Extreme Value Theory in Engineering

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To whoever does not suffer from uncertainty

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Preface

Before initiating the task of writing a book, one needs an encouraging motivation. After several years of work with engineers in the area of extremes, several Ph. D. Theses of my students on this topic and the contact, personal or through their many publications, with mathematicians and statisticians, I discovered the lack of close communication and collaboration between practitioners and theorists. This fact is the main cause, on the one hand, for engineers to be unaware of the very many recent advances in the area of extremes, and, on the other hand, for the mathematicians and statisticians to find practical applications and motivations for their theoretical work. The exponentially increasing number of publications on any subject makes it impossible to become a specialist in a wide area of knowledge. Thus, the need for collaboration and joint work. All this has increased my motivation for writing the present book and clearly defined its goal. However, to be a bridge between engineers and mathematicians or statisticians is not an easy task, as experience demonstrates; their daily languages and main concerns are far enough apart to make it difficult. In addition, everything must be done at the risk of being criticized from both sides. In this context, the present book will be only a grain of sand in a beach still to be built.

In order to make it more readable for engineers and to avoid unnecessary repetition, most of the proofs have been intentionally omitted, and only new or very simple ones are included. With the aim of facilitating the understanding of the definitions and theorems, many clarifying examples have been inserted. A collection of 14 sets of data has also been included and many techniques described throughout the book have been illustrated by their direct application to them. Some computer codes, which appear in appendix B, were prepared with the aim of facilitating the use of the above mentioned techniques to daily practice problems. The language Basic was selected to make this possibility of a wider range. Fortran or Pascal users will not find any trouble in making a translation of codes.

The book can be used as a textbook or as a consulting book. In the first case the examples and computer codes will facilitate the work of students, individually or in groups, by its application to fictitious or real problems taken from imagination or daily practice. The extensive bibliography, included in the book, can also serve as the basis for some student additional work.

The book covers a wide range of the available material to date. Engineers can find useful results on extremes and other order statistics and motivating examples and ideas. Mathematicians and statisticians can find some motivating practical ideas and examples to illustrate their theoretical aspects.

Some theoretical parts of the book are completely original, as is chapter 9 and parts of chapters 3, 4 and 6. The rest is a recapitulation of existing material with added personal views of the author. However, throughout the book, a very large percentage of the included examples are original and many of them refer to engineering problems.

One of the main contribution of the book is the emphasis on the tail behaviour as the basis for the analysis. This is indicated mainly in chapters 3, 4, 5 and 6, and leads to a change in the recommended techniques of analysis, as probability paper, estimation of parameters or selection of domains of attraction techniques, for example.

I want to publicly thank Professor J. Galambos for his extremely valuable help. He made possible my sabbatical period at Temple University, during which a great deal of the work included in this book was completed. He also contributed the section on characterization of distributions and many suggestions and corrections to this book. This was initially planned to be a joint book, but previous engagements with the editor of the second edition of his book on extreme order statistics made it impossible.

I want also to thank the US-Spain Joint Committee for Research and the University of Santander, who financed my sabbatical leave, and the Computer Center and the Department of Mathematics of Temple University, who provided me the use of computers and the library and gave me room space.

Thanks also to my family for being generous in allowing me to dedicate part of its time to make the book possible, to my wife for her collaboration in

preparing the bibliography and index, to Alberto Luceño for some valuable comments on the manuscript, to Agustin Manrigue for drawing all figures, and to Deborah Beaumont for her careful proofreading of the manuscript.

Finally, I want to mention the scientific community, mainly those included in the Bibliography and those who were, surely unintentionally, omitted. They, through their life work, made this book a possibility. To all of them my sincere thanks.

I Introduction

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Chapter 1 Introduction and Motivation

1.1. Introduction

Experienced engineers know how successful design depends on the adequate selection of the method of analysis. The modeling or idealization of the problem under consideration (water supply system, structure, road, harbour, dam, etc.) should be sufficiently simple, logically irrefutable, admit of mathematical solution, and, at the same time, reproduce sufficiently well the actual problem. As in any other branch of knowledge, the selection of the idealized model should be achieved by detecting and reproducing the essential first-order factors, and discarding or neglecting the inessential second-order factors. The new elements under design are represented by the simplest geometrical, structural, social, etc. elements and the relationships between them are idealized.

In many situations, the final design is a compromise between the capacity of the element (strength, service capacity, production capacity, etc.) and the actual operating conditions (actual loads, service, traffic, demand, etc.). Classical methods of analysis lie quite far from the operating conditions of the actual elements. The most important reason for this is that the properties of the elements and external conditions are assumed to be known with complete certainty. However, all these properties are under the influence of such amount of incontrollable external factors that they become random variables. Two extreme results of this classical design are those related to under- or over-design which lead to failure of the designed element or to a large waste of resources.

The aim of the engineering design of an element is to guarantee that none of the catastrophic limiting states occur during its lifetime. To be more specific, the element shall satisfy the following requirement

$$O_t \leq C_t \qquad \text{for any } t \tag{1.1}$$

where O_t and C_t are the actual operating conditions (loads, demands, traffic, etc.) acting on the element and the real capacity of the element at time t, respectively.

Both components are clearly random variables whose distribution functions may be established only by systematically analyzing the available history for similar elements and conditions. Fortunately, since operating conditions, O_t, and capacities, C_t, are statistically independent random variables, their distributions can be examined separately. This avoids a great deal of difficulty.

In some cases, condition (1.1) is replaced by

$$O_{\max} \leq C_{\min} \tag{1.2}$$

where $O_{\max}$ and $C_{\min}$ are some upper and lower bounds, respectively, that can under no condition be exceeded. However, conditions (1.1) and (1.2) are not equivalent.

The random nature of the fundamental design parameters is taken into account by the so-called safety factor. This factor summarizes, in some simple way, the random character of design parameters. Consequently, capacities are lessened (diminished) and operating conditions strengthened (increased). This means that under classical design techniques expression (1.1) is replaced by

$$S_o O \leq \frac{C}{S_c} \tag{1.3}$$

where S_o and S_c are two safety factors or coefficients whose values are larger than unity and O and C are values given by design codes.

However, the safety factor technique is so simple that it becomes completely insufficient for the analysis of many problems whose solution is possible only on the basis of Probability Theory and Mathematical Statistical methods.

In order to achieve satisfactory behaviour of the element, condition (1.1) must hold for the entire lifetime of the element and because many values of O_t and C_t occur, we must have

$$\mathrm{Max}(O_t - C_t) \leq 0 \tag{1.4}$$