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AGUSTÍN RAYO

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For Carmen

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Preface

What is logical space? And in what sense is it constructed?

Let me start with the notion of truth. To set forth a statement is to make a *distinction* amongst ways for the world to be, and to single out one side of this distinction; for the statement to be *true* is for the region singled out to include the way the world actually is. To set forth the statement that snow is white, for example, is to distinguish between white-snow and non-white-snow ways for the world to be, and to suggest that the world falls on the white-snow side of this distinction; for the statement to be true is for the world to actually fall on the white-snow side of the distinction—for it to actually be the case that snow is white.

Our search for truth is therefore inextricably linked to our search for distinctions amongst ways for the world to be. *Logical space*, as I shall understand it here, is the set of all such distinctions. (To specify a *region* in logical space is to take sides with respect to some of these distinctions; to specify a *point* in logical space is to take sides with respect to every such distinction.)

In order to develop a useful conception of logical space, it is not enough to come up with a set of distinctions. One must also get clear about which pairs of distinctions coincide, and which ones do not. Consider the distinction between water-containing and water-free ways for the world to be, on the one hand, and the distinction between H_2O-containing and H_2O-free ways for the world to be, on the other. Do these distinctions come to the same thing? We think that to be composed of water *just is* to be composed of H_2O, so we think that they do. But if we had different views about the chemical composition of water, we would think they do not.

One of the main themes of this book is the idea that our conception of logical space is shaped by our acceptance or rejection of 'just is'-statements—that it is by accepting or rejecting a 'just is'-statement that one settles the question of when distinctions coincide, and when they do not.

To accept a 'just is'-statement is to close a theoretical gap. By accepting 'to be composed of water *just is* to be composed of H_2O', for example,

one closes the theoretical gap between being composed of water and being composed of H_2O. A bit more colorfully: one comes to think that when God created the world and made it the case that the Earth was filled with water, she *thereby* made it the case that the Earth was filled with H_2O; and that when God made it the case that the Earth was filled with H_2O, she *thereby* made it the case that the Earth was filled with water. There was nothing *extra* that God needed to do, or refrain from doing. For there is *no difference* between creating water and creating H_2O.

The acceptance of a 'just is'-statement comes with costs and benefits. The benefit is that there are less possibilities to rule out in one's search for the truth, and therefore less explanatory demands on one's theorizing; the cost is that one has less distinctions to work with, and therefore fewer theoretical resources. In deciding whether to accept a 'just is'-statement one strives to find a balance between these competing considerations. Different 'just is'-statements can be more or less hospitable to one's scientific or philosophical theorizing. So the decision to accept them should be grounded on their ability to combine with the rest of one's theorizing to deliver a fruitful tool for scientific or philosophical inquiry. And because of the crucial role of 'just is'-statements in shaping one's conception of logical space, this yields the result that one's conception of logical space cannot be constructed independently of the rest of one's theorizing.

Philosophy of Mathematics

One reason to be interested in this way of thinking about logical space is that it can help us address a family of stubborn problems in the philosophy of mathematics. It opens the door to new ways of defending Mathematical Platonism (the view that mathematical objects exist), and to new ways of thinking about the epistemology of mathematics.

Much of the material in this book can be thought of as an extended argument for *Trivialist Platonism*, a form of Mathematical Platonism according to which the following 'just is'-statement is true:

DINOSAURS

> For the number of the dinosaurs to be Zero *just is* for there to be no dinosaurs.

and, more generally,

NUMBERS
For the number of the Fs to be *n just is* for there to be exactly *n* Fs.

My argument for Trivialist Platonism has three main components. The first is a thesis in the philosophy of language, and is developed in Chapter 1. I defend a conception of reference according to which there is no *linguistic* obstacle for the truth of a 'just is'-statement such as DINOSAURS. I then argue that the resulting conception of language won't lead to an unattractive metaphysics.

The second component of my argument is an account of 'just is'-statements, and is developed in Chapter 2. I argue that 'just is'-statements play a central role in shaping our conception of logical space, as suggested above, and defend an account of the sorts of considerations that might ground the acceptance or rejection of a 'just is'-statement.

The resulting picture is one according to which there are significant theoretical pressures for accepting NUMBERS. For although the acceptance of NUMBERS comes with costs, they are far outweighed by the benefits. Significantly, NUMBERS allows one to dismiss the following as an illegitimate demand for explanation:

> I can see that there are no dinosaurs. What I want to understand is whether it is *also* the case that the number of the dinosaurs is Zero.

For in accepting NUMBERS one rejects the idea that there is a theoretical gap between there being no dinosaurs and their number being Zero. One thinks there is *no difference* between the number of the dinosaurs being Zero and there being no dinosaurs.

The third component of my argument is a defense of the claim that Trivialist Platonism can be developed into a viable philosophy of mathematics. In Chapter 3 I argue that one can construct a semantics for mathematical discourse whereby 'just is'-statements such as NUMBERS turn out to be true. In Chapter 4 I show that the resulting proposal has the resources to explain how mathematical knowledge is possible.

Metaphysics without Foundationalism

Like many of my peers, I see the work of David Lewis as a great source of inspiration. At the same time, I have become increasingly skeptical of some of the foundationalist assumptions that guided his philosophy.

Lewis's foundationalism is based on three main theses:

Property Fundamentalism
: Some properties are, metaphysically speaking, 'fundamental': they 'carve the world at the joints'. Fundamental properties are intrinsic, and render their instances 'perfectly similar'. (Lewis 1983*a*, 2009)

Humean Supervenience
: No two possible worlds just alike in their spatio-temporal distributions of point-sized instantiations of fundamental properties could differ in any other way. (Lewis 1986*b*, 2009)

Modal Reductionism
: Modally rich claims can be analyzed as claims about the intrinsic features of possible worlds, given a suitable counterpart relation. (Lewis 1986*a*)

These three claims come together to ensure that a modally rich claim like 'Humphrey didn't win the election but might have' is ultimately grounded on spatio-temporal distributions of fundamental properties. By Modal Reductionism, our modally rich claim can be broken down into claims that are 'modally flat': they each depend only on how matters stand in one particular world. Relative to a suitable counterpart-relation, the original claim might be analyzed as the requirement that: (*a*) the actual world verify the modally flat claim that there is an election-losing Humphrey, and (*b*) that there be a possible world verifying the modally flat claim that there is a Humphrey-counterpart that wins a counterpart election. By Humean Superveniece, each modally flat component of the analysis can be cashed out as a claim about the distribution of fundamental properties across the space-time manifold of the relevant world. Property Fundamentalism completes the picture by delivering the requisite notion of fundamentality.

Lewis's foundationalist assumptions deliver a tidy metaphysics. But they are also potentially problematic. A first problem is emphasized by Lewis himself: the identities of the fundamental properties turn out to be unknowable (Lewis 2009). For Lewis is committed to a combinatorial principle of possibility: if you cut out any region of any world, keeping its intrinsic properties fixed, and put it together with any other such regions, intrinsic properties also fixed, you get a genuine possibility. This yields the result that connections between fundamental properties and their effects are always contingent. So we have no way of distinguishing the

fundamental properties that are actually responsible for the phenomena we observe from rival fundamental properties that could have had the same effects. Like Kant's noumena, Lewis's fundamental properties turn out to be irremediably beyond our reach. The price of Lewisian foundationalism is that one must remain ignorant of the identities of the relevant foundations.

To see what an alternative picture might look like, consider the property of having negative charge. Particles with negative charge have certain dispositional properties: they are, for example, disposed to repel other particles with negative charge. Lewis's foundationalism commits him to the claim that such dispositions can ultimately be cashed out as distributions of purely intrinsic properties across the space-time manifolds of various worlds. And because of his combinatorialism, he also thinks that these intrinsic properties can be rearranged in arbitrary ways. So the intrinsic properties of negatively charged particles float free of their dispositional properties. On an alternative picture, the property of negative charge cannot be separated from its theoretical role: part of *what it is* to be a negatively charged particle is to be disposed to repel other negatively charged particles. This threatens the foundationalism, since it is no longer clear that the property of being negatively charged can be factored into modally flat components, and it is no longer clear that such components would be decomposable into spatio-temporal distributions of intrinsic properties. But there is no longer any reason to think that the property is beyond our reach.

A second problem with Lewis's foundationalist picture is that Humean Supervenience may well be inconsistent with our best physical theorizing. Consider the relation of quantum entanglement. In order for Humean Supervenience to be true, such a relation must supervene on the distributions of point-sized instantiations of intrinsic properties. But, as Maudlin (1998) points out, it is far from obvious that this is so. Lewis's foundationalist picture gets us metaphysical tidiness, but it does so by incurring empirical liabilities. (Lewis is aware of this as a potential problem, and partly for that reason thinks of Humean Supervenience as a contingent truth; see Lewis (1986*b*: *x*.)

A third problem concerns Property Fundamentalism. Notions of metaphysical fundamentality such as Lewis's have played an increasingly important role in contemporary metaphysics (Fine 2001; Schaffer 2009; Sider 2012). But, as we will see in Sections 1.2 and 2.1.4, reliance on such

notions comes at a cost. When one buys into a notion of metaphysical fundamentality one gains access to additional distinctions, and therefore to additional theoretical resources. But the additional distinctions also give rise to potentially awkward questions. One is forced, for example, to elucidate the connection between objective fundamentality and fundamentality by the lights of a particular theoretical framework. To my mind, at least, it is not obvious that this is a price worth paying.

A fourth problem concerns Modal Reductionism. By presupposing that no property resists factorization into modally flat components, Lewis is making a non-trivial assumption—an assumption that one might have good reason to resist for reasons that go beyond any concerns one might have about Property Fundamentalism or Humean Supervenience. This is a point that will be developed in detail in Section 5.2.

Because of these worries, I will make a concerted effort to steer clear of Lewis's foundationalist assumptions in this book. In so doing, I will be developing a brand of metaphysics that is distinctly unlewisian. But the book is certainly not an attempt to produce *anti*-Lewisian metaphysics. What I hope to show is that one can do interesting philosophy without committing oneself to Lewis's foundationalist assumptions, not that the assumptions are mistaken.

Rather than assuming from the outset that there is sense to be made of metaphysical fundamentality—as Property Fundamentalism requires—I will remain *neutral* with respect to matters of metaphysical fundamentality. Rather than assuming from the outset that every truth about the world supervenes on a particular kind of foundation—as Humean Supervenience requires—I will let supervenience relations be determined by whatever 'just is'-statements are suggested by our best overall theorizing. And rather than assuming from the outset that there must be a way of factoring modally rich claims into modally flat components—as Modal Reductionism requires—I will develop a conception of modality that does not depend on a reduction of the modal to the non-modal.

How to Read this Book

I have divided the book into three parts:

Part I: Main Texts (Chapters 1–4)
Part II: Detours (Chapters 5–8)
Part III: Appendices (A–C)

All the of main themes of the book are developed in Part I. If you'd like to see the overall picture while limiting your time-commitment, what I recommend is that you focus on Chapters 1–4. (If you have time for only one chapter, I recommend Chapter 1.)

Part II is for enthusiasts. It discusses issues arising from Part I that are important for a detailed understanding the project, but may be skipped by less committed readers. These detours should not be thought of as extended footnotes, though. Each of them develops a self-standing idea of independent interest. In Chapter 5, I explore the connection between 'just is'-statements and metaphysical possibility, and argue that a limited class of 'just is'-statements can be used to fix the truth-value of every sentence in a first-order modal language. In Chapter 6, I characterize the *dot-function*: a device for simulating quantification over merely possible objects. In Chapter 7, I describe the expressive resources that would be needed to supply nominalistic paraphrases for the language of arithmetic. In Chapter 8, I offer an account of linguistic stipulation in mathematics.

If you are tempted by the material in Part II, what I recommend is that you read it in conjunction with other parts of the book. You'll find plenty of pointers in the text, but here are some suggestions:

Table 0.1 How to Read This Book

Read ...	in conjunction with ...
Chapter 5	Section 2.2.1
Chapter 6	Section 3.3.1 or Section 5.2.2
Chapter 7	Section 1.4 or Section 3.3.1
Chapter 8	Section 1.3 or Chapter 3

Part III consists of technical appendices, and is intended only for the true aficionado.

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- An earlier version of Chapter 6 and Appendix B, is forthcoming as 'An Actualist's Guide to Quantifying-In', *Crítica: Revista Hispanoamericana de Filosofía* (forthcoming), and is reprinted here by permission of the editor.
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PART I

Main Texts

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1

Language and Metaphysics

Consider the following 'just is'-statements:

1. Sibling
   For Susan to be a sibling *just is* for her to share a parent with someone else.
2. Water
   For the glass to be filled with water *just is* for it to be filled with H_2O.
3. Physicalism
   For such-and-such a mental state to be instantiated *just is* for thus-and-such brain states and environment conditions to obtain.
4. Properties
   For Susan to instantiate the property of running *just is* for Susan to run.
5. Death
   For Socrates's death to take place *just is* for Socrates to die.
6. Tables
   For there to be a table *just is* for there to be some things arranged tablewise.
7. Dinosaurs
   For the number of the dinosaurs to be Zero *just is* for there to be no dinosaurs.

Statement 1 is utterly uncontroversial. Statement 2 should be pretty uncontroversial too, at least if we ignore certain complications (such as the possibility of impurities). Statement 3 is somewhat controversial (Chalmers 1996), but it seems to be the dominant view amongst contemporary analytic philosophers.

Statements 4–7, on the other hand, are all highly controversial metaphysical theses. My own view is that they are all true, but I won't try to

convince you of that in this chapter. The aim of the chapter is to argue that they shouldn't be rejected *on general linguistic or metaphysical grounds*. I will proceed by defending a conception of language I call 'compositionalism', and showing that it makes room for Statements 4–7. I will then argue that a compositionalist who accepts Statements 4–7 won't thereby be left with an unattractive metaphysics.

The plan for the chapter is as follows. I will start by explaining how I think the 'just is'-operator should be understood (Section 1.1). I will then introduce my foil: a view I refer to as *metaphysicalism*. Metaphysicalists believe that in order for an atomic sentence to be true, there needs to be a certain kind of correspondence between the logical form of the sentence and the 'metaphysical structure' of reality. I will explain why I think metaphysicalism is bad philosophy of language (Section 1.2), and develop compositionalism as an alternative (Section 1.3). Attention will then turn to metaphysics. I will argue that compositionalism does not lead to untoward metaphyical consequences, even if one accepts 'just is'-statements such as 4–7 (Sections 1.4 and 1.5).

1.1 The 'Just is'-Operator

Before mounting my defense of compositionalism, it will be useful to say something about how I will be understanding the 'just is'-operator, as it occurs in Statements 1–7.

Consider SIBLING as an example. What it takes for SIBLING to be true is for there to be *no difference* between Susan's having a sibling and Susan's sharing a parent with someone else. If Susan is a sibling it is *thereby* the case that she shares a parent with someone else, and if she shares a parent with someone else it is *thereby* the case that she is a sibling. More colorfully: when God created the world, and made it the case that Susan shared a parent with someone else, there was nothing *extra* she had to do, or refrain from doing, in order to ensure that Susan was a sibling. She was already done. And when God created the world, and made it the case that Susan was a sibling, there was nothing *extra* she had to do, or refrain from doing, in order to ensure that Susan shared a parent with someone else. She was already done.