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Marine Turbulence

PROCEEDINGS OF THE 11th INTERNATIONAL LIÈGE COLLOQUIUM
ON OCEAN HYDRODYNAMICS

Edited by

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FOREWORD

The concepts of mixing of water masses, dispersion of anomalies, transport of momentum and vorticity and the dissipation of kinetic energy by turbulent motions in the ocean, play a central role in almost every aspect of Oceanography. The action of turbulence appears not only in the circulation models of physical oceanographers but equally in the models of chemical distributions, of biological production and of sedimentation.

The interaction between atmosphere and ocean - with its important consequences for climate variation - can only be modelled by taking into account the transport of heat, salt and momentum by turbulent motions in the wind-mixed surface layer and the underlying thermocline.

The major advances that have been made in the past decade have revolutionized our thinking about turbulence in the ocean. The bulk effects of turbulence that were known fifty years ago can now for the first time be discussed in terms of models of the responsible flow patterns based on an understanding of the underlying dynamics and physics.

The IAPSO General Assembly in Grenoble (1976) accepted a proposal submitted by the British National Committee to sponsor a second Symposium on Turbulence in the Ocean, at which the emphasis would be on seeking a synthesis across the whole spectral band from millimetres to megametres. This proposal was subsequently discussed by SCOR which agreed to co-sponsor the meeting and by the IOC which offered support. A number of member states recommended that the Symposium should be recognised as a IDOE meeting. It was therefore designated the IAPSO-SCOR-IDOE Second Symposium on "Turbulence in the Ocean", with support from the IOC.

It was decided to make the first week an open meeting, during which both invited and contributed papers could be presented and discussed in open sessions to be attended by all comers. The plan was to follow this by a closed session during the second week. In order to accomodate the idea of an open first week, it was agreed that this would be combined with the annual Colloquium on Ocean Hydrodynamics held in Liège University.*

*J.D. Woods : Report to the IOC.

The international Liège Colloquia on Ocean Hydrodynamics are organized annually. Their topics differ from one year to another and try to address, as much as possible, recent problems and incentive new subjects in physical oceanography.

Assembling a group of active and eminent scientists from different countries and often different disciplines, they provide a forum for discussion and foster a mutually beneficial exchange of information opening on to a survey of major recent discoveries, essential mechanisms, impelling question-marks and valuable suggestions for future research.

The papers presented at the Eleventh International Liège Colloquium on Ocean Hydrodynamics report theoretical and experimental research and they address such different scales of motions as synoptic eddies, fronts, mesoscale blinis, and three-dimensional microscale fluctuations. Their unity resides in a common approach to the variability of the seas, based on the profound understanding of non-linear processes which the theory of turbulence provides.

Jacques C.J. NIHOUL.

The Scientific Organizing Committee of the Eleventh International Liège Colloquium on Ocean Hydrodynamics and all the participants wish to express their gratitude to the Belgian Minister of Education, the National Science Foundation of Belgium, The University of Liège, the Intergovernmental Oceanographic Commission and the Division of Marine Sciences (UNESCO) and the Office of Naval Research for their most valuable support.

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THE TURBULENT OCEAN

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ABSTRACT

The variability of the ocean over a wide range of scales, from the megameter to the millimeter, is examined in the light of turbulence theory.

The geophysical constraints which arise from the Earth's rotation and curvature and from the stratification are discussed with emphasis on the role they can play at different scales in inducing instabilities and a transfer of energy to other scales of motion.

INTRODUCTION

Turbulence in the ocean is still a very controversial subject. An innocent physicist, emboldened by a solid background in the theory of turbulence, who would approach the subject relying on his good understanding of, say, turbulent channel flow, might soon find himself confronted with a maze of conflicting experimental data and a nightmarish farrago of theories where he recognizes very little of what he regards as "turbulence".

Assuming that he is able to muster enough of his high school latin and greek to find his way among iso-halines, iso-pycnals and other proliferating peculiar surfaces which seem to fill the ocean with complexity, he may still require some time to adjust to new cabalistic concepts such as enstrophy, red energy cascade, eddies "which do not overturn", double diffusion or fossil turbulence.

Used to regard a turbulent flow as the superposition of a mean motion and turbulent fluctuations, he must face the fact that, if such things exist in the ocean, they are several thousands orders of magnitude apart in scales and apparently separated by a jungle of complicated movements which one refers to as the "variability of the ocean".

One of the most intriguing aspect of ocean variability is that, while the macroscale dynamics is claimed to be governed by a cascade of enstrophy to smaller scales, studies of mesoscale and microscale variability seem totally unconcerned with it as if it had disappeared somewhere on the way. (Planetary oceanographers talk about enstrophy "dissipation" but surely they mean something else like "annihilation"; even if one of them, answering a question at the Eleventh Liège Colloquium on Ocean Hydrodynamics, expressed the somewhat surprising view that "dissipated" enstrophy turns into heat).

For somebody trained in classical turbulence theory, it is not immediately obvious that the variability of the ocean is a form of turbulence.

If it is turbulence, it is clearly very different from the type of turbulence with which mechanical engineers, say, are familiar.

The reason why it should be so is found in the work of those oceanographers who, taking a quite opposite view, insist on describing ocean hydrodynamics in terms of non-turbulent theories such as linear wave propagation or molecular effects.

That they succeed in explaining some of the observations - after removing turbulence from the experimental signals in an operation which they gallantly call "decontamination" - is an indication of the mechanisms which are particular to geophysical flows and which are liable to modify, more or less drastically, geophysical turbulence.

The Coriolis force and the density stratification - which allow wave motions that do not exist in non-rotating non-stratified fluids -, the variations of density not only with temperature but also with salinity, - with different molecular diffusivities for momentum, heat and salt -, and their interference in other processes related to bottom or coastal topography and air-sea interactions, provide a great variety of mechanisms which eventually combine with mechanical effects to determine the stability or the instability of oceanic motions and the subsequent constraints on turbulence at different scales.

Including such geophysical constraints, it becomes possible to understand some essential characteristics of ocean variability and to build an image of what turbulence in the ocean may be.

TIME SCALES, LENGTH SCALES AND LINEAR WAVE THEORY

The equations of Geophysical Fluid Dynamics admit linear wave solutions of different kinds (e.g. Monin et al, 1977). Whether these waves can be observed in the ocean depends on a series of factors. Very small amplitude waves will not be affected by non-linear interactions but, on the other hand, they may be masked by stronger motions and remain unnoticed. Interactions between larger amplitude waves may create an intricated field of waves and wave packets of all scales, wave breaking and turbulence, from which the ideal individual wave of linear theory cannot be sorted out.

The theory of linear waves is however always a useful mathematical exercise as it helps to identify the dominant length scales (wave numbers) and time scales (frequencies) of motions.

From this point of view, it is convenient to divide ocean waves into three categories (Nihoul, 1979) :

A. Macroscale waves

These waves have frequencies (ω) in the range

$$10^{-8}s^{-1} \lesssim \omega \lesssim 10^{-5}s^{-1} \tag{1}$$

and operate over a range of horizontal wave-numbers (k)

$$10^{-6}m^{-1} \lesssim k \lesssim 10^{-3}m^{-1} \tag{2}$$

(Waves of larger periods could be considered but the limiting frequency 10^{-8} corresponding to periods larger than 10 years, they are presumably rather irrelevant to the present discussion). Macroscale waves are directly related to the spatial variations of the Coriolis parameter f ($f = 2\,\Omega\,\sin\phi$ where Ω is the angular velocity of the earth's rotation and ϕ the latitude) i.e. to the parameter

$$\beta = \|\underset{\sim}{\nabla} f\| \sim 10^{-11}m^{-1}s^{-1} \tag{3}$$

Macroscale waves include (e.g. Rhines, 1977 ; Nihoul, 1979)

(i) very slow baroclinic waves ($\omega k << \beta$) for which

$$\omega \sim \frac{N^2\beta H^2}{f^2}\,k \tag{4}$$

where H is the depth of the ocean and N the Brunt-Väisälä frequency.

(The condition $\omega k << \beta$ yields $L \sim k^{-1} >> R$ where $R = NHf^{-1} \sim 10^5$ m is the so-called "Rossby internal scale". These waves are thus very slow large scale small amplitude waves which, if excited, are likely to break rapidly under the effect of bottom slope and general baroclinic instability).

(ii) barotropic Rossby waves ($\omega k \sim \beta$) for which

$$\omega \sim \beta k^{-1} \sim f \gamma H^{-1} k^{-1} \tag{5}$$

where γ is the non-dimensional mean bottom slope chosen here of the order $\gamma \sim 10^{-4}$ such that $f \gamma H^{-1} \sim \beta \sim 10^{-11}$.

(In a forced problem, with wind blowing accross the ocean surface, these waves constitute the most important mode at "weatherlike" time scales over horizontal scales greater than R).

(iii) fast baroclinic waves confined within a layer of thickness $f L N^{-1}$ above the sloping bottom, for which

$$\omega \sim \gamma N \tag{6}$$

(If, in the dispersion relation of topographic Rossby waves $\omega \sim f \gamma H^{-1} k^{-1}$, one replaces the depth H by the "penetration height" $f L N^{-1}$, one obtains the dispersion relation (6). These waves may thus be regarded as topographic Rossby waves where density stratification provides a lid for vortex stretching).

B. Mesoscale waves

These waves have frequencies in the range

$$10^{-5} s^{-1} \lesssim \omega \lesssim 10^{-2} s^{-1} \tag{7}$$

and include gyroscopic waves and inertial oscillations, tides, and internal gravity waves (e.g. Tolstoy, 1963 ; Monin et al, 1977).

The effect of the Earth's curvature becomes here negligible and the essential factors in the dispersion relations are the Coriolis parameter f and the Brunt-Väisälä frequencies N_{min} and N_{Max} .

C. Microscale waves

These waves have frequencies

$$10^{-2} s^{-1} \lesssim \omega \tag{8}$$

They are essentially surface waves and acoustic waves. The former only affect the upper layer of the ocean and may be regarded as an indispensable - unfortunately rather complicated - way of transferring momentum and energy directly from the wind to the sea ; the latter are marginally important in Ocean Hydrodynamics from which they are customarily excluded by the Boussinesq approximation.

MACROSCALE TURBULENCE

Macroscale motions in the ocean include large scale currents (gyres) and quasi-geostrophic or "synoptic" eddies which appear, from observational studies, to contain a large fraction of the ocean's kinetic energy.

The dynamics of the synoptic eddies is dominated by the earth's curvature - parameterized in terms of β - and their horizontal length scale is of the order of the Rossby internal scale R.

The spectral characteristics of the synoptic eddies, wave number κ_β, frequency ω_β and energy level $\kappa_\beta E(\kappa_\beta)$ where $E(\kappa)$ is the horizontal kinetic energy spectral density, can be estimated by turbulence similarity arguments. One finds

$$\kappa_\beta \sim R^{-1} \sim 10^{-5} m^{-1} \tag{9}$$

$$\omega_\beta \sim \beta \kappa_\beta^{-1} \sim 10^{-6} s^{-1} \tag{10}$$

$$\kappa_\beta E(\kappa_\beta) \sim \beta^2 \kappa_\beta^{-4} \sim 10^{-2} m^2 s^{-2} \tag{11}$$

These estimates appear to be in good agreement with the observations (e.g. Koshlyakov and Monin, 1978).

The use of the term "synoptic" emphasizes the physical analogy between these eddies and the synoptic eddies of the atmosphere (cyclones and anticyclones, quasi-geostrophic motions at the Rossby scale).

The synoptic variability of the atmosphere, however, has time scales of the order of a week and is shaped by pressure lows and highs with characteristic horizontal scales of the order of the thousand of kilometers and one must exclude the hypothesis of the generation of synoptic ocean eddies by direct resonant interactions. Atmospheric disturbances - lows and highs - generate large-scale currents in the ocean and it is the barotropic and essentially baroclinic instability of these currents which provide the energy for the synoptic eddies (Kosklyakov and Monin, 1978).

(The kinetic - and approximately equal potential-energy of the synoptic eddies is essentially higher than the kinetic energy of the large scale currents and at the same time much smaller than the available potential energy of the latter. This constitutes strong experimental evidence of eddy generation through baroclinic instability of the large scale oceanic currents).

Large scale ocean experiments (Polygon, Mode, ...) give evidence of synoptic eddies of two kinds, "frontal eddies" produced by the cut-off of meanders from such frontal currents as the Gulf Stream and the Kuroshio, and much weaker "open-ocean eddies". The kinetic energy of the frontal eddies can be two orders of magnitude larger than the kinetic energy of the typical ocean eddies, the rotation velocity in the upper part of frontal eddies can reach meters per second. (e.g. Kosklyakov and Monin, 1978 ; Nihoul, 1979).

The vertical length scale of the synoptic eddies is of the order of the depth (e.g. Rhines, 1977 ; Woods, 1977 ; Nihoul, 1979) and it is very tempting to regard them as constituting a form of two-dimensional turbulence.

Potential vorticity would be conserved in such motion and, in the words of Gill and Turner (1979), "patches of marked particles would be teased out, into spindly shapes, leading to an enstrophy ('mean square vorticity') cascade to smaller scales".

Such a cascade predicted by the mathematical theory of homogeneous two-dimensional turbulence (e.g. Kraichnan, 1967 ; Batchelor, 1969) implies that the flow of kinetic energy is from smaller to larger scales (the "red cascade").

There must be however processes - presumably different at different levels and in different regions - which are not quasi-geostrophic and which limit the extension of potential vorticity contours.

Turbulent energy transfer to small scale may jump over an eventual synoptic valley via boundary turbulence, intermittent internal turbulence or non-local cascade into internal waves. Bottom roughness provides a permanent mechanism for the conversion from large to small scales. An initial cluster of eddies, surrounded by quiet fluid may cascade to longer scales but eventually the energetic patch will contain too few eddies to act as turbulence. Another obstacle to the 2D red cascade is the restoring force provided by the β-effect or its topographic equivalent. No matter how intense or how small the initial eddies, the red cascade carries the flow into the regime of linear waves. The red cascade is then not only blocked by Rossby wave propagation but it is reversed near western boundaries as fast long weak westward-propagating Rossby waves reflect at a western boundary into slow short strong eastward-propagating waves (Rhines, 1977).

Instabilities of fronts could play an important part. Such fronts can be formed by the same mechanisms which produce atmospheric fronts but according to Woods (1977, 1978), it is possible that they reach a limiting equilibrium form well before the larger scale velocity field which produced them has changed significantly.

The transfer of energy to internal waves has been strongly advocated. According to Müller (1976), for instance, internal waves could extract energy from synoptic eddies at about the same rate as that at which they gain energy from baroclinic instability of the wind - generated Sverdrup flow . Desquieting evidence against such a scheme was presented by Ruddick and Joyce (1979) from direct measurements of the vertical eddy momentum flux, due to internal waves, with moored current-meters and temperature sensors. They found no significant correlation with the mean shear and estimated an upper bound for the vertical eddy viscosity more than one order of magnitude smaller than Müller's suggestion (Garrett, 1979).

Following Panchev (1976), one can estimate the rate of energy transfer ε_β from the synoptic eddies to smaller turbulent oceanic scales as being of the order of

$$\varepsilon_{\_} \sim 10^{-} \, m^{-} s \tag{12}$$

i.e. of the same order as the atmospheric energy input into the largest oceanic scales (e.g. Ozmidov, 1965) but apparently one or two orders of magnitude smaller than the rate of energy transfer to larger scales through the red cascade (Panchev, 1976).

In the macroscale range, one expects - as a result of the two-dimensional turbulence enstrophy cascade - the energy spectral function $E(\kappa)$ to fall off as κ^{-3} i.e.

$$\kappa^3 E(\kappa) \sim \kappa_\beta^3 \, E(\kappa_\beta) \sim 10^{-12} s^{-2} \tag{13}$$

Hence, at a scale of a few kilometers ($\kappa_w \sim 3 \; 10^{-4} m^{-1}$, say) characteristic of eddies and intrusive layers which may emanate from fronts (Woods, 1978), the energy level would be, in the mean (all space and time intermittencies taken into account)

$$\kappa_w \, E(\kappa_w) \sim \kappa_\beta^3 \, E(\kappa_\beta) \kappa_w^{-2} \sim 10^{-5} m^2 s^{-2} \tag{14}$$

Now, the rate of energy transfer from these scales, - which belong to the frontier districts between macro- and mesoscales -, into mesoscale turbulence can be estimated from turbulence similarity arguments. One finds