis** (Chapter 3), is termed **empiricism**. **Environmental fluid mechanics** also concerns the **dispersion of pollutants** in the atmosphere and in the sea, rivers, lakes, etc.

Some of the most advanced theoretical and experimental work in fluid mechanics has been associated with the development of aircraft, spacecraft, and missiles. There have been remarkable advances in aviation since December 1903, when Orville Wright flew a powered, heavier-than-air, machine some 260 m in 59 s. For example, we now take for granted passenger aircraft such as the turbofan-powered Airbus A380-800 with a passenger-carrying capacity up to about 850, a maximum take-off weight of 575,000 kg, a wingspan of 80 m, a cruising speed of 945 km/h (just below soundspeed), and a range of 15,700 km. Although taken out of service in 2003, just as impressive was the performance of the **turbojet**-powered (see Section 10.3) British Aerospace Corporation/Aérospatiale supersonic transport aircraft, **Concorde**, which routinely carried about 130 passengers at twice **soundspeed** (a flight speed of about 2130 km/h) in the **stratosphere** (see Section 4.13). Although, as we see from Figure 1.10, the atmospheric temperature at **cruise altitude** (about 18 km) is about $-56.5\text{ }^{\circ}\text{C}$, the skin of Concorde reached a temperature of about $120\text{ }^{\circ}\text{C}$, due to **frictional heating**, causing the length of the aircraft to increase by about 0.3 m. Modern combat aircraft, such as the **Lockheed Martin F-22 Raptor**, again turbofan powered, can fly at **Mach numbers** above two (about 2500 km/h). Although

manned flights into space are now regarded as almost routine, in reality each flight represents an extraordinary engineering achievement. For example, the speed required to escape the earth's gravitational pull is about 11 km/s (i.e. 40,000 km/h or a Mach number above 30) and, on re-entry into the earth's atmosphere, the air surrounding the space shuttle becomes so hot (6000 °C plus) that the craft is surrounded by a glowing plasma.

One of the ways we distinguish between different flight regimes is through the Mach number, which is the ratio of the flight speed of an aircraft to the speed of sound at the flight altitude (discussed further in Section 3.12 and Chapter 11). As the Mach number increases, the fluid mechanics becomes more complicated because an increasing number of physical phenomena have to be taken into account. If the Mach number is considerably less than unity (0.3 is the value usually taken), changes in fluid density are negligible and the flow is said to be **incompressible**. For higher Mach numbers, **compressibility** effects (i.e. density changes) become increasingly important but can be accounted for in a relatively straightforward way using the **perfect-gas law** to relate temperature, pressure, and density (see Section 2.4), together with the first law of thermodynamics (Chapter 11). Once the Mach number exceeds about five, however, very high temperatures develop near surfaces, and the air properties change due to chemical breakdown of the molecules and the subsequent reaction of free atoms. At this point, physical chemistry also comes into play, but beyond the scope of this book.

The preceding paragraphs suggest an important aspect of the subject of fluid mechanics which students often find difficult to understand: even at relatively low flowspeeds, there are few problems we can solve completely, usually because the mathematics involved becomes far too complicated, even if we understand all the physics involved and know the relevant equations. To a degree, computers can take over at some stage in the analysis of a problem to provide a numerical rather than an analytical (i.e. algebraic) solution. Unfortunately, even the largest and fastest computers available at the present and in the foreseeable future are inadequate to solve most practical problems and we have no choice but to introduce approximations, assumptions, and simplifications. In fact, this 'engineering' approach represents common sense. For example, if we are dealing with a low-speed gas flow where we know that the fluid density remains practically constant, there is no point in making our task more difficult (and more expensive) than necessary by not introducing this simplification from the outset. Of course, it is usually a matter of experience, or even hindsight, which tells us what simplifications are justified. In this textbook, we approach problems using the simplest possible physics and mathematics, with the aim of deriving approximate solutions which provide some insight into the interplay between fluid properties, flow geometry, and flowspeed. The reader needs to bear in mind that our approach often represents only a start to, rather than a complete treatment of, the solution of problems of fluid flow.

Even land vehicles have now reached speeds where air-density variations must be accounted for. The land-speed record, held by the turbofan-powered car **Thrust SSC** since 1997, is 1228 km/h, which corresponds to a Mach number of 1.018, i.e. just **supersonic** (Mach numbers in the range close to unity are termed **transonic**). A new turbofan-powered car, **Bloodhound SSC**, is being developed with a target speed of about 1700 km/h (Mach 1.9). Somewhat slower is the Japanese Tōhoku Shinkansen '**Bullet Train**' which has a top speed of about 320 km/h or 89 m/s, corresponding to a Mach number of 0.26, so that compressibility effects are largely

insignificant. However, some racing cars can achieve speeds where compressibility effects cannot be neglected: the highest speed reached at the California Speedway track in Fontana, California, is about 400 km/h or 111 m/s, which corresponds to a Mach number of 0.33. Although this figure is close to the 0.3 ‘cutoff’, it must be the case that on the bodywork of the cars there would have been regions where the airspeed was considerably higher. It has to be said that normal cars, buses, and lorries have considerably lower top speeds and the airflow around them can safely be considered to be incompressible (i.e. to have constant density).

Although the speeds of even the fastest marine vehicles are much lower than for most land vehicles, the fluid mechanics involved is complicated by **wave motion** which arises due to the tendency for gravitational pull to overcome any disturbance to a water surface. We are all familiar with the surface **gravity waves** which propagate radially outwards when we throw a stone into a pond, whereas the forward movement of a ship creates a vee-shaped pattern of surface waves. Although invisible to the eye, a submarine travelling deep below the surface also generates gravity waves as it disturbs water layers of different densities which occur due to variations with depth of salt content and temperature. The energy required to generate waves has to be provided by the propulsion system of the ship or submarine and so corresponds to an additional contribution to the drag force, so-called **wave drag**.

1.4 Internal flows

Most of the flow situations dealt with in this textbook are concerned with internal flows through **pipes, ducts, nozzles**, engines, **turbomachines**, etc. In one sense, internal flows are easier to deal with than external flows because the flow is confined within solid boundaries unlike the flow over an **aerofoil** (Figure 1.6), for example, where the region of flow is practically unlimited.

The most common man-made device through which flow occurs is a metal, plastic, or glass pipe of circular cross section. Pipes of this kind allow oil and gas to flow to the earth’s surface from reservoirs which may be many kilometres below, often deep below the seabed, and then hundreds of kilometres across land, directly to refineries or to ports for transfer to ships. Oil and gas pipelines, and also the pipes which convey water into the turbines of a **hydroelectric power plant**, may be a metre or more in diameter. The enormous capital cost involved means that careful consideration has to be given to the design of such pipelines including all the associated **valves, bends, contractions, expansions, pumps**, monitoring equipment, etc. Smaller diameter pipes connect the pumps, separators, boilers, distillation columns, burners, filters, etc., of oil refineries and other chemical-processing plant. Such pipes allow gas and water to be transported to the homes where we live and to the offices and factories where we work. Fluid flow through a straight pipe is resisted by **friction** between the fluid and the internal surface of the pipe, which arises due to the viscosity of the fluid (see Section 2.8) and has to be overcome by a pressure difference created by a pump or compressor, or by gravitational effects. Friction also causes the fluid temperature to rise, the fluid density to decrease, and the average fluid velocity to increase. Much like the situation of an external flow, a boundary layer develops and grows in thickness with downstream distance so that, in an internal flow, if the flow channel is long enough, it is inevitable that eventually fluid across the entire cross section of the channel

is affected by viscosity (see Chapter 16). Pipe flow of compressible fluids is considered in detail in Chapter 13. Due to surface friction or external heating, in the case of a gas, the fluid velocity in a pipe may even reach the speed of sound, causing an effect called **choking** (see Chapters 11 and 13), which limits the volume of gas which can be pumped through the pipe. Clearly, even a flow which at first sight probably appears to be the simplest we can think of turns out to be rather complicated. In fact, the situation is even more complicated than we have indicated so far because it is only for low flowrates or small-diameter pipes or highly viscous fluids (all of these influences are accounted for by a non-dimensional parameter termed the **Reynolds number**, which we discuss further in Chapters 3 and 15 to 18) that the flow remains smooth and steady (so-called **laminar flow**) and we are able to analyse it completely. The majority of flows of engineering interest exhibit a high degree of random unsteadiness which we call **turbulence** (see Chapter 18) and, even today, we are able to calculate turbulent pipe flow from first principles only through the use of supercomputers. Fortunately, the principles of dimensional analysis apply whether a flow is laminar or turbulent, and this enables us to generalise experimental data for use in engineering-design calculations.

In industrial applications, pipes rarely stay straight or keep the same diameter for long (see Section 18.11). Often more important than understanding the details of the flow within a pipe or pipe system is the ability to calculate the **hydrodynamic forces** which arise when a pipe changes direction and, perhaps, also diameter, as illustrated by the pipe bend in Figure 1.14 (see Section 10.7).

The calculation of hydrodynamic forces is one of the main topics of Chapter 10, which brings together many of the concepts and principles introduced in previous chapters, particularly those in Chapters 6, 7, and 9.

Combustion chambers, furnaces, boilers, jet pumps, control valves, guidevanes, cyclone separators, radiators, oil coolers, fuel-injection systems, carburettors, rocket engines, and the coolant channels within the core of a nuclear reactor or the block of a petrol or diesel engine

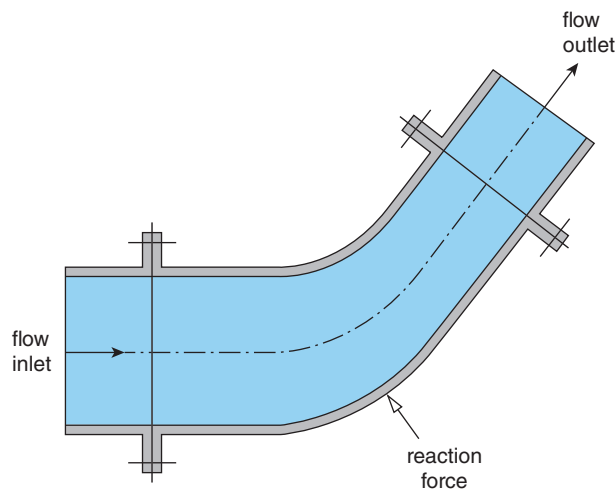


Figure 1.14 Hydrodynamic reaction force exerted on a pipe bend

are all examples involving internal fluid flow. As we show in Chapter 10, the flow characteristics which underlie the design of many of these devices, including the **rocket engine**, **jet pump**, and **cascade of guide vanes**, shown in Figures 1.15, 1.16, and 1.17, respectively, can be determined using the principles of fluid mechanics that we cover in this textbook. The analysis of most of the other cases requires more advanced aspects of fluid mechanics and may also involve considerations of heat transfer, thermodynamics, and chemistry, all of which are beyond the scope of this book.

The turbojet and turbofan engines shown in Figures 10.3(a) and 1.8, respectively, are examples of a class of devices called **turbomachines**, derived from the Latin word *turbo*, which has the meanings 'whirlwind' and 'spinning top'. Other examples of turbomachines are pumps, fans, compressors, steam turbines, gas turbines, hydraulic turbines (see Figure 1.9), turbochargers, and superchargers. A common feature of all turbomachines is a central rotating

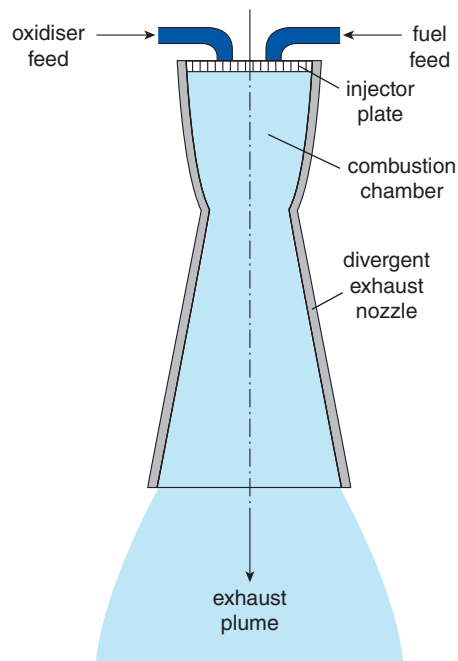


Figure 1.15 Thrust of a liquid-propellant rocket

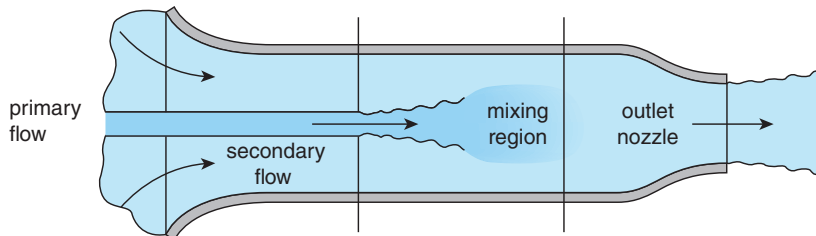


Figure 1.16 Performance of a jet pump

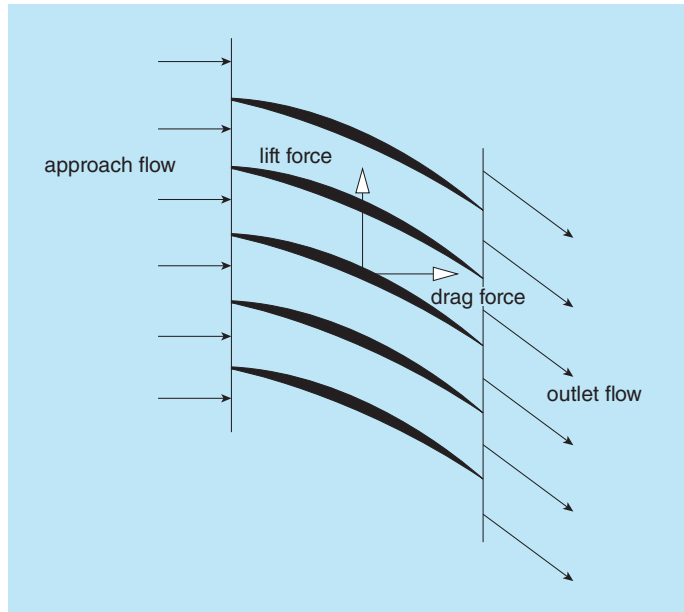


Figure 1.17 Hydrodynamic forces on a cascade of guide vanes

shaft which carries blades (the **rotor** or **impeller**) to transfer momentum and work either to or from the fluid which flows through them by causing changes in the direction of fluid flow. Most turbomachines also incorporate stationary blades (called **stators** or **nozzle rings**) attached to the casing to guide the flow to and from the rotor stages. As we show in Chapter 10, we can learn a considerable amount about the performance of these complex machines simply by considering the state of flow at inlet and outlet. The basic flow within a **stator** or **rotor stage** can be analysed in much the same way as that through a stationary **cascade of guide vanes**; but, to take the analysis further, as we do in Chapter 14, requires that we use more advanced aspects of fluid mechanics, often together with considerations of thermodynamics.



1.5 SUMMARY

In this chapter, we have indicated the wide array of engineering devices, from the kitchen tap (a valve) to supersonic aircraft, for which the basic design depends upon considerations of the flow of gases and liquids. Much the same is true of most natural phenomena, from our weather to ocean waves and the movement of sperm and other bodily fluids. This textbook introduces a number of the concepts, principles, and procedures which underlie the analysis of any problem involving fluid flow. In this **Introduction**, we have selected a number of examples for which, by the end of the book, the student should be in a position to make practically useful engineering-design calculations. We emphasise that simply attending lectures or reading this book is not sufficient: it is absolutely essential for the student to spend at least twice the amount of lecture time attempting to solve the self-assessment problems which follow most chapters.

2

Fluids and fluid properties

Wet. Sticky. Viscous. Viscid. Gelatinous. Slippery. Greasy. Oily. Lubricious. Slimy. Oleaginous. Oozy. Soapy. Thick. Thin. Runny. Syrupy. Treacly. Tacky. Claggy. Muddy. Gummy. Goopy. Mucilaginous. Glutinous. These are among the many adjectives commonly used to describe liquids, to convey something about how liquids feel, how they flow, or how they respond to being stirred or mixed. The list of words available to describe gases is far more limited: viscous, viscid, heavy, and dense. We could also include smelly in both lists, although in the case of liquids what is sensed is the vapour form. In contrast to these adjectives, which primarily give us a qualitative tactile impression, in this chapter we introduce the properties used to quantify the physical characteristics of liquids and gases: **dynamic** and **kinematic viscosity**, **density**, **specific volume**, **relative density**, **bulk modulus of elasticity** and **compressibility**, **speed of sound** (or **soundspeed**), **vapour pressure**, and **surface tension**, together with the **perfect-gas law** and an **equation of state for liquids**. We discuss how and why fluids and solids are different both on a molecular and on a macroscopic scale. We show that central to the definition of the physical properties of fluids, and the way in which we go on to analyse fluid flow, is the **continuum hypothesis**, which allows us to define properties on a scale which is far smaller than any scale of engineering interest but still far larger than the underlying molecular scale.

2.1 Fluids and solids

The state of any substance can be classified as **solid** or **fluid**, with the term fluid including **liquids**, **gases**, and **vapours**. From an engineering viewpoint, the essential difference between a fluid and a solid is the way in which the substance resists **shear stress**. In the case of a solid, the shear stress is resisted by a static deformation, the magnitude of which (for a given shear stress) depends upon a material property called the **modulus of rigidity**. For a fluid, no matter how low the shear stress, the deformation increases without limit as long as the shear stress is applied. The rate of deformation of a fluid is determined by a property called the **dynamic viscosity** (or just the **viscosity**). A fluid for which the viscosity is zero is said to be **inviscid**, whereas a fluid with non-zero viscosity is said to be **viscous**⁴. A fluid with vanishingly small viscosity is also termed a **perfect fluid**, the only known example of which is liquid helium cooled to 2.17 K, at which critical temperature a fraction of the liquid becomes an inviscid **superfluid**.

We can begin to quantify the statements in the first sentences of the preceding paragraph as follows. Suppose we have a solid rectangular block subjected to a shear (i.e. tangential) force F ,

⁴ The term **viscid** is also used.

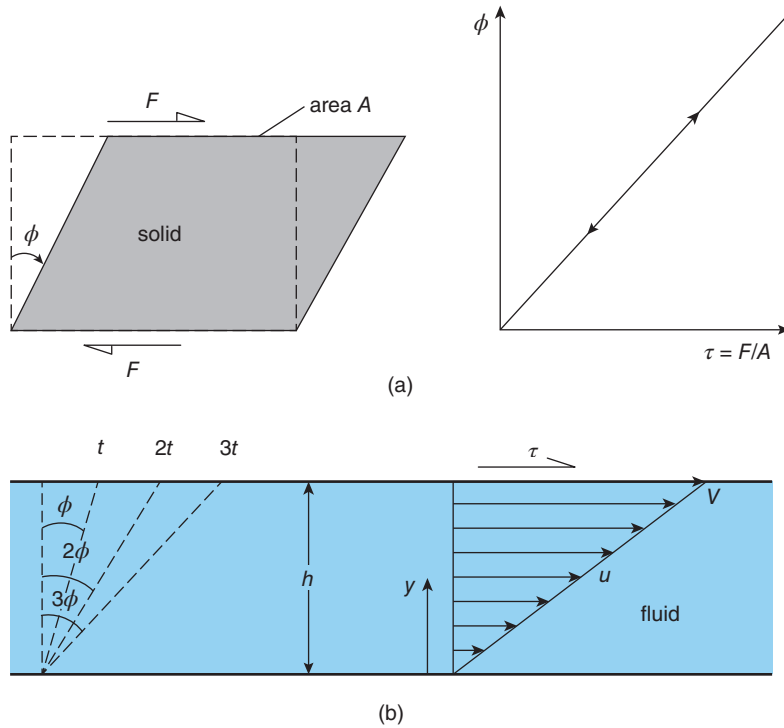


Figure 2.1 (a) Shear force applied to an elastic solid (b) Shear stress applied to a fluid

as illustrated in Figure 2.1(a). Unless the magnitude of the force is so great that the material fractures or deforms plastically (in a sense, behaving like a liquid), the solid resists the force F by a static deformation which we can measure by the angle ϕ (the Greek letter *phi*). In the case of an elastic solid, according to **Hooke's law**, the deformation is proportional to the applied force, so we can write

$$\frac{F}{A} = \tau = G\phi \quad (2.1)$$

where A is the surface area over which F is distributed, τ (the Greek letter *tau*) is the shear stress (i.e. the shear force per unit area), and the constant of proportionality G is called the **modulus of rigidity** or **shear modulus**.

Consider now the situation illustrated in Figure 2.1(b), which shows a fluid between two parallel plates separated by a short distance h , with the lower plate stationary and the upper plate moving at velocity V . A fundamental concept of the flow of a viscous fluid, called the **no-slip condition** (see Sections 6.4 and 15.3), is that fluid in contact with a solid surface adheres to it and moves at the speed of the surface. Thus, the fluid in the immediate vicinity of the upper surface moves forwards at velocity V , the fluid in contact with the lower surface is at rest, and the fluid in-between moves as though in infinitesimally thin layers with velocity u , which increases progressively with distance y from the lower surface, i.e.

$$u = \frac{Vy}{h}. \quad (2.2)$$

If we imagine a line normal to the plate surfaces and marking the fluid at some instant of time, at time t later the line will have rotated through an angle ϕ , as shown in Figure 2.1(b), so that

$$\tan \phi = \frac{Vt}{h}. \quad (2.3)$$

If the time t is short, the angle ϕ will be small and negligibly different (measured in radians) from $\tan \phi$, so that

$$\phi = \frac{Vt}{h}, \quad (2.4)$$

from which we see that if t doubles, ϕ also doubles; if t triples, ϕ also triples; and so on. Rather than think of progressive deformation in this way, it is far more convenient to think in terms of the **rate of change of deformation**, which is given by

$$\frac{d\phi}{dt} = \frac{V}{h}. \quad (2.5)$$

From equation (2.2) and Figure 2.1(b) we can see that the quantity V/h is the gradient of the velocity u with respect to distance y , i.e.

$$\frac{du}{dy} = \frac{V}{h}. \quad (2.6)$$

Because gradients of velocity within a fluid occur due to the effects of shear stress, the **rate of deformation** du/dy is referred to as the **shear rate**. For a fluid, the statement equivalent to equation (2.1) can now be written as

$$\tau = \mu \frac{d\phi}{dt} = \mu \frac{du}{dy} \quad (2.7)$$

where the symbol μ (the Greek letter *mu*) represents the fluid property known as **dynamic viscosity** (usually just referred to as the **viscosity**). In some books, the symbol η (the Greek letter *eta*) is used rather than μ . Viscosity is the principal property which distinguishes a fluid from a solid, and many of the adjectives listed at the beginning of this chapter are qualitative descriptions of the viscous nature of fluids. For many simple fluids, including air and water, μ is a thermodynamic property which depends only upon temperature and pressure but not on the shear rate. As mentioned in Chapter 1, such fluids are known as **Newtonian**. One of Newton's many contributions to scientific understanding was the recognition that the resistance to relative motion between two 'layers' of a fluid is proportional to the velocity difference between the layers, as represented by equation (2.7).

It is easy to find descriptive distinctions between the four states (solid, liquid, gas, vapour) in which matter occurs. Solids are hard and not easily deformed. A liquid has no inherent shape and is so easily deformed that under the influence of gravity it takes up the shape of any container into which it is poured without a change in volume. A gas is even easier to deform than a liquid and increases in volume without limit unless constrained by a closed container, which it then fills completely. The volume of a fixed mass of gas is decreased by any increase in pressure, whereas to decrease the volume of a liquid by a measurable amount requires very high pressures (see Section 2.6). These and other differences between the gas, liquid, and solid states can be explained on the basis of their molecular structures. Movement of the molecules

of a solid is highly restricted because they are closely packed in a fixed lattice structure with large **intermolecular cohesive forces** between them. The molecules of a liquid have freedom of movement and are further apart (though the typical spacing is still only 10^{-10} m or 0.1 nm) so the intermolecular forces are smaller. In fact, the molecules are in a continual state of interaction with their neighbours and never move very far. Gas molecules, on the other hand, move about randomly but in straight lines at high speed (about 1.2 to 1.5 times the speed of sound), occasionally colliding with each other or the surfaces of a confining container. For both liquids and gases the continual bombardment of any surface by molecules gives rise to a stress which is normal to the surface and which we call **pressure**.

Since many substances can exist in any one of the three basic states, the differences in molecular structure are largely a matter of degree, and there is the possibility of transition between these states. For example, the volume of a fixed mass of gas is easily decreased by increasing its pressure, a process termed **compression**, while **expansion** is the opposite process. At very high levels of compression the gas molecules are forced so close together that the gas becomes indistinguishable from a liquid and is said to liquefy. **Liquefaction** can also be achieved by cooling a gas to a temperature below its **critical temperature**. The free surface of any liquid is always in contact with its gaseous state, called a **vapour**. At sufficiently high temperature many solids melt and become liquid and, with further increase in temperature, increasing amounts of vapour are produced until all the material is in the gaseous state. These different states are identified thermodynamically as **phases** which represent forms of matter which are physically and chemically stable.

2.2 Fluid density ρ

The **density** ρ (the Greek letter *rho*) of a fluid (or a solid), sometimes referred to as its **mass density**, is the ratio of the mass m of a given volume of that substance to its volume $\mathcal{V}$, i.e.

$$\rho = \frac{m}{\mathcal{V}}. \quad (2.8)$$

In the SI system of units, which we use exclusively in this textbook and present in some detail in Chapter 3, the unit of mass is the kilogram (symbol kg), that of volume is the cubic metre (m^3), and the unit of density is kilogram per cubic metre (kg/m^3). As we indicated in Section 2.1, we can decrease the volume of a fixed mass of gas by increasing its pressure. According to equation (2.8), the consequence of compression is an increase in the gas density. The pressure of the air flowing through the core of a jet engine, such as that illustrated in Figure 1.8, is increased progressively as the air passes through the compressor stages and so the air density also increases (there is an accompanying increase in temperature).

As may be evident, our definition of density in the previous paragraph is incomplete: the idea that the density of air can vary with location as it flows through a compressor implies that we regard density as having a value at a given point, as is the case for all fluid properties. A more complete definition of density requires that the volume $\mathcal{V}$, and hence the mass m , is so small that there is no appreciable variation of density within it. At the extreme, we could define a volume so small that at any instant of time it contained a single molecule but this

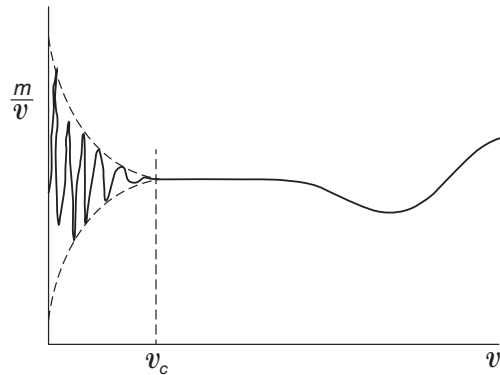


Figure 2.2 Variation of the ratio mass: volume with volume

does not lead to a sensible definition of density, not least because the molecule would not have a fixed location. However, by progressively increasing the volume above such a low value, we eventually reach a situation where, although molecules are continuously moving into and out of the volume at its boundary, the net number of molecules within the volume at any instant is practically constant. The effect on the ratio $m:v$ of progressively increasing v is shown schematically in Figure 2.2. The horizontal scale is compressed to the right of the vertical line, and expanded to its left. Once v exceeds the critical value v_c , we can define a density as a thermodynamic property (i.e. a physical property which depends only upon temperature and pressure) which is independent of volume and which can vary smoothly and continuously throughout the entire body of fluid. We shall quantify the order of magnitude of v_c in Section 2.5. The densities and other properties of pure water and dry air at a pressure of 1 atm are given in Tables A.3 and A.4, respectively⁵. For other fluids of engineering interest the physical properties are given in Tables A.5 (liquids) and A.6 (gases), also at a pressure of 1 atm.

There are two principal ways in which the density of a fluid influences flow. The most important stems from **Newton's second law of motion**, which tells us that the acceleration of a given mass is proportional to the net force applied to it. We shall discuss in detail the application of Newton's second law to fluid flow in many of the chapters in this book. For the time being it is sufficient to realise that to produce a change in the velocity of a high-density fluid, such as a liquid, involves much larger forces (per unit volume) than is the case for a fluid of low density, such as a gas. For example, the power required to propel a submerged **submarine** would be about a thousand times greater than for an **airship** of the same size and speed flying through the air. The second way in which density plays a role involves gravity and the associated decrease in atmospheric pressure with altitude or increase in pressure with liquid depth. These and other **hydrostatic** effects are the subject of Chapters 4 and 5. **Compressible flow**, in which there can be very large, and even discontinuous, changes in density, is the subject of Chapters 11, 12, and 13.

⁵ Table A.1 lists some atomic and molecular weights, and Table A.2, some universal constants. The physical properties of the 1976 Standard Atmosphere (see Section 4.13) are given in Table A.7. Tables A.1 to A.7 form Appendix 2.

2.3 Atoms, molecules, and moles

All matter is made up of a limited number of elementary substances, the **chemical elements** (as of November 2011, 118 had been identified: 94 naturally occurring and 24 synthetic). So far as this textbook is concerned, the basic building block for any chemical substance is the **atom**, a tiny (typically with a radius less than 1 nm) particle which cannot be split without losing the properties of the element. Each element has a **relative atomic mass (atomic weight)** based on a scale in which the mass of the carbon-12 (^{12}C) atom, the most abundant (almost 99%) isotope of carbon, is 12. Most substances consist of **molecules** in which atoms are bound together by interatomic forces. In a way similar to that of atomic weight, the **molecular weight $\mathcal{M}$ (relative molecular mass, or molecular mass)**, with the units kg/kmol, of these compounds is defined relative to the mass of ^{12}C . The atomic weights and molecular weights of some common substances are listed in Table A.1 in Appendix 2, together with the symbols used for atoms or the molecular formulae for molecules.

Although molecular weight is defined as a ratio, and so is a non-dimensional quantity (see Chapter 3) which has no units, it is useful to express molecular weights in terms of a unit called the **mole** (symbol mol), 1 mol being the amount of a substance in grams numerically equal to its molecular weight, or the **kilomole** (symbol kmol), which is the amount of substance in kilograms. In the case of methane, for example, $\mathcal{M} = 16.04 \text{ kg/kmol}$. The number of molecules in 1 kmol of any substance is given by the **Avogadro number**, N_A , a fundamental physical constant the value of which is 6.022×10^{26} molecules/kmol. If we have N molecules of a substance with molecular weight $\mathcal{M}$, the amount of that substance $n = N/N_A$ kmol, and the corresponding mass is $m = n\mathcal{M} = N\mathcal{M}/N_A$ kg.

2.4 Perfect-gas law

At very high temperatures (above about 1000°C) the molecular structure of a gas breaks down (a process known as **dissociation**) and at very high pressures or low temperatures, as we have already indicated in Section 2.1, gases can liquefy. Away from these extremes, most gases are in good agreement with a **thermal equation of state** known as the **perfect-gas⁶ law**

$$p = \rho RT, \quad (2.9)$$

where p is the gas pressure in pascal ($\text{Pa} = \text{N/m}^2$), T is the absolute temperature of the gas in degrees kelvin ($\text{K} = 273.15 + ^\circ\text{C}$), and R is a constant of the gas called the **specific gas constant** (with units $\text{m}^2/\text{s}^2 \cdot \text{K}$ or $\text{kJ/kg} \cdot \text{K}$). A gas which obeys the equation of state $p = \rho RT$ is a **thermally perfect gas⁷**. The unit $\text{m}^2/\text{s}^2 \cdot \text{K}$ suggests a connection between R and a speed which we shall show in Section 2.12 is that for the propagation of sound through the gas, i.e. the **speed of sound**. The specific gas constant is related to the **universal (or molar) gas constant $\mathcal{R}$** as follows

⁶ The term perfect gas should not be confused with perfect fluid, which is an idealised fluid lacking both viscosity and thermal conductivity.

⁷ Such a gas is sometimes termed an **ideal gas** rather than a perfect gas.

$$\mathcal{R} = \mathcal{M}R, \quad (2.10)$$

where $\mathcal{M}$ (with the unit kilogram per kilomole) is the molecular weight of the gas. The universal gas constant is defined in terms of the **Boltzmann constant** k_B and the Avogadro number N_A as

$$\mathcal{R} = k_B N_A. \quad (2.11)$$

Boltzmann's constant has the value 1.3807×10^{-23} J/K, and the universal gas constant has the value 8.31451 kJ/kmol · K (or 8314.51 J/kmol · K).

The specific gas constant R is equal to the difference between the **specific heats** at constant pressure C_P and constant volume C_V , i.e.

$$R = C_P - C_V. \quad (2.12)$$

For a range of gases, values for the molecular weight $\mathcal{M}$, the specific gas constant R , and the ratio of the specific heats,

$$\gamma = \frac{C_P}{C_V} \quad (2.13)$$

are tabulated in Table A.6. A perfect gas for which C_P and C_V , and hence γ , are constant is called a **calorically perfect gas**. It is usual to refer to a calorically perfect and thermally perfect gas obeying $p = \rho RT$ simply as a **perfect gas**. The quantities in Table A.6 play an important role in compressible-flow theory (see Chapters 11, 12, and 13). Although values for the corresponding gas density ρ at STP (20 °C, 1 atm) are also tabulated, this is not essential, since the density of any of the gases listed can be calculated from equation (2.9).

ILLUSTRATIVE EXAMPLE 2.1

Calculate the density of nitric oxide (NO) at 20 °C and 1 atm and also at 500 °C and 5 bar.

Solution

$\mathcal{M} = 30.01$ kg/kmol (from Table A.6); $p_1 = 1.01325 \times 10^5$ Pa; $T_1 = 293.15$ K; $p_2 = 5 \times 10^5$ Pa; $T_2 = 773.15$ K.

From equation (2.10)

$$R = \mathcal{R}/\mathcal{M} = 8314.51/30.01 = 277.1 \text{ m}^2/\text{s}^2\cdot\text{K}.$$

From equation (2.9)

$$\rho_1 = \frac{p_1}{RT_1} = \frac{1.01325 \times 10^5}{277.1 \times 293.15} = 1.248 \text{ kg/m}^3$$

and

$$\rho_2 = \frac{p_2}{RT_2} = \frac{5 \times 10^5}{277.1 \times 773.15} = 2.334 \text{ kg/m}^3.$$

Comments:

- (a) It is generally unnecessary to carry so many significant figures (s.f.) in an engineering calculation; 4 s.f. for $\mathcal{R}$ and 3 for other quantities are usually sufficient.
- (b) As they should be, the values calculated here for R and ρ_1 for NO are precisely the same as those in Table A.6.
- (c) The first step in the solution was to restate the data given (in this case for temperature and pressure) in standard SI units. The student should develop the habit of converting given data to standard SI form in this way.

2.5 Continuum hypothesis and molecular mean free path

In Section 2.1 we discussed some of the qualitative differences between the molecular structures of liquids and gases. As we shall now see, these differences have a direct influence on the size of the critical volume $\mathcal{V}_C$ introduced in Section 2.4.

We consider first a gas with molecular weight $\mathcal{M}$ which obeys the perfect-gas law, to calculate the average number of molecules contained in a cube (the choice of a cube is arbitrary, and we could just as well have chosen another shape, such as a sphere) of gas of side length L m. If the fluid density is ρ , from equation (2.8) the mass of the cube will be ρL^3 , since the cube volume $\mathcal{V} = L^3$. From equations (2.9) and (2.10) we have

$$\rho = p\mathcal{M}/\mathcal{R}T \quad (2.14)$$

so that the mass of our cube is given by

$$m = \rho\mathcal{V} = p\mathcal{M}\mathcal{V}/\mathcal{R}T. \quad (2.15)$$

From Section 2.3 we know that the molecular weight $\mathcal{M}$ is the mass in kg of 1 kmol of that substance, so that our cube contains $p\mathcal{V}/\mathcal{R}T$ kmol (the unit kmol is often written as kg mole). Since the number of molecules in 1 kmol of any substance is given by the **Avogadro number**, N_A , the value of which is 6.022×10^{26} molecules/kmol, we see that the average number of molecules N in the cube must be given by

$$N = p\mathcal{V}N_A/\mathcal{R}T. \quad (2.16)$$

Equation (2.16) can be rearranged as

$$\mathcal{V} = N\mathcal{R}T/pN_A \quad (2.17)$$

from which we can calculate the volume $\mathcal{V}$, which contains N molecules of a gas at temperature T (K) and pressure p (Pa).

In terms of the gas density ρ , equation (2.17) becomes

$$\mathcal{V} = NM/\rho N_A. \quad (2.18)$$

Equation (2.16) shows that, since $\mathcal{R}$ is a universal constant, the same for all gases, the average number of molecules N in a volume $\mathcal{V}$ of any gas depends only upon its pressure p and

Table 2.1 Number N of gas molecules in a volume $\mathcal{V}$.

L	$\mathcal{V}$ (m^3)	N
1 mm	10^{-9}	2.7 E+16
1 μm	10^{-18}	2.7 E+7
100 nm	10^{-21}	2.7 E+4
50 nm	1.25×10^{-22}	3362
33.4 nm	3.73×10^{-23}	1000
20 nm	9×10^{-24}	215
10 nm	10^{-24}	27
3.34 nm	3.73×10^{-26}	1

absolute temperature T . This equation can therefore be regarded as a quantitative form of Avogadro's law: equal volumes of two gases, at the same temperature and pressure, contain the same number of molecules. Table 2.1 shows the results obtained for N using equation (2.16), for 0°C and 1 atm (standard temperature and pressure, or STP).

As we shall see in Section 4.13, the air density in the atmosphere decreases with altitude. At the lower limit of the **stratosphere** (an altitude of about 20 km), according to Table A.7, the temperature is about 217 K, and the pressure is 5475 Pa (the corresponding density is 0.0880 kg/m^3), while at the outer limit of the mesosphere (about 80 km) the values are 196.7 K and 0.886 Pa, respectively, so that, according to equation (2.17), a cube of air containing 1000 molecules would have a side length of 81.7 nm at an altitude of 20 km, and $0.797\text{ }\mu\text{m}$ at 80 km, both of which are negligibly small compared with the dimensions of any object likely to be flying at such altitudes. Equation (2.18) also shows that the density of air would have to fall to $4.8 \times 10^{-14}\text{ kg/m}^3$ (which would correspond to an altitude of about 1600 km) for the cube size to reach 1 mm.

We cannot give a precise value but would probably not want the number of molecules over which to form an average to be any lower than 1000 and so conclude that for a gas at STP the concept of fluid density begins to fail if the cube size $\mathcal{V}_C$ is below about 30 nm (i.e. $0.3\text{ }\mu\text{m}$ or $3 \times 10^7\text{ m}$). To put this in perspective, the diameter of a human hair is typically about $100\text{ }\mu\text{m}$, and the wavelength of visible light is about 589 nm. There are few, if any, practical situations involving gas-flow channels with dimensions which come anywhere close to 30 nm. Even devices known as **microchannels** typically have dimensions in the range 1 to $500\text{ }\mu\text{m}$. Gases for which the number of molecules in a $1\text{ }\mu\text{m}$ cube fall below about 1000 are said to be **rarified** and are normally encountered only in outer space.

Because the molecular structure of a liquid is generally more complex than that of a gas, the number of molecules per unit volume, N , which is termed the **molecular number density**, varies from liquid to liquid. For a cube of side length L of liquid with density ρ the mass is again given by $m = \rho L^3$. The number of kilomoles of liquid is then $\rho L^3/\mathcal{M}$, and the number of molecules is $\rho L^3 N_A/\mathcal{M}$. Table 2.2 shows values of N for several liquids, with $L = 1\text{ }\mu\text{m}$ ($\mathcal{V} = 10^{-18}\text{ m}^3$).

Table 2.2 Number N of molecules in a liquid cube of side length $L = 1 \mu\text{m}$

Liquid	N
Petrol, C_8H_{18}	4.4 E+9
Carbon tetrachloride, CCl_4	6.3 E+9
Liquid oxygen, O_2	7.7 E+9
Pure glycerol, $\text{C}_3\text{H}_8\text{O}_3$	8.2 E+9
Ethyl alcohol, $\text{C}_2\text{H}_5\text{OH}$	1.0 E+10
Water, H_2O	3.35 E+10
Mercury, Hg	4.07 E+10

If we compare Table 2.2 with Table 2.1, we see that the molecular number density N for liquids far exceeds that for gases. We conclude from the foregoing that, except in extreme circumstances, $\mathcal{V}_C$ will always be far smaller than any volume of engineering interest and can be regarded as defining what we mean by a **point** in a fluid. Although we have specifically discussed the property density, the same considerations apply to any physical property and enable us to define point values of these properties, which vary smoothly and continuously throughout a fluid. Although these ‘large-scale’ (or **macroscopic**) properties reflect the underlying molecular structure, it is generally the case that we can treat the majority of problems of fluid flow without the need to consider molecular structure directly. The idea that both fluid properties and flow properties can be treated in this way is known as the **continuum hypothesis**.

If molecules in a fluid are considered to be hard spheres of effective diameter σ in random motion constantly colliding with each other elastically, **kinetic theory** leads an approximate expression for the average distance Λ between successive collisions

$$\Lambda = \frac{1}{\sqrt{2}\pi N_V \sigma^2} \quad (2.19)$$

where N_V is the number of molecules per unit volume or the **volume number density**. The quantity Λ is termed the **molecular mean free path**, and equation (2.19) is usually attributed to James Clerk Maxwell. The equation for Λ can be written in terms of other quantities as follows. The number of moles n in a mass of gas m of molecular mass (or molecular weight) $\mathcal{M}$ is given by

$$n = m/\mathcal{M} \quad (2.20)$$

and so the number of molecules in the mass of gas N_M is

$$N_M = nN_A = mN_A/\mathcal{M} \quad (2.21)$$

where N_A is the Avogadro number (see Section 2.3). It follows that

$$N_V = N_M/\mathcal{V} = mN_A/\mathcal{M}\mathcal{V} = \rho N_A/\mathcal{M} \quad (2.22)$$

and, from equation (2.19),

$$\Lambda = \mathcal{M}/\sqrt{2\pi}\rho N_A\sigma^2. \quad (2.23)$$

Since $\mathcal{M}$ and σ are fixed for a given gas, and N_A is a universal constant, equation (2.23) leads to the conclusion that Λ is inversely proportional to the gas density ρ . As we shall see in Section 4.13, the density of the air in the earth's atmosphere decreases with altitude (as can be seen from Table A.7). Although at an altitude of 71 km ρ has fallen to about 0.01% of its value at sea level, the corresponding value of Λ is still less than 1 mm.

If we introduce the perfect-gas equation (2.9), $p = \rho RT$, equation (2.23) may be written as

$$\Lambda = \mathcal{R}T/\sqrt{2\pi}pN_A\sigma^2 \quad (2.24)$$

where we have made use of equation (2.10) to introduce the universal gas constant $\mathcal{R}$. Since the ratio $\mathcal{R}/N_A$ defines the Boltzmann constant k_B , the equation for Λ may also be written as

$$\Lambda = \frac{k_B T}{\sqrt{2\pi}p\sigma^2}. \quad (2.25)$$

Table 2.3 includes values of the effective molecular diameter σ and molecular mean free path Λ for some common gases at 0 °C and 1 atmosphere⁸.

Table 2.3 Effective molecular diameter σ and molecular mean free path Λ for some common gases at 0 °C and 1 atmosphere

Gas	σ (pm)	Λ (nm)
Air	366	69.1
Argon	342	62.6
Carbon dioxide	390	39.0
Carbon monoxide	371	58.6
Chlorine	440	27.4
Ethylene	423	34.3
Helium	258	173.6
Hydrogen	297	110.6
Methane	380	48.1
Neon	279	124.0
Nitrogen	375	58.8
Nitrous oxide	388	38.7
Oxygen	354	63.3
Sulphur dioxide	429	27.4

⁸ With the exception of those for air, the values for σ and Λ have been taken from Kaye and Laby online. The values for air are from the *CRC Handbook of Chemistry and Physics*. Many of the values from these two sources differ by as much as 20%.

For the gases in the table, the arithmetic average value for $\Lambda = 66.8$ nm. A cube with this side length 66.8 nm would have a volume of about $3 \times 10^{-22} \text{ m}^3$ and so contain about 8000 molecules, another indication of the validity of the continuum hypothesis.

ILLUSTRATIVE EXAMPLE 2.2

Calculate the molecular mean free path for a gas with molecular weight 28.96 kg/kmol, density 1.28 kg/m³, and effective molecular diameter 366 pm.

Solution

$\mathcal{M} = 28.96$ kg/kmol; $\rho = 1.28$ kg/m³; $\sigma = 3.66 \times 10^{-10}$ m; $N_A = 6.022 \times 10^{26}$ molecules/kmol. We use equation (2.23) to find Λ

$$\begin{aligned}\Lambda &= \mathcal{M} / \sqrt{2} \pi \rho N_A \sigma^2 \\ &= \frac{28.96}{\sqrt{2} \times \pi \times 1.28 \times 6.022 \times 10^{26} \times (3.66 \times 10^{-10})^2} \\ &= 6.33 \times 10^{-8} \text{ m or } 63.3 \text{ nm.}\end{aligned}$$

Comment:

The value for Λ calculated from equation (2.23) represents the result of kinetic theory for a gas with the properties of dry air. This value differs by about 8% from the experimentally determined value of 69.1 nm.

2.6 Equation of state for liquids

Although equation (2.12) for the ratio of specific heats γ , has no generally valid equivalent applicable to liquids, it is usually adequate to assume that $\rho = \text{constant}$, and $C_P = C_V = \text{constant}$, so that

$$\gamma = \frac{C_P}{C_V} = 1. \quad (2.26)$$

An approximate equation, cited by Batchelor (2000), for the influence of extreme pressure (typically in excess of 1000 bar) on the density of water is

$$\ln \left(\frac{\rho}{\rho_0} \right) = \frac{1}{n} \ln \left(\frac{p/p_0 + C}{1 + C} \right) \quad (2.27)$$

where p is the static pressure measured in bar, ρ is the corresponding density, $p_0 = 1$ bar (i.e. approximately equal to atmospheric pressure), $\rho_0 = 1000$ kg/m³, $C = 3000$, and $n = 7$. For $p/p_0 = 1000$, approximately equal to the pressure at a water depth of 10 km, the equation gives $\rho/\rho_0 \approx 1.04$, confirming that the effect of pressure on water density can be considered practically negligible.

A more general equation for the influence of pressure on the density of a range of liquids is the modified **Tait equation**

$$1 + \frac{\rho}{\rho_0} = A \ln \left(\frac{p/p_0 + D}{1 + D} \right) \quad (2.28)$$

where p_0 is a low pressure (usually the barometric pressure B or 1 bar), ρ_0 is the liquid density at pressure p_0 , and A and D are constants for the given liquid.

2.7 Specific volume ν , relative density σ , and specific weight w

In thermodynamics it is often more convenient to work in terms of **specific volume** ν than density ρ . The word ‘specific’ here means ‘per unit mass’, i.e.

$$\nu = \frac{v}{m} \quad (2.29)$$

from which we see that the unit of ν is m^3/kg .

Relative density σ (Greek letter *sigma*) is the ratio of the density of a fluid to that of a standard **reference fluid** ρ_{REF} , i.e.

$$\sigma = \frac{\rho}{\rho_{REF}} \quad (2.30)$$

Because it is defined as the ratio of two physical quantities with the same unit, relative density⁹ has a purely numerical value without unit and is again non-dimensional (see Chapter 3).

For liquids, the reference fluid is usually taken to be pure water at 4 °C and 1 atm when it has a density $\rho_{REF} = 1000 \text{ kg/m}^3$. Water shows anomalous behaviour in that between 0 °C and 4 °C its density increases to a maximum of 999.972 kg/m^3 at 4 °C. Below 0 °C water solidifies to become ice. The temperature for the reference fluid is sometimes taken as 20 °C at which the density of water is 998.20 kg/m^3 .

For gases the reference fluid is usually pure air (although hydrogen is sometimes used), which has a density of 1.204 kg/m^3 at 20 °C and 1 atm. In practice, relative density is little used for gases.

Specific weight w , which should not be confused with specific gravity (i.e. relative density), is the weight per unit volume of a substance. Since density ρ is mass per unit volume, it follows that

$$w = \rho g \quad (2.31)$$

where g is the acceleration due to gravity and has the value 9.807 m/s^2 , usually rounded to three significant figures as 9.81 m/s^2 . The units of w can be shown to be N/m^3 because, as we shall see in Chapter 3, 1 newton (symbol N) = 1 $\text{kg} \cdot \text{m/s}^2$.

⁹ Particularly in older texts, the term **specific gravity** is sometimes used instead of relative density.

ILLUSTRATIVE EXAMPLE 2.3

Calculate the density and specific weight for liquid oxygen, which has a relative density of 1.46 at -252.7°C and 1 atm.

Solution

$\sigma = 1.46 = \rho/\rho_{REF}$; so, with $\rho_{EF} = 1000 \text{ kg/m}^3$, for a liquid, $\rho = \sigma\rho_{REF} = 1.46 \times 1000 = 1460 \text{ kg/m}^3$.

$$w = \rho g = 1460 \times 9.81 = 14\,320 \text{ N/m}^3.$$

Comments:

- (a) In any problem where either relative density or specific weight is specified, the first step should always be to calculate the fluid density in SI units.
- (b) It is almost always advisable to work through any problem using algebraic symbols and to substitute numerical values as late as possible.

2.8 Dynamic viscosity (viscosity) μ

In Section 2.1, we introduced dynamic viscosity (symbol μ) as the property which provides the link between the shear stress applied to a fluid and the resulting rate of deformation. For the simple case of a fluid confined between two parallel plates, one fixed, the other moving, we showed that the rate of deformation was equal to the velocity gradient within the fluid. In most flows the spatial variation of velocity is more complicated than the linear variation shown in Figure 2.1. In more general situations, such as that shown in Figure 2.3, the continuum hypothesis allows us to relate the shear stress τ at any point in a fluid to the velocity gradient (often termed the **shear rate**) du/dy at that point according to

$$\tau = \mu \frac{du}{dy}. \quad (2.32)$$

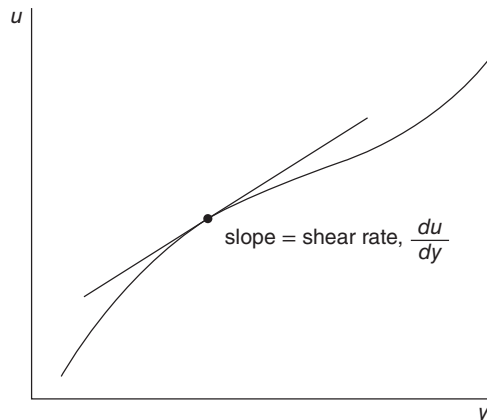


Figure 2.3 Velocity versus normal distance to illustrate velocity gradient