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PROCEEDINGS OF THE 10th INTERNATIONAL LIÈGE COLLOQUIUM
ON OCEAN HYDRODYNAMICS

Edited by

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FOREWORD

The International Liège Colloquia on Ocean Hydrodynamics are organized annually. Their topics differ from one year to another and try to address, as much as possible, recent problems and incentive new subjects in physical oceanography.

Assembling a group of active and eminent scientists from different countries and often different disciplines, they provide a forum for discussion and foster a mutually beneficial exchange of information opening on to a survey of major recent discoveries, essential mechanisms, impelling question-marks and valuable suggestions for future research.

Basic studies of atmospheric processes continuously feed a science called Meteorology and a public service called Meteorological Forecasting. For a long time, ocean sciences have remained more descriptive in nature, more concerned with the understanding of the basic processes and mathematical models were often designed with the main purpose of elucidating particular aspects of the ocean dynamics.

However, the rapid advancement, in the recent years, of both the physical sciences of the ocean and the mathematical techniques of marine modelling have made possible the development, in the field of marine hydrodynamics and air-sea interactions, of prognostic models serving a new science and initiating a public service : Marine Forecasting.

The papers presented at the Tenth International Liège Colloquium on Ocean Hydrodynamics report fundamental or applied research and they address such different fields as storm surges, mixing in the upper ocean layers, surface waves, cyclogenesis and other air-sea or sea-air interactions. Their unity resides in a common approach, seeking a better understanding (by modellers and users) of the scientific maturity and of the incentive new prospects of Marine Forecasting.

Jacques C.J. NIHOUL.

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The Scientific Organizing Committee of the Tenth International Liège Colloquium on Ocean Hydrodynamics and all the participants wish to express their gratitude to the Belgian Minister of Education, the National Science Foundation of Belgium, the University of Liège and the Office of Naval Research for their most valuable support.

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REVIEW OF THE THEORIES OF WIND-MIXED LAYER DEEPENING

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ABSTRACT

One considers here the time evolution of the oceanic surface boundary layer in relation with the synoptic variability of atmospheric processes. Attention is restricted to situations where the major responsability for the short-period variability of the vertical structure of the surface boundary layer lies on the local thermal and dynamic interactions between the atmosphere and the ocean and on the internal thermocline - supported transfer processes. Emphasis is laid on theoretical and experimental results which can be interpreted by means of simple one-dimensional vertical mixing models.

INTRODUCTION

The description of the dynamic of wind mixing in oceanic surface layers (e.g. Kitaigorodskii, 1970) is based on the assumption that the main sources of turbulent energy are

i) the breaking of wind waves which produces turbulence in a relatively thin surface layer (having a thickness of the order of the amplitude of the breaking waves) which extends into the fluid by turbulent energy diffusion effects (Kitaigorodskii and Miropolskii, 1967 ; Kalatskiy, 1974) ;

ii) the velocity shear associated with drift currents responsible for turbulent energy production throughout the turbulent layer and, primarily, in those parts of it where the velocity shear is large.

In oceanic surface layers, the two mechanisms can act simultaneously. However, in laboratory conditions, it is possible to explore each of them individually.

To study the wind wave breaking effect, the initial stirring of the thin surface layer can be simulated by means of a vertically oscillating grid placed in the vicinity of the fluid surface (Turner, 1973 ; Linden, 1975). The mixing caused by drift currents can be modelled by experiments in which a constant stress is applied at the surface of the fluid (Kato and Phillips, 1969 ; Kantha et al, 1977).

The laboratory experiments (Turner, 1973 ; Linden, 1975 ; Kato and Phillips, 1969 ; Kantha et al, 1977 ; Moore and Long, 1971) explicitly show that all the mechanisms of turbulence production create a thin region of large vertical density gradient in the initially continuously stratified fluid. This region, referred to as the "turbulent entrainment layer", normally lies below a well-mixed layer, the so-called "upper homogeneous layer". Beneath the turbulent entrainment layer, lies a relatively unperturbed region of the fluid in which internal waves and irregular irrotational perturbations may exist. In laboratory test conditions, the intensity of the fluctuations below the turbulent entrainment layer is found rather insignificant and such motions do not appear to contribute to the vertical momentum, heat and energy transfer processes.

When a steady stress acts on the free surface, a layer of considerable velocity shear (of thickness δ) is formed at the top of the mixed layer. If one excepts the very beginning of the entrainment process, the thickness of the shear layer is always much smaller than the depth D of the mixed layer ($\delta \ll D$). Large mean velocity gradients are also observed in the turbulent entrainment layer (Kato and Phillips, 1969 ; Kantha et al, 1977 ; Moore and Long, 1971) and they may extend to the lower part of the mixed layer (Moore and Long, 1971).

At very large values of the Richardson number (based on the variation of density accross the turbulent entrainment layer) a certain amount of heat and momentum transfer in the core of the entrainment layer can be attributed to molecular diffusion (Kantha et al, 1977 ; Crapper and Linden, 1974 ; Wolanski and Brush, 1975 ; Phillips, 1977). However, in cases of well-developed turbulence in the mixed layer, the molecular effects in the turbulent entrainment layer are obviously negligible. (Molecular diffusion can only play a role in the one-centimeter thick layer of water immediately below the surface).

In situ observations show that the thickness h of the turbulent entrainment layer reaches several meters in storm conditions. The ratio $\frac{h}{D}$ is then of the order of 10^{-1}. Detailed measurements made in laboratory test conditions, (Crapper and Linden, 1974 ; Wolanski and Brush, 1975) show that $\frac{h}{D}$ does not depend on the density variation accross the turbulent entrainment layer (provided the density jump is large enough). Beside, it has become evident that with increasing Peclet number $(Pe = \frac{wD}{\lambda}$ where w is the root mean square of the horizontal fluctuating velocity at the upper boundary of the entrainment layer and λ the molecular diffusivity of heat or salt) $\frac{h}{D}$ decreases and tends to a constant value $\sim 1.5\ 10^{-1}$. Measurements by Moore and Long (1971), in experiments where turbulence was generated by a velocity shear, lead to $\frac{h}{D} \sim 0.8\ 10^{-1}$. Finally, laboratory experiments by Wolanski and Bush (1975) also showed that $\frac{h}{D} \sim O(10^{-1})$ and is independent of the Richardson number $(Ri = \frac{g\,\Delta\rho\,D}{\rho_o w^2})$, where g is the acceleration of gravity and $\Delta\rho$ the density difference accross the entrainment layer.

In modelling the deepening process of the upper homogeneous layer, in the ocean as well as in laboratory experiments, one may thus assume*

$$\frac{h}{D} \sim 10^{-1} \quad ; \quad \frac{\delta}{D} \sim 10^{-1}$$

EQUATIONS DESCRIBING THE EFFECT OF WIND MIXING ON THE DEEPENING OF THE UPPER HOMOGENEOUS LAYER IN A STRATIFIED FLUID

The basic features of an oceanic wind-mixed layer can be simulated by one-dimensional models, disregarding advection, horizontal diffusion and large scale vertical motions. It will be assumed here, for simplicity, that the water density is a function of temperature only (the introduction of variations of salinity or horizontal non-homogeneity is not a major difficulty). It will be further assumed that the short-wave radiation is absorbed at the sea surface. A simple technique to account for the volume absorption of solar radiation has been described by Kraus and Turner (1967) and Denman (1973). The corrections introduced thereby have been found to be not very significant since the thickness of the effective absorption layer is, on the average, about one order of magnitude smaller than D (Denman,1973).

*This assumption provides a good approximation in modelling local one-dimensional vertical mixing processes but may not be applicable to the study of the evolution of the seasonal thermocline (Kitaigorodskii and Miropolskii, 1970). The analysis of the whole year development of the temperature field in the active layer of the ocean (200 - 400 m) must take into account the universal temperature profiles below the upper homogeneous layer. These profiles were found first by Kitaigorodskii and Miropolskii (1970) and were confirmed later by numerous observations of the vertical distributions of temperature and salinity in many parts of the ocean (Moore and Long, 1971 ; Miropolskii et al, 1970 ; Nesterov and Kalatskiy, 1975 ; Reshetova and Chalikov, 1977).

With these assumptions, the equations describing the non-steady, one-dimensional vertical heat, momentum and turbulent energy transfers in a stratified rotating fluid can be written

$$\frac{\partial \Theta}{\partial t} = - \frac{\partial s}{\partial z} \tag{1}$$

$$\frac{\partial \underset{\sim}{u}}{\partial t} = - \frac{\partial \underset{\sim}{\tau}}{\partial z} - f \, \underset{\sim}{e}_z \wedge \underset{\sim}{u} \tag{2}$$

$$\frac{\partial e}{\partial t} = - \underset{\sim}{\tau} \cdot \frac{\partial \underset{\sim}{u}}{\partial z} - g \beta s - \varepsilon - \frac{\partial M}{\partial z} \tag{3}$$

where Θ, $\underset{\sim}{u}$ and e denote respectively the mean temperature, the mean horizontal velocity and the mean turbulent energy and where s, $\underset{\sim}{\tau}$ and M are the corresponding fluxes (normalized with respect to the mean thermal capacity $\rho_o C_p$ and the mean density ρ_o respectively). f is equal to twice the vertical component of the earth's rotation vector, g is the acceleration of gravity, β the thermal expansion coefficient and ε is the rate of turbulent energy dissipation. The frame of reference is sinistrorsum and such that the x-axis is in the direction of the surface wind and the z-axis is vertical pointing downwards.

At the upper boundary of the mixed layer ($z = 0$), one must prescribe the fluxes. The fluxes depend on the atmospheric conditions and they are normally parameterized in terms of the meteorological data. In general, they are functions of time. However, in the following, the discussions will be restricted to the steady case, for the sake of simplicity.

If ϕ stands for any of the variables Θ, u, v, e, one defines

$$\overline{\phi} = \frac{1}{D} \int_0^D \phi \, dz \quad ; \quad \tilde{\phi} = \frac{1}{h} \int_D^{D+h} \phi \, dz \tag{4)(5}$$

$$\phi_o = \phi(0,t) \quad ; \quad \phi_- = \phi(D,t) \quad ; \quad \phi_+ = \phi(D+h,t) \tag{6)(7)(8}$$

Obviously, one has

$$\|\tilde{\underset{\sim}{u}}\| \leq \|\underset{\sim}{u}_-\| \leq \|\overline{\underset{\sim}{u}}\| \quad ; \quad \tilde{\Theta} < \Theta_- \leq \overline{\Theta}$$

$$\tilde{e} \leq \overline{e} \quad ; \quad \tilde{q} \leq \overline{q}$$

Integrating eqs. 1 - 3 over the upper homogeneous layer and the turbulent entrainment layer, one derives a system of equations for the depth-averaged variables $\overline{\phi}$ and $\tilde{\phi}$.

Combining these equations and neglecting small terms of relative magnitude $\frac{h}{D}$ (in the hypothesis of a "thin interface" $\frac{h}{D} \ll 1$)* , one obtains, after some calculations,

$$\frac{d}{dt}(\bar{\Theta}D) = s_o - s_+ + \frac{dD}{dt}\Theta_+ \tag{9}$$

$$\frac{d}{dt}(\bar{\underset{\sim}{u}}D) + f\,\underset{\sim}{e}_z \wedge \bar{\underset{\sim}{u}}D = \underset{\sim}{\tau}_o - \underset{\sim}{\tau}_+ + \frac{dD}{dt}\bar{\underset{\sim}{u}}_+ \tag{10}$$

$$\frac{d}{dt}(\bar{e}D) + g\,\beta\,\bar{s}D = M_o + \Pi_D + \Pi_h - \bar{\varepsilon}D - \tilde{\varepsilon}h - M_+ + \frac{dD}{dt}e_+ \tag{11}$$

where

$$\Pi_D = -\int_o^D \underset{\sim}{\tau}\cdot\frac{\partial \underset{\sim}{u}}{\partial z}\,dz \tag{12}$$

$$\Pi_h = -\int_D^{D+h} \underset{\sim}{\tau}\cdot\frac{\partial \underset{\sim}{u}}{\partial z}\,dz \tag{13}$$

The calculation of Π_D can be most easily done with the assumption that the velocity shear in the upper homogeneous layer is concentrated in the constant stress layer δ . Then

$$\Pi_D \sim \Pi_\delta = -\int_o^\delta \underset{\sim}{\tau}\cdot\frac{\partial \underset{\sim}{u}}{\partial z}\,dz \sim \underset{\sim}{\tau}_o\cdot(\underset{\sim}{u}_o - \underset{\sim}{u}_\delta) \tag{14}$$

where $\underset{\sim}{u}_\delta$ is the velocity at the lower boundary of the constant stress layer of thickness δ .

From eq.(2) and its scalar product by $\underset{\sim}{u}$, one gets, after some rearrangement and neglecting small terms involving h

$$\underset{\sim}{\tau}_- = \underset{\sim}{\tau}_+ + \frac{dD}{dt}(\underset{\sim}{u}_- - \underset{\sim}{u}_+) \tag{15}$$

$$\Pi_h \sim \frac{1}{2}\frac{dD}{dt}\|\underset{\sim}{u}_- - \underset{\sim}{u}_+\|^2 + \underset{\sim}{\tau}_+\cdot(\underset{\sim}{u}_- - \underset{\sim}{u}_+) \tag{16}$$

It can be shown that the turbulent energy production in the upper homogeneous layer and in the turbulent entrainment layer is not very sensitive to the detailed velocity distribution in the main part of the upper homogeneous layer.. In a first approach, it seems thus reasonable to make the so-called "slab model approximation" where the vertical velocity distribution is assumed homogeneous for $\delta \leq z \leq D$ so that

*Even, in the hypothesis $\frac{h}{D} \ll 1$, such simplification is difficult to justify because the remaining terms can partially cancel each other and sum up to be comparatively small. It must be regarded as a first approximation liable to revision. The term $\tilde{\varepsilon}h$ is retained in the absence of a clear-cut evaluation of the respective orders of magnitude of $\bar{\varepsilon}$ and $\tilde{\varepsilon}$.

$$\underset{\sim}{u}_- = \overline{\underset{\sim}{u}} = \underset{\sim}{u}_\delta \tag{17}$$

In this particular case, one can write

$$\Pi_D = \Pi_\delta = \underset{\sim}{\tau}_o \cdot (\underset{\sim}{u}_o - \overline{\underset{\sim}{u}}) = \tau_o (u_o - \overline{u}) \tag{18}$$

$$\Pi_h = \frac{1}{2} \frac{dD}{dt} \| \overline{\underset{\sim}{u}} - \underset{\sim}{u}_+ \|^2 + \underset{\sim}{\tau}_+ \cdot (\overline{\underset{\sim}{u}} - \underset{\sim}{u}_+) \tag{19}$$

$$\Pi_{D-\delta} = 0 \tag{20}$$

Velocity shear layers are thus taken into account as velocity jumps $(\underset{\sim}{u}_o - \overline{\underset{\sim}{u}})$ and $(\overline{\underset{\sim}{u}} - \underset{\sim}{u}_+)$ in thin layers of thickness $\delta << D$ and $h << D$, respectively.

There is some experimental evidence that one can assume

$$u_o - \overline{u} \sim \alpha\, \tau_o^{1/2} \tag{21}$$

where α is an empirical constant.

Then, if one sets

$$G_\delta = M_o + \Pi_\delta = M_o + \alpha\, \tau_o^{3/2} \tag{22}$$

and restrict attention to the case of steady fluxes at the air-sea interface, G_δ does not depend on time and may be used successfully as one of the external characteristic parameters ($G_\delta\ \delta^{1/3}$ is the velocity scale) of turbulence in the wind mixed layer of the ocean.

Along the same line, one may assume that the temperature is uniform in the upper homogeneous layer. Then, integrating eq.(1) in the mixed layer and over the turbulent entrainment layer, one gets

$$s(z) = s_o - \frac{s_o - s_-}{D} z \tag{23}$$

$$s_- = s_+ + \frac{dD}{dt} (\overline{\theta} - \theta_+) \tag{24}$$

Hence

$$g\, \beta\, \overline{s}\, D = \frac{1}{2}\left(g\, \beta\, s_o D + g\, \beta\, s_+ D + g\, \beta \frac{dD}{dt} (\overline{\theta} - \theta_+) D \right) \tag{25}$$

The time scale of turbulent energy dissipation $\frac{\overline{e}}{\overline{\varepsilon}}$ does not exceed a few minutes whereas the deepening of the mixed layer has a characteristic time of several hours. One may thus regard the turbulence as adapting itself instantaneously to the modifications of the mixed layer and following the "slow" evolution of the latter. At the scale of turbulence, this slow evolution is not noticeable and the turbulent energy may be regarded as quasi steady, i.e.

$$\frac{\partial}{\partial t} (\overline{e}\, D) \sim \overline{e} \frac{dD}{dt} \tag{26}$$

In this case, one can usually assume

$$\overline{e} = c_o \tau_o \tag{27}$$

where c_o *is a constant of order unity* ($c_o \sim 2 - 3$, in the atmospheric boundary layer) or, more generally

$$\overline{e} = c_\delta \; G_\delta^{2/3} \tag{28}$$

which is valid also in the case of turbulence generated by a turbulent energy flux M_o in the absence of momentum flux.

The values of the variables at the lower boundary of the turbulent entrainment layer depend on the characteristics of the layer of fluid below. These can be affected by different factors : molecular diffusion (Mellor and Durbin, 1975), internal waves generated inside the turbulent entrainment layer,but also below it,by turbulent disturbances at the bottom of the mixed layer and radiating momentum and energy away, turbulent diffusion caused by the instability and breaking of the internal waves or other mechanisms not directly related with the local wind mixing process (Garnich, 1975).

The effect of both molecular and turbulent diffusion below the turbulent entrainment layer is not perceivable at the time scale of the mixed layer deepening (of the order of a day) and it may accordingly be neglected.

The contribution, to the energy balance, of internal waves excited by the upper turbulent layer is difficult to evaluate (Linden, 1975 ; Thorpe, 1973 ; Kantha, 1977).

In the absence of vertical density stratification below the turbulent entrainment layer $\frac{\partial \Theta}{\partial z} = 0$ for $z \geq D + h$) , the fluid there may be regarded as being at rest. The internal waves generated by the upper turbulent layer do not propagate downwards and, in the non-viscous approximation which is actually made here, the fluid motion reduces to weak irrotational fluctuations which are rapidly attenuated as they move deeper from the bottom of the upper homogeneous layer and which contribute very little to e_+ and not at all to τ_+ , s_+ and M_+ .

In that case, one may take

$$e_+ = u_+ = \tau_+ = M_+ = s_+ = 0 \tag{29}$$

This assumption is not quite valid when the fluid below the turbulent entrainment layer is stratified but, in a first approximation, when the density variation accross the turbulent entrainment layer is large, perturbations below it can be regarded as insignificant and eq.(29) can be used.

The temperature at the lower boundary of the turbulent entrainment layer can be written quite generally

$$\Theta_+ = \Theta_{oo} - \gamma D \tag{30}$$

where Θ_{oo} is a constant and γ the temperature gradient below the turbulent entrainment layer.

With the approximations described above (18, 19, 21, 22, 24, 25, 26, 28, 29, 30), the basic equations (9)-(11) can be written

$$\frac{d}{dt}\left(\Delta\Theta D - \frac{1}{2}\gamma D^2\right) = s_o \tag{31}$$

$$\frac{d}{dt}(D\bar{u}) - f\bar{v}D = \tau_o \tag{32}$$

$$\frac{d}{dt}(D\bar{v}) + f\bar{u}D = 0 \tag{33}$$

$$\frac{dD}{dt}\left(\frac{g\beta\Delta\Theta D}{2} + c_\delta G_\delta^{2/3}\right) = G_\delta + \frac{1}{2}\frac{dD}{dt}\|\bar{\underset{\sim}{u}}\|^2 - (\bar{\varepsilon}D + \tilde{\varepsilon}h) - \frac{Dg\beta s_o}{2} \tag{34}$$

where

$$\Delta\Theta = \bar{\Theta} - \Theta_+ \tag{35}$$

Similarly the entrainment fluxes reduce to*

$$\tau_- = \frac{dD}{dt}\underset{\sim}{u}_- \tag{36}$$

$$s_- = \frac{dD}{dt}\Delta\Theta \tag{37}$$

$$M_- = \tilde{\varepsilon}h + \frac{dD}{dt}e_- - \Pi_h \tag{38}$$

$$= \tilde{\varepsilon}h + \frac{dD}{dt}\left(e_- - \frac{1}{2}\|\underset{\sim}{u}\|^2\right)$$

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*Attention is restricted here to the case $\frac{dD}{dt} > 0$. As a result of eq.(29), eqs.(36)-(38) cannot be used when $dD/dt < 0$ ("reverse entrainment"). As suggested by Krauss and Turner (1967), one should write

$$s_- = 0 \quad , \quad \tau_- = 0 \quad \text{for} \quad \frac{dD}{dt} \leq 0 \tag{39),(40}$$

When one considers the upper homogeneous layer throughout a long period when strong storms may give way to perfectly still weather, it is important to describe the reverse entrainment process (Garnich, 1975 ; Kosnizev et al, 1976).

It is important to note that, for $h \to 0$, $\tilde{\varepsilon}h$ has a finite but small value. Then when the turbulent kinetic energy e_- is smaller than the mean flow kinetic energy $\frac{1}{2}\|\underset{\sim}{u}\|^2$, the flux of turbulent energy can be directed from the turbulent entrainment layer into the upper homogeneous layer, contributing to the turbulent mixing in that layer. This is confirmed by experiments in the laboratory (Kato and Phillips, 1969 ; Kantha et al, 1977).

Eqs.(31)-(33) can be integrated easily since s_o and t_o are known functions of time. In particular, when they are constant, one gets

$$\Delta\Theta D - \frac{1}{2}\gamma D^2 = \Delta\Theta(o)D(o) - \frac{1}{2}\gamma D^2(o) + s_o t \tag{41}$$

$$D\overline{u} = \left(\frac{\tau_o}{f} + D(o)\overline{v}(o)\right) \sin ft + D(o)\overline{u}(o) \cos ft \tag{42}$$

$$D\overline{v} = - D(o)\overline{u}(o) \sin ft + \left(\frac{\tau_o}{f} + D(o)\overline{v}(o)\right) \cos ft - \frac{\tau_o}{f} \tag{43}$$

where

$D(o)$, $\overline{u}(o)$, $\overline{v}(o)$ are the initial values.

To close the system of equations (31)-(34), one needs an estimate of the integrated viscous dissipation in the turbulent energy balance equation. A question arises : what portion of G_δ and Π_h is used for mixing and what portion dissipates into heat ? This is probably the most important problem in one-dimensional modelling of mixed layer deepening and the models can be classified according to their particular parameterization of the integral energy dissipation.

Let*

$$\overline{\varepsilon}D + \tilde{\varepsilon}h = (1 - \Phi_1)G_\delta + (1 - \Phi_2)\Pi_h \tag{44}$$

where Φ_1 and Φ_2 are non-dimensional functions depending on the external parameters of the problem. Their dependence will be examined later, using similarity theory together with experimental data obtained in well controlled external conditions.

However, by simple inspection of eq.(34) and (44), one can get some information about the functions Φ_1 and Φ_2 .

If the deepening of the mixed layer is produced only by the diffusion of turbulent energy down from the surface into the stratified fluid ($M_o \neq 0$, $\tau_o = 0$, $\Pi_h = 0$), then obviously

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* Attention is restricted here to $s_o \geq 0$. When the ocean surface is cooling $\Pi_q = \frac{g\beta s_o D}{2} < 0$ acts as a complementary source of energy and (44) must generalized to

$$\overline{\varepsilon}D + \tilde{\varepsilon}h = (1 - \phi_1)G_\delta + (1 - \phi_2)\Pi_h + (1 - \phi_3)|\Pi_q| \tag{45}$$

$$0 \leq \Phi_1 \leq 1 \tag{46}$$

On the other hand, in the case of shear generated turbulence ($M_o << \Pi_D$, Π_h ; $G_\delta \sim \Pi_D = \Pi_\delta$) deepening of the upper homogeneous layer is possible only if

$$(1 - \Phi_1)G_\delta + (1 - \Phi_2)\Pi_h < G_\delta + \Pi_h - \Pi_q \tag{47}$$

where

$$\Pi_q = \frac{g\ \beta\ s_o D}{2} \geq 0 \tag{48}$$

Hence

$$\frac{\Phi_1}{\Phi_2} > \frac{\Pi_q}{G_\delta\ \Phi_2} - \frac{\Pi_h}{G_\delta} \tag{49}$$

If one considers, to begin with, a situation where the effects of rotation and surface heating can be neglected, in the asymptotic case $\frac{h}{D} \to 0$, there is one non-dimensional number characterizing the problem, the overall Richardson number defined as

$$R_{iG} = \frac{g\beta\Delta\Theta D}{G_\delta^{2/3}} \tag{50}$$

The functions Φ_1 and Φ_2 should then be functions of R_{iG} only.

In the following, one shall write for simplicity u, v, ... *instead of* $\bar{u}$, $\bar{v}$, ...

FUNDAMENTAL RESULTS OF LABORATORY EXPERIMENTS

In laboratory studies of the entrainment process in a stratified fluid, two essentially different mechanisms were used to generate the turbulence.

In the first type of experiments, (e.g. Turner, 1973 ; Thorpe, 1973 ; Long, 1974) turbulence is produced by an oscillating grid placed at a given depth Z_1 below the free surface in the tank. The depth of the mixed layer is

$$D = Z_1 + Z_D \tag{51}$$

where Z_D is the distance from the grid to the turbulent entrainment layer.

The most detailed studies of entrainment in this case have been performed in a fluid consisting of two homogeneous layers of different density. The results have been interpreted, as a rule, using a relationship of the form

$$\frac{1}{\omega_o}\frac{dD}{dt} = C\left(\frac{\omega_o^2\rho_o}{g\,\Delta\,\rho}\right)^n \tag{52}$$

where C is some appropriate dimensional factor independent of the grid oscillation frequency ω_o and of the density jump $\Delta\rho$ accross the turbulent entrainment layer (ρ_o is the mean density in the mixed layer).

The exponent n is a function of the Peclet number which tends asymptotically to the value 1.5 for large Peclet numbers (Turner, 1973 ; Crapper and Linden, 1974 ; Wolanski, 1972).

Hence, in the asymptotic case

$$\frac{1}{\omega_o}\frac{dD}{dt} = C\,\frac{\omega_o^3}{\left(g\,\frac{\Delta\rho}{\rho_o}\right)^{3/2}} \tag{53}$$

Long (1974), using experimental values of ω_o and $g\,\frac{\Delta\rho}{\rho_o}$, suggested, on the basis of dimensional arguments, the following form for C

$$C = Z_o^{5/2}\,\Psi\left(\frac{Z_o}{D}\,,\,\frac{Z_1}{D}\right) \tag{54}$$

where Z_o is the amplitude of the grid oscillations kept constant during the experiment, Ψ is some non-dimensional function tending to a constant value $\Psi(o,o)$ for $\frac{Z_o}{D} \to 0$ and $\frac{Z_1}{D} \to 0$.

The flux of turbulent energy must be proportional to the third power of ω_o and Z_o, i.e.

$$M_o = C_1(\omega_o Z_o)^3 \tag{55}$$

where C_1 is a non-dimensional constant*.

Combining (51), (53), (54) and (55), one obtains

$$\frac{1}{M_o^{1/3}}\frac{dD}{dt} = C_2\,R_{iM}^{-3/2} \tag{56}$$

where C_2 is another non-dimensional constant defined by

$$C_2 = \frac{\Psi}{C_1^{4/3}}\left(\frac{Z_1 + Z_o}{Z_o}\right)^{3/2} \tag{57}$$

and where the overall Richardson number R_{iM} is defined by

$$R_{iM} = \frac{g\beta\Delta\Theta D}{M_o^{2/3}} \tag{58}$$

The results can also be presented using a "turbulent" Richardson number (Turner, 1973)

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*At least, C_1 is a constant for each particular geometry of the grid.