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The Law of Non-Contradiction

New Philosophical Essays

edited by

GRAHAM PRIEST, JC BEALL, AND BRADLEY ARMOUR-GARB

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The book is divided into parts, with the chapters in each section centering on a particular aspect of the given (part-) issue. The structuring is, as ever, to a certain extent arbitrary, and a contribution in one part may well contain material that bears on the topic of other parts. Following Beall's introductory essay, the book leads off with Priest's chapter at the suggestion of a reader from Oxford University Press, since that is the one that sets the scene for much of what follows. We are grateful for the advice from such readers.

This volume began in 1999 with discussions between Beall and Priest in Australia (when Beall was still living there), with Armour-Garb joining the project a little later. We are grateful for the help and support of those directly involved, and also for those who gave suggestions and encouragement early on. We are particularly grateful to both internal and external referees whose careful comments have made for a better volume.

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Melbourne, Storrs, Albany G.P.
2003 J.C.B.
B.A.-G.

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Introduction: At the Intersection of Truth and Falsity

JC Beall

'Now we will take another line of reasoning. When you follow two separate chains of thought, Watson, you will find some point of intersection which should approximate to the truth.'—Sherlock Holmes, in 'The Disappearance of Lady Frances Carfax'.

1. TOWARDS THE INTERSECTION

Suppose that we have (at least) two categories $\mathcal{X}$ and $\mathcal{Y}$ for any meaningful, declarative sentence $\mathcal{A}$ of our language.[1] Pending further information about $\mathcal{X}$ and $\mathcal{Y}$, there seem to be four options for an arbitrary sentence $\mathcal{A}$:

» $\mathcal{A}$ is only in $\mathcal{X}$
» $\mathcal{A}$ is only in $\mathcal{Y}$
» $\mathcal{A}$ is in both $\mathcal{X}$ and $\mathcal{Y}$
» $\mathcal{A}$ is in neither $\mathcal{X}$ nor $\mathcal{Y}$

Whether each such 'option' is logically possible depends not only on our logic (about which more below) but on the details of $\mathcal{X}$ and $\mathcal{Y}$.

Suppose that $\mathcal{X}$ comprises all (and only) sentences composed of exactly six words, and $\mathcal{Y}$ those with exactly nineteen words. In that case, only the third option is ruled out: $\mathcal{X}$ and $\mathcal{Y}$ are *exclusive*—their intersection $\mathcal{X} \cap \mathcal{Y}$ is empty—since no $\mathcal{A}$ can be composed of exactly six words and also be composed of exactly nineteen words.[2] Despite being exclusive, $\mathcal{X}$ and $\mathcal{Y}$ are *not exhaustive*—their union $\mathcal{X} \cup \mathcal{Y}$ does not exhaust all sentences—since some $\mathcal{A}$ may fall into neither $\mathcal{X}$ nor $\mathcal{Y}$. (Just consider 'Max sat on Agnes'.)

Consider another example. Let $\mathcal{X}$ comprise all sentences of your favourite novel and $\mathcal{Y}$ your all-time favourite sentences. In that case, exclusion is not ruled out; the intersection of $\mathcal{X}$ and $\mathcal{Y}$ may well be non-empty. (Suppose that your favourite sentence is the first sentence of your favourite novel.) Presumably, $\mathcal{X}$ and $\mathcal{Y}$ are not

[1] Henceforth, 'sentence' is used for meaningful, declarative sentences.
[2] Actually, even this is a bit contentious, since there are inconsistent (but non-trivial) arithmetics in which 19 and 6 'collapse'. (See [29].)

exhaustive, since (presumably) there are sentences that are neither your favourite nor in your favourite novel.

2. AT THE INTERSECTION

Now for the interesting question. Assuming that *truth* and *falsity* are categories of sentences, we can let $\mathcal{X}$ be the former and $\mathcal{Y}$ the latter. Let us assume, following standard practice, that one constraint on *falsity* is that, by definition, falsity is truth of negation, that is, that $\mathcal{A}$ is false if and only if its negation $\neg\mathcal{A}$ is true. The question, then, is this: Are $\mathcal{X}$ and $\mathcal{Y}$ both exclusive and exhaustive categories?

For present purposes, the question of exclusion is central.[3] Are truth and falsity exclusive? The question is intimately connected with others:

» Is there any a priori (or empirical) reason to think that truth and falsity are exclusive?

» If truth and falsity are exclusive, how is the non-exclusivity to be formulated? If truth and falsity are not exclusive, how is that to be formulated?

» How would we decide whether truth and falsity are (non-)exclusive? Can there be any non-question-begging debate?

» Is there any a priori (or empirical) reason to think that truth and falsity are not exclusive?

» Even if truth and falsity are not exclusive, is it rational to believe anything that lies in the intersection of truth and falsity?

I will not (here) address all of those questions; they are discussed in depth, in one form or another, in the following chapters.[4] Here my aim is to (briefly) cover a few topics that serve as background to the rest of the book. I give indications for further reading along the way.[5]

3. 'THE' LAW OF NON-CONTRADICTION

The classic source of much thought about contradiction comes from Aristotle's Book Γ of the *Metaphysics*. To this day, many of Aristotle's views have been widely rejected; the conspicuous exception, despite the work of Dancy [21] and Łukasiewicz [28], are his views on contradiction. That no contradiction is

[3] The two questions, as RESTALL, BRADY, and VARZI emphasize, are closely related, but I will concentrate on the question of exclusion in this introductory essay. MCGEE's essay also brings out the very tight connection between the questions of exclusion and exhaustion.

[4] In fact, the questions roughly correspond to the five parts of the volume.

[5] In giving further reading, I also highlight the chapters in this volume by using UPPERCASE for names of contributors.

true remains an entrenched 'unassailable dogma' of Western thought—or so one would think.[6]

In recent years, due in no small measure to progress in paraconsistent logic (more on which in ss. 4 and 7), the 'unassailable dogma' has been assailed. As Priest's detailed discussion shows [32], neither Aristotle's arguments for (non-)contradiction nor modifications of those arguments [3, 41, 45] have produced strong arguments for the thesis that no contradiction could be true—that the intersection of truth and falsity is necessarily empty. Moreover, there seem to be reasons for thinking that at least some contradictions are true (see s. 5). At the very least, the issue is open for debate—the main motivation behind this volume.

But what exactly *is* the so-called law of (non-)contradiction? Unfortunately, 'the' so-called law is not one but many—and perhaps not appropriately called a 'law'. Aristotle distinguished a number of principles about (non-)contradiction, and the correct exegesis of his views remains an issue among historians. For present purposes, I will simply list a few principles, and then briefly fix terminology concerning 'contradiction'.[7]

» SIMPLE (NON-)CONTRADICTION: No contradiction is true

» ONTOLOGICAL (NON-)CONTRADICTION: No 'being' can instantiate contradictory properties

» RATIONALITY (NON-)CONTRADICTION: It is irrational to (knowingly) accept a contradiction

The principles, so formulated, are hardly precise, but they indicate different (not to say logically independent) versions of 'the' target principle. For present purposes, I will focus almost entirely on Simple (Non-)Contradiction, though some of what follows will also indirectly touch on the other principles.[8]

What needs to be clarified is the sense of 'contradiction' at play (at least in this introductory chapter). I will discuss two uses of the term, the *explosive* and the *formal* usage.[9]

Explosive Usage

Some philosophers use the term 'contradiction' to mean an *explosive sentence*, a sentence such that its truth entails triviality—entails that all sentences are true.

[6] Despite showing the holes in Aristotle's various arguments on (non-)contradiction, Łukasiewicz [28] concludes that Aristotle was right to preach (as it were) the 'unassailable dogma', as Łukasiewicz called it.

[7] Chapters by BRADY, RESTALL, and VARZI are particularly relevant to the issue of formulating 'the' relevant 'law'.

[8] The chapters by KROON, COGBURN, and TENNANT are particularly relevant to all three principles, as is BROWN's.

[9] GRIM's chapter is particularly useful for gaining a sense of the divergent uses of 'contradiction', as is that by WEIR.

A familiar example of such a sentence is 'Every sentence is true.' That sentence is apparently explosive, since if 'every sentence is true' is true, then every sentence is true, in which case triviality abounds.

Could a contradiction in the explosive sense be true? The question is tricky, as tricky as the modality 'could'. Suppose that by 'could' we mean *logically possible.* Then the question is: Is it logically possible that a contradiction (in the explosive sense) be true?

The answer, of course, depends on the given logic. Does classical logic afford the logical possibility of true contradictions (in the explosive sense)? Interestingly, there is a sense in which classical logic—or, at least, an intuitive account of classical consequence—does afford the logical possibility of true (explosive) contradictions.[10] Intuitively, an argument is classically valid if and only if there is no 'world' in which the premisses are true but the conclusion is untrue. Such worlds, on the classical account, are complete and consistent, in the sense that for any world w and any sentence $\mathcal{A}$, either $\mathcal{A}$ or its negation $\neg\mathcal{A}$ is true at w, but not both $\mathcal{A}$ and $\neg\mathcal{A}$ are true at w. What the classical approach demands, of course, is that if *both* $\mathcal{A}$ and $\neg\mathcal{A}$ are true at some world w, then so too is $\mathcal{B}$, for *any* $\mathcal{B}$. But, then, there is nothing in the classical account, at least intuitively understood, that precludes recognizing an exceptional 'trivial world', the world in which *every* sentence is true. In that respect, even classical consequence affords the logical possibility of true (explosive) contradictions: it is just the 'logical possibility' in which every sentence is true—the 'logical possibility' in which explosion happens!

Be that as it may, classical consequence is usually understood in terms of 'classical interpretations'. A classical interpretation is—or is usually modelled by—a function ν from sentences into $\{1, 0\}$ (intuitively, The True and The False) such that $\nu(\neg\mathcal{A}) = 1$ exactly if $\nu(\mathcal{A}) = 0$. But, then, there is no classical interpretation on which a contradiction (in the explosive sense) is true.

The upshot is that if classical logic dictates the space of logical possibility, there is at best only a remote and trivial sense in which contradictions, in the *explosive* sense, could be true. But there is another sense of 'contradiction', to which I now turn—and classical logic, of course, is only one among many logical theories.

Formal Usage

The explosive usage is not the only prevalent usage of 'contradiction', and for present purposes, it is not the target usage. The *formal* usage of 'contradiction' has it that contradictions are sentences *of the form* $\mathcal{A} \wedge \neg\mathcal{A}$, where $\wedge$ is conjunction and, as above, $\neg$ is negation. In other words, a contradiction, on the formal usage, is the conjunction of a sentence and its negation.

Tradition distinguishes between (among others) sub-contraries and contradictories. $\mathcal{A}$ and $\mathcal{B}$ are *contraries* if they both cannot be true. $\mathcal{A}$ and $\mathcal{B}$ are *subcontraries*

[10] Here, I assume single-conclusion classical semantics. As Greg Restall pointed out (in conversation), the issue is slightly more complicated in a so-called multiple-conclusion framework.

if they cannot both be false. $\mathcal{A}$ and $\mathcal{B}$ are *contradictories* if they are both contraries and sub-contraries.

For present purposes, all that is required of a contradiction, at least on the *formal usage* (as here specified), is that it be of the form $\mathcal{A} \wedge \neg\mathcal{A}$. In particular, there is no further requirement that $\mathcal{A} \vee \neg\mathcal{A}$ be logically true, or that $\neg(\mathcal{A} \wedge \neg\mathcal{A})$ be logically true.[11]

The target sense of 'contradiction' is the formal one.[12] Could such a contradiction be true? At this stage, the question of logic becomes pressing. If we let classical logic dictate the constraints of 'could' (in whatever sense might interest us), then we have already been through the question at hand. After all, if classical logic dictates the constraints of (say) logical possibility, then any *formal contradiction* is an explosive contradiction, as the famous 'independent argument' shows. (See s. 4 for further discussion.) But, as above, classical logic is just one among many different theories of consequence (validity). In addition to classical logic, and particularly relevant to the present volume, is so-called paraconsistent logic, to which I turn.[13]

4. WEAK AND STRONG PARACONSISTENCY

The question at the intersection of truth and falsity is whether it (the intersection) could be non-empty but non-trivial—whether *some but not all* contradictions could be true. Classical logic, and intuitionistic logic, for that matter, give a swift answer: No.[14] In each such logic, the so-called 'independent argument' goes through:[15]

(1) Assume that $\mathcal{A} \wedge \neg\mathcal{A}$ is true

(2) By (1) and Simplification, $\mathcal{A}$ is true

[11] Of course, one might argue—and some [40] have—that an operator φ is *negation* (or a negation) only if $\mathcal{A} \vee \varphi\mathcal{A}$ and $\varphi(\mathcal{A} \wedge \varphi\mathcal{A})$ are logically true. If that is right, then $\mathcal{A} \wedge \neg\mathcal{A}$ is a contradiction only if $\mathcal{A}$ and $\neg\mathcal{A}$ are sub-contraries and $\neg(\mathcal{A} \wedge \neg\mathcal{A})$ is logically true—since otherwise $\neg$ wouldn't be a negation. (Recall that on the formal usage, a contradiction is of the form $\mathcal{A} \wedge \neg\mathcal{A}$, where $\neg\mathcal{A}$ is the negation of $\mathcal{A}$.) But, again, I will leave this issue aside, not because it is not important but, rather, because a full discussion would be too full for present purposes. Useful discussion of negation is in Brady's paper, as well as Sainsbury's, and also in the volumes [23, 47] and Routley and Routley [44].

[12] Henceforth, I use 'contradiction' along the formal usage, unless otherwise specified.

[13] I will say nothing here about 'revisions of logic' or the like, due only to space considerations. My own view is along Quine-the-good lines, according to which *any* 'logical principle' may be revised in the face of appropriate 'evidence'. (Quine-the-bad, of course, imposed exceptions—notably, the 'unassailable dogma' of which Aristotle and Łukasiewicz spoke.) Resnik's chapter, in addition to those by Bueno and Colyvan and Brown, discuss these issues along various lines. The two letters by Lewis are also relevant.

[14] Priest [38] and Beall and van Fraassen [18] provide introductory presentations of intuitionistic logic, in addition to the sample paraconsistent framework discussed in s. 7. Priest's text also discusses more mainstream approaches to so-called relevant (-ance) logic.

[15] The 'proof' is often ascribed to C. I. Lewis, who rediscovered it for contemporary readers, but Medieval logicians were apparently aware of the proof (like so many other 'recent discoveries'). I am grateful to Graham Priest on the historical point.

(3) By (2) and Addition, $\mathcal{A} \vee \mathcal{B}$ is true

(4) By (1) and Simplification, $\neg\mathcal{A}$ is true

(5) But, then, by (3), (4), and Disjunctive Syllogism, $\mathcal{B}$ is true

The upshot is that any contradiction is explosive if each of the foregoing steps is valid.

Paraconsistent logics, by definition, are not explosive. A consequence relation $\vdash$, however defined, is said to be *explosive* if $\mathcal{A}, \neg\mathcal{A} \vdash \mathcal{B}$ holds for arbitrary $\mathcal{A}$ and $\mathcal{B}$. A consequence relation is said to be *paraconsistent* if and only if it is not explosive.[16]

A sample paraconsistent logic is presented in s. 7. That sample is one among various approaches to paraconsistent logic, and by no means decidedly 'the right one'. One approach, for example, due to Da Costa [19, 20], is to let negation fail to be truth-functional. Without truth-functionality, there is no a priori reason that $\mathcal{A}$ and $\neg\mathcal{A}$ could not both be true. Other approaches filter out explosion while retaining as many familiar features of the logical connectives as possible. And there are yet other approaches.[17]

Paraconsistent logic, regardless of the details, affords the 'possibility' of inconsistent but non-trivial theories—theories according to which both $\mathcal{A}$ and $\neg\mathcal{A}$ are true (for some $\mathcal{A}$) but not every sentence is true. Such logics, in other words, open up the 'possibility' in which *some but not all* contradictions 'could' be true.

The matter (again, regardless of the formal details) is delicate. Paraconsistentists, those who construct or use or rely on some paraconsistent logic, usually divide into (at least) three classes:

» Weak Paraconsistentist: a paraconsistentist who rejects that there are 'real possibilities' in which a contradiction is true; paraconsistent models are merely mathematical tools that prove to be useful but, in the end, not representative of real possibility

» Strong Paraconsistentist: a paraconsistentist who accepts that there are 'real possibilities' in which contradictions are true, and more than one such 'real possibility' (and, so, not only the trivial one); however, no contradiction is in fact true

» Dialetheic Paraconsistentist: a paraconsistentist who accepts that there are true contradictions—and, so, that there could be (since our world is a 'real possibility' in which there are some)[18]

Most contemporary paraconsistentists, including so-called relevantists [1, 2, 43], fall into the first class. The minority position, but the position of most relevance

[16] That account of paraconsistent consequence is not ideal, but it is the standard one. Priest and Routley [39, 40] provide a nice discussion of the issue.

[17] For a discussion, see Priest [35].

[18] Depending on the details of the given logic, strong paraconsistentists sometimes collapse into dialetheic paraconsistentists. For discussion see Restall [42] and Beall and Restall [17].

to the current volume, is the third class: dialetheic paraconsistentists. What is important to note is that 'paraconsistency' and 'dialetheism' are *not* synonyms. Any rational version of the latter will require the former, but the converse seems not to hold.

Many of the contributions in this volume revolve around dialetheism. PRIEST's chapter argues that there are no good arguments against dialetheism.[19] Suppose that Priest's arguments are sound. Even so, an immediate question arises: Is there any reason to think that dialetheism is correct? Is there any reason to think that some contradictions are true? To that question I now (very briefly) turn.

5. TOWARDS A NON-EMPTY INTERSECTION

Let us suppose, as above, that *truth* and *falsity* are categories of sentences, with at least the constraint that $\neg\mathcal{A}$ is true if and only if $\mathcal{A}$ is false. Consider the following sentence (a 'Liar'):

» The first displayed sentence in s. 5 is false

Does that sentence go in category *truth* or in *falsity*? Given the way we use 'true', the first displayed sentence in s. 5 goes in *truth* only if it goes in *falsity*. But, given the way we use 'true', the first displayed sentence in s. 5 goes in *falsity* only if it goes in *truth*. What we seem to have, then, is a sentence that goes into the one category (truth) exactly if it goes into the other (falsity).

True contradiction? It depends. Suppose that *truth* and *falsity* are not exhaustive—that some sentences are in neither category, that there are 'truth value gaps'. Then we have no true contradiction, at least not via the first displayed sentence.

A question arises: When we say that the first displayed sentence is neither true nor false, what are we saying? One thing we are saying, it seems, is that the negation of the first displayed sentence is *not* false. But falsity is truth of negation, in which case we seem to be saying something of the form $\neg\neg\mathcal{A}$. (If T is our truth predicate and $\langle\mathcal{A}\rangle$ a name of $\mathcal{A}$, then we seem to be saying something of the form $\neg\mathsf{T}\langle\neg\mathcal{A}\rangle$, which is to say that $\neg\mathcal{A}$ is false, which seems to be equivalent to $\neg\neg\mathcal{A}$.) But, now, assuming Double Negation-Elimination, that entails $\mathcal{A}$. We seem to be back to the apparent true contradiction.

One natural suggestion is that we have at least two negations—one $\sim$ being a 'gap-closer', the other $\neg$ affording gaps. The idea is that we use the 'gap-closer' (sometimes called 'exclusion') when we say that the first displayed sentence in s. 5 is *not* false (or true). While that suggestion will avoid the problem above, it also

[19] Of course, PRIEST's contribution was written prior to the others in this volume. Debate will tell whether some of those considerations work against dialetheism.

returns us to the appearance of true contradiction:

> The second displayed sentence in s. 5 is not true

It seems that the non-exhaustiveness of *truth* and *falsity* does little to avoid the apparent emergence of contradiction: The second displayed sentence seems to be true if and only if it is not. A simple lesson to draw is the dialetheic one: The second displayed sentence is *in the intersection* of both truth and falsity—or the intersection of truth and 'untruth' (if one adds that category to accommodate gaps).

Anyone familiar with contemporary work on the Liar will know that, in an effort to avoid 'true contradictions', many different non-dialetheic avenues have been pursued.[20] Some of the given avenues are ingenious attempts to avoid the apparent inconsistency, and most are mathematically or logically interesting frameworks for thinking about language. In the end, though, none of the given approaches are as simple as a dialetheic response, which simply accepts that the intersection of truth and falsity is non-empty. And given some suitable paraconsistent logic, the dialetheist may accept that some *but not all* contradictions are true—the non-empty intersection may be approached and enjoyed without explosive traffic.[21]

Simple or not, one might think, it seems downright irrational to accept that the intersection of truth and falsity is non-empty—that there are truths with true negations, that there are 'true contradictions' (even if they don't explode). Such a sentiment remains prominent—a residual vestige, perhaps, of the 'unassailable dogma' of (non-)contradiction. But it really is just dogma, at least as far as I can tell (and notwithstanding some of the contributions in this volume), but you (the reader) can judge for yourself.

One issue that should be emphasized is that nothing in dialetheism requires the existence of *observable* contradictions—true contradictions that have observable (but inconsistent) consequences. *That*, despite considerations to the contrary [7, 33], is difficult to understand. But one might, as some suggest,[22] restrict dialetheism to the purely semantic fragment of the language. In that case, the charge of 'irrationality' or even 'incredulous stares' are difficult to appreciate,

[20] For a discussion of contemporary approaches, see Beall [11, 12]. Priest [31] gives extended arguments against many such approaches, and also gives one of the earliest and most extended arguments for a dialetheic approach. Beall [10] presents arguments for a different (non-Priestly, as it were) version of dialetheism.

[21] Priest [31] has launched various arguments for dialetheism. The case from semantic paradox, by Priest's lights, is not as strong as the overall case from what he calls 'the inclosure schema' and 'principle of uniform solutions' [37]. Given that Priest's work is largely responsible for the 'spread of dialetheism' (slow as the spread may be), many of the chapters in this volume discuss a variety of Priest's arguments. My own thinking is that, regardless of 'inclosure' or the like, simplicity and preservation of naïve appearance is sufficient for accepting some version or other of dialetheism. But that too, in the pages to come, is challenged by various contributors. Zalta and Goldstein, for example, offer direct challenges by proposing alternative responses to various apparent inconsistencies. Armour-Garb discusses whether, and in what sense, dialetheism offers a solution to *paradox*.

[22] See the chapters by Beall and Mares.

as the only 'true contradictions' are grammatical residue (like the first or second displayed sentences) that carry no observational import. All that is claimed, at least on such restricted dialetheic positions, is that the intersection of truth and falsity contains various peculiar—but none the less grammatically inevitable—sentences that carry no observational consequences. Provided, as above, that a suitable paraconsistent logic is in place, there seems to be little to back worries of irrationality or instability or the like—little, again, beyond the dogma.

6. BEYOND THE SEMANTIC PARADOXES?

One would be misled to think that the *only* considerations towards true contradictions involve semantic paradoxes. Are there reasons to think that some contradictions, having nothing at all to do with the semantic paradoxes, are true? Debate will tell, but I briefly mention two considerations towards the possibility.[23]

Naïve Extensions

Priest [31] argues that the paradoxes of set theory, and in particular Russell's paradox, calls out for a dialetheic solution. Part of Priest's argument turns on his 'inclosure schema' and 'principle of uniform solutions' [37]. In effect, the argument is that Russell's paradox and the semantic paradoxes have the *same basic structure*—what Priest calls 'inclosure'—and, hence, ought to receive the same solution. While I am sympathetic with Priest's argument, I leave its details and merits to the reader.

By my lights, 'Russell's paradox' is ambiguous. On one hand, it denotes a type of paradox that arises in *set theory*, a discipline within mathematics. Sets were originally constructed within and for mathematics. If mathematics wishes to remain consistent, then Russell's *set*-theoretic paradox may be resolved as it has been—by stipulating it out (via axioms or the like).[24] Whether a set-theory is mathematically sufficient is governed by the pragmatic issue of whether it does the job—whether sets, so specified, do the trick for which they were constructed. In that respect, Russell's paradox may have a simple, consistent solution, at least for purposes of mathematics. And the same would go, of course, for mathematical versions of the Liar—stipulate them out, so long as the job is still fully achieved.

[23] One would likewise be misled to think that the following two points exhaust the considerations, or are even the strongest. Priest [37] covers a wide variety of other areas that arise, as he puts it, 'at the limits of thought and language'. Priest [31] also discusses the apparent inconsistency involved in *change, motion, legal contexts*, and much else.

[24] Arguments towards, and explorations of, *inconsistent mathematics*, may be found in Mortensen [29] (and references therein).

But there is another Russell's paradox, the paradox of (naïve) *extensions*, which arises not in the restricted confines of mathematics but in natural language. Semanticists and philosophers of language have long recognized the need for *extensions* of predicates (and expressions, in general). A look down the corridor reveals the mathematician's *sets*—and we have since been off running. The trouble is that there is no a priori reason to think that *sets* (the entities constructed within and for mathematics) will sufficiently play the role of extensions; indeed, there is reason to think otherwise. At least initially, with an aim on natural language, we want to have extensions for *every* predicate of the language. In particular, we want to have an extension not only for 'is a philosopher' and 'is a cat' but also for 'is an extension' and 'is not in its own extension' (i.e. 'χ is not in the extension of χ'). The simple idea, of course, is that our extension theory should not only be unrestricted but also should satisfy what seems plainly correct: that the denotation of a is in the extension of $\mathcal{F}$ iff $\ulcorner \mathcal{F}a \urcorner$ is true.[25] But having that calls for dialetheism, at least if one is to accept one's own theory.

I have not given an argument for true contradictions that arise from extensions, but it is an area in which true contradictions may well arise. While inconsistency in *set theory* can be resolved by axiomatizing away, the same is not clearly the case with respect to extensions. Extensions, unlike mathematical sets (at least on the picture I've suggested), are constrained not only by their role in our overall theories, but also by our 'intuitions' about them. Whether such a role or our given 'intuitions' yield true contradictions is something that, as always, debate will tell.

Borderline Cases

Another potential area in which true contradictions might arise is at the 'limits' of vagueness. Not a lot of work has been done on this topic, but a few considerations run as follows.[26]

So-called *tolerance conditionals* that appear in soritical paradoxes appear to be true. If b is a child at t_n, then b is a child at t_{n+1} (for some minuscule measure of time). Rejecting such conditionals, it seems, reveals an incompetence with respect to how the predicate 'is a child' (or any other vague predicate) is used. But the sorites paradox seems to challenge that appearance. Indeed, virtually all known approaches to the sorites reject at least one tolerance conditional, holding that it is

[25] Likewise, of course, one wants to have an *extension* of 'is a truth', something that comprises all truths. The mathematicians' *sets*, as Grim [24, 25] argued, seem not to do the trick. All the more reason for an *extension* theory that does the trick.

[26] Dominic Hyde [26] has advanced a paraconsistent, though not clearly dialetheic, approach to vagueness. For something closer to a dialetheic approach see Beall [6] and Beall and Colyvan [15, 16].

not rationally or competently assertable.[27] The trouble with such responses is that one none the less 'feels' that such conditionals *are* true.

One avenue towards resolving the issue is to recognize true contradictions at the 'limits' of vagueness. The suggestion, for example, is that all of the tolerance conditionals are true, but some of them are also false: they reside at the intersection of truth and falsity. In particular, the 'penumbra' is awash with true contradictions. A semantics that affords such an approach is covered below (LP, s. 7).

Of course, if vagueness affords true contradictions, then there may well be 'observable contradictions', and that may be a heavy cost to bear. But that issue deserves debate. In the end, it seems initially as reasonable to think that a 'vague language' is *overdetermined* as it is to think it *underdetermined*. But that issue, like others, is one that must here be left open.

Further discussion of dialetheism (both for and against), of course, may be found in the following chapters. For now, and for purposes of giving the reader a basic framework in which to think about some of the foregoing (and forthcoming) issues, I turn to a brief sketch of a common paraconsistent framework associated with dialetheism—Priest's 'logic of paradox', LP.

7. A SAMPLE PARACONSISTENT LOGIC

As above (s. 4), there are various standard approaches to paraconsistent semantics. Because of its 'classical' appearances (and, hence, familiarity), and also its historical tie to dialetheism, the focus here will be on a basic many-valued, truth-functional approach. The logic typically associated with dialetheism is Priest's 'logic of paradox', LP [30]. For purposes of generality, I present FDE but highlight LP in due course.

Propositional Semantics

The syntax is that of classical logic. The semantics arises by letting interpretations be functions ν from sentences into $\mathcal{V} = \wp(\{1, 0\})$. Hence, where $\mathcal{A}$ is any sentence, $\nu(\mathcal{A}) = \{1\}$, $\nu(\mathcal{A}) = \{0\}$, $\nu(\mathcal{A}) = \{1, 0\}$, or $\nu(\mathcal{A}) = \emptyset$. Given that $\nu(\mathcal{A})$ is a set (comprising either 1, 0, or nothing), we may (by way of informal interpretation) say that $1 \in \nu(\mathcal{A})$ iff $\mathcal{A}$ is (at least) true under ν, and $0 \in \nu(\mathcal{A})$ iff $\mathcal{A}$ is (at least) false under ν. In the case where $\nu(\mathcal{A}) = \emptyset$, we may (informally) say that $\mathcal{A}$ is neither true nor false (under ν); and when $1 \in \nu(\mathcal{A})$ and $0 \in \nu(\mathcal{A})$, we may (informally) say that $\mathcal{A}$ is both true and false (under ν).

[27] For recent work on the sorites, see Beall [9] and the references therein. (That volume also contains recent work on various semantic paradoxes.)

$\mathcal{D}$, our designated values, comprises $\{1\}$ and $\{1, 0\}$. (Intuitively, and informally, we designate all and only those sentences that are 'at least true'.)

We say that an interpretation ν is *admissible* just in case it 'obeys' the following clauses:[28]

» $1 \in \nu(\neg\mathcal{A})$ iff $0 \in \nu(\mathcal{A})$

» $0 \in \nu(\neg\mathcal{A})$ iff $1 \in \nu(\mathcal{A})$

» $1 \in \nu(\mathcal{A} \wedge \mathcal{B})$ iff $1 \in \nu(\mathcal{A})$ and $1 \in \nu(\mathcal{B})$

» $0 \in \nu(\mathcal{A} \wedge \mathcal{B})$ iff $0 \in \nu(\mathcal{A})$ or $0 \in \nu(\mathcal{B})$

Logical consequence (semantic consequence) is defined as 'truth preservation' over all (admissible) interpretations, that is, if every premise in Σ is at least true, then so too is $\mathcal{A}$:

» $\Sigma \Vdash \mathcal{A}$ iff $\nu(\mathcal{A}) \in \mathcal{D}$ if $\nu(\mathcal{B}) \in \mathcal{D}$, for all $\mathcal{B}$ in Σ

A sentence $\mathcal{A}$ is *valid* (a tautology, logical truth) exactly if $\emptyset \Vdash \mathcal{A}$.

Remarks

The foregoing semantics yields the propositional language of FDE (first degree entailment) [1, 2]. There are a few notable features of the current semantics.

» There are no valid sentences: Just consider the admissible interpretation according to which every sentence is neither true nor false. (Compare Kleene's 'strong' semantics K_3.)

» Suppose that we restrict the (admissible) interpretations to those interpretations the range of which is $\wp(\{1, 0\}) - \{\{1, 0\}\}$. In that case, we have K_3, a simple 'gappy' semantics that is *not* paraconsistent.

» Suppose that we restrict the (admissible) interpretations to those interpretations the range of which is $\wp(\{1, 0\}) - \{\emptyset\}$. In that case, we have LP, a simple 'glutty' semantics which *is* paraconsistent. As one can easily show, the valid sentences of LP and those of classical logic are precisely the same. (The consequence relation, of course, is different: LP-consequence is weaker, since it is not explosive.)

» Suppose that we restrict the (admissible) interpretations to those interpretations the range of which is $\wp(\{1, 0\}) - \{\{1, 0\}\} \cup \{\emptyset\}$. In that case, we have classical semantics, which admits neither 'gluts' nor 'gaps' and is explosive.

[28] Disjunction $\vee$ and the hook $\supset$ (the 'material conditional') are defined in the usual way.

Quantification

The syntax, as in the propositional case, is that of classical (predicate) logic. Algebraic techniques for extending a many-valued propositional language to a quantified one are available; however, a straightforward, and perhaps more familiar, technique is available in the (non-algebraic) current case.

We let an interpretation be a pair $\langle \mathcal{O}, \delta \rangle$, where $\mathcal{O}$ is a non-empty set of objects (the domain of quantification) and δ a function that does two things:[29]

» δ maps the constants into $\mathcal{O}$

» δ maps every n-ary predicate $\mathcal{P}^n$ into a *pair* $\langle \mathcal{E}_{\mathcal{P}^n}, \mathcal{A}_{\mathcal{P}^n} \rangle$, where $\mathcal{E}_{\mathcal{P}^n} \subseteq \mathcal{O}^n$ and $\mathcal{A}_{\mathcal{P}^n} \subseteq \mathcal{O}^n$

$\mathcal{E}_{\mathcal{P}^n}$ is said to be the *extension* of $\mathcal{P}^n$ and $\mathcal{A}_{\mathcal{P}^n}$ the *anti-extension*. (The extension of $\mathcal{P}^n$, informally, comprises all the objects of which $\mathcal{P}^n$ is at least true, and the anti-extension the objects of which $\mathcal{P}^n$ is at least false.)

Atomic sentences are assigned 'truth values' (elements of $\mathcal{V}$) according to the familiar clauses:

» $1 \in \nu(\mathcal{P}^n c_1, \ldots, c_n)$ iff $\langle \delta(c_1), \ldots, \delta(c_n) \rangle \in \mathcal{E}_{\mathcal{P}^n}$

» $0 \in \nu(\mathcal{P}^n c_1, \ldots, c_n)$ iff $\langle \delta(c_1), \ldots, \delta(c_n) \rangle \in \mathcal{A}_{\mathcal{P}^n}$

Non-quantified compound sentences, in turn, are assigned values as per the propositional case (negation, conjunction, and, derivatively, disjunction, material implication, etc.). The clauses for quantifiers run thus:[30]

» $1 \in \nu(\forall \chi \mathcal{A})$ iff $1 \in \nu(\mathcal{A}(\chi/c))$, for every $c \in \mathcal{O}$

» $0 \in \nu(\forall \chi \mathcal{A})$ iff $0 \in \nu(\mathcal{A}(\chi/c))$, for some $c \in \mathcal{O}$

» $1 \in \nu(\exists \chi \mathcal{A})$ iff $1 \in \nu(\mathcal{A}(\chi/c))$, for some $c \in \mathcal{O}$

» $0 \in \nu(\exists \chi \mathcal{A})$ iff $0 \in \nu(\mathcal{A}(\chi/c))$, for every $c \in \mathcal{O}$

Logical consequence is defined as per usual: 'truth preservation' over all (admissible) interpretations.

Remarks

Not surprisingly, classical semantics (and, similarly, strong Kleene 'gappy' semantics) may be 'regained' by imposing appropriate constraints on the foregoing semantics, and in particular on what counts as an admissible interpretation. Example: By imposing the constraint that $\mathcal{E}_{P^n} \cup \mathcal{A}_{P^n} = \mathcal{O}^n$ and $\mathcal{E}_{P^n} \cap \mathcal{A}_{P^n} = \emptyset$

29 For simplicity, assume that every element of $\mathcal{O}$ has a name, and in particular that elements of $\mathcal{O}$ name themselves and, thus, function as constants.

30 One of the quantifiers is taken to be defined (per usual) but, despite redundancy, clauses for both quantifiers are given here. $\mathcal{A}(\chi/c)$ is $\mathcal{A}$ with every free occurrence of χ replaced by c. (Usual caveats about bondage are in place! And recall that $c \in \mathcal{O}$ serves as a name of itself.)

(for any predicate P^n), one 'regains' classical semantics. As in the propositional case, the upshot is that any classical (first-order) interpretation is a (first-order) FDE-interpretation, and so the former is a (proper) extension of the latter.

The foregoing semantics can be (and have been) augmented to include function symbols, identity, and modal operators (and also extended to second-order). For present purposes, I leave those extensions aside.[31]

8. BUT WHAT OF THE APPARENT LOSS?

Suppose that for purposes of adopting dialetheism we accept LP. We may then enjoy a simple response to the intersection of truth and falsity: it is non-empty, but no explosive traffic ensues.

But what about the apparent loss? We avoid explosion, to be sure; however, we thereby lose Disjunctive Syllogism (DS)—the inference from $\mathcal{A} \vee \mathcal{B}$ and $\neg\mathcal{A}$ to $\mathcal{B}$.[32] But we reason with DS all the time, and it is not clear whether we could do without it. If not, the 'gain' of simple dialetheism is too expensive to bear.

The concern is an important and natural one, one that frequently emerges in early discussion of dialetheism. I will not dwell on the issue here, but it is important to say something on the matter.[33]

In the first instance, the response is (of course) that there is no genuine loss. If dialetheism is true and LP the appropriate logic, then DS was never really truth-preserving. (One cannot lose something that was not there.) Moreover, if (as it appears to me) Liar-like sentences are the only root of the invalidity, it is not surprising that we would think DS to be valid, since Liars are easy to overlook.

There is more to say. In particular, it is not abundantly clear that we really do employ DS in our standard reasoning, as opposed to a closely related 'rule of inference'. The dialetheist, as Priest [31] emphasizes, is free to follow the rationality-version of 'Disjunctive Syllogism':

» If one accepts $\mathcal{A} \vee \mathcal{B}$ and one rejects $\mathcal{A}$, then one ought *rationally* accept $\mathcal{B}$

Provided that acceptance and rejection are exclusive (though they needn't be exhaustive), the 'rationality version' is a principle by which one can regain the

31 See Priest [34, 35] for details (and also a suitable proof theory). LITTMANN AND SIMMONS's chapter raises interesting issues involving descriptions in a dialetheic setting.

32 The reader familiar with 'material modus ponens' will recognize that that 'also' is lost—as it is little more than DS in disguise. Accordingly, a detachable conditional must be added to the language. A variety of conditionals is available. Priest [31] contains discussion, and recent work on 'restricted quantification' by Beall, Brady, Hazen, Priest, and Restall [14] introduces a new option. Because of lack of space, I leave that (admittedly important) topic aside.

33 And, of course, a paraconsistent logic in which DS is preserved but some other 'classic' inference is gone is one for which precisely the same issue arises. There is nothing peculiar about DS, except that its 'loss' is often associated with dialetheism.

reasoning that often passes for (the invalid) DS. If that is right, then the 'loss' of DS seems not to be a great loss, after all.[34]

Finally, it is important to note that a dialetheist has no reason to reject consistency as a default assumption, or as a high theoretical virtue, in general. That some contradictions are true does not imply that most contradictions are true—especially if such true contradictions turn out to be only the peculiar paradoxical sentences. (Even if other sorts of sentences, beyond the paradoxical ones, yield true contradictions, the point still applies.) All that the dialetheist requires is that the default aim of consistency is just that: it is *default*, not absolute.[35]

9. BUT WHAT OF TRUTH?

Beyond the concern about 'losing' DS, there are (regrettably) few other articulated objections against dialetheism. The few standard worries—epistemic, belief revision, and the like—are discussed in Priest's chapter, and I leave them to that essay.[36] I close by mentioning one topic that philosophers tend to worry about when the notion of 'true contradiction' is raised: Truth.[37]

Some philosophers might think that there is something in the 'nature' of truth that rules out the existence of true contradictions. But on reflection, the thought seems not to pan out. Consider, for example, the two main approaches to truth: correspondence and deflationism. (I don't say the *only* two, but the two main contenders.) The latter, as Priest [36], Beall, and Beall and Armour-Garb [4, 13] have argued, seems to yield dialetheism quite naturally. After all, there is no 'nature' to bar the grammatically inexorable true contradictions; there are simple rules of dis-quotation and en-quotation (or simply inter-substitution)—and that's it. Deflationists might well seek to avoid true contradictions, but (again) one wonders why such avoidance is sought—especially when, as it appears, the avoidance-procedures make for a much more complicated position.

34 Shapiro's chapter challenges the current move to some extent, in as much as it challenges the dialetheist's ability to give a coherent notion of *exclusion*. I leave the reader to weigh the merits of Shapiro's arguments against the proposed move. (I should also point out that, as far as I can see, Shapiro's chief objections may not affect a version of dialetheism underwritten by a logic other than LP (or, for that matter, FDE). For one such alternative approach, see Beall [10].)

35 See the appendix of Beall's chapter for brief discussion and references on 'default consistency'.

36 There are other, more technical worries that I will omit here. One such is Curry's paradox, but that depends on which conditional is in play—a topic that I have omitted here. (A dialetheic response to the 'material conditional' version of Curry is precisely the same as the general response to Liars. A detachable conditional, as above, is where the issue arises. See [31] for discussion.) A similar issue concerns so-called Boolean negation. Restall's chapter, as with Brady's, Priest's, and Sainsbury's, touch on that issue.

37 Many of the contributions in this volume presuppose one stance or another on truth, but the chapters by Garfield, Cogburn, and Tennant have direct bearing on the topic, as does Kroon's. Beall's chapter specifically focuses on (one conception of) truth.

More interesting are concerns that arise from correspondence. While there remains no clear account of 'correspondence', the basic idea is clear enough. The idea (not formulated as such by all 'robust theorists', but common enough for present purposes) is that any truth has a truth-maker—that any truth is 'made true' by 'the facts', by some actual 'something' in the world without which a putative truth would fail to correspond and, hence, fail to be a truth. Now suppose, as per dialetheism, that there are truths of the form $\mathcal{A} \wedge \neg\mathcal{A}$. Such a truth would require truth-makers for both $\mathcal{A}$ and $\neg\mathcal{A}$. But how could that be?

The worry, in the end, is not substantial. Whether correspondence is the right approach to truth remains an open (and much debated) question [22]. Suppose, though, that correspondence is the right approach, and that each truth requires a truth-maker. What, exactly, is the worry about having truth-makers for both $\mathcal{A}$ and $\neg\mathcal{A}$? On the surface, no particular problem presents itself, at least not one that is peculiar to dialetheism. To be sure, dialetheism requires that there be 'negative truth-makers', since at least one 'negative truth' is true if both $\mathcal{A}$ and $\neg\mathcal{A}$ are true. But that is a general problem for correspondence, not one peculiar to dialetheism. Moreover, the problem of accommodating 'negative truths' is not particularly difficult; there are standard models available, due to van Fraassen [46], Barwise [5], and others.[38] The worry, as said, seems not to be substantial—at least pending further details.

10. AT THE CROSSROADS: CLOSING REMARKS

Unfortunately, and despite the enormous activity in paraconsistent logic over the last thirty years, there has been little debate centred on non-contradiction—or, at least, little by way of *defense*. Perhaps many have echoed Łukasiewicz in thinking that, while Aristotle's arguments are (at best) insubstantial, Simple (Non-)Contradiction, or perhaps Rationale (Non-)Contradiction, are 'unassailable dogmas' that need only be entrenched, as opposed to defended.[39] Such a thought is philosophically suspect. The incredulous stare was an insufficient 'reply' to modal realism; and it is an insufficient 'reply' to dialetheism.

The hope behind the current volume is that debate may move forward, and that the attitude of unassailable dogma swiftly slides into the past. The intersection is before you; the question is whether it is empty.[40]

[38] Note that van Fraassen's given work was not intended to yield 'negative facts', but it yields a suitable framework for them none the less. For further discussion and details of suitable frameworks, see Beall [8].

[39] What is interesting is that Łukasiewicz's student Jaskowski [27] was an early pioneer of contemporary paraconsistent logic.

[40] I am grateful to Brad Armour-Garb, Mark Colyvan, and Dave Ripley for discussion and comments. Special thanks to Graham Priest and Greg Restall for discussion over the last few years, especially early on in Oz, where this volume was conceived—back in 1999! Thanks, finally, to Katrina Higgins for her support, and also for her patience with this and related projects.

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I

Setting up the Debate

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1

What's So Bad About Contradictions?

Graham Priest

In this chapter[1] I will address the title question; and the answer I shall give is 'maybe nothing much'. Let me first explain how, exactly, the question is to be understood. I shall interpret it to mean 'what is wrong with believing *some* contradictions?' I emphasize the 'some'; the question 'what is wrong with believing *all* contradictions' is quite different, and, I am sure, has a quite different answer. It would be irrational to believe that I am a fried egg. (*Why*, we might argue about, but *that* this is so is not contentious.) A fortiori, it is irrational to believe that I am both a fried egg and not a fried egg. It is important to emphasize this distinction right at the start, since the illicit slide between 'some' and 'all' is endemic in discussions of the question, as we will see.

I think that there is nothing wrong with believing some contradictions. I believe, for example, that it is rational (rationally possible—indeed, rationally obligatory) to believe that the Liar sentence is both true and false. I shall not argue for this directly here, though. I have discovered, in advocating views such as this, that audiences suppose them to be a priori unacceptable. When pressed as to why, they come up with a number of arguments. In what follows, I shall consider five of the most important, and show their lack of substance.

The five objections that we will look at can be summarized as follows:

1. Contradictions entail everything.
2. Contradictions can't be true.
3. Contradictions can't be believed rationally.
4. If contradictions were acceptable, people could never be rationally criticized.
5. If contradictions were acceptable, no one could deny anything.

I am sure that there must be other possible objections; but the above are the most fundamental that I have encountered. I will take them in that order. What I have to say about the first objection is the longest. This is because it lays the basis for all the others.

[1] The chapter is a written version of a lecture that was given at universities in South Africa, Canada, and the United States in 1996 and 1997. I am grateful to many audiences for their lively discussions. It is reprinted with only minor modifications from the *Journal of Philosophy*, 95 (1998), 410–26. I am grateful for permission to reprint.

OBJECTION 1: CONTRADICTIONS ENTAIL EVERYTHING

The first objection is as follows. Rational belief is closed under entailment, but a contradiction entails everything. Hence, if someone believed a contradiction, they ought to believe everything, which is too much.

I certainly agree that believing everything is too much: I have already said that there is an important difference between *some* and *all* here. Still, I take the argument to be unsound. For a start, it is not at all obvious that rational belief is closed under entailment. This seems to be the lesson of the 'paradox of the preface'. You write a (non-fictional) book on some topic—history, karate, cooking. You research it as thoroughly as possible. The evidence for the claims in your book, $\alpha_1, \ldots, \alpha_n$, is as convincing as empirically possible. Hence, you endorse them—rationally. None the less, as you are well aware, there is independent inductive evidence of a very strong kind that virtually all substantial factual books that have been written contain some false claims. Hence, you also believe $\neg(\alpha_1 \wedge \ldots \wedge \alpha_n)$—rationally. However, you do not believe $(\alpha_1 \wedge \ldots \wedge \alpha_n) \wedge \neg(\alpha_1 \wedge \ldots \wedge \alpha_n)$, a simple contradiction, even though this is a logical consequence of your beliefs. Rational belief is not, therefore, closed under logical consequence.

This is all just softening-up, though. The major problem with objection number one is the claim that contradictions entail everything: $\alpha, \neg\alpha \models \beta$, for all α and β. The Latin tag for this is *ex contradictione quodlibet.* I prefer the more colourful: *Explosion.* It is true that Explosion is a valid principle of inference in standard twentieth-century accounts of validity, such as those of intuitionism and the inappropriately called 'classical logic'. But this should be viewed in an historical perspective.

The earliest articulated formal logic was Aristotle's syllogistic. This was not explosive. To see this, merely consider the inference:

Some men are mortals.
No mortals are men.
Hence all men are men.

This is not a valid syllogism, though the premisses are inconsistent. According to Aristotle, some syllogisms with inconsistent premisses are valid, some are not (*An.Pr.* 64^a15). Aristotle had a propositional logic as well as syllogistic. It was never clearly articulated, and what it was is rather unclear. However, for what it is worth, this does not seem to have been explosive either. In particular, a contradiction, $\alpha \wedge \neg\alpha$, does not entail its conjuncts.[2]

The Stoics did have an articulated propositional logic. But whilst one might try to extract Explosion from some of the theses that they endorsed, it is notable that it is not to be found in anything that survives from that period—and one would expect any principle as striking as this to have been made much of by the most

[2] See Priest (1999*a*).

notable critic of Stoicism, Sextus Empiricus. Presumably, then, Explosion was not taken to be correct by the Stoics.

So if Explosion is not to be found in Ancient Logic, where does it come from? The earliest appearance of the principle that I am aware of seems to be in the twelfth-century Paris logician, William of Soissons. At any rate, William was one of a school of logicians called the Parvipontinians, who were well known, not only for living by a small bridge, but also for defending Explosion.[3] After this time, the principle appears to be a contentious one in Medieval logic, accepted by some, such as Scotus; rejected by others, such as the fifteenth-century Cologne School.

The entrenchment of Explosion is, in fact, a relatively modern phenomenon. In the second half of the nineteenth-century, an account of negation—now often called 'Boolean negation'—was championed by Boole, Frege, and others. Boolean negation is explosive, and was incorporated in the first contemporary formal logic. This logic, now usually called classical logic (how inappropriate this name is should now be evident), was so great an improvement on traditional logic that it soon became entrenched. Whether this is because it enshrined the Natural Light of Pure Reason, or because it was the first cab off the rank, I leave the reader to judge.

There is, in fact, nothing sacrosanct about Boolean negation. One can be reminded of this, by the fact that intuitionists, who gave the second contemporary articulated formal logic, provide a different account of negation. Despite this, intuitionist logic is itself explosive. Logics in which Explosion fails have come to be called 'paraconsistent'. The modern construction of formal paraconsistent logics is more recent than anything I have mentioned so far. The idea appears to have occurred to a number of people, in very different countries, and independently, after the Second World War. There are now a number of approaches to paraconsistent logic, all with well-articulated proof-theories and model-theories.

I do not intend to go into details here. I will just give a model-theoretic account of one propositional paraconsistent logic, so that those unfamiliar with the area may have some idea of how things might work.[4] I assume familiarity with the classical propositional calculus. Consider a language with propositional parameters, $p, q, r, \ldots$ and connectives $\wedge$ (conjunction), $\vee$ (disjunction) and $\neg$ (negation). In classical logic, an evaluation is a *function* that assigns each formula one of 1 (true) or 0 (false). Instead of this, we now take an evaluation to be a *relation*, R, between formulas and truth values. Thus, given any formula, α, an evaluation, R, may relate it to just 1, just 0, both, or neither. If $R(\alpha, 1)$, α may be thought of as true under R; if $R(\alpha, 0)$, it may be thought of as false. Hence formulas related to both 1 and 0 are both true and false, and formulas related to neither, are neither true nor false.

[3] For references and more details of the following history of paraconsistency, see part 3 of Priest (2002).

[4] The logic is that of First Degree Entailment. For further details of all the approaches to paraconsistency, see Priest (2002).

As in the classical case, evaluations of propositional parameters are extended to all formulas by recursive conditions. The conditions for $\neg$ and $\wedge$ are as follows. (The conditions for $\vee$ are dual to those for $\wedge$, and may safely be left as an exercise.)

$R(\neg\alpha, 1)$ iff $R(\alpha, 0)$
$R(\neg\alpha, 0)$ iff $R(\alpha, 1)$

$R(\alpha \wedge \beta, 1)$ iff $R(\alpha, 1)$ and $R(\beta, 1)$
$R(\alpha \wedge \beta, 0)$ iff $R(\alpha, 0)$ or $R(\beta, 0)$

Thus, $\neg\alpha$ is true iff α is false, and vice versa. A conjunction is true iff both conjuncts are true; false iff at least one conjunct is false. All very familiar.

To complete the picture we need a definition of logical consequence. This also presents no surprises. An inference is valid iff whenever the premisses are true, so is the conclusion. Thus, if Σ is a set of formulas:

$\Sigma \models \alpha$ iff for all R (if $R(\beta, 1)$ for all $\beta \in \Sigma$, $R(\alpha, 1)$)

It is now easy to see why the logic is paraconsistent. Choose an evaluation, R, that relates p to both 1 and 0, but relates q only to 0. Then it is easy enough to see that both p and $\neg p$ (and $p \wedge \neg p$) are true under R (and false as well, but at least true), whilst q is not. Hence $p, \neg p \not\models q$. For future reference, note that the same evaluation refutes the disjunctive syllogism: $p, \neg p \vee q \vdash q$.

The logic given here should look very familiar. It *is* very familiar. It is exactly the same as classical logic, except that one does not make the assumption, usually packed into textbooks of logic without comment, that truth and falsity in an interpretation are exclusive and exhaustive. The difference between classical logic and the above logic can therefore be depicted very simply. In classical logic, each interpretation partitions the set of formulas (Fig. 1.1). In the paraconsistent logic, an interpretation may partition in this way: classical interpretations are, after all, simply a special case. But in general, the partitioning looks like Fig. 1.2.

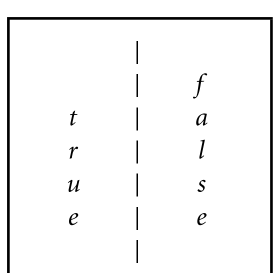

Fig. 1.1.

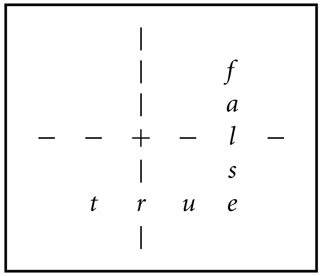

FIG. 1.2.

The crucial question now, is: assuming that all the other assumptions packed into the story are right, should we, or should we not, countenance interpretations that correspond to the second picture? There is no quick way with this question. Each logic encapsulates a substantial metaphysical/semantical *theory*. It should be noted that a paraconsistent logician does not have to hold that truth itself behaves as in the second picture. They have to hold only that in defining validity one has to take into account interpretations that do. And though the claim that truth itself behaves like this is one argument for this conclusion, it is not the only one. If we think of interpretations as representing situations about which we reason, then interpretations of the second kind might be thought to represent 'impossible' situations that are inconsistent or incomplete, such as hypothetical, counterfactual, or fictional situations, or as situations about which we have incomplete or inconsistent information. One may well suppose that there are, in some relevant sense, such situations, and that they play an important metaphysical and/or semantical role.

More boldly, one may suppose that truth itself behaves according to the second picture, and hence that there must be at least one interpretation that does, namely, that interpretation which assigns truth values in accord with the actual. One cannot simply *assume* that it does not. Here, again, lie profound metaphysical issues. Even the founder of Logic, Aristotle, did not think that truth satisfies the first picture. According to him, statements about future contingents, such as the claim that there will be a sea battle tomorrow, are neither true nor false (unless you live in Bolivia).[5] The top left square of Fig. 1.2 is therefore occupied. And modern logic has provided many other possible candidates for this square: statements employing non-denoting terms, statements about undecidable sentences in science or mathematics, category mistakes and other 'nonsense', and so on.

The thought that the bottom right corner might also have denizens is one much less familiar to modern philosophers. Yet there are plausible candidates. Let me give two briefly.[6] The first concerns paradoxes of self-reference. Let us take the

[5] *De Interpretatione*, ch. 9. He seems to think that this is consistent with the Law of Excluded Middle, however. At least, he defends this law in *Metaphysics* Γ.

[6] These and others are discussed at much greater length in Priest (1987).

Liar as an example. The natural and most obvious principle concerning truth is encapsulated in the T-schema: for any sentence, α: $T\langle\alpha\rangle \leftrightarrow \alpha$. I use '$T$' here as a truth predicate, and angle brackets as a name-forming device. With standard self-referential techniques, we can now produce a sentence, β, that says of itself that it is not true: $\neg T\langle\beta\rangle \leftrightarrow \beta$. Substituting β in the T-scheme and juggling a little gives $\beta \wedge \neg\beta$. Prima facie, then, β is a sentence that is both true and false, and so occupies the bottom right corner.

Another example: I walk out of the room; for an instant, I am symmetrically poised, one foot in, one foot out, my centre of gravity lying on the vertical plane containing the centre of gravity of the door. Am I in or not in the room? By symmetry, I am neither in, rather than not in, nor not in, rather than in. The Pure Light of Reason therefore countenances only two answers to the question: I am both in and not in, or neither in nor not in. Thus, we certainly appear to have a denizen of either the top left or the bottom right quarter. But wait a minute. If I am neither in nor not in, then I am not (in) and not (not in). By the law of double negation, I am both in and not in. (And even without it, I am both not in and not not in, which is still a contradiction.) Hence we have a denizen of the bottom right.

There is, of course, much more to be said about both these examples. But I do not intend to say anything further here.[7] The point is simply to illustrate some of the semantic/metaphysical issues that must be hammered out even to decide whether truth itself satisfies the first or the second picture. To suppose that the answer is obvious, or that the issue can be settled by definition is simple dogmatism.

There is a famous defence of classical logic, by Quine, that comes very close to this, in fact. Someone who takes there to be interpretations corresponding to the second picture just 'doesn't know what they are talking about': to change the logic is to 'change the subject'. It is changing the subject only if one assumes *in the first place* that validity is to be defined in terms only of interpretations that satisfy the first picture—which is exactly what is at issue here. Two logicians who subscribe to different accounts of validity are arguing about the same subject, just as much as two physicists who subscribe to different accounts of motion.[8]

[7] Though since the second example is not as familiar as the first, let me add one comment. Let us represent the sentence 'GP is in the room' by α. An obvious move at this point is to suggest that α is, in fact, a denizen of the top left quarter, but that one cannot express this fact by saying that I am neither in nor not in the room. What one has to say is that neither α nor its negation is true, $\neg T\langle\alpha\rangle \wedge \neg T\langle\neg\alpha\rangle$. This is certainly not an explicit contradiction. Unfortunately, it, too, soon gives one. The T-schema for α and $\neg\alpha$ tell us that $T\langle\alpha\rangle \leftrightarrow \alpha$ and $T\langle\neg\alpha\rangle \leftrightarrow \neg\alpha$. Contraposing and chaining together gives: $\neg T\langle\neg\alpha\rangle \leftrightarrow T\langle\alpha\rangle$, and we are back with a contradiction. A natural move here is to deny the T-schema for α or $\neg\alpha$ (presumably these stand or fall together). But on what ground can one reasonably do this? 'GP is in the room' is a perfectly ordinary sentence of English. It is meaningful, and so must have truth conditions. (In fact, most of the time it is simply true or false.) These (or something equivalent to them) are exactly what the T-schema gives. Compare this with the case of the Liar. Many have been tempted to reject the T-schema for the Liar sentence on the ground that the sentence is semantically defective in some way. No such move seems to be even a prima facie possibility in the present case.

[8] For references to Quine, with further discussion, see Priest (2003).

And now, finally, to return to the main point. I have not shown that Explosion fails, that one ought to take into the scope of logic situations that are inconsistent and/or incomplete, though I do take it that when the dust settles, this will be seen to be the case, and that even truth itself requires the second picture.[9] The point of the above discussion is simply to show that the failure of Explosion is a plausible logico-metaphysical one, and that one cannot simply *assume* otherwise without begging the question.

OBJECTION 2: CONTRADICTIONS CAN'T BE TRUE

Let us turn now to objection number two. This is to the effect that contradictions can't be true. Since one ought to believe only what is true, contradictions ought not to be believed.

This argument appeals to the Law of Non-Contradiction (LNC): nothing is both true and false. The first thing we need to do is distinguish clearly between the LNC and Explosion. They are very different. For a start, as we have seen, Explosion is a relative newcomer on the logical scene. The LNC is not. It is true that some have challenged it: some Presocratics, such as Heraclitus; some Neoplatonists, such as Cusanus; and some dialecticians, such as Hegel. But since the time of Aristotle, it is a principle that has been very firmly entrenched in Western philosophy. (Its place in Eastern philosophy is much less secure.) The view that the LNC fails, that some contradictions are true, is called *dialetheism*. As we have already seen, one does not have to be a dialetheist to subscribe to the correctness of a paraconsistent logic, though if one is, one will. As we also saw, though, there are arguments that push us towards accepting dialetheism. Is there any reason why one should reject these a priori? Why, in other words, should we accept the LNC?

The *locus classicus* of its defence is Aristotle's *Metaphysics*, Γ4. It is a striking fact about the Law that there has not been a sustained defence of it *since* Aristotle (at least, that I am aware of). Were his arguments so good that they settled the matter? Hardly. There are about seven or eight arguments in the chapter (it depends how you count). The first occupies half the chapter. It is long, convoluted, and tortured. It is not at all clear *how* it is supposed to work, let alone *that* it works. The other arguments in the chapter are short, often little more than throw-away remarks, and are at best, dubious. Indeed, most of them are clearly aimed at attacking the view that *all* contradictions are true (or even that someone can *believe* that all contradictions are true). Aristotle, in fact, slides back and forth between 'all' and 'some', with gay abandon. His defence of the LNC is therefore of little help.[10]

[9] Though, as a matter of fact, I think that its top left quarter is empty. See Priest (1987), ch. 4.

[10] For a detailed analysis of Aristotle's arguments, see Priest (1998).

So what other arguments are there for the LNC? Very few that I am aware of, and none that survive much thought. Let me mention four here. The first two, some have claimed, are to be found in Aristotle. I doubt it, but let us not go into this here.

According to the first argument, contradictions have no content, no meaning. If so, then, a fortiori, they have no true content: contradictions cannot be true. The first thing to note about this objection is that it is not only an objection against dialetheism, but also against classical logic. For in classical logic, contradictions have *total* content, they entail everything. One who subscribes to orthodox logic cannot, therefore, wield this objection.

There have been some who endorsed different propositional logics, according to which contradictions do entail nothing, and so have no content.[11] But the claim that contradictions have no content does not stand up to independent inspection. If contradictions had no content, there would be nothing to disagree with when someone uttered one, which there (usually) is. Contradictions do, after all, have meaning. If they did not, we could not even understand someone who asserted a contradiction, and so evaluate what they say as false (or maybe true). We might not understand what could have brought a person to assert such a thing, but that is a different matter—and the same is equally true of someone who, in broad daylight, asserts the clearly meaningful 'It is night.'

A second objection (to be found e.g. in McTaggart) is to the effect that if contradictions could be true, *nothing* could be meaningful. The argument here appeals to the thought that something is meaningful only if it *excludes* something (*omnis determino est negatio*): a claim that rules out nothing, says nothing. Moreover, it continues, if α does not rule out $\neg\alpha$, it rules out nothing. An obvious failing with this argument is, again, the slide from 'some' to 'all'. Violation of the LNC requires only that some statements do not rule out their negations (whatever that is supposed to mean). The argument depends on the claim that *nothing* rules out its own negation.

But there is a much more fundamental flaw in the argument than this. The premiss that a proposition is not meaningful unless it rules something out is just plain false. Merely consider the claim 'Everything is true.' This rules nothing out: it entails everything. Yet it is quite meaningful (it is, after all, false). If you are in any doubt over this, merely consider its negation 'Something is not true.' This is clearly true—and so meaningful. And how could a meaningful sentence have a meaningless negation?

A third argument for the LNC, and one that is typical of many, starts from the claim that the correct truth conditions for negation are as follows:

$\neg\alpha$ is true iff α is not true.

[11] See Priest (1999*a*).

Now suppose that $\alpha \wedge \neg\alpha$ is true. Then assuming that conjunction behaves normally, α is true, and $\neg\alpha$ is true. Hence by the truth conditions of negation, α is both true and not true, which is impossible.

It is not difficult to see what is wrong with this argument. For a start, the truth conditions of negation are contentious. (Compare them with those given in the previous section.) More importantly, why should one suppose that it is impossible for α to be both true and not true? Because it is a contradiction. But it is precisely the impossibility of having true contradictions that we were supposed to be arguing for. The argument, therefore, begs the question, as do many of the other arguments that I am aware of.[12]

The fourth, and final, argument I shall mention is an inductive one. As we review the kinds of situations that we witness, very few of them would seem to be contradictory. Socrates is never both seated and not seated; Brisbane is firmly in Australia, and not not in it. Hence, by induction, no contradictions are true. Note that one does not have to suppose that logical principles are a posteriori for this form of argument to work. One can collect a-posteriori evidence even for a priori principles. For example, one verifies $\alpha \vee \neg\alpha$ every time one verifies α.

The flaws of this argument are apparent enough, though. It is all too clear that the argument may be based on what Wittgenstein called 'an inadequate diet of examples'. Maybe Socrates *is* both sitting and not sitting sometimes: at the instant he rises. This, being instantaneous, is not something we observe. We can tell it to be so only by a-priori analysis. Worse, counter-examples to the principle are staring us in the face. Think, for example, of the Liar. Most would set an example such as this aside, and suppose there to be something wrong with it. But this may be short-sighted. Consider the Euclidean principle that the whole must be larger than its parts. This principle seemed to be obvious to many people for a long time. Apparent counter-examples were known from late Antiquity: for example, the set of even numbers appeared to be the same size as the set of all numbers. But these examples were set aside, and just taken to show the incoherence of the notion of infinity. With the nineteenth century all this changed. There is nothing incoherent about this behaviour at all: it is paradigmatic of infinite collections. The Euclidean principle holds only for finite collections; and people's acceptance of it was due to a poor induction from unrepresentative cases. In the same way, once one gets rid of the idea, in the form of Explosion, that inconsistency is incoherent, the Liar and similar examples can be seen as paradigm citizens of a realm to which our eyes are newly opened (we can call it, by analogy with set-theory, the transconsistent). In any case, the inductive argument to the LNC is simply a poor one.

It is sometimes said that dialetheism is a position based on sand. In fact, I think, it is quite the opposite: it is the LNC that is based on sand. It appears to have no

[12] In particular, one may argue for the LNC from Explosion, assuming that not all contradictions are true. But an appeal to Explosion would beg the question, as we have already seen.

rational basis; and the historical adherence to it is simply dogma. Hence—and finally to return to the second objection—it fails.

OBJECTION 3: CONTRADICTIONS CAN'T BE BELIEVED RATIONALLY

The third objection is that even if contradictions could be true, they can't be believed rationally, consistency being a constraint on rationality; hence one ought not to believe a contradiction since this would be irrational.

We have already seen, in answer to the first objection, that this objection fails. The paradox of the preface shows that it can be quite rational to have inconsistent beliefs. Hence, consistency is not an absolute constraint on rationality. The rational person apportions their beliefs according to the evidence; and if the evidence is for inconsistent propositions, so be it.

There is, of course, more to the story than this. To approach it, let me take what will appear to be a digression for a moment. Have you ever talked to a flat-earther, or someone with really bizarre religious beliefs—not one who subscribes to such a view in a thoughtless way, but someone who has considered the issue very carefully? If you have, then you will know that it is virtually impossible to show their view to be wrong by finding a knock-down objection. If one points out to the flat-earther that we have sailed round the earth, they will say that one has, in fact, only traversed a circle on a flat surface. If one points out that we have been into space and seen the earth to be round, they will reply that it only *appears* round, and that light, up there, does not move in straight lines, or that the whole space-flight story is a CIA put-up, etc. In a word, their views are perfectly consistent. This does not stop them being irrational, however. How to diagnose their irrationality is a nice point, but I think that one may put it down to a constant invoking of ad hoc hypotheses. Whenever one thinks one has a flat-earther in a corner, new claims are pulled in, apparently from nowhere, just to get them out of trouble.

What this illustrates is that there are criteria for rationality other than consistency, and that some of these are even more powerful than consistency. The point is, in fact, a familiar one from the philosophy of science. There are many features of belief that are rational virtues, such as simplicity, problem-solving ability, non-adhocness, fruitfulness, and, let us grant, consistency. However, these criteria are all independent, and may even be orthogonal, pulling in opposite directions. Now what should one do if, for a certain belief, all the criteria pull towards acceptance, except consistency—which pulls the other way? It may be silly to be a democrat about this, and simply count the number of criteria on each side; but it seems natural to suppose that the combined force of the other criteria may trump inconsistency. In such a case, then, it is rational to have an inconsistent belief.

The situation I have outlined is an abstract one; but it seems to me that it, or something like it, already obtains with respect to theories of truth. Since the abstract point is already sufficient answer to the objection we are dealing with, I do not want to defend the example in detail here; still, it will serve to put some flesh on the abstract bones. The following is a simple account of truth. Truth is a principle that is characterized formally by the T-schema: for every sentence, α, $T\langle\alpha\rangle \leftrightarrow \alpha$ (for a suitable conditional connective). And that's an end on't. (There may be more to be said about truth, but nothing that can be captured in a formalism.) This account is inconsistent: when suitable self-referential machinery is present, say in the form of arithmetic, the Liar paradox is forthcoming. Yet the inconsistencies are isolated. In particular, it can be shown that, when things are suitably set up, inconsistencies do not percolate into the purely arithmetic machinery. In fact, it can be shown that any sentence that is grounded (in Kripke's sense) behaves consistently.[13] What are the alternatives to such an account? There is a welter of them: Tarski's, Kripke's, Gupta and Herzberger's, Barwise and Perry's, McGee's, etc., etc. These may all have the virtue of consistency, but the other virtues are thinly distributed amongst them. They often have strong ad hoc elements; they are complex, usually involving transfinite hierarchies; they have a tendency to pose just as many problems as they solve; and it is not clear that, in the last instance, they really solve the problem they are supposed to: they all seem subject to extended paradoxes of some kind.[14] It seems to me that rationality speaks very strongly in favour of the simple inconsistent theory. This is exactly a concrete case of the abstract kind I have described.

Naturally, it may happen that someone, a hundred years hence, will come up with a consistent account of truth with none of these problems, in which case, what it is rational to believe may well change. But that is neither here nor there. Rational belief about anything is a fallible matter. It is a mistake to believe where the evidence does not point; but it is equally a mistake not to believe where the evidence points.

I have argued that it may well be rational to believe a contradiction, and shown how this may arise. If there is sufficient evidence that something is true, one ought, rationally, to accept it. Let me consider just one reply. It is natural to suppose that there is a dual principle here: if there is sufficient evidence that something is false, one ought, rationally, to reject it. If, therefore, there is strong evidence that contradictories, α and $\neg\alpha$, are both true, there is evidence that both are also false. One ought, then, to reject both.

No. In the appropriate sense, truth trumps falsity. Truth is, by its nature, the aim of cognitive processes such as belief. (This is the 'more' to truth that I referred to above.) It is constitutive of truth that that is what one ought to accept. Falsity, by contrast, is merely truth of negation. It has no independent epistemological

[13] For a proof of this, see Priest (2002), s. 8.

[14] See Priest (1987), ch. 1.