METHODS OF Experimental Physics

VOLUME 19

ULTRASONICS

Methods of Experimental Physics

VOLUME 19

ULTRASONICS

METHODS OF EXPERIMENTAL PHYSICS:

L. Marton and C. Marton, Editors-in-Chief

Volume 19

Ultrasonics

Edited by

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ACADEMIC PRESS

A Subsidiary of Harcourt Brace Jovanovich, Publishers

New York London Toronto

Sydney San Francisco

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ACADEMIC PRESS, INC. 111 Fifth Avenue, New York, New York 10003

United Kingdom Edition published by ACADEMIC PRESS, INC. (LONDON) LTD. 24/28 Oval Road, London NW1 7DX

Library of Congress Cataloging in Publication Data Main entry under title:

Ultrasonics.

(Methods of experimental physics; v. 19) Includes bibliographical references and index. 1. Ultrasonics. I. Edmonds, Peter D. II. Series. QC244.UA3 534.5'5 80-28270 ISBN 0-12-475961-0

PRINTED IN THE UNITED STATES OF AMERICA

81 82 83 84 98 76 54 32 1

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FOREWORD

This volume, edited by Dr. Peter Edmonds, is the first of the Methods to be devoted to acoustics. Future volumes will deal with the more classical aspects of acoustics, a field that has been adopted by our colleagues in engineering as well.

Ultrasonics plays many roles, ranging from physical and bioengineering applications to the study of the fundamental properties of materials. Dr. Edmonds and his contributors cover these areas in a manner that should make this volume a definitive reference work on the subject. We expect that researchers in a given specialization will find much useful information and have their imaginations stimulated by going through the book as a whole.

> L. MARTON C. MARTON

PREFACE

This volume offers detailed and comprehensive treatments of a number of important topics in the broad field of ultrasonics. It is intended to serve the needs of graduate students and also of specialists in other fields who may desire an assessment of the capabilities of ultrasonics as a technique with the potential for solving specific problems.

Ultrasonics interfaces with many fields, including optics, low temperature and solid state physics, chemical kinetics, cavitation, viscoelasticity, lubrication, nondestructive evaluation, medical diagnostic imaging, signal processing, and materials processing. The authors of one or more of the following parts discuss these fields. However, other important topics have been omitted, e.g., ultrasonics in gaseous media, plasma- and magneto-acoustics, and phonon phenomena in general. Ultrasonic scattering in noncrystalline media proved to be insufficiently developed for treatment in this treatise. (Seekers of information on these topics should consult the excellent treatise "Physical Acoustics," edited by W. P. Mason and R. N. Thurston, published by Academic Press.)

I wish to thank all authors for their cooperation and hard work in writing and many supplementary tasks. The essential contributions made by the secretarial assistants to the authors and by their institutions are also acknowledged.

I am grateful to several anonymous reviewers of parts of this volume, whose excellent advice has been freely given and usually heeded. Valuable support provided by the management and staff of SRI International is acknowledged with thanks.

All who have contributed to this volume profoundly regret that one of its editors-in-chief, Dr. Ladislaus Marton, did not live to see its publication. In his absence, the functions of editor-in-chief have been admirably fulfilled by Mrs. Claire Marton.

PETER D. EDMONDS

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0. INTRODUCTION: PHYSICAL DESCRIPTION OF ULTRASONIC FIELDS†

By Peter D. Edmonds and F. Dunn

List of Symbols

a	subscript denoting amplitude
A	magnitude of α/f^2 contributed by a relaxation process (except
	viscosity) at $f \ll f_r$
В	magnitude of α/f^2 contributed by the viscosity relaxation process
C	at $j > j_v$
	specific heat of medium at constant pressure
	specific heat per unit mass of medium at constant pressure
C _v	specific neat of medium at constant volume
D	logarithmic decrement
E_0	acoustic energy density = energy stored per unit volume
ΔE_0	energy loss per cycle due to absorption
f	frequency
f_{v}	relaxation frequency for viscosity
$g_1(\tau)$	distribution function for relaxation times
G	shear modulus of elasticity
$G^*(j\omega)$	complex shear modulus of elasticity
$G'(\boldsymbol{\omega}), G''(\boldsymbol{\omega})$	real and imaginary parts of G^*
G^{∞}	infinite frequency asymptote of G'
I	intensity
I_{1+}, I_{1-}, I_{2+}	intensities of incident, reflected, and transmitted waves
j	$\sqrt{-1}$
k	wave vector
К	magnitude of k
K	bulk modulus of elasticity
n	integer
p	acoustic pressure
Г Р	amplitude of n
Р.	ambient pressure
- 0	

[†] Portions of this introduction have been adapted with permission from W. J. Fry and F. Dunn, Ultrasound: Analysis and experimental methods in biological research, *in* "Physical Techniques in Biological Research," (W. L. Nastuk, ed.), Vol. 4, pp. 265–275. Academic Press, New York, 1962; and from F. Dunn, P. D. Edmonds, and W. J. Fry, Absorption and dispersion of ultrasound in biological media, *in* "Biological Engineering" (H. P. Schwan, ed.), pp. 207–233. McGraw-Hill, New York, 1969.

P_{+}, P_{-} $P_{1+}, P_{1-}, P_{2+}, P_{2-}$ P_{max}, P_{min} $P_{1i}, P_{1r}, P'_{1i}, P'_{1r}$ P_{9i}, P'_{9i}		see Table IV
q Q Q _m e	,	general field variable amplitude of q quality factor = π/D general spatial coordinate
$r_{2/1}, r_{3/1}, r_{2/3}$	}	see Table IV
r_{11} , r_{21} \mathcal{R}_{a} \mathcal{R}_{I} S SWR t t t T T T T T \hat{T} \hat{T} \hat{T}	J	amplitude reflection coefficient intensity reflection coefficient condensation = $(\rho - \rho_0)/\rho_0$ amplitude of condensation; entropy (as subscript) standing wave ratio time decay time of field amplitude parameter in an absorbing medium absolute temperature shear stress initial value of shear stress asymptotic final value of shear stress amplitude of sinusoidal shear stress amplitude of sinusoidal shear stress
Т ^о Ф		amplitude transmission coefficient
ла Г,		intensity transmission coefficient
v		wave propagation velocity; sound speed
v ^o		limiting sound speed at zero frequency (compressional wave)
vi		compressional wave speed
v_s^0		limiting shear wave speed at zero frequency
v_1, v_2, v_3	ł	see Table IV
A_{11} , b_{12} , b_{11} , b_{22}	,	velocity dispersion
x		spatial coordinate
<i>x</i> ₃		see Table IV
Z_0, Z_1, Z_2		characteristic acoustic impedance
α		amplitude absorption coefficient
$\alpha_{\rm r}$		generalized relaxational contribution to α
α_v		adiabatic compressibility
PS Br		isothermal compressibility
γ		ratio of specific heats = $C_{\rm p}/C_{\rm y}$
δ		phase lag between acoustic pressure and particle velocity in an ab- sorbing medium
η(ω)		shear viscosity coefficient
η°		limiting shear viscosity as frequency tends to zero
η*(jω)		complex shear viscosity
η (ω)		real part of η^{-1}
θ1. θ. θ'. θ'		see Table IV

θ	amplitude of temperature peturbation
λ	wavelength
λ ₃	see Table IV
Ę	"particle" (elemental volume) displacement
ξx	particle displacement in x direction
È	particle velocity
È.	particle velocity in x direction
Ëx	particle acceleration in x direction
⊒∓	amplitudes of particle displacement for waves in the positive and negative directions
宮	amplitude of particle velocity
営	amplitude of particle acceleration
ρ	density
ρο	mean density
ρ_1, ρ_2, ρ_3	see Table IV
au	relaxation time
τ_a, τ_b	limits of relaxation time distribution
$ au_{v}$	relaxation time for viscosity
Ŷ	instantaneous temperature increment in medium
φ	scalar displacement potential (irrotational)
Φ	vector displacement potential (rotational)
ψ	scalar velocity potential
ω	angular frequency
(dot over symbol)	differentiation with respect to time

0.1. Development of Propagation Relations

The propagation of an acoustic disturbance or the presence of an acoustic field in an elastic medium is characterized by changes in a number of the physical variables that describe the state of the system or medium. Examples of these variables are pressure, temperature, and density.

For a traveling, sinusoidal, plane wave propagating in the positive direction of the x axis (when no attenuation of the waves occurs because we assume absorption of energy by the medium is absent), the changes in the physical variables can each be expressed in the form of Eq. (0.1.1), provided that the medium responds linearly to the stresses imposed upon it.

$$q = Q \cos \omega (t - x/v) \quad \text{or} \quad q = \operatorname{Re}\{Q \exp[j\omega(t - x/v)]\}. \quad (0.1.1)$$

In this equation q designates any one of the variables that undergoes sinusoidal change owing to the presence of the disturbance in the medium and Q designates the amplitude of the cyclic change in that variable; t and x are the time and space coordinates, respectively, ω is the angular frequency ($\omega = 2\pi f$), f the frequency, and v the free-field sound speed, i.e.,

the propagation speed of a plane wave traveling through a liquid medium of infinite extent. Equation (0.1.1) is one solution, namely, that representing a wave traveling in the positive x direction, of the one-dimensional elastic wave equation as it applies to an ideal, linear, homogenous, perfectly elastic (dissipationless), fluid medium

$$\partial^2 q / \partial t^2 = (1/v^2) \ \partial^2 q / \partial x^2. \tag{0.1.2}$$

In this equation q could represent the instantaneous displacement ξ of an element of volume of the medium. This approximation to the more general hydrodynamical equation is valid under conditions that permit linearization, that is, when the velocity amplitude $\Xi = (\partial \xi / \partial t)_{max}$ of the elementary volume is small in comparison with the speed of sound v and when the adiabatic compressibility β_s , which is the reciprocal of the adiabatic elastic bulk modulus K, is not significantly dependent on pressure over the range of pressure variations present in the acoustic field.

Since sound propagation is very close to an adiabatic process at most frequencies of interest, the adiabatic compressibility is a significant parameter in the description of sound propagation. It is related to the free-field sound speed for compressional waves as follows:

$$v^{2} = v_{l}^{2} = \frac{1}{\rho_{0}\beta_{\rm S}} = \frac{\gamma}{\rho_{0}\beta_{\rm T}} = \frac{C_{\rm p}/C_{\rm v}}{\rho_{0}\beta_{\rm T}},$$
 (0.1.3)

where β_s is the adiabatic compressibility of the medium and ρ_0 the mean density of the medium. The sound speed can be expressed, as indicated in Eq. (0.1.3), in terms of the isothermal compressibility β_T by introducing the ratio of specific heats $\gamma = C_p/C_v$, where C_p and C_v are the specific heats of the medium at constant pressure and constant volume, respectively. Clearly, a measurement of the speed of a plane compressional wave can be interpreted immediately to yield the adiabatic compressibility of the medium if the density is known; and if the value of γ is also known, the isothermal compressibility can be determined.

Equation (0.1.2) is a special case of the more general wave equation that is applicable to three-dimensional propagation:

$$\frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} = \frac{1}{\rho_0 \beta_{\rm S}} \, \nabla^2 \boldsymbol{\xi}. \tag{0.1.4}$$

Solutions of Eq. (0.1.4) include not only waves propagating in the positive r direction away from the origin but also those propagating in the negative r direction toward the origin. All are represented when the \pm sign is placed in the exponent for one-dimensional propagation, e.g.,

$$\boldsymbol{\xi} = \boldsymbol{\Xi}_{\boldsymbol{\tau}}(\boldsymbol{r}) \exp[j(\boldsymbol{\omega}\boldsymbol{t} \pm \mathbf{k} \cdot \mathbf{r})]. \qquad (0.1.5)$$

The wave vector \mathbf{k} that appears in the solution is related to the angular frequency and the sound speed as

$$\mathbf{k} = \mathcal{H}\mathbf{n};$$
 $\mathcal{H} = -\omega/v = 2\pi/\lambda;$ $v = f\lambda.$ (0.1.6)

Equation (0.1.4) is itself a specialization, applicable to fluids of the type indicated, of the following wave equation describing propagation of disturbances in a dissipationless, isotropic, elastic solid:

$$\frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} = \frac{K + 4G/3}{\rho_0} \, \boldsymbol{\nabla} \boldsymbol{\nabla} \cdot \boldsymbol{\xi} - \frac{G}{\rho_0} \, \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{\xi}, \qquad (0.1.7)$$

where K and G are, respectively, the bulk and shear moduli of elasticity of the medium.

It is possible to express the displacement vector as the sum of terms involving a scalar potential ϕ and a vector potential Φ as

$$\boldsymbol{\xi} = \boldsymbol{\nabla}\boldsymbol{\phi} + \boldsymbol{\nabla} \times \boldsymbol{\Phi} \tag{0.1.8}$$

For irrotational motion, such as in a spherical wave, the vector potential $\Phi = 0$ and only the scalar displacement potential ϕ remains; that is,

$$\boldsymbol{\xi} = \boldsymbol{\nabla}\boldsymbol{\phi}. \tag{0.1.9a}$$

The time derivative of the displacement potential is the velocity potential ψ , i.e.,

$$\partial \phi / \partial t = \psi; \quad \dot{\boldsymbol{\xi}} = \nabla \psi. \quad (0.1.9b)$$

These potentials are fundamental functions (analogous to electric field potentials) in terms of which acoustic field parameters may be expressed. The specialization of Eq. (0.1.7) for fluids is obtained when the modulus of shear rigidity G is set equal to zero, which is true for lossless fluids, since the latter are characterized by an inability to support an elastic shear strain, and $\Phi = 0$.

Returning to a consideration of the simple plane wave propagating in an ideal isotropic elastic medium in the positive x direction, we can express the sinusoidally varying acoustic parameters in terms of the displacement potential or velocity potential and in terms of one another.

$$p = -\rho_0 \, \partial \psi / \partial t, \qquad \dot{\boldsymbol{\xi}}_x = (\boldsymbol{\nabla} \psi)_x, \qquad (0.1.10)$$

$$s = (\rho - \rho_0)/\rho_0 = \beta_s p,$$
 (0.1.11)

$$\mathbf{Y} = (T\theta/\rho_0 C'_p)p = (\gamma - 1)(\beta_s/\theta)p, \qquad (0.1.12)$$

where s is the condensation or the fractional change in density, ρ the instantaneous density, Υ the instantaneous temperature increment resulting from adiabatic compression of the medium, T the absolute temper-

TABLE L

Parameter	Parameter symbol q	Amplitude symbol Q	Р	S
Pressure	p	Р		${}^{\pm} ho_0 v^2$
Condensation	\$	S	$\frac{1}{\rho_0 v^2}$	_
Particle dis- placement	Ę	Ξ	$\frac{1}{j\omega\rho_0 v}$	$\ddagger \frac{v}{j\omega}$
Particle velocity	È	Ż	$\frac{1}{\rho_0 v}$	‡ <i>v</i>
Particle ac- celeration	Ë	Ë	$\frac{j\omega}{\rho_0 v}$	‡jωv
Temperature	Ŷ	θ	$\ddagger \frac{1}{\theta} \left(\beta_{\mathrm{T}} - \frac{1}{\rho_0 v^2} \right)$	$\frac{+\rho_0 v^2}{\theta} \left(\beta_{\rm T} - \frac{1}{\rho_0 v^2} \right)$

^a Multiply expression in the table by the column heading to obtain the relations equal to the amplitude quantities tabulated in the amplitude symbol column. Note that $j = \sqrt{-1}$. The relations apply to plane waves traveling in either direction. The upper sign applies to waves traveling in the positive direction and the lower sign to the negative direction [see Eq. (0.1.5)]. The amplitude of a change in any one physical parameter is equal to the amplitude of the change in any other physical parameter multiplied by the absolute value of the appropriate quantity in the table. A self-consistent set of units is used throughout the table (e.g., mks or cgs).

ature of the medium, θ the isobaric thermal expansion coefficient, and C'_{p} the heat capacity at constant pressure per unit mass. The interrelation of the acoustic field parameters is shown in Table I.

The method of detection and description of the field, in any specific case, may depend on the measurement of one or several of these parameters. The quantity $\rho_0 v$, the product of density and sound speed, which appears in many relations in the table, is known as the characteristic acoustic impedance of the medium Z_0 ; that is,

$$Z_0 = \rho_0 v. \tag{0.1.13}$$

For plane traveling waves, Z_0 is numerically equal to the specific acoustic impedance, which is defined as the ratio of the pressure p to the particle velocity $\dot{\xi}$ at any point in the field. For other field configurations, including plane standing waves, the specific acoustic impedance differs numerically from $\rho_0 v$ and is, in general, a function of position. It should also be noted that the characteristic acoustic impedance is dependent on

<u> </u>	主	ä	θ
± <i>jω</i> ρ₀v	$\pm \rho_0 v$	+ <u>ρ₀ν</u> jω	$\ddagger \frac{\theta}{(\beta_{\rm T} - (1/\rho_0 v^2))}$
$\pm \frac{j\omega}{v}$	$\pm \frac{1}{v}$	$\ddagger \frac{1}{j\omega v}$	$\frac{\theta}{\rho_{0}v^{2}[\beta_{T}-(1/\rho_{0}v^{2})]}$
-	$\ddagger \frac{1}{j\omega}$	$=\frac{1}{\omega^2}$	$\frac{1}{j\omega\rho_0 v [\beta_{\rm T} - (1/\rho_0 v^2)]}$
‡ <i>jω</i>	-	$\ddagger \frac{1}{j\omega}$	$\frac{1}{\rho_0 v [\beta_T - (1/\rho_0 v^2)]}$
Ξω²	‡jω	-	$\frac{-\frac{j\omega\theta}{\rho_0 v [\beta_{\rm T} - (1/\rho_0 v^2)]}}{\rho_0 v [\beta_{\rm T} - (1/\rho_0 v^2)]}$
$\frac{\pm j\omega\rho_0 v}{\theta} \left(\beta_{\rm T} - \frac{1}{\rho_0 v^2}\right)$	$\frac{+\underline{\rho_0 v}}{\theta} \left(\beta_{\mathrm{T}} - \frac{1}{\rho_0 v^2} \right)$	$\frac{+\underline{\rho_0 v}}{j\omega\theta} \left(\beta_{\rm T} - \frac{1}{\rho_0 v^2} \right)$	

Relations between Amplitudes of the Various Physical Parameters^a

the type of wave that is propagating, since the speed of shear waves is different from that of compressional waves.

The intensity I of the sound wave is defined as the time average of the rate of propagation of energy through unit area normal to the direction of propagation; for plane traveling waves, I is related to field-parameter amplitudes by

$$I = P^2 / 2Z_0 = P \Xi / 2 = Z_0 \Xi^2 / 2. \qquad (0.1.14)$$

The energy density E_0 of the wave motion at a specific position in the field is the sum of the kinetic energy per unit volume of the moving volume element and the potential energy per unit volume of compression (or expansion) of the element. For plane traveling waves, it is equal to the ratio of the intensity to the sound speed, i.e.,

$$E_0 = \rho_0 \dot{\Xi}^2 / 2 = I / v. \qquad (0.1.15)$$

Root mean square (rms) quantities are not employed in the majority of publications in acoustics, and consequently the symbols in Eqs. (0.1.14) and (0.1.15) are the amplitudes of the acoustic field parameters. If rms values had been used, the factors 2 would have been eliminated from the equations.

As stated previously, linearizing of the hydrodynamical equations depends on two assumptions which can now be expressed symbolically as

$$\dot{\Xi}/v \ll 1; \quad [(\beta_{\rm S})_{P_0+P} - (\beta_{\rm S})_{P_0-P}]/(\beta_{\rm S})_{P_0} \ll 1, \quad (0.1.16)$$

Material	f	T	P_0	I	Р	s	E	 主	Ë	θ	Ξ/v
To obtain results in: Multiply figures in table by: To obtain results in: Multiply figures in table by:	MHz 1	°C 1	atm 1 N/m ^{2 a} 1.013 × 10 ⁵	W/cm ² 1 W/m ² 10 ⁴	atm 1 N/m ^{2 a} 1.013 × 10 ⁵	10 ⁻⁵	cm 10 ⁻⁶ m 10 ⁻⁸	cm/sec 1 m/s 10 ⁻²	cm/sec ² 10 ⁶ m/s ² 10 ⁴	°C 10 ⁻⁴	10-5
Water degassed and distilled	1	30	1	0.01 1 100	0.171 1.71 17.1	0.762 7.62 76.2	0.183 1.83 18.3	1.15 11.5 115	7.22 72.2 722	3.82 38.2 382	0.762 7.62 76.2

TABLE II. Numerical Example of Physical Parameters for Water

^a N/m² \equiv Pascal (Pa).

where P_0 represents the ambient pressure in the absence of a sound wave. Nonlinear or second-order effects still may be of importance for values of Ξ/v smaller, for example, than 0.01, but the linearized equations constitute a good first approximation for calculating values of the physical parameters when this numerical limit is placed on the interpretation of the symbol $\ll 1$.

Table II shows values of the numerical magnitudes of the acoustic field parameters for a plane traveling wave, when the propagation medium is water, for representative intensity values of the wave spanning four orders of magnitude. It may be noted in particular that the temperature excursion in water is small and that this parameter is entirely unrelated to the monotonic rise in temperature of the specimen that occurs when energy is absorbed by the specimen. However, even for low-amplitude ultrasonic waves, which may be used as a probe to measure the response of a system to an extremely small perturbation, the pressure amplitude may be comparable to one atmosphere, and the amplitude of the particle acceleration can be exceedingly high and give rise to significant local stresses.

Table III lists values for the various characteristic constants of a number of materials of general utility. These data may be used in connection with the relations appearing in Table I to obtain numerical values of field parameters such as those listed in Table II. It is usually convenient to express the intensity in watts per square centimeter and the acoustic pressure amplitude in atmospheres. However, for calculations using the expressions of Table I, the intensity should be expressed in ergs per square centimeter per second and the pressure amplitude in dynes per square centimeter if the other parameters are expressed in the indicated units. Equivalent mks units may also be used.

0.2. Reflection and Refraction

Reflection and refraction of acoustic waves occur in a manner analogous to that for electromagnetic waves, and many of the concepts that arise in the theory of transmission lines are applicable in "onedimensional" situations. The formulas listed in Table IV are for media within which no acoustic absorption occurs and for which the normals to the planar wave fronts and the normals to the interfaces lie in the same plane.

Case 1. Reflection and transmission occur at a single interface between two media. The reflection coefficient \mathcal{R}_a , the transmission coefficient \mathcal{T}_a , and the standing wave ratio (SWR) for waves incident on the interface

Material	T	<i>P</i> ₀	ρ_0	υ	ρου	$C_{\rm p}/C_{\rm v}$	β _T	θ	α
To obtain results in:	 °C	atm	g/cm ³	cm/sec	$g/(cm^2 sec)$		cm²/d	(°C)-1	Np/cm
Multiply figures in table by:	1	1	1	10 ⁵	10 ⁵	1	10-12	10-5	1
To obtain results in:		N/m^{2a}	kg/m³	m/s	$kg/(m^2 s)$		m^2/N^a		Np/m
Multiply figures in table by:		1.013×10^{5}	10 ³	10 ³	106		10-11		10²
Water									
Degassed, distilled	0	1	0.999841	1.4027	1.4025	1.000583	50.86	-5.89	
α proportional to f^{2b}	10	1	0.999701	1.4476	1.4472	1.001085	47.79	+9.45	
	20	1	0.998207	1.4827	1.4800	1.00656	45.86	21.19	25×10^{-5}
	30	1	0.995651	1.5094	1.5028	1.01526	44.76	30.75	
	40	1	0.992220	1.5292	1.5173	1.02575	44.20	38.93	
	0	136	0.9941	1.4245	1.4161	1.00012	49.58	2.01	
	10	136	0.9946	1.4700	1.4621	1.00356	46.69	15.09	
	20	136	0.9961	1.5057	1.4998	1.01041	44.74	25.10	
	30	136	0:9986	1.5329	1.5308	1.01827	43.40	34.05	
	40	136	1.0019	1.5531	1.5560	1.02672	42.48	40.92	
Water Solutions									
0.9% normal saline ^c	0	1	1.00668	1.4134	1.4228			1.98	
α proportional to f^{2b}	10	1	1.00631	1.4582	1.4674			8.46	
	20	1	1.00460	1.4932	1.5001			23.89	25×10^{-5}
	30	1	1.00189	1.5198	1.5268			29.94	
	40	1	0.99837	1.5394	1.5369			40.07	

TABLE IIIA. Physical Constants of Various Materials

Ous						
Castor, at 30°C	0	1	0.972	1.580	1.536	0.26
α proportional to $f^{5/3b,d}$	10	1	0.960	1.536	1.474	0.16
	20	1	0.952	1.494	1.422	0.096
	30	1	0.946	1.452	1.374	0.057
	40	1	0.941	1.411	1.328	0.037
Phenylated silicone	0	1	1.124	1.446	1.625	
Dow-Corning No. 710	10	1	1.112	1.409	1.567	0.135
α proportional to f^2 to ~ 20 MHz ^{b,e}	20	1	1.102	1.378	1.518	0.070
	30	1	1.095	1.349	1.477	0.040
	40	1	1.089	1.321	1.438	0.024
Aluminum (rolled)			2.70	6.42	17.3	
Ceramics (approximate range)			2.5-3.4	4.6-6.8	12-18	
Glasses						
Borate crown (light)			2.24	5.10	11.4	
Pyrex (702)			2.32	5.64	13.1	
Silicate flint (heavy)			3.88	3.98	15.4	
Silica (fused)			2.2	5.97	13.1	
Stainless steel (347)			7.91	5.79	45.8	

- **

Material	Т	Po	ρ	v	$ ho_0 v$	α
To obtain results in:	°C	atm	g/cm ³	cm/sec	g/(cm ² sec)	Np/cm
Multiply figures in table by:	1	1	1	105	105	1
To obtain results in:		N/m^{2a}	kg/m³	m/sec	$kg/(m^2 sec)$	Np/m
Multiply figures in table by:		1.013×10^{5}	103	10 ³	106	10 ²
Central nervous system ^{g-i}					_	
Brain (average)	37	1	1.03	1.51	1.56	
Soft parenchymal tissues,						See Table IIIC
e.g., liver, kidney (average)	37	1	1.05	1.56	1.64	
Muscle (skeletal) ^{b,g,h}	37	1	1.07	1.57	1.68	0.13
Fat ^{b,h}	37	1	0.97	1.44	1.40	0.05
Bone						
Skull (human) ⁱ	37	1	1.7	3.36	6.0	
Frequency (MHz) 0.6		1				0.4
0.8		1				0.9
1.2		1				1.7
1.6		1				3.2
1.8		1				4.2
2.25		1				5.3
3.5		1				7.8

TABLE IIIB. Physical Constants of Biological Media⁴