The Physics of Nanoelectronics

Transport and Fluctuation Phenomena at Low Temperatures

Tero T. Heikkilä





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Preface

I started to write this book in autumn 2005 when I presented a course of lectures on nanoelectronics at Helsinki University of Technology. I had presented the course once before with three other people, and could not find a book with which I would have been satisfied. Initially, I intended to write only lecture notes, but after running the course again in 2007 and 2009 I thought that the 250 pages of lecture material could easily be published as a book. Little did I know. After being late for my initial deadline by two years, I now finally dare to submit the text to the publisher.

There are other books on electron transport and nanoelectronics. Nevertheless, I found them too narrow in their topic, outdated, too deep for a textbook, containing too much formalism, or having other deficiencies. In other words, I wanted to write my own book that would contain a wide selection of topics and explain them without too difficult mathematics or formalism, and which would tell both about experiments and theory. So here it is.

Because of my aim of avoiding too heavy formalism I have chosen not to describe non-equilibrium Green's function approaches to transport phenomena. Courses detailing these approaches usually spend half the time on the formalism, finding the poles of the various Green's functions, and figuring out analytic continuations and so on. Whereas this is a necessary tool for theorists in the field, most of the transport phenomena as such can be explained without using these tools. Because of this my book concentrates on different types of transport phenomena, without forgetting about the main theoretical approaches (but at a lower level than non-equilibrium Green's functions) to describe them: Boltzmann equations, scattering theory, master equations and so on. On the other hand, I also do not discuss the different fabrication or measurement techniques that are the bread and butter of nanoelectronics experimentalists. However, I try to tell what and how things are measured on a general level. Whereas many of the basic theories presented in the book date back to the 1980s or 1990s (or even before), I have tried to include references and especially pictures of more recent experiments.

Some may complain about the selection of phenomena discussed in this book. Nanoelectronics is a vast field and I could have made different choices. In particular, I have decided to omit the quantum Hall effect altogether. Neither do I discuss, for example, Luttinger liquids or other strongly correlated phenomena, or topological insulators. Spintronics is only a section in the chapter on semiclassical theory, whereas a whole book could be written about the topic. My choice is obviously strongly affected by my own background. Because of this, I have subtitled the book 'Transport and fluctuation phenomena at low temperatures'. Low temperatures are not a necessity for everything I describe, but many of the effects are in practice accessible only at cryogenic temperatures (below a few Kelvin). That is where the interesting physics lies.

Use as a textbook

This book is intended for advanced undergraduate and starting graduate students who have passed courses on quantum mechanics, solid state physics and some statistical physics. In Appendix A I summarize some of the main technical tools needed when reading the book, but I assume that the reader has already encountered most of them before.

I have tried to arrange the book such that as a rule of thumb every chapter could be lectured in one two-hour lecture. The exception to this is Ch. 6, which requires two lectures. It is probable that not all aspects of every chapter can be discussed during such lectures, so the lecturer should decide which details to omit. This pace means that the whole book could be lectured on a course containing 12–14 lectures, which is, at least in my university, quite a typical length of a single-semester course having one lecture each week. Many of the chapters have been written such that they form almost independent entities, so that some of them can be bypassed without too big problems. I would not advise skipping Chs. 2, 3 and 7, which are needed in many other chapters as well, although Secs. 2.7, 2.8, 3.6 and 7.6 are not entirely necessary. Moreover, the concepts of noise and the fluctuation-dissipation theorem discussed in Sec. 6.1 are also needed in other chapters. Chapter 5 is a precursor to Ch. 9, but it can be skipped if the students know the basics of the theory of superconductivity. Part of the text has also been organized as examples and complements, which aim to give some deeper insight on the topic at hand. The difference between the two is that the examples are intended for all readers, whereas the complements are more for the advanced students. The appendices contain first a set of technical tools that students usually learn in basic courses on quantum mechanics or solid state theory, but which can be used as reminders. Also, some of the long derivations are presented in the appendices to improve the flow of the main text.

Each chapter contains several exercises, that are directly related to the text or offer ideas for further development. A solutions manual will be available upon request once the book is published. In addition, every chapter contains one or two questions on recent scientific papers, by which the students are expected to take a particular recent experimental paper and to understand what was measured.

While every effort has been made to carefully check and edit this book, corrections to any errors that may have crept in will be posted on the book website at http://www.thephysicsofnanoelectronics.info.

Happy reading!

Acknowledgements

Over the course of writing this book I have benefited from comments from many people. Foremost, I would like to thank the assistants of my course, Pauli Virtanen, Matti Laakso, Janne Viljas and Francesco Massel, for various helpful remarks and suggestions. I also thank all the students for enduring the draft versions of this text. Special thanks are extended to Philip Jones, Ville Kauppila, Raphaël Khan and Andreas Uppstu for detailed comments on the text.

Parts of this text have also been read and commented upon by other researchers in nanoelectronics. I would like to thank Norman Birge, Guido Burkard, Sebastiaan van Dijken, Francesco Giazotto, Andreas Isacsson, Nikolai Kopnin, Peter Liljeroth, Sorin Paraoanu, Jukka Pekola, Peter Samuelsson, Mika Sillanpää, Janine Splettstösser and Fabio Taddei for helpful suggestions and comments. Thanks are also due to Jukka Pekola, Edouard Sonin and Pertti Hakonen, with whom we started the lectures on nanoelectronics. I also apologize to those whose names I have forgotten to include, but who contributed in different ways. Needless to say, responsibility for any errors is mine. Finally, I would like to thank Sonke Adlung of Oxford University Press for his infinite patience and encouragement over the past three years.

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Tero Heikkilä

In Aalto University Espoo, June 2012

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List of symbols

 λ_F $\ell_{\rm el}$ $\ell = v_F \tau. \sqrt{D\tau}$ $\tau_{\varphi}, \ell_{\varphi}$ τ_{e-ph}, ℓ_{e-ph} τ_{e-e}, ℓ_{e-e} $\tau_{\rm sf}, \ell_{\rm sf}$ $\ell_m = \sqrt{\Phi_0/B}$ N_F p_F, \mathbf{p}_F v_F, \mathbf{v}_F E_F $\mu_{L/R}, T_{L/R}$ $f(E), f^{0}(E)$ $n(E), n^0(E)$ $D = v^2 \tau / d$ T_n $\overline{T}(E), \overline{R}(E), M(E);$ $\bar{T}(E) + \bar{R}(E) = M(E)$ $s_{nm}^{\dot{\alpha}\beta}$ E I_y, I_r T $R_K = 1/G_K = h/e^2 = 25812.8 \ \Omega$ $\Phi_0 = h/e = 4.13567 \times 10^{-15} \text{ Wb}$ $\Phi_S = h/2e$ $\sigma, \bar{\sigma} \in \{\uparrow, \downarrow\}$ $F(\vec{r}) = \langle \psi_{\uparrow}(\vec{r})\psi_{\downarrow}(\vec{r})\rangle$ Δ λ λ_p $S(\omega)$ P(E) $\partial_x = \frac{\partial}{\partial x}$

Fermi wavelength elastic scattering length relation of a scattering length to a scattering time in the ballistic, diffusive limit dephasing time, length electron-phonon scattering time, length electron-electron scattering time, length spin-flip scattering time, length length scale related to the magnetic field Bdensity of states at the Fermi level Fermi momentum (magnitude and the vector) Fermi velocity (magnitude and the vector) Fermi energy chemical potential (Fermi level) and temperature for left/right leads electron energy distribution function, Fermi function boson (phonon, photon) distribution function, Bose function diffusion constant for dimensionality dtransmission coefficient for channel ntotal transmission probability (summed over modes), total reflection probability, number of modes scattering matrix or its element connecting mode m in lead β to mode *n* in lead α energy or Young's modulus bending moduli temperature or tension (mass) density quantum of resistance tunnelling resistance flux quantum flux quantum for Cooper pairs spin and the opposite spin pairing amplitude superconducting pair potential, BCS gap interaction constant in the theory of superconductivity magnetic field penetration depth into a superconductor noise power spectral density (Fourier transform of a two-operator correlator) (in the dynamical Coulomb blockade theory) probability for an environment to absorb energy Eshort-hand for derivative

Introduction

This book describes electron transport phenomena in small, *mesoscopic* systems. The word mesoscopic comes from the Greek word *mesos*, which means middle, indicating that the mesoscopic world resides between the microscopic and the macroscopic. Typically this means that the phenomena take place in systems consisting of a large number of atoms and electrons, but they only occur if the system is small enough, i.e., the size of the system is smaller than some length scale characterizing the transition from the microscopic to the macroscopic world.

Let us take a few examples that are discussed in the remainder of the book. In a perfect metallic crystal, electron transport is essentially dissipationless, i.e., the conductivity is infinite. But there are no everydayscale perfect crystals. The dislocations, vacancies and other impurities distort the crystal periodicity and lead to a finite conductivity. Also, the presence of lattice vibrations, phonons, induce scattering that further decreases conductivity. These processes come with a length scale, a mean free path, that characterizes the average distance between the impurities, or between subsequent scatterings of conduction electrons from them, or from the phonons. A metallic wire which is smaller than this mean free path is said to be ballistic. In a ballistic wire, the resistance does not depend on the length of the wire, and therefore the conductivity can be said to be infinite. As explained in Ch. 3 of this book, however, even with this infinite conductivity, the measured resistance of such a wire is non-zero, as the finite number of quantum channels in the wire leads to a finite resistance. In this example the crossover from mesoscopic (where a macroscopic observable such as resistance is well defined, but it is determined by the quantum nature of the wire) to macroscopic (where resistance scales with the length of the wire) is characterized by the mean free path.

Another example concerns the definition of the electron temperature. From statistical physics we learn that temperature characterizes the width of the electron energy distribution function turning from one below the chemical potential μ to zero above μ . This temperature is a parameter in the Fermi-Dirac distribution function. In a non-equilibrium setting, an applied voltage V giving rise to a current I leads to the power P = VI applied to the sample, and the sample heats up. But as discussed in Ch. 2, the resultant distribution function may not always be of the Fermi-Dirac form, and one can have many different types of definition for the electron temperature. This non-equilibrium form persists in the wire until after some distance the electrons relax into the

1

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Length Fermi wavelength (metals) Fermi wavelength (2DEG) Elastic scattering length (metals)	$\begin{array}{c} \text{Symbol} \\ \lambda_F \\ \lambda_F \\ \ell_{\text{el}} \\ \ell \end{array}$	order of magnitude 0.1 1 nm 10 100 nm 10 100 nm
Elastic scattering length (2DEG) Energy relaxation length at $T = 1$ K Dephasing length at $T = 1$ K	$\ell_{ m el} \ \ell_{ m en} \ \ell_{arphi}$	$\begin{array}{c} \dots \text{ hundreds } \mu \mathbf{m} \\ 1 \dots \text{ few tens } \mu \mathbf{m} \\ 0.1 \dots 10 \ \mu \mathbf{m} \end{array}$

I

Table 1.1 Orders of magnitude for some length scales in typically studied mesoscopic systems. These should be compared to the structure sizes with dimensions some tens of nanometres, fabricated with standard lithography techniques (at minimum, larger conductors can obviously also be made), which also set the scale for the potential profiles in two-dimensional electron gas (2DEG) systems. With more advanced techniques, such as mechanically controllable break junctions, electromigration, atomic layer deposition, manipulations with atomic force microscopes, etc., one may reach wires with atomic widths and thicknesses. Moreover, such ultra-thin conductors may be reached with 'bottom to top' approaches, e.g., from carbon nanotubes.

usual equilibrium shape. Such a relaxation is characterized by a relaxation length, and this again describes a crossover between mesoscopic and macroscopic regimes.

Especially the length scales concerning energy relaxation are strongly temperature dependent. For example, for a typical copper wire,¹ the relaxation length at room temperature is of the order of a few nanometres, at T = 10 K it is close to a micrometre, at T = 1 K it already reaches a few tens of micrometres, and for the minimum typically achievable electron temperature, T = 10 mK, it is a 'macroscopic' length, of the order of centimetres. This shows that mesoscopic effects are best seen at low temperatures. Reaching temperatures below 1 K is nowadays an everyday task in many physics laboratories. As many of the effects discussed in this book require a large enough wire so that level quantization effects are small within the phase breaking or energy relaxation lengths, finding some of those effects really requires cooling the sample to low temperatures. This also shows up in the assumptions made of the operating temperatures in the remainder of this book.

Other typical length scales relevant for mesoscopic conductors are summarized in Table 1.1.

The opposite end of mesoscopic effects, i.e., turning of microscopics into mesoscopics, is a matter of convention. Molecular electronics measuring charge transport through single molecules—is rarely included in the strict definition of mesoscopic physics, but apart from the energy scales many of the observed effects are quite similar to those seen in quantum dot systems consisting of millions of atoms and electrons. One characteristic feature of mesoscopics is that the systems are open: the small system under interest is coupled to a larger system, and this coupling may modify the properties of the small system, for example, inducing finite lifetimes of the electronic states. For these reasons, the def-

¹See details in Sec. II.C.2 of the review on thermal effects (Giazotto *et al.*, 2006); in this particular case the electron–phonon scattering length is proportional to $T^{-3/2}$.

inition of the mesoscopic realm can be extended down to the molecular level, when concentrating on those effects where the materials-dependent features affect only quantitative details.

1.1 Studied systems

This book concentrates on electron transport phenomena in small conducting structures, rather than the detailed (often materials-dependent) characteristics of those structures themselves. But in order to acquire the correct mindset on where these phenomena are found, let us shortly discuss those structures and their main characteristics as well.

Mesoscopic phenomena are studied in four different types of conducting system depending on the main materials used for their construction: metallic wires, semiconductor structures, molecules and graphene.² In addition, different kinds of nanowires typically fall within all of these categories, but often have properties characteristic to those nanowires rather than the bulk material of which they are made. These systems are described in some detail in the following.

Common to the studies of any nanostructure, one also has to fabricate the contact from this structure to the macroscopic measuring devices (see the large-scale picture in Fig. 1.1). Typically this is done by structuring much wider electrodes than the studied wires in contact with them (see Figs. 1.2, 1.6, 1.8 and 1.9 for examples). Such an electrode ideally works as a *reservoir* (heat and particle bath) of electrons: once the electrons enter the electrode, they quickly thermalize with the lattice. In practice, this means that their energy distribution function obtains the Fermi– Dirac form³

$$f^{0}(E) = \frac{1}{\exp[(E-\mu)/(k_{B}T)] + 1},$$
(1.1)

where μ is the chemical potential and T the lattice temperature of the reservoir. In the ideal case, the reservoir is then unperturbed by what happens in the nanostructure. These wide electrodes continue for some hundreds of microns and connect to contact pads, the latter with dimension of the order of a millimetre. To these the experimentalist connects the external wires, which then connect the sample to the measurement apparatus. At this point there are often a few signal amplification stages. As the mesoscopic nanoelectronics experiments are often carried out at low temperatures, the thermal noise in the measurement apparatus (residing at room temperature) and the electronic heat current through the wires typically heat up the electrons in the sample. Therefore, one typically needs a good thermal contact of the wires with the low-temperature equipment, and several electronic filters for the noise between the sample and the room-temperature equipment. All this is of extreme importance when measuring the mesoscopic effects in small conductors. However, this is a book concentrating on phenomena, so I do not dwell on the detailed problems faced when fabricating and measuring nanoelectronic samples.

 2 Recently, quite a lot of attention has also been paid to a fifth class of mesoscopic systems, fabricated on systems called topological insulators, which have a conducting surface because of the surface states formed at the interface between a topological insulator and a conventional insulator or vacuum. For a recent review, see (Qi and Zhang, 2011).



Fig. 1.1 Photograph of a GaAs chip used for measurements of spin qubits

(see Sec. 8.5.4). The chip (in the centre) is about 5 mm \times 5 mm. It is bonded to the external measurement equipment via the many aluminium

wires, which supply the control voltages to the qubit. The (sub-nanosecond) qubit driving is carried out via the two coaxial transmission lines at the left, and the readout via the surface mount copper inductor (in the background).

Courtesy of F. Kuemmeth, Marcus Lab, Harvard University.

³Below, when I want to emphasize that the distribution function has this form, I use the superscript 0.



Fig. 1.2 Scanning electron micrograph (SEM) of a single-junction thermometer structure, consisting of aluminium wires attached to each other via aluminium oxide tunnel contacts. This sample can be used for a very precise measurement of the temperature via the Coulomb blockade, as discussed in Ch. 7, see (Pekola et al., 2008). This structure was fabricated in NEC Nanoelectronics Research Laboratories in Japan, and measured in the Low Temperature Laboratory of the Aalto University, Finland. Courtesy of

Jukka Pekola and Matthias Meschke.



Fig. 1.3 SEM figure of a normal-metal (Au)-superconductor (W) loop attached to a superconducting resonator (made of Nb, only a small part shown). This structure was fabricated in the University of Paris Sud for the measurements of the high-frequency impedance of a superconductornormal-metal-superconductor junction (Chiodi et al., 2011). Courtesy of F. Chiodi and H. Bouchiat.

1.1.1 Metallic wires and metal-to-metal contacts

Perhaps the simplest studied systems in nanoelectronics are small metallic wires and metal-to-metal contacts fabricated with the help of a lithographically patterned mask through which the metals are evaporated. Typically used metallic materials are based on aluminum, copper, gold, niobium, silver, platinum or palladium, but also other metals are used. As aluminum and niobium become superconducting at sub-Kelvin temperatures (the critical temperature of Al is 1.1 K and for Nb it is 9.3 K), a combination of superconducting and normal-metallic (nonsuperconducting) effects can be studied using them (see Chs. 5 and 9). Most single-electron transistors (Ch. 7) and superconducting Josephson junctions (Ch. 9) are fabricated with metals. For an example of an ultrasmall system consisting of metallic wires and many tunnel contacts, see Figs. 1.2, and for one not so small but containing many different metals, see Fig. 1.3.

In the following I discuss very briefly the simplest approach to the electronic structure of metals. For most purposes this is a sufficient description. Besides being needed for describing the transport phenomena in metals, it is also needed for understanding the electronic transport through other types of material, as the samples made from them are eventually always contacted to metal electrodes for the measurements.

'Free' fermions in metals

Electrons interact strongly via the Coulomb interaction. However, it turns out that the low-energy properties of the conduction electrons in metals or doped semiconductors can be well described within an independent electron description. This means that for most bulk properties we need to consider the Schrödinger equation for a single particle wave function ψ ,

$$i\hbar\frac{\partial\psi(\vec{r})}{\partial t} = \left(-\frac{\hbar^2}{2m^*}\nabla^2 + U(\vec{r})\right)\psi(\vec{r}),\tag{1.2}$$

where $U(\vec{r})$ describes the possibly random local variations of a meanfield potential felt by the electron. That such a description is possible is explained by Landau's Fermi liquid theory.⁴ We do not dwell on this theory here, and it suffices to state that this theory is valid only at excitation energies much smaller than the Fermi energy, and in this case the effect of electron–electron interactions is captured mostly in the effective mass m^* . In fact, the first term on the right-hand side of eqn (1.2) is the lowest-order term in the expansion of the bulk electron energy vs. the momentum around the Fermi level.

There is one remaining effect of the electron–electron interactions even in this case: the collisions between the electrons serve as a route for relaxation towards an equilibrium state. This effect is described in Ch. 2.

⁴See, e.g., Ch. 17 in (Ashcroft and Mermin, 1976) or Sec. 11.2.1 in (Mahan, 2000). In fact, the free electrons in this theory are replaced by *Landau quasiparticles*, which are elementary excitations of the interacting electron system. These quasiparticles are the single electrons 'dressed' with the interactions.

In restricted geometries the electron–electron interactions may again become more relevant, and a mean-field type of a description is no longer sufficient. For transport through zero-dimensional islands, this shows up in the Coulomb blockade phenomenon (see Chs. 7 and 8). In one-dimensional wires the (sufficiently weak) interactions result in the formation of a Luttinger liquid.⁵ Finally, if the effective interaction between electron pairs is attractive, the electronic system may turn into a superconducting state. The consequences of this are discussed in further detail in Chs. 5 and 9.

Besides the neglect of electron–electron interactions, there is another relevant approximation usually applicable when describing the transport properties of metals. Namely, typical Fermi energies of metals are in the range of electron volts, or in temperature units, tens of thousands of Kelvin. Typical excitation energies in transport studies and the operating temperatures of the devices are much lower than this scale. In other words, the relevant electron momenta lie within $\delta \mathbf{p}$ from the Fermi momentum \mathbf{p}_F . In this case the electron dispersion relation (relation between the momentum and the kinetic energy of an electron) for the relevant excitations can be often approximated via

$$\epsilon_p = \frac{\mathbf{p}^2}{2m^*} = \frac{(\mathbf{p}_F + \delta \mathbf{p})^2}{2m^*} = \frac{\mathbf{p}_F^2}{2m^*} + \frac{\mathbf{p}_F}{m^*} \cdot \delta \mathbf{p} + o(\delta p^2) \approx E_F + \mathbf{v}_F \cdot \delta \mathbf{p}, \quad (1.3)$$

where $\mathbf{p}_F = \hat{p}p_F$, $p_F = \sqrt{2m^*E_F} = m^*v_F$ is the Fermi momentum and \mathbf{v}_F is the Fermi velocity. This is called the semiclassical approximation, and it implies that for many purposes electrons can be viewed as classical particles moving with the Fermi velocity. Such an approach is employed in Ch. 2 to study the semiclassical limit of transport, widely applicable for metals.

Linearizing the dispersion relation around the Fermi energy has also another important consequence: it makes the density of states constant. Assume we want to calculate the value of some energy-dependent observable $F(\epsilon_p)$ given as an integral over the momentum states, i.e.,

$$\int \frac{d^3p}{4\pi^3\hbar^3} F(\epsilon_p) = \int_0^\infty \frac{p^2 dp}{\hbar^3\pi^2} F(\epsilon_p) = \int_0^\infty d\epsilon N(\epsilon) F(\epsilon).$$
(1.4)

On transforming from the momentum integral to the energy integral we introduced the density of states

$$N(\epsilon) = \frac{1}{\pi^2 \hbar^3} \frac{p^2 dp}{d\epsilon_p} \approx \frac{2m^* E_F}{\pi^2 \hbar^3 v_F} = \frac{m^* p_F}{\pi^2 \hbar^3} \equiv N_F, \qquad (1.5)$$

where the last three forms are independent of energy. This approximation is used throughout the text when dealing with metals. Moreover, for typical observables related with electron transport, $F(\epsilon)$ is non-zero only within a small energy window around the Fermi energy E_F much larger than this energy window.⁶ Therefore the integral over the energies in eqn (1.4) can be extended to start from $\epsilon = -\infty$.

⁶This applies for all 'transport' observables, such as average current, which vanish in the absence of a bias (voltage), or thermal noise, which vanishes at T = 0.

⁵Luttinger liquids are outside the scope of this book. For an introduction, see, for example, Sec. 19.4 in (Bruus and Flensberg, 2004).



Fig. 1.4 Band-bending diagram of modulation doped $GaAs/Al_xGa_{1-x}As$ heterostructure. A 2DEG is formed in the undoped GaAs at the interface with the *p*-type doped AlGaAs.



Fig. 1.5 Single electron transistor fabricated starting from InAs/InP nanowires deposited on a SiO2/Si substrate. Devices are contacted by a Ti/Au metallization and local nanogates at the sides of the wire can be used to tune the spectrum and filling of electron orbitals. Device fabricated in Scuola Normale Superiore, Pisa, Italy. Courtesy of Stefano Roddaro and Francesco Giazotto.

1.1.2 Semiconductor systems

Fabricating semiconducting materials requires more massive and expensive techniques than with metals, but the resultant systems are also generally more controllable. Besides replacing metals with heavily doped semiconductor materials, for mesoscopic transport studies semiconductors are most often used in two distinct flavors: metal-gated two-dimensional electron gases (2DEGs) formed between semiconductors with different levels of doping, or nanowires grown out of different types of semiconductor material.

Figure 1.4 shows the energy diagram of a 2DEG forming at the interface between non-doped GaAs and Al-doped GaAs. To adjust to the variation in the doping, there is charge transfer at the interface. As a result, the band energies are bent as shown in the figure. With suitable doping, the conduction band drops to just below the Fermi level within a small region near the interface. As a result, excitations to higher-order states in the perpendicular direction to the interface require a large energy, whereas in the direction parallel to the interface electrons can move almost freely. The electron gas is thus effectively two-dimensional, and the electron density can further be controlled *in situ* by attaching metal gates on top of the heterostructure. Moreover, patterning these gates suitably, one can locally change the electron density (also to zero) and thereby produce different types of confining potential for the electrons. With such a scheme one can realize even lower- (than two-)dimensional objects, such as one-dimensional wires, quantum point contacts, or zerodimensional quantum dots (see Fig. 1.6 for an example).⁷ These systems are described in this book.

Different types of semiconductor can nowadays be quite controllably grown into thin nanowires (see an example in Fig. 1.5). Typical materials used for these wires are Si and different III-V semiconductor compounds, such as InAs, InP and GaAs. The thickness of these wires is typically a few tens of electron wavelengths, i.e., they contain a few tens of electron modes contributing to the charge transport. In effect, semiconductor nanowires have many similar properties as (multi-wall) carbon nanotubes, but their different materials parameters allow for studying a wider range of topics. Especially, some of them have interesting magnetic properties, such as strong spin-orbit scattering and large effective electron g-factor for Zeeman splitting, which may make them interesting for spintronics applications.

Depending on the particular semiconductor concerned, on the magnitude of the excitation energies and the observable studied, doped semi-

⁷Nearly all systems in condensed-matter physics are strictly speaking three-dimensional. However, often systems can be described by being effectively lower-dimensional: in this case one or more of the dimensions (width, thickness, length) is smaller than some characteristic length scale describing certain type of physics. For example, 2DEGs are thin in the z-direction compared to the Fermi wavelength, and therefore excitation to higher states in that direction are costly. In some cases there are also other relevant length scales, such as those related to screening of interactions, dephasing of interference effects (see Ch. 4) and energy relaxation. Therefore, a given conductor may be three-dimensional in one respect and, for example, one-dimensional in another. Moreover, often systems showing gradients of variables in only one (or two) directions are called quasi-one-(or two-)dimensional, even though otherwise they would be three-dimensional.





conductors and low-dimensional structures based on them may often still be described with the free-electron picture similar to that of metals—at least when constrained in the relevant number of dimensions. However, semiconductors have typically a more complicated Fermi surface than metals, and this often shows up in the applications. Besides the possibility of realizing low-dimensional structures, such materials-specific features of semiconductor nanostructures are not discussed in this book.

1.1.3 Carbon nanotubes and molecules

There are techniques to contact molecules between metallic leads and study transport through them. Two commonly used techniques are depicted in Fig. 1.7 showing a schematic way of making extremely small break junctions into which molecules can be placed; a third alternative is to use scanning tunnelling microscopy.⁸ Many different types of Fig. 1.6 Atomic force microscope (AFM) image of a double quantum dot fabricated on a GaAs/AlGaAs heterostructure by a local oxidation. The double quantum dot (QD1 and QD2) is formed in the centre of the structure. The radius of the QD1–QD2 system is of the order of 100 nm. The two quantum point contacts (QPC1 and QPC2) can be used to monitor the charge state of the quantum dots, and the whole system can be controlled by applying voltages to the two gates (G1 and G2)and between source S and drain D. This structure was used to measure the charge dynamics of the quantum dot. see (Gustavsson et al., 2007). Courtesy of Klaus Ensslin, Swiss Federal Institute of Technology, ETH, Zürich, Switzerland.

Fig. 1.7 Two techniques used for making molecular break junctions: mechanically (b) and electronically (c) bro-These techniques alken junctions. low reaching ultra-narrow gaps between the electrodes, so that molecules can be deposited in the free space between them as illustrated in (a). In reality, the molecules are typically in a solution everywhere near the junction. and only a transport measurement reveals whether one of them links the two electrodes. In this setup, usually the transport takes place dominantly only through one molecule. (van der Molen and Liljeroth, J. Phys. Condens. Matter 22, 133001 (2010), Fig. 6.)

⁸For a review of these techniques, see (van der Molen and Liljeroth, 2010).

⁹Note that the viewpoint in molecular electronics is often different from the one in this book: there the aim is often, besides the generic picture of the transport process, to connect the chemical valence to the charge transport characteristics. That means that the exact positions of the energy levels matters rather than the consequences of the mere level quantization.



Fig. 1.8 Double-gated suspended ultra-clean single-wall carbon

nanotube (SWNT) between two metal electrodes. This type of a sample was used for the measurements of the

coupling of the spin and orbital motion of electrons in SWNTs (Kuemmeth *et al.*, 2008). Courtesy of Shahal Ilani, Weizmann Institute of Science, Israel.

¹⁰However, CNTs are often also used to make quantum dots.

 11 I do not dwell on the electronic structure of the carbon nanotubes here. For those interested, there are books written on them; see for example (Ando, 2005).

 $^{12}{\rm When}$ adding spin, the total degeneracy is thus four.

molecule have been studied, for example the electric current through a single Hydrogen molecule has been measured (Smit *et al.*, 2002). The transport through the molecules show behaviour reminiscent of quantum wires (Ch. 3) or quantum dots (Ch. 8). In this respect the most notable difference to semiconductor quantum dots is that the energy level spacing in molecules tends to be larger.⁹

For ordinary molecules, the bulk band structure is no longer an applicable concept. Rather, typically the relevant topic of study is the discrete electronic spectrum on the molecule, and the dependence of this spectrum on the number of electrons. Therefore, the energy scales encountered in molecular electronics are closely related to the chemical properties of these molecules.

In terms of transport studies, carbon nanotubes (CNTs) are quite different from ordinary molecules. They are typically long enough so that their studies generally do not require break-junction techniques. and they are long enough so that a description as a bulky object is often justified.¹⁰ In them the nature of the extended electron states depends strongly on how the nanotubes are wound into the tubes. Therefore, some nanotubes are insulating whereas some are metallic, depending on the exact microscopic structure. Because of this, the transport properties of different carbon nanotubes vary considerably.¹¹ Moreover, nanotubes exist in multiple forms: Perhaps the best understood are the single-wall nanotubes (SWNTs), which contain only a single carbon sheet rolled into a tube. What makes the SWNTs special is that they contain two degenerate orbital states at the Fermi energy,¹² corresponding to the possible two directions of the chiral orbits around and along the tubes. In contrast, multi-wall tubes (MWNTs) contain many more states, and in some cases they can be likened to thin metal wires. In some cases the tubes form bundles through which the transport can also be studied, but the properties of the bundles are less controllable than those of the individual tubes.

1.1.4 Graphene

In 2004, physicists at the University of Manchester attached scotch tape to a piece of graphite, then removed it and placed it on top of a Si substrate. After peeling off the tape, they discovered that some flakes of graphite were left on the substrate. They also found that the optical response of these flakes depends on the number of carbon layers in the flakes. Closer inspection showed that some of these flakes had only a few carbon layers, and some only one. Such single-layer graphene had been studied since the 1940s (Wallace, 1947) as a mathematical model electronic system with properties very different from ordinary conductors or insulators. Namely, besides being the thinnest-known two-dimensional material, the electrons in graphene seem to behave like massless Dirac fermions. As discussed shortly below and with more detail in Ch. 10, this property is related to the graphene honeycomb lattice structure. Since its initial finding, many laboratories have been able to start fabricating graphene samples (see examples in Figs. 1.9 and 1.10). Besides the curious behaviour of the electrons, there are many properties of graphene that are attractive for the electronics industry: for example, graphene samples can be made extremely clean, the transport can be tuned via gate voltages, the charge and heat conductivities in graphene are large and optical properties qualitatively different from other materials.

Massless Dirac fermions in graphene

Graphene is a two-dimensional layer of graphite. It is formed from carbon atoms that are arranged in a honeycomb lattice, shown in Fig. 1.10. The fact that the unit cell contains two similar atoms has an important consequence for the electronic structure: each electron is described by a Dirac spinor wave function, with components describing the amplitude of occupying one of the two atoms in the unit cell. The electronic dispersion relation in graphene is derived in Ch. 10. According to this derivation, one gets at low energies an effective Hamiltonian

$$H_g \approx \hbar v_F \begin{pmatrix} 0 & \pm k_x - ik_y \\ \pm k_x + ik_y & 0 \end{pmatrix} = \hbar v_F \left(\pm k_x \sigma_x + k_y \sigma_y \right).$$
(1.6)

The two signs are for two non-equivalent valleys in the reciprocal space. This Hamiltonian has the form of the Dirac Hamiltonian for massless¹³ particles, but the speed of light has been replaced by $v_F \approx 10^6$ m/s.

Besides the single-layer graphene, bilayer graphene has also been studied actively. In the bilayer case, the electrons are still described by two-component spinors, but the Hamiltonian has the form

$$H = -\frac{\hbar^2}{2m^*} \begin{pmatrix} 0 & (k_x \mp ik_y)^2 \\ (k_x \pm ik_y)^2 & 0 \end{pmatrix}.$$
 (1.7)

Hence, the excitations of bilayer graphene are still spinors with two indices for the atoms in the unit cell, but contrary to the case in singlelayer graphene, they have a finite mass m^* , which is roughly 5% of the electron mass. However, the particular Hamiltonian of the bilayer depends on the orientation of the two layers with respect to each other, and in some cases it is possible to induce a finite gap in the bilayer density of states. This is discussed in more detail in Ch. 10.

The particular dispersion relations and the presence of the 'pseudospin' (corresponding to the two similar atoms in the unit cell) result in many exotic electronic properties seen only in graphene.¹⁴ This is why the successful fabrication and measurements of single graphene sheets have resulted in massive activity in the detailed study of its properties.

1.2 Classical vs. quantum transport

The forthcoming chapters discuss quantum-mechanical transport properties of nanostructures. These show deviations from the classical Ohm's



Fig. 1.9 Atomic force microscope (AFM) image of a suspended graphene sheet between four metal electrodes residing at the corners of the image, fabricated in the Low Temperature Laboratory, Aalto University, Finland. Courtesy of Jayanta Sarkar and Peter Liljeroth.

¹³Masslessness here refers to the picture of general relativity: the dispersion relation does not contain the usual mass term mc^2 , which would correspond to an energy gap in the dispersion relation.



Fig. 1.10 High-resolution scanning tunnelling micrograph of graphene. The carbon atoms form a single layer

honeycomb lattice, residing at the edges of the honeycomb as shown in

the schematic figure. The unit cell encloses two neighbouring atoms of the honeycomb. (Stolyarova *et al.*, *PNAS* **104**, 9209 (2007), Fig. 2.) \bigcirc 2007

National Academy of Sciences, USA.

 14 For a short review see (Geim and Novoselov, 2007), and for a more thorough one (Castro Neto *et al.*, 2009).

law I = U/R, stating that the current I through a given sample is a linear function of the voltage U, the coefficient R being the resistance. In the classical case, this resistance scales linearly with the wire length,

$$R = \frac{L}{\sigma A},\tag{1.8}$$

A being the cross-section of the wire and σ its conductivity. In Ch. 2 we derive this law using semiclassical arguments: ignoring the interference effects (Ch. 4), which often are weak, and single-electron effects (Ch. 7), which arise in strongly interacting systems.

1.2.1 Drude formula

The Boltzmann-equation analysis of Ch. 2 yields for the conductivity in the semiclassical limit

$$\sigma = e^2 N_F D = e^2 N_F v_F \ell_{\rm el} / d, \qquad (1.9)$$

where e is the electric charge, N_F is the density of states at the Fermi level, D is the diffusion constant, v_F is the Fermi velocity, $\ell_{\rm el}$ is the elastic mean free path and d is the dimensionality of the wire.

Besides the semiclassical derivation of eqn (1.9), quite a similar result was found by (Drude, 1900*a*; Drude, 1900*b*), before the microscopic theories of solid state. Drude's derivation goes as follows. Let us consider an electron in a solid where one has applied an electric field \vec{E} . The electric field produces a Lorentz force $-e\vec{E}$, accelerating the electron. Assume that a time *t* has passed since the electron last collided with the lattice. The velocity of the electron has increased to $\vec{v}_0 - e\vec{E}t/m$ due to the Lorentz force, \vec{v}_0 being the initial velocity after the last collision. If that electron collides in random directions from the impurities, the initial velocity has no contribution to the average velocity of the electron, which must therefore be given as the average of $-e\vec{E}t/m$. Denote the average time between collisions as 2τ .¹⁵ This implies the average electron velocity given by

$$\vec{v}_a = -\frac{e\vec{E}\tau}{m}.\tag{1.10}$$

The average current carried by the electrons is given by $\vec{j} = -en\vec{v}_a$, where *n* is the electron density. Therefore, we get $\vec{j} = \sigma_D \vec{E}$ with the Drude conductivity

$$\sigma_D = \frac{ne^2\tau}{m}.\tag{1.11}$$

This is the same as eqn (1.9) (up to a prefactor of order unity) after we identify $n \sim N_F v_F^2 m/2 = N_F E_F$. This is why also the result (1.9) is often referred as the Drude conductivity.

1.2.2 Quantum effects

Typical quantum effects encountered in nanoelectronic systems arise due to the energy and/or charge quantization effects, tunnelling, and interference effects. A large-scale quantum phenomenon is the transition to

 15 The factor 2 does not matter much, but is included here to get the original Drude result in eqn (1.11). the superconducting state taking place in many materials at low temperatures. This transition is not a mesoscopic effect, but also in nanoelectronic circuits it gives rise to many types of phenomena which would not be present in simple normal-metallic systems.

There are many possible reasons for violating Ohm's law, i.e., either linearity between current and voltage or the scaling of the resistance with the length of the wire. Often in these cases both of them are simultaneously violated, but there are exceptions to this. Such nonclassical effects are, for example:

- **Tunnelling** through a thin insulating region or a vacuum gap between two metals. Such *tunnel junctions* are an important part of mesoscopic electronics, and they can be used to realize many types of structure. A single tunnel junction fabricated between two normal (non-superconducting) metals is typically a linear object, i.e., its resistance is independent of voltage up to very large voltages of the order of the work function difference between the tunnel barrier and the metals. However, the scaling of the resistance with the thickness of the tunnel junction is exponential, not linear. Calculating the current due to tunnelling through a single insulating barrier is illustrated in Example 1.1 below.
- In low-dimensional systems, such as quantum dots, electron **energy level quantization** within the system shows up in the current through the system. This is discussed in detail in Ch. 8.
- Single-electron effects. When tunnel barriers have a small capacitance C, the energy required for charging the capacitance with the charge of a single electron, $E_C = e^2/(2C)$, may become relevant. In this case, to get a finite current through the tunnel contact, the external circuit must provide this charging energy E_C to the electrons crossing the barrier. It turns out that in a double-junction system ('single-electron transistor' SET, see Ch. 7), this energy has to be provided by the bias voltage or the temperature. Otherwise no current can flow, and the system is in the state of a *Coulomb blockade*.
- In metal or semiconductor wires, carbon nanotube or graphene structures whose size is smaller than the *phase relaxation length* ℓ_{φ} , **interference effects** between different electron paths within the wire may alter the conductivity. Such effects are Aharonov– Bohm effects and persistent currents in multiply connected systems under an applied magnetic field, and localization and universal conductance fluctuations in disordered systems. These effects are discussed in Ch. 4.
- In **ballistic wires**, i.e., conductors whose size is smaller than the mean free path, the resistance no longer scales with the length of the wire, but it becomes quantized, depending on the ratio between the width of the wire and the Fermi wavelength λ_F of electrons (see Ch. 3).



Fig. 1.11 Schematic model of a tunnel junction with transparency τ .

- At a high magnetic field, *Landau levels* are formed within the conductor, and the resistance through the system is strongly field dependent. The resultant **quantum Hall effects** are not discussed in this book, but for a general introduction see for example (Imry, 2002) or (Datta, 1995).
- In superconducting wires and junctions, and superconductornormal metal heterostructures, quantum-mechanical phase coherent effects show up, and Ohm's law ceases to be valid. Some of these phenomena are discussed in Chs. 5 and 9.

Before embarking on the multitude of transport phenomena I would like to point the reader to Appendix A, which contains a set of necessary theoretical tools, concepts and formulas employed in the book. An advanced or impatient reader may skip these appendices, and if necessary return to them when they are needed.

Example 1.1 Current through a tunnel junction

As an example of calculating the current through a low-dimensional object. consider the case of a tunnel junction depicted in Fig. 1.11. The proper microscopic calculation of the current in this case is carried out in Ch. 7, but the formula for the current can also be argued based on a simple picture. With an applied voltage V through the junction, the Fermi levels of the electron systems on the two sides of the junctions are shifted by eV, i.e., they are μ_L and $\mu_R = \mu_L - eV$. Assume that the average transmission probability for an electron on the left side of the junction to enter the right side is τ (taken for simplicity independent of energy). According to elementary quantum mechanics, this depends on the junction parameters as $\tau \sim \exp(-2d\sqrt{2mU}/\hbar)$, where d is the width and U is the height (work function difference between the metal and the tunnel barrier) of the tunnel barrier, and m is the electron mass. At a vanishing temperature, the current from the left to the right at energy Eis then $eAc\tau N_L(E-\mu_L)N_R(E-\mu_R)$, where A is the area of the junction, $N_{L/R}(E)$ is the density of states in the left/right side of the junction and c is a constant fixing the dimensions of this expression to (current/energy). In what follows, we identify the prefactor as the resistance, and eliminate c. At non-zero temperature we have to include the occupation numbers (distribution functions) $f_i(E)$ of electrons in reservoir i: the initial state has to be filled and the final state has to be empty. Therefore, the total current (current from the left to the right minus that from the right to the left) is

$$I = ecA\tau \int_{-\infty}^{\infty} dE N_L (E - \mu_L) N_R (E - \mu_R) \{ f_L(E) [1 - f_R(E)] - f_R(E) [1 - f_L(E)] \}$$
(1.12)

$$= ecA\tau \int_{-\infty}^{\infty} dE N_L (E - \mu_L) N_R (E - \mu_R) [f_L(E) - f_R(E)], \qquad (1.13)$$

where $\mu_R = \mu_L - eV$. When the two sides of the tunnel junction are at a local equilibrium, $f_{L/R}(E) = f^0(E; \mu_{L/R}, T)$, where $f^0(E; \mu, T)$ is the Fermi-Dirac distribution function, eqn (1.1). The integrand is non-vanishing within the window of width $\sim \max(k_B T, eV)$ around μ_L and μ_R , i.e., around the Fermi energies E_F of the two leads.¹⁶ If one of the reservoirs, say the one on the right, is made of a normal metal and the voltage is much smaller than E_F/e , its density of states is almost constant within this window (see

¹⁶In the limit $k_B T \ll E_F$, E_F is the same as the chemical potential; see Appendix A.5.

the discussion in Sec. 1.3). It can therefore be replaced by its value at E_F , $N_R(E) \approx N_R(E_F) \equiv N_F^R$. In this case, assuming Fermi–Dirac distributions and equal temperatures on both sides of the tunnel junction, and taking the voltage derivative of the current yields

$$\frac{dI}{dV} = e^2 c A \tau N_F^R \int dE \frac{N_L(E - \mu_L - eV)}{4T \cosh^2\left(\frac{\epsilon - \mu_L}{2T}\right)} \stackrel{T \to 0}{\to} e^2 c A \tau N_F^R N_L(-eV). \quad (1.14)$$

The latter part of the equation was taken in the limit of a low temperature. The differential conductance of the junction at a given voltage is directly proportional to the density of states at energy E = -eV. This way the tunnelling current can be used for measuring the energy dependence of the local density of states on one side of the tunnel junction. An example of such a measurement is presented in Fig. 5.8.

If both reservoirs are normal metals, the current is given by

$$I = ecA\tau N_F^L N_F^R \int_{-\infty}^{\infty} dE[f^0(E;\mu_L,T_L) - f^0(E;\mu_R,T_R)]$$

= $ecA\tau N_F^L N_F^R(\mu_L - \mu_R) = e^2 cA\tau N_F^L N_F^R V \equiv V/R_T.$ (1.15)

Here we used the integral given in eqn (A.64a). The resistance R_T of a tunnel junction, defined in this formula, is thus independent of temperature and voltage.

Current through a tunnel junction can also be used to actually measure the distribution function or (in the equilibrium case) the temperature, provided the densities of states are energy dependent and their form is known. Such measurements are explained in Example 2.4 and in Sec. 9.2.

We may also write for the *heat current* carried by the electrons from the left reservoir

$$\dot{Q}_L = \frac{1}{e^2 R_T} \int_{-\infty}^{\infty} dE (E - \mu_L) [f^0(E; \mu_L, T_L) - f^0(E; \mu_R, T_R)]$$
(1.16)

and from the right reservoir

$$\dot{Q}_R = \frac{1}{e^2 R_T} \int_{-\infty}^{\infty} dE (E - \mu_R) [f^0(E; \mu_R, T_R) - f^0(E; \mu_L, T_L)].$$
(1.17)

In the absence of a bias voltage, $\mu_L = \mu_R$, these heat currents are opposite, i.e., $\dot{Q}_L = -\dot{Q}_R$. In the presence of a voltage, the sum $\dot{Q}_L + \dot{Q}_R = IV$ results from the Joule power dissipated into the reservoirs.

Further reading

There are a few other books detailing some of the topics discussed in this book. For example:

• (Imry, 2002) contains a discussion on many basic mesoscopic effects, such as quantized conductance, many of the interference effects, noise and meso-

scopic superconductivity. It also includes the quantum Hall effect, which is not discussed here.

• (Nazarov and Blanter, 2009) is a recently published book on quantum transport in mesoscopic systems for advanced readers.