



# INTEGER PROGRAMMING

Volume 76

Harold Greenberg

# INTEGER PROGRAMMING

This is Volume 76 in  
MATHEMATICS IN SCIENCE AND ENGINEERING  
A series of monographs and textbooks  
Edited by RICHARD BELLMAN, *University of Southern California*  
A complete list of the books in this series appears at the end of this volume.

# INTEGER PROGRAMMING

---

**HAROLD GREENBERG**

*Naval Postgraduate School  
Monterey, California*

---

---

**ACADEMIC PRESS**



*New York · London*

COPYRIGHT © 1971, BY ACADEMIC PRESS, INC.

ALL RIGHTS RESERVED

NO PART OF THIS BOOK MAY BE REPRODUCED IN ANY FORM,  
BY PHOTOSTAT, MICROFILM, RETRIEVAL SYSTEM, OR ANY  
OTHER MEANS, WITHOUT WRITTEN PERMISSION FROM  
THE PUBLISHERS.

ACADEMIC PRESS, INC.

111 Fifth Avenue, New York, New York 10003

*United Kingdom Edition published by*  
ACADEMIC PRESS, INC. (LONDON) LTD.  
Berkeley Square House, London W1X 6BA

LIBRARY OF CONGRESS CATALOG CARD NUMBER: 73-137596

AMS (MOS) 1970 Subject Classification: 90C10

PRINTED IN THE UNITED STATES OF AMERICA

*For Eleanor*

This page intentionally left blank

# Contents

## **PREFACE**

xi

## **Chapter 1 Introduction to Integer Programming**

1 Presentation of the Problem	1
2 Pilot Scheduling	2
3 A Quadratic Assignment Problem	4
4 The Knapsack Problem	5
5 The Traveling Salesman Problem	6
6 The Fixed-Charge Problem	8
7 Nonlinear Approximation	9
8 Dichotomies	10

## **Chapter 2 Linear Programming**

1 The General Linear Program	13
2 Recognition of Optimality	14
3 The Simplex Method	21
4 Tableau Form	27
5 The Inverse Matrix Method	28
6 Variables with Upper Bounds	33
7 The Lexicographic Dual Simplex Method	38



**Chapter 3 All-Integer Methods**

1 Optimality Theory for Integer Programming	48
2 Improving a Nonoptimal Solution	49
3 Equivalent Integer Programs	55
4 Convergence to Optimality	59
5 Bounded Variable Problems	62
6 Negative $c_j$ Values	66
7 The Use of Bounding Forms	67
8 A Primal Integer Method	72

**Chapter 4 Solving Integer Programs by Enumeration**

1 A Direct Enumeration Method	81
2 Solution to the Integer Program	86
3 An Accelerated Enumeration	91
4 A Dynamic Programming Method	95
5 Knapsack Functions	97

**Chapter 5 Continuous Solution Methods**

1 A Continuous Solution Method	104
2 Improving a Nonoptimal Solution	106
3 Convergence in the Algorithm	111
4 Reducing the Continuous Solution to an All-Integer Format	115
5 Bounding the Determinant Value	121
6 Bounded Variable Problems	123
7 The Mixed Integer Problem	128

**Chapter 6 Number Theory Results**

1 The Euclidean Algorithm	135
2 Linear Diophantine Equations	141
3 Linear Congruences	145
4 The Solution of a System of Linear Congruences	151

**Chapter 7 Dynamic Programming Solutions**

1 A Dynamic Programming Solution	156
2 Reducing the Number of Congruences	162
3 An Accelerated Dynamic Programming Solution	169

**Chapter 8 Branch and Bound Procedures**

1 A Branch and Bound Method	177
2 Tightening the Bounds	187
3 The Mixed Integer Problem	188

AUTHOR INDEX	193
SUBJECT INDEX	194

This page intentionally left blank