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General Systems Theory: Mathematical Foundations

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GENERAL SYSTEMS THEORY: MATHEMATICAL FOUNDATIONS

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To

Gordana and Mitsue

with affection and gratitude

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PREFACE

This book reports on the development of a mathematical theory of general systems, initiated ten years ago. The theory is based on a broad and ambitious program aimed at formalizing all major systems concepts and the development of an axiomatic and general theory of systems. The present book provides the foundations for, and the initial steps toward, the fulfillment of that program. The interest in the present volume is strictly in the mathematical aspects of the theory. Applications and philosophical implications will be considered elsewhere.

The basic characteristics and the role of the proposed general systems theory are discussed in some detail in the first chapter. However, the unifying power of the proposed foundations ought to be specifically singled out; within the same framework, using essentially the same mathematical structure for the specification of a system, such diverse topics are considered and associated results proven as: the existence and minimal axioms for state-space construction; necessary and sufficient conditions for controllability of multivalued systems; minimal realization from input–output data; necessary and sufficient conditions for Lyapunov stability of dynamical systems; Goedel consistency and completeness theorem; feedback decoupling of multivariable systems; Krohn–Rhodes decomposition theorem; classification of systems using category theory.

A system can be described either as a transformation of inputs (stimuli) into outputs (responses)—the so-called input–output approach (also referred to as the causal or terminal systems approach), or in reference to the fulfillment of a purpose or the pursuit of a goal—the so-called goal-seeking or decision-making approach. In this book we deal only with the input–output approach. Originally, we intended to include a general mathematical theory of goal-seeking, but too many other tasks and duties have prevented us from carrying out that intention. In fairness to our research already completed, we ought to point out that the theory of multilevel systems which has been reported elsewhere† although aimed in a different direction, does contain the elements of a general theory of complex goal-seeking systems. For the sake of completeness we have given the basic definition of a goal-seeking system and of an open system (another topic of major concern) in Appendix II.

We have discussed the material over the years with many colleagues and students. In particular the advice and help of Donald Macko and Seiji Yoshii were most constructive. The manuscript would have remained a scribble of notes on a pile of paper if it were not for tireless and almost nondenumerable series of drafts retyped by Mrs. Mary Lou Cantini.

† M. D. Mesarovic, D. Macko, and Y. Takahara, "Theory of Hierarchical Multilevel Systems." Academic Press, New York, 1970.

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