The Bond and Money Markets:
Strategy, Trading, Analysis

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Moorad Choudhry

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Butterworth-Heinemann
Linacre House, Jordan Hill, Oxford OX2 8DP
225 Wildwood Avenue, Woburn, MA 01801-2041
A division of Reed Educational and Professional Publishing Ltd

A member of the Reed Elsevier plc group

First published 2001

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication Data
A catalogue record for this book is available from the Library of Congress

ISBN 0 7506 4677 2

Produced by ReadyText Publishing Services, Bath, Great Britain
Printed and bound in Great Britain by the Bath Press

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Foreword

The world's bond markets have a value of more than $30 trillion. They form a vital source of finance both for governments and companies. For investors they provide an invaluable home for capital, offering a range of risks and rewards, and yet they are little understood outside the arcane spheres in which bankers and brokers move. Further, the bond markets have grown enormously both in size and complexity in the last quarter of the twentieth century. Long gone are the days when Galsworthy's fictional Forsyte family were content to lodge their fortunes in "Consols", earning interest at 3 per cent a year, secure in the belief that both their capital and income was safe.

The development of the international bond markets, the advent of swaps and the arrival of sophisticated computer bond trading programs, are only a few of the changes over the last 25 years which have made the bond markets more intricate. More volatile interest rates and the abandonment of fixed exchange rates have introduced greater turbulence into the markets, requiring ever more agility from participants. The relative decline of bond issuance by governments –for the most part, at least among developed nations, regarded as highly unlikely to default on their debts –and the expansion of borrowing by corporate entities, has added a greater element of credit risk to the equation.

Indeed, a recent substantial and authoritative report by the investment bank Merrill Lynch concludes that the traditional risk-free asset presents challenges to investors and issuers alike. The report[1] is a most comprehensive analysis, which shows that the world bond market has experienced dramatic growth in its size and major shifts in its structure. The report highlights the extent of change in the current bond market world-wide, the US Fixed Income market, the divergent trends in government and corporate bonds and emerging market local currency debt securities.

Some of the most significant facts and developments in recent years have included the following:

- the global bond market maintains a robust rate of growth, while undergoing a dramatic shift in its composition;
- the world bond market grew 8.1% in 1999, due to a sharp increase in the non-government debt arena;
- the size of the world bond market at the end of 1999 was estimated to be US$ 31.1 trillion;
- government bonds continue to see their share of the world market fall, to a new low of 54% at the end of 1999, while corporate bonds and Eurobonds volumes rose to 42% of the world bond market capitalisation at the same time;
- bonds in the world's three largest currencies (US$, euro and Japanese yen) account for 88% of the total size of the world bond market;
- emerging market local currency debt securities are estimated at $1.2 trillion as of year-end 1999.

It is thus a particularly apposite time for the publication of this book. *The Bond and Money Markets* provides a wide-ranging and detailed examination of the global markets for debt capital. Starting from first principles, and proceeding to explain the technicalities of bond valuation and trading strategies, this book will be of particular benefit to newcomers to the subject as well as to more advanced students, and experienced practitioners.

Moorad Choudhry has the experience of a number of years working in the bond markets, which has given him the knowledge to write this book. I have known Moorad since 1992, when we worked together at Hoare Govett Securities Limited. Although initially a sterling bond house, it later became an UK gilt-edged market maker, thus becoming one of the few houses to cover all sterling bonds. Moorad traded the short-dated gilt-edged market as well as the Treasury book, a possibly almost unique experience and exposure to money markets, repo, off-balance sheet and bond trading all at the same time. From there he went on to proprietary trading at Hambros Bank Limited. This background is evident in the style and accessibility of his book, which has been written from a practitioner viewpoint, with an emphasis on clarity of approach. Readers will find much of interest and value within the following pages.

Sean Baguley
Director, Merrill Lynch

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[1] *Size and structure of the World Bond Market: The Decline of the Risk-Free Asset*, Merrill Lynch, April 2000.

Preface

This book is about the bond and money markets. This means that it is about the instruments used in the world's debt capital markets, because bonds are debt capital market products. One could stock an entire library with books about the fixed income markets and about how bonds are traded and analysed. In such a library one would also expect to find related books dealing with derivative instruments, bond portfolio management, technical analysis, financial market mathematics, yield curve modelling and so on. The subject matter is indeed a large one. In bringing together all the different strands into one volume one is forced to sacrifice some of the depth afforded by dedicated texts. However the purpose of this book is to present a fairly comprehensive and in-depth look at all aspects of the debt capital markets. Therefore while we start right from the beginning we hope, by the book's conclusion, to have presented the reader with most of the information required to be fully conversant with bonds and related derivatives, whether this is the terminology, analytical techniques, financial mathematics, or trading strategy.

The bond markets, also known as the fixed interest or *fixed income* markets, are an important part of the global financial markets, and a vital conduit through which capital is raised and invested.[1] Over the last two decades the growth in trading volumes has been accompanied by the introduction of ever more sophisticated financial engineering techniques, such that the bond markets today are made up of a large variety of structures. Banks can tailor packages to suit the most esoteric of requirements for their customers, so that bond cash flows and the hedging instruments available for holders of bonds can be far removed from the conventional fixed interest instruments that originally made up the market. Instruments are now available that will suit the needs of virtually all users of the financial markets, whether they are investors or borrowers.

The purpose of this book is to provide an introductory description and analysis of the bond markets as a whole. However we seek to leave the reader with sufficient information and worked examples to enable him or her to be at ease with all the different aspects of the markets. Hence we begin by considering conventional bonds and elementary bond mathematics, before looking at the array of different instruments available. This includes an overview of off-balance sheet instruments and their uses. We also consider the analytical techniques used by the markets, valuation of securities and basic trading and hedging strategy. We then develop the concepts further and look at constructing and managing portfolios, speculation and arbitrage strategies and hedging strategies. The basic principles apply in all bond markets around the world, but there are details differences across countries and currencies and so we also provide brief descriptions of some of the major bond markets. The exception to this is the United Kingdom government bond market, which is called the Gilt market, and which we look at in some detail.

One of the objectives behind writing this book was to produce something that had a high level of application to real-world situations, but maintained analytical rigour. We hope this objective has been achieved. There is no shortage of books in the market that are highly academic, perhaps almost exclusively so. Certain texts are essentially a collection of advanced mathematics. We have attempted to move seamlessly between academic principles and actual applications. Hence this book seeks to place every issue in context, and apply the contents to real-world matter. Where possible this is backed up by worked examples and case studies. Therefore the aim of this book is to be regarded as both an academic text as well as a practical handbook.

The capital markets

Capital markets is the term used to describe the market for raising and investing finance. The economies of developed countries and a large number of developing countries are based on financial systems that contain investors and borrowers, *markets* and trading arrangements. A market can be one in the traditional sense such as an exchange where *financial instruments* are bought and sold on a trading floor, or it may refer to one where participants deal with each other over the telephone or via electronic screens. The basic principles are the same in

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[1] The term "fixed income" is something of a misnomer. Originally bonds were referred to as fixed income instruments because they paid a fixed rate of interest on the nominal value or "face amount" of the bond. In the sterling markets bonds were called "fixed interest" instruments. For some time now many instruments in the market have not paid a fixed coupon, for example floating-rate notes and bonds where the pay-off is linked to another reference rate or index. Although the term is still used, the fixed income markets are in reality the entire debt capital markets, and not just bonds that pay a fixed rate of interest.

any type of market. There are two primary users of the capital markets, lenders and borrowers. The source of lenders' funds is, to a large extent, the personal sector made up of household savings and those acting as their investment managers such as life assurance companies and pension funds. The borrowers are made up of the government, local governments and companies (called corporates). There is a basic conflict in the financial objectives of borrowers and lenders, in that those who are investing funds wish to remain *liquid*, which means they have easy access to their investments. They also wish to maximise the return on their investment. A corporate on the other hand, will wish to generate maximum net profit on its activities, which will require continuous investment in plant, equipment, human resources and so on. Such investment will therefore need to be as long-term as possible. Government borrowing as well is often related to long-term projects such as the construction of schools, hospitals and roads. So while investors wish to have ready access to their cash and invest short, borrowers desire as long-term funding as possible. An economist referred to this conflict as the "constitutional weakness" of financial markets (Hicks 1946), especially when there is no conduit through which to reconcile the needs of lenders and borrowers. To facilitate the efficient operation of financial markets and the price mechanism, intermediaries exist to bring together the needs of lenders and borrowers. A bank is the best example of this. Banks accept deposits from investors, which make up the *liability* side of their balance sheet, and lend funds to borrowers, which form the *assets* on their balance sheet. If a bank builds up a sufficiently large asset and liability base, it will be able to meet the needs of both investors and borrowers, as it can maintain liquidity to meet investors' requirements as well as create long-term assets to meet the needs of borrowers. The bank is exposed to two primary risks in carrying out its operations, one that a large number of investors decide to withdraw their funds at the same time (a "run" on the bank), or that a large numbers of borrowers go bankrupt and default on their loans. The bank in acting as a financial intermediary reduces the *risk* it is exposed to by spreading and pooling risk across a wide asset and liability base.

Corporate borrowers wishing to finance long-term investment can raise capital in various ways. The main methods are:

- continued re-investment of the profits generated by a company's current operations;
- selling shares in the company, known as equity capital, equity securities or *equity*, which confirm on buyers a share in ownership of the company. The shareholders as owners have the right to vote at general meetings of the company, as well as the right to share in the company's profits by receiving dividends;
- borrowing money from a bank, via a bank loan. This can be a short-term loan such as an overdraft, or a longer term loan over two, three, five, years or even longer. Bank loans can be at either a fixed or more usually, variable rate of interest;
- borrowing money by issuing debt securities, in the form of *bonds* that subsequently trade in the debt capital market.

The first method may not generate sufficient funds, especially if a company is seeking to expand by growth or acquisition of other companies. In any case a proportion of annual after-tax profits will need to be paid out as dividends to shareholders. Selling further shares is not always popular amongst existing shareholders as it dilutes the extent of their ownership; there are also a host of other factors to consider including if there is any appetite in the market for that company's shares. A bank loan is often inflexible, and the interest rate charged by the bank may be comparatively high for all but the highest quality companies. We say comparatively, because there is often a cheaper way for corporates to borrow money: by tapping the bond markets. An issue of bonds will fix the rate of interest payable by the company for a long-term period, and the chief characteristic of bonds –that they are *tradeable* – makes investors more willing to lend a company funds.

Bond markets play a vital and essential role in raising finance for both governments and corporations. In 1998 the market in dollar-denominated bonds alone was worth over $11 trillion, which gives some idea of its importance. The basic bond instrument, which is a loan of funds by the buyer to the issuer of the bond, in return for regular interest payments up to the termination date of the loan, is still the most commonly issued instrument in the debt markets. Nowadays there is a large variety of bond instruments, issued by a variety of institutions. An almost exclusively corporate instrument, the international bond or Eurobond, is a large and diverse market. In 1998 the size of the Eurobond market was over $1 trillion.

In every capital market the first financing instrument that was ever developed was the bond; today in certain developing economies the government bond market is often the only liquid market in existence. Over time as financial systems develop and corporate debt and equity markets take shape, bond market retain their importance due to their flexibility and the ease with which (in theory!) transactions can be undertaken. In the advanced financial markets in place in developed countries today, the introduction of *financial engineering* techniques has greatly expanded the range of instruments that can be traded. These products include instruments used for *hedging* positions held in bonds and other *cash* products, as well as meeting the investment and *risk management* needs of a whole host of market participants. The debt capital markets have been and continue to be tremendously important to the economic development of all countries, as they have been the form of *intermediation* that allowed governments and corporates to finance their activities. In fact it is difficult to imagine long-term capital intensive projects such as those undertaken by say, petroleum, construction or aerospace companies, as well as sovereign governments, taking place without the existence of a debt capital market to allow the raising of vital finance.

Efficient markets

We often come across the term "free market", and economists refer to "the price mechanism". The role of the market in an economy is to allocate resources between competing interests in the most efficient way, and in a way that results in the resources being used in the most productive way. Where this takes place the market is said to be *allocatively efficient*. The term *operationally efficient* is used to describe a market where the *transaction costs* involved in trading are set competitively. Intermediaries in the capital markets do indeed determine their prices in relation to the competition, and because they depend on profits to survive there is always a cost associated with transacting business in the market. A market is described as *informationally efficient* if the price of any asset at any time fully reflects all available information that is available on the asset. A market that is allocatively, operationally and informationally efficient at the same time is *perfectly efficient*.

The concept of the efficient market was first described by Fama (1970). The *efficient markets hypothesis* is used to describe a market where asset prices fully reflect all available information. There are three types of the efficient markets hypothesis, which are:

- the *weak form*, which describes a situation where market prices reflect only historical data on the asset or security in questions;
- the *semi-strong form*, where prices reflect all publicly available information;
- the *strong form*, where prices reflect all known information, whether this information is publicly known or not.

The *weak-form efficient markets hypothesis* states that current market prices for assets fully reflects all information contained in the past history of asset prices. This implies therefore that historical prices provide no information on future prices of value to an investor seeking to make excess returns over the returns being earned by the market. Empirical evidence from market trading suggests that markets are indeed weak-form efficient, and that security prices incorporate virtually instantaneously all information reflected in past prices to enable investors to acquire any advantage.

The *semi-strong efficient markets hypothesis* states that current asset prices fully reflect all publicly available information about markets. If this is correct it means that any new information entering the public domain is incorporated almost instantaneously into the current price of the relevant security.[2] Once the security price has reacted to the new information there will be no more price changes as a result of that information. If markets are semi-strong efficient then this means that an investor who waits for the release of data before deciding which way to trade will be too late to make any gains. The evidence suggests that markets are also semi-strong efficient, and in fact security prices often change *before* the official release of information: this is because the markets have anticipated the content of the new data, through reading, say, media or brokers' reports, and have "priced in" the information accordingly. An example of this is where a central bank is expected to move interest rates a certain way; if the markets anticipate interest rates to be lowered and this view subsequently proves to be correct, the change in asset prices is much less than if the markets had guessed wrongly or were not expecting any change at all. So markets are not only weak-form but also semi-strong form efficient, and operating an investment policy of reacting to publicly available information will not generate returns that exceed those of the market itself.

[2] If it is macro-economic information the relevant securities could be all the stocks on the exchange, including the government bonds. If it is company specific information then it will probably be only that company's shares, or companies in the same sector.

The *strong-form efficient markets hypothesis* states that securities prices reflect all known information about the securities and the market, including information that is available only privately. If this is true it implies that market prices respond so quickly that an investor with private (that is, *inside*) information would not be able to trade and generate excess returns above the market rate. This will not usually be the case, since someone armed with inside information can usually generate excess profits. However insider trading is illegal in most countries, so this suggests that strong-form inefficiency exists only through illegal activity. In recent years evidence has indicated that markets may be strong-form efficient as well, in the activities of fund managers. Where a portfolio of assets is actively managed by a fund manager with the objective of outperforming the market but does not, it indicates that even the possession of privately held information is insufficient to generate excess returns. We say "private" because fund managers undertake a large amount of research on companies and markets, the results of which are not available publicly. Over the years certain "active" fund managers have been much criticised for failing to outperform or even underperforming, the market. This has resulted in the popularity of "passive" fund managers, who simply structure their portfolios to replicate the constituents of the market, hoping to match overall market performance.[3] So the growth of passive fund managers would seem to indicate that more and more investors believe markets to be strong-form efficient, and that it is impossible, over the long-term, to outperform the market.

We hope that this initial discussion on capital markets and market efficiency has set the scene for the discussions that follow. It is always worth keeping in mind the context within which the bond markets operate, and that debt capital trading exists in order to facilitate the efficient allocation of resources.

Intended audience

This book is aimed at a wide readership, from those with little or no previous understanding of or exposure to the bond markets to experienced practitioners. The subject matter is wide ranging and this makes the book useful for undergraduate and postgraduate students on finance or business courses. The second half of the book will be valuable for advanced level students and first-year researchers. Undergraduate students are recommended to tackle the book after initially studying the principles of finance, however the basic concepts required (such as present and future value) are covered and serve to make a complete volume. While most of the mathematics assumes a knowledge of basic algebra, some of the contents, particularly in the chapters dealing with derivatives and fixed income analytics, will require slightly higher level mathematical ability. It is not necessary to have degree-level or even A-level maths in order to understand the basic principles; however those with only elementary maths may find some of the chapters, particularly those on yield curve modelling, somewhat difficult. Complete beginners may wish to review first an elementary text on financial market mathematics. Nevertheless this book is intended to serve as a complete text, and takes readers from the first principles to advanced analysis. Note that by this we mean analysis of the bond and related derivatives markets; the budding rocket scientists among you may wish to consider books specifically concerned with say, option pricing, stochastic calculus or programme trading. For students wishing to enter a career in the financial services industry this book has been written to provide sufficient knowledge and understanding to be useful in their first job and beyond, thus enabling anyone to hit the ground running. It is also hoped that the book remains useful as a reference handbook.

The book is primarily aimed at people who work in the markets, including front office, middle office and back office banking and fund management staff who are involved to some extent in fixed interest markets. This includes traders, salespersons, money markets dealers, fund managers, stockbrokers and research analysts. Others including corporate and local authority treasurers, risk management personnel and operations staff will also find the contents useful, as will professionals who work in structured finance and other market sectors, such as accountants, lawyers and corporate financiers. As a source of reference the book should be valuable reading for management consultants and financial sector professionals, such as tax, legal and corporate finance advisors, and other professionals such as financial sector auditors and journalists.

The book is also aimed at postgraduate students and students sitting professional exams, including MBA students and those specialising in financial markets and financial market economics. Undergraduate students of business, finance or the securities markets will hopefully find the book to be a useful source of reference, while practitioners sitting the exams of various professional bodies may observe that there is much useful practical information that will help them to apply their studies to their daily work.

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[3] The rise of so-called "tracker" funds.

Comments on the text are welcome and should be sent to the author care of Butterworth-Heinneman. We apologise for any errors that are lurking in the text, and would appreciate being made aware of any that the reader might find. The author also welcomes constructive suggestions for improvement which we hope to incorporate in a second edition.

Organisation of the book

This book is organised into 12 parts. Each part introduces and then develops a particular aspect of the debt capital markets. Part I is the introduction to bonds as a debt market instrument. The ten chapters in Part I cover the basics of bonds, bond pricing and yield measurement, and interest-rate risk. There is also an initial look at the different types of bond instruments that trade in the market. For beginners, there is also a chapter on financial markets mathematics. Part II looks in detail at two government bond markets, the United Kingdom gilt market and the United States Treasury market. There is also a chapter looking briefly at selected government bond markets around the world. The type of subjects covered include market structure, the way bonds are issued and specific detail on the structure of the different markets.

The seventeen chapters in Part III look in some detail at the corporate debt markets. This sector of the bond markets is extremely diverse, and it is often in corporate markets that the latest and most exciting innovations are found. Some of the instruments used in the corporate markets demand their own particular type of analysis; to this end we review the pricing and analytics of callable bonds, asset-backed bonds and convertibles, among others. There is also a chapter on credit analysis.

In Part IV we review the money markets. The money markets are part of the debt markets and there is a relationship between the two, although as we shall see the money market yield curve sometimes trades independently of the bond yield curve. We cover in detail some related issues, which would probably be at home more in a book about banking than bond trading; these includes bank capital requirements, asset and liability management (ALM) and the bond repurchase or *repo* market. Derivative instruments such as futures contracts play an important part in the money markets, and it was decided to include the chapter on money market derivatives in Part IV rather than in the section on derivatives, as it was felt that this would make Part IV complete in its own right.

In the capital markets and banking generally, risk management is a keenly-debated topic. A book dealing with capital markets trading would not be complete without a look at this topic, which is considered in Part V. We also cover one of the main risk management tools used today, the measurement methodology known as *value-at-risk*.

Part VI is a comprehensive review of derivative instruments. There are separate chapters on futures, swaps and options. Readers who have had only an introduction to this subject may find some of the chapters a little trying, particularly those dealing with stochastic processes and option pricing. We recommend perseverance, as the subject has been reviewed and summarised in a way that should be accessible to most, if not all. The mathematics has been kept to a minimum, and in most cases proofs and derivations are taken as given and omitted. The interested reader is directed to relevant texts that supply this detail in a bibliography at the end of each chapter.

Part VII is composed of a single chapter only, which deals with elementary trading and hedging strategy.

Part VIII on advanced fixed income analytics is the author's favourite and deals with a particularly exciting subject, interest-rate models and yield curve modelling. The main models that have emerged from leading academic writers are introduced, explained and summarised. We also cover fitting the yield curve, and there are additional chapters dealing with advanced analytics regarding index-linked bonds and the pricing of long-dated bonds.

The remainder of the book deals with related topics. In Part IX we consider portfolio management, essentially only the main strategies and techniques. There is also a chapter on constructing bond indices. Part X is another one that contains just one chapter; it sits on its own as it deals with technical analysis or "charting". In Part XI we introduce credit derivatives, which are relatively new instruments but are rapidly becoming an important part of the bond markets. The content is introductory however, and in fact there are a number of excellent texts on credit derivatives appearing in the market. The final part of the book looks at emerging markets and Brady bonds, and the additional considerations involved in investing across international markets. We conclude with a look at some likely future developments.

Study materials

Where possible the main concepts and techniques have been illustrated with worked examples and case studies. The case studies are examples of actual real-world happenings at a number of investment banks.

Questions and exercises are provided at the end of each chapter, which are designed to test readers' understanding of the material. Most of the questions can be answered using the content of the preceding chapters. Answers to the questions are available from the publishers for those involved in a teaching or lecturing capacity. Where appropriate a bibliography and list of selected references is provided so that the interested reader has a starting point for further research.

Some of the content in this book has been used to form part of bond market courses taught at a number of professional bodies and teaching institutions. This includes the material on the introduction to bonds, the gilt market, the repo market, value-at-risk, and advanced fixed income analytics. For these topics a number of Microsoft PowerPoint slides are available for use as teaching aids, and may be downloaded from the author's Web site at **www.mchoudhry.co.uk**. This Web site also contains details of training courses that are available on advanced bond market topics, run by the author and his associates.

Acknowledgements

As you might expect with a work such as this, various individuals have helped out both before and during the endeavour and I'd like to take this opportunity to thank them. First up there's Sherif who provided all that IT consulting advice, especially with regard to the kit this book was written on. Thanks, I really appreciated it. I've no benchmark to compare you with this time though, no Maurice Deebank, but I'm sure the advice was tops. And by the way, your work on "The Day After Tomorrow" was your best ever, it brings a smile to my face every time I hear it. I am equally indebted to Anuk Teasdale for her help with the graphics and illustrations used in the text, and also for her example of an hypothetical bond offering circular. John Paul George Bank indeed! Why not Ringo? Without her help many of the chapters would not have been possible.

My very best, as always, to Olga the Scottish wildcat, both elusive and exclusive…

I would like to thank Martin Barber for his continued help and support, and for inspiring me to do better. Thanks for everything Barberman, and also for agreeing with me that Mark Ramprakash should be England captain. For reviewing the book's manuscript, making constructive comments for improvement and being general all-round good eggs I'd like to thank Rod Pienaar at Deutsche Bank, and Richard Pereira at Dresdner Kleinwort Wasserstein, two top lads, and John Lenton, a true gent, excellent teacher and top Treasury man, latterly with King & Shaxson discount house and Woolwich plc. In particular Richard helped out with some of the more complex mathematics and structures, particularly the chapters on option pricing, yield curve modelling and stochastic calculus, and this was much appreciated. Any remaining errors are of course my own.

A very special thank you and best wishes to Sean Baguley, a truly splendid chap, for all his help in the past (including recruiting me into the market!) and for writing the excellent foreword.

For all kinds of assistance over the years I'd like to thank Derek Taylor at King & Shaxson Bond Brokers Limited –who knows everything there is to know about the sterling market, thanks very much Del-boy for all your help, (by the way, say hello to David Franklin for me, I did enjoy the morning joust with him and his team), and Ray Bieber at Garban ICAP, another sterling markets expert.

I'd like to thank Martin Davies for his help during the time he was at GNI, and later when he was at Yamaichi Securities, when together we were "the dream team," especially as I've since lost touch with him. If you're reading this Martin, please give me a call. (I'm still hoping for that invite to go and watch the Palace!) Thanks also to the head of GNI's gilts booth on the LIFFE floor in those days, Big John Howell, the chap in their short sterling booth, Ed Hardman, and Mr Tony Beard (now retired, previously at King & Shaxson discount house). Thanks to Ian Lincoln, now at Tullet & Tokyo I believe, for starting the trend in the sterling market of calling me "the Doctor", years before I ever started research towards my PhD! He was taking the mick, I'm sure…

For academic help in the past and currently I'd like to thank Matthew McQueen at the Department of Economics, University of Reading, Kerstin Barkman, Vicki Randall and Robin Theobald at the University of Westminster and Dr Ian Turner at Henley Management College. Mr McQueen especially first got me interested in writing so he's ultimately responsible! Thanks to Dr Christine Oughton at Birkbeck College, University of London for helping me to introduce some academic rigour into my research work.

A special thanks to Mike Cash and Nicki Kear, two very pleasant and charming individuals, and everyone at Butterworth-Heinemann. A big thank you to Graham Douglas, or "Captain Quirk", who did the editorial project management, a mammoth task. He's a perfectionist in the George Martin mould. In addition, Graham gets my thanks for deriving the Choudhry panic function, which describes emotional state leading up to publication.[4]

My appreciation to the following people for general assistance and providing prompt permission to reproduce various items in the book: Nimish Thakker at LIFFE, Ian Collett and Alison James at Garban InterCapital, Laura MacIntyre at Bridge Telerate, Peter Thomas at Reuters, Kim Pompilii at the Bank of England, Joyce Ruby at Bridge-Telerate, Andy Albert at Asset-Backed Alert, Bill Curtin at the US Bureau of the Public Debt, Christina Papavasiliou at the Copyright Clearance Centre in the USA, Linda Nicol at Cambridge University Press, and Mark Deacon and Steve Whiting at the United Kingdom Debt Management Office. Also at the DMO, Gurminder Bhachu helped out with detail on the gilts market, for which many thanks. The people at Bloomberg were also very prompt, especially Tibor Szigeti. But a collective thanks to all who work at that impressive organisation.

Parts of this book were originally written for the bond markets course run by the Securities Institute in London. My thanks to Zena Deane at the Institute for giving me the opportunity to teach this course, and for pointing out the possibility of getting some of my material published. I must mention Stafford Bent, because I said I would!

Thanks to Phil "the Mod" Broadhurst, who can write better than anyone I know. I'd like to thank big time Alan Fulling for expert football coaching, he knows how to motivate a team, and my man Tony Holloway, always inspired to aim higher. Respect. Additional thanks to two long-standing friends, Clax, and Nik Slater.

A very big thank you, best wishes and warm regards to Dr Didier Joannas, the finest rocket scientist in the markets today (its "Goldfinger", remember Dids!), Peter Matthews, from whom I learnt a great deal, Tony Alderman (ditto), Peter Capel, "Harry" Cross, for composing that desk song on icing stock (and who knows the long end of the curve as well as anyone I've seen), Kevin Gaffney, a top chap to be a junior trader to, Graham Leach and Jim Harrison (your anecdote about the jobber on the old stock exchange floor is surely the best), all of whom were formerly with ABN AMRO Hoare Govett Sterling Bonds Limited or Hoare Govett Securities Limited, for all the times that they pointed out intricacies of bond trading that one can't learn by reading a book! As for Nigel Marsh, well, just stay away from those relative value trades! My very best wishes gentlemen.

For inspiration I'd like to thank Derek Wanless, Stephan Harris, Luke Ding, Mr Frank Fabozzi, Kal Patel, Jonathan Ingersoll, Sheldon Ross, Bruce Foxton, Martin Hewes, Andy Rourke, Mike Joyce, and Simon Barnes' writing in *The Times*. Out in a world of excellence and emotion of their own, thanks for inspiration to Lloyd Cole, Neil Clarke, Blair Cowan, Lawrence Donegan and Stephen Irvine.

Finally thanks to you for taking the time to read this book. The bond markets are an incredibly fascinating and exciting subject, as well as being extremely dynamic, and I have had tremendous fun writing about them and all the market instruments. It is certainly a subject that one could spend endless fascinating days discussing and debating about. I hope my enthusiasm has carried over onto the pages and that, having digested the contents, the reader will carry on his or her research and knowledge gathering to greater heights. I sincerely hope that this book has contributed to a greater understanding of and familiarity with the debt capital and derivatives markets for all of you, for both the sterling markets and the global debt markets. If readers spot any errors (and there will be a fair few I'm sure!) or have any other comments do please write to me care of the publishers and let me know – I very much look forward to hearing from you.

Moorad Choudhry
Surrey, England
May 2000

[4] $P(t, Publication\ date) = \lambda_{Moorad} e^{1/(Publication\ date\ - t)}$, where λ_{Moorad} is a constant.

About the author

Moorad Choudhry is a vice-president with JPMorgan in London. He began his career in 1989 at the London Stock Exchange, before joining the sterling Eurobond desk at Hoare Govett Fixed Interest. He was later employed as a gilt-edged market maker and treasury trader at ABN Amro Hoare Govett Sterling Bonds Limited, where he ran the short-dated gilt book, the gilt repo book and the sterling money markets book, and was also responsible for stock lending and interbank funding. From there he moved on to Hambros Bank Limited, where he set up and ran the Treasury division's sterling proprietary trading desk. He then worked as a strategy and risk management consultant to some of the world's leading investment banks, before joining JPMorgan in March 2000.

Moorad has an MA in Econometrics from the University of Reading and an MBA from Henley Management College. He has taught courses on bond and money markets subjects for organisations both in the City of London and abroad, including the International Faculty of Finance and FinTuition Limited, and has lecturered at City University Business School. He is a Fellow of the Securities Institute and a member of the Global Association of Risk Professionals, and previously sat on the supervisory committee of the Co-operative Society's "rainbow" credit union. He currently sits on the Securities Institute Diploma examination panel.

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Moorad was born in Bangladesh but moved at an early age to Surrey in the United Kingdom, where he lives today.

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Part I Introduction to the Bond Markets

We begin by describing the main instruments that go to make up the bond markets. So in Part I we explain the structure of bonds, and the variety of instruments available. This includes bond pricing and yield, and an initial look at the yield curve. Chapter 6 on the yield curve is a fairly long one and looks not only at the different types of yield curve that may be encountered, but also the issue of spot and forward interest rates, and how to interpret the shape of the yield curve. The remaining four chapters consider interest-rate risk, namely duration, modified duration and convexity, and how these measures are used to analyse and manage bond market risk.

"What's the secret, Sean?"
"Buy cheap, sell dear…!"

1 The Debt Capital Markets

Readers will be familiar with the cursory slot on evening television news, where the newscaster informs viewers where the main stock market index closed that day and where key foreign exchange rates closed at. In the United States most bulletins go one better and also tell us at what yield the Treasury long bond closed at. This is because bond prices are affected directly by economic and political events, and yield levels on certain government bonds are fundamental indicators of the economy. The yield level on the US Treasury long bond reflects the market's view on US interest rates, inflation, public sector debt and economic growth. Reporting the bond yield level reflects the importance of the bond market to a country's economy, as important as the level of the equity stock market.

Bond and shares form part of the *capital markets*. Shares are *equity capital* while bonds are *debt capital.* So bonds are a form of debt, much like how a bank loan is a form of debt. Unlike bank loans however bonds can be *traded* in a market. A bond is a debt capital market instrument issued by a borrower, who is then required to repay to the lender/investor the amount borrowed plus interest, over a specified period of time. Bonds are also known as *fixed income* instruments, or *fixed interest* instruments in the sterling markets. Usually bonds are considered to be those debt securities with terms to maturity of over one year. Debt issued with a maturity of less than one year is considered to be *money market* debt. There are many different types of bonds that can be issued. The most common bond is the *conventional* (or *plain vanilla* or *bullet*) *bond.* This is a bond paying regular (annual or semi-annual) interest at a fixed rate over a fixed period to maturity or redemption, with the return of *principal* (the par or nominal value of the bond) on the maturity date. All other bonds will be variations on this.

A bond is therefore a financial contract, in effect an IOU from the person or body that has issued the bond. Unlike shares or equity capital, bonds carry no ownership privileges. An investor who has purchased a bond and thereby lent money to an institution will have no voice in the affairs of that institution and no vote at the annual general meeting. The bond remains an interest-bearing obligation of the issuer until it is repaid, which is usually the maturity date of the bond. The issuer can be anyone from a private individual to a sovereign government.[1]

There is a wide range of participants involved in the bond markets. We can group them broadly into borrowers and investors, plus the institutions and individuals who are part of the business of bond trading. Borrowers access the bond markets as part of their financing requirements; hence borrowers can include sovereign governments, local authorities, public sector organisations and corporates. Virtually all businesses operate with a financing structure that is a mixture of debt and equity finance. The debt finance almost invariably contains a form of bond finance, so it is easy to see what an important part of the global economy the bond markets are. As we shall see in the following chapters, there is a range of types of debt that can be raised to meet the needs of individual borrowers, from short-term paper issued as part of a company's cash flow requirements, to very long-dated bonds that form part of the financing of key projects. An example of the latter was the recent issue of 40-year bonds by London and Continental Railways to finance the Channel Tunnel rail link, and guaranteed by the United Kingdom government. The other main category of market participant are investors, those who lend money to borrowers by buying their bonds. Investors range from private individuals to fund managers such as those who manage pensions funds. Often an institution will be active in the markets as both a borrower and an investor. The banks and securities houses that facilitate trading in bonds in both the *primary* and *secondary* markets are also often observed to be both borrowers and investors in bonds. The bond markets in developed countries are large and *liquid*, a term used to describe the ease with which it is possible to buy and sell bonds. In emerging markets a debt market usually develops ahead of an equity market, led by trading in government *bills* and bonds. This reflects the fact that, as in developed economies, government debt is usually the largest volume debt in the domestic market and the highest quality paper available.

The different types of bonds in the market reflect the different types of issuers and their respective requirements. Some bonds are safer investments than others. The advantage of bonds to an investor is that they represent a

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1 The musician David Bowie has issued bonds backed with royalties payable from purchases of his back catalogue. Governments have issued bonds to cover expenditure from early times, such as the issue by King William in 1694 to pay for the war against France, in effect the first United Kingdom gilt issue. That was also the year the Bank of England was founded.

fixed source of current income, with an assurance of repayment of the loan on maturity. Bonds issued by developed country governments are deemed to be guaranteed investments in that the final repayment is virtually certain. In the event of default of the issuing entity, bondholders rank above shareholders for compensation payments. There is lower risk associated with bonds compared to shares as an investment, and therefore almost invariably a lower return over the longer term.

We can now look in more detail at some important features of bonds.

1.1 Description

We have said that a bond is a debt instrument, usually paying a fixed rate of interest over a fixed period of time. Therefore a bond is a collection of cash flows and this is illustrated at Figure 1.1. In our hypothetical example the bond is a six-year issue that pays fixed interest payments of C% of the *nominal* value on an annual basis. In the sixth year there is a final interest payment and the loan proceeds represented by the bond are also paid back, known as the maturity proceeds. The amount raised by the bond issuer is a function of the price of the bond at issue, which we have labelled here as the issue proceeds.

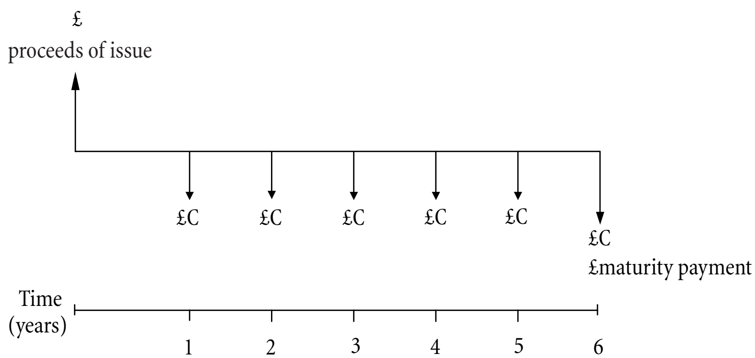

Figure 1.1: Cash flows associated with a six-year annual coupon bond.

The upward facing arrow represents the cash flow paid and the downward facing arrows are the cash flows received by the bond investor. The cash flow diagram for a six-year bond that had a 5% fixed interest rate, known as a 5% *coupon*, would show interest payments of £5 per every £100 of bonds, with a final payment of £105 in the sixth year, representing the last coupon payment and the redemption payment. Again, the amount of funds raised per £100 of bonds depends on the price of the bond on the day it is first issued, and we will look into this later. If our example bond paid its coupon on a semi-annual basis, the cash flows would be £2.50 every six months until the final redemption payment of £102.50.

Let us examine some of the key features of bonds.

- **Type of issuer** A primary distinguishing feature of a bond is its issuer. The nature of the issuer will affect the way the bond is viewed in the market. There are four issuers of bonds: sovereign governments and their agencies, local government authorities, supranational bodies such as the World Bank and corporations. Within the corporate bond market there is a wide range of issuers, each with differing abilities to satisfy their contractual obligations to lenders. The largest bond markets are those of sovereign borrowers, the government bond markets. The United Kingdom government issues bonds known as *gilts*. In the United States government bonds are known as *Treasury Notes* and *Treasury Bonds*, or simply Treasuries.
- **Term to maturity** The *term to maturity* of a bond is the number of years after which the issuer will repay the obligation. During the term the issuer will also make periodic interest payments on the debt. The *maturity* of a bond refers to the date that the debt will cease to exist, at which time the issuer will redeem the bond by paying the principal. The practice in the market is often to refer simply to a bond's "term" or "maturity". The provisions under which a bond is issued may allow either the issuer or investor to alter a bond's term to maturity after a set notice period, and such bonds need to be analysed in a different way. The term to maturity is an important consideration in the make-up of a bond. It indicates the time period over which the bondholder can expect to receive the coupon payments and the number of years before the principal will be paid in full. The

bond's *yield* is also depends on the term to maturity. Finally, the price of a bond will fluctuate over its life as yields in the market change and as it approaches maturity. As we will discover later, the *volatility* of a bond's price is dependent on its maturity; assuming other factors constant, the longer a bond's maturity the greater the price volatility resulting from a change in market yields.

- **Principal and coupon rate** The *principal* of a bond is the amount that the issuer agrees to repay the bondholder on the maturity date. This amount is also referred to as the *redemption value, maturity value*, *par value*, *nominal value* or *face amount*, or simply *par*. The *coupon rate* or *nominal rate* is the interest rate that the issuer agrees to pay each year. The annual amount of the interest payment made is called the *coupon.* The coupon rate multiplied by the principal of the bond provides the cash amount of the coupon. For example a bond with a 7% coupon rate and a principal of £1,000,000 will pay annual interest of £70,000. In the United Kingdom, United States and Japan the usual practice is for the issuer to pay the coupon in two semi-annual instalments. For bonds issued in European markets and the Eurobond market coupon payments are made annually. On rare occasions one will encounter bonds that pay interest on a quarterly basis. Certain bonds pay monthly interest. All bonds make periodic interest payments except for *zero-coupon bonds.* These bonds allow a holder to realise interest by being sold substantially below their principal value. The bonds are redeemed at par, with the interest amount then being the difference between the principal value and the price at which the bond was sold. We will explore zero-coupon bonds in greater detail later.
- **Currency** Bonds can be issued in virtually any currency. The largest volume of bonds in the global markets are denominated in US dollars; other major bond markets are denominated in euros, Japanese yen and sterling, and liquid markets also exist in Australian, New Zealand and Canadian dollars, Swiss francs and other major currencies. The currency of issue may impact on a bond's attractiveness and liquidity which is why borrowers in developing countries often elect to issue in a currency other than their home currency, for example dollars, as this will make it easier to place the bond with investors. If a bond is aimed solely at a country's domestic investors it is more likely that the borrower will issue in the home currency.

1.2 Bond issuers

In most countries government expenditure exceeds the level of government income received through taxation. This shortfall is made up by government borrowing and bonds are issued to finance the government's debt. The core of any domestic capital market is usually the government bond market, which also forms the benchmark for all other borrowing. Figure 1.2 illustrates UK gilt price and yield quotes as listed in the *Financial Times* for 23 July 1999.

Gilts are identified by their coupon rate and year of maturity; they are also given names such as *Treasury* or *Exchequer*. There is no significance attached to any particular name, all gilts are equivalent irrespective of their name. From Figure 1.2 we see that the 5¾% 2009 stock closing price from the day before was 104.79, which means £104.79 of par value. (Remember that par is the lump sum paid at maturity.) This price represents a *gross redemption yield* of 5.15%. If we pay £104.79 per £100 of stock today, we will receive £100 per £100 of stock on maturity. At first sight this appears to imply we will lose money, however we also receive coupon payments every six months, which for this bond is £2.875 per £100 nominal of stock. Also from Figure 1.2 we see the change in price from the day before for each gilt; in the case of the 5¾% 2009 the price was up 0.18 from the previous day's closing price.

Government agencies also issue bonds. Such bonds are virtually as secure as government bonds. In the United States agencies include the Federal National Mortgage Association. Local authorities issue bonds as part of financing for roads, schools, hospitals and other capital projects.

Corporate borrowers issue bonds both to raise finance for major projects and also to cover ongoing and operational expenses. Corporate finance is a mixture of debt and equity and a specific capital project will often be financed as a mixture of both.

UK GILTS PRICES

	Notes	Yield Int	Yield Red	Price £	+ or –	52 week High	52 week Low
Shorts" (Lives up to Five Years)							
Treas 6pc 1999		6.00	4.91	100.05	–.01	100.50	98.70
Treas 2½pc 1999	✠	1.26	–	198.90	–.02	199.60	191.47
Conv 10¼pc 1999		10.07	4.79	101.76	–.02	105.37	101.76
Treas 8½pc 2000	✠	8.35	4.81	101.85xd	–.03	103.54	101.81
Conv 9pc 2000		8.78	4.77	102.52	–.02	104.38	102.52
Treas 13pc 2000		12.08	4.91	107.61	–.05	113.28	107.61
Treas 8pc 2000		7.70	5.05	103.86	–.06	105.84	102.24
Treas Fltg Rate 2001		–	0.32	100.35	–.09	100.82	100.30
Treas 10pc 2001		9.34	5.28	107.12	–.05	110.37	107.11
Conv 9½pc 2001	✠	8.81	5.26	107.84	–.05	110.89	106.73
Conv 9¾pc 2001	✠	8.98	5.25	108.63	–.05	111.80	107.58
Treas 7pc 2001		6.78	5.46	103.25	–.06	106.51	100.83
Conv 10pc 2002	✠	9.03	5.66	110.79	–.08	116.41	110.79
Treas 7pc 2002		6.74	5.52	103.89	–.06	108.13	101.52
Conv 9½pc 2002	✠	8.63	5.65	110.14	–.08	115.64	109.81
Treas 9¾pc 2002		8.74	5.64	111.53	–.08	117.29	111.19
Exch 9pc 2002	✠	8.18	5.63	110.07	–.08	115.62	108.87
Conv 9¾pc 2003	✠	8.56	5.61	113.94	–.08	120.19	112.78
Treas 8pc 2003		7.37	5.53	108.51	+.05	114.74	106.70
Treas 10pc 2003		8.62	5.60	116.02	+.04	123.52	115.58
Treas 13¾pc 2000–3	✠	12.66	4.90	108.58xd	–.05	115.00	108.58
Treas 6½pc 2003		6.26	5.50	103.84	+.06	110.21	101.10
Treas 11½pc 2001–4		10.49	5.34	109.64	–.04	113.90	109.64
Treas 10pc 2004	✠	8.44	5.59	118.42	+.04	126.55	117.36
Five to Fifteen Years							
Treas 5pc 2004		5.10	5.45	98.10	+.07	98.80	98.03
Funding 3½pc 1999–4		3.77	5.17	92.76	+.05	98.98	91.80
Conv 9½pc 2004		8.06	5.53	117.90	+.08	126.43	116.58
Treas 6¾pc 2004		6.38	5.49	105.75	+.09	113.28	103.09
Conv 9½pc 2005		7.97	5.54	119.25	+.13	128.16	117.84
Exch 10½pc 2005	✠	8.36	5.53	125.62	+.13	135.46	124.50
Treas 12½pc 2003–5		9.90	5.58	126.26	+.04	135.55	126.22
Treas 8½pc 2005		7.32	5.48	116.06	+.10	125.31	113.85
Conv 9¾pc 2006	✠	7.76	5.46	125.57	+.16	136.21	123.98
Treas 7¾pc 2006		6.85	5.49	113.19	+.12	122.40	110.47
Treas 8pc 2002–6		7.49	5.66	106.75	–.10	111.73	105.05
Treas 7½pc 2006		6.68	5.46	112.27	+.13	121.62	109.33
Treas 11¾pc 2003–7	✠	9.85	5.61	119.25xd	–.08	126.29	119.25
Treas 8½pc 2007		7.12	5.46	119.44	+.16	129.86	116.89
Treas 7¼pc 2007		6.46	5.41	112.27	+.17	122.55	109.27
Treas 13½pc 2004–8	✠	10.19	5.52	132.47	+.08	142.61	132.39
Treas 9pc 2008		7.12	5.34	126.37	+.12	138.52	123.05
Treas 8pc 2009		6.59	5.25	121.43	+.19	132.91	117.03
Treas 5¾pc 2009		5.49	5.15	104.79	+.18	114.67	99.36
Treas 6¼pc 2010		5.74	5.21	108.83	+.02	118.76	103.24
Conv 9pc Ln 2011		6.77	5.25	132.99	–.01	145.31	127.66
Treas 9pc 2012		6.66	5.24	135.19	–.05	147.74	129.19
Treas 5½pc 2008–12		5.38	5.18	102.29	+.22	112.26	97.60
Treas 8pc 2013		6.25	5.18	128.08	–.10	139.64	120.83
Treas 7¾pc 2012–15		6.36	5.32	121.95xd	+.04	133.56	116.74
Over Fifteen Years							
Treas 8pc 2015		6.04	5.05	132.54	–.29	144.12	123.43
Treas 8¾pc 2017		6.06	5.00	144.36	–.41	156.08	133.09
Exch 12pc 2013–17	✠	7.17	5.26	167.45	–.01	183.80	161.32
Treas 8pc 2021		5.63	4.85	142.11	–.67	153.21	127.51
Treas 6pc 2028		4.94	4.65	121.48	–.63	131.17	104.17
Undated							
Consols 4pc		5.13	–	77.98	–.54	87.19	64.75
War Loan 3½pc		4.95	–	70.73	–.55	79.83	59.16
Conv 3½pc '61 Aft.		4.23	–	82.73	–.55	95.68	76.16
Treas 3pc '66 Aft.		5.39	–	55.70	–.42	61.93	46.98
Consols 2½pc		4.93	–	50.70	–.42	56.93	41.98
Treas. 2½pc		5.00	–	50.00	–1.18	55.17	41.20

	Notes	Yield (1)	Yield (2)	Price £	+ or –	52 week High	52 week Low
Index–Linked	(b)						
2½pc '01	(78.3)	2.78	3.33	204.26	–.06	206.40	198.10
2½pc '03	(78.8)	2.38	2.68	205.29	–.13	207.83	195.70
4⅜pc '04	(135.6)	1.88	2.12	133.34	–.08	134.77	126.55
2pc '06	(69.5)	1.62	1.79	237.40xd	–.23	239.80	212.21
2½pc '09	(78.8)	1.75	1.88	218.00	–.28	221.45	192.71
2½pc '11	(74.6)	1.88	1.99	230.05	–.37	235.62	202.19
2½pc '13	(89.2)	1.88	1.97	194.19	–.31	199.07	168.63
2½pc '16	(81.6)	1.86	1.94	215.25xd	–.31	221.10	183.37
2½pc '20	(83.0)	1.84	1.91	215.71	–.25	221.68	178.82
2½pc '24	(97.7)	1.81	1.87	187.39	–.26	193.74	151.30
4⅛pc '30	(135.1)	1.76	1.81	185.99xd	–.28	193.23	149.11

Prospective real redemption rate on projected inflation of (1) 5% and (2) 3%. (b) Figures in parentheses show RPI base for indexing (ie 8 months prior to issue) and have been adjusted to reflect rebasing of RPI to 100 in February 1987. Conversion factor 3.945. RPI for November 1998: 164.4 and for June 1999: 165.6.

Other Fixed Interest

	Notes	Yield Int	Yield Red	Price £	+ or –	52 week High	52 week Low
Asian Dev 10¼pc 2009		7.89	6.11	129⅞	....	140 27/32	128¼
B'ham 11½pc 2012		7.93	6.40	145	....	158¾	145
Leeds 13½pc 2006		9.78	–	138	....	152	138
Liverpool 3½pc Irred.		5.83	–	60	....	70½	55
LCC 3pc '20 Aft.		5.88	–	51	....	59	47
Manchester 11½pc 2007		8.58	6.28	134	....	151¾	132
Met. Wtr. 3pc 'B'		3.41	6.45	88	....	95	85
N'wide Anglia 3⅞pc IL 2021		–	3.17	188 9/16xd	....	192 13/32	172¼
4¼pc IL 2024		–	3.13	182	....	187½	164⅞

● 'Tap' stock. All UK Gilts are tax-free to non-residents on application. E Auction basis. xd Ex dividend. Closing mid-prices are shown in pounds per £100 nominal of stock. Prospective real Index-Linked redemption yields are calculated by HSBC Greenwell from Gemma closing prices. ✠ Indicative price.

Figure 1.2: UK gilts prices page. Reprinted from *The Financial Times*, 23 July 1999. © *Financial Times*. Used with permission.

1.3 Capital market participants

The debt capital markets exist because of the financing requirements of governments and corporates. The source of capital is varied, but the total supply of funds in a market is made up of personal or household savings, business savings and increases in the overall money supply. Growth in the money supply is a function of the overall state of the economy, and interested readers may wish to consult the bibliography at the end of this chapter which includes several standard economic texts. Individuals save out of their current income for future consumption, while business savings represent retained earnings. The entire savings stock represents the capital available in a market. As we saw in the preface however the requirements of savers and borrowers differs significantly, in that savers have a short-term investment horizon while borrowers prefer to take a longer term view. The "constitutional weakness" of what would otherwise be *unintermediated* financial markets led, from an early stage, to the development of financial intermediaries.

1.3.1 Financial Intermediaries

In its simplest form a financial intermediary is a *broker* or *agent*. Today we would classify the broker as someone who acts on behalf of the borrower or lender, buying or selling a bond as instructed. However intermediaries originally acted between borrowers and lenders in placing funds as required. A broker would not simply on-lend funds that have been placed with it, but would accept deposits and make loans as required by its customers. This resulted in the first banks. A *retail bank* deals mainly with the personal financial sector and small businesses, and in addition to loans and deposits also provides cash transmission services. A retail bank is required to maintain a minimum cash reserve, to meet potential withdrawals, but the remainder of its deposit base can be used to make loans. This does not mean that the total size of its loan book is restricted to what it has taken in deposits: loans can also be funded in the wholesale market. An *investment bank* will deal with governments, corporates and institutional investors. Investment banks perform an agency role for their customers, and are the primary vehicle through which a corporate will borrow funds in the bond markets. This is part of the bank's corporate finance function; it will also act as wholesaler in the bond markets, a function known as *market making*. The bond issuing function of an investment bank, by which the bank will issue bonds on behalf of a customer and pass the funds raised to this customer, is known as *origination*. Investment banks will also carry out a range of other functions for institutional customers, including export finance, corporate advisory and fund management.

Other financial intermediaries will trade not on behalf of clients but for their own *book*. These include *arbitrageurs* and speculators. Usually such market participants form part of investment banks.

1.3.2 Investors

There is a large variety of players in the bond markets, each trading some or all of the different instruments available to suit their own purposes. We can group the main types of investors according to the time horizon of their investment activity.

- **Short-term institutional investors.** These include banks and building societies, money market fund managers, central banks and the treasury desks of some types of corporates. Such bodies are driven by short-term investment views, often subject to close guidelines, and will be driven by the total return available on their investments. Banks will have an additional requirement to maintain *liquidity*, often in fulfilment of regulatory authority rules, by holding a proportion of their assets in the form of easily tradeable short-term instruments.
- **Long-term institutional investors.** Typically these types of investors include pension funds and life assurance companies. Their investment horizon is long-term, reflecting the nature of their liabilities; often they will seek to match these liabilities by holding long-dated bonds.
- **Mixed horizon institutional investors.** This is possibly the largest category of investors and will include general insurance companies and most corporate bodies. Like banks and financial sector companies, they are also very active in the primary market, issuing bonds to finance their operations.
- **Market professionals.** This category includes the banks and specialist financial intermediaries mentioned above, firms that one would not automatically classify as "investors" although they will also have an investment objective. Their time horizon will range from one day to the very long term. They include the proprietary trading desks of investment banks, as well as bond market makers in securities houses and banks who are providing a service to their customers. Proprietary traders will actively position themselves in the market in order to gain trading profit, for example in response to their view on where they think interest rate levels are headed. These participants will trade direct with other market professionals and investors, or via brokers. Market makers or *traders* (also called *dealers* in the United States) are wholesalers in the bond markets; they make two-way prices in selected bonds. Firms will not necessarily be active market makers in all types of bonds, smaller firms often specialise in certain sectors. In a two-way quote the *bid price* is the price at which the market maker will buy stock, so it is the price the investor will receive when selling stock. The *offer price* or *ask price* is the price at which investors can buy stock from the market maker. As one might expect the bid price is always lower than the offer price, and it is this *spread* that represents the theoretical profit to the market maker. The bid–offer spread set by the market maker is determined by several factors, including supply and demand and liquidity considerations for that particular stock, the trader's view on market direction, *volatility* of the stock itself and the presence of any market intelligence. A large bid-offer spread reflects low liquidity in the stock, as well as low demand.

As mentioned above *brokers* are firms that act as intermediaries between buyers and sellers and between market makers and buyers/sellers. Floor-based stock exchanges such as the New York Stock Exchange (NYSE) also feature a *specialist*, members of the exchange who are responsible for maintaining an orderly market in one or more securities. These are known as *locals* on the London International Financial Futures and Options Exchange (LIFFE)[2]. Locals trade securities for their own account to counteract a temporary imbalance in supply and demand in a particular security; they are an important source of *liquidity* in the market. Locals earn income from brokerage fees and also from pure trading, when they sell securities at a higher price than the original purchase price.

1.3.3 Markets

Markets are that part of the financial system where capital market transactions, including the buying and selling of securities, takes place. A market can describe a traditional stock exchange, a physical trading floor where securities

[2] Since this was written, trading on LIFFE has moved off the exchange floor and is conducted electronically via screens.

trading occurs. Many financial instruments are traded over the telephone or electronically over computer links; these markets are known as *over-the-counter* (OTC) markets. A distinction is made between financial instruments of up to one year's maturity and instruments of over one year's maturity. Short-term instruments make up the *money market* while all other instruments are deemed to be part of the *capital market*. There is also a distinction made between the *primary market* and the *secondary market*. A new issue of bonds made by an investment bank on behalf of its client is made in the primary market. Such an issue can be a *public* offer, in which anyone can apply to buy the bonds, or a *private* offer where the customers of the investment bank are offered the stock. The secondary market is the market in which existing bonds and shares are subsequently traded.

A list of selected world stock exchanges is given in Appendix 1.1.

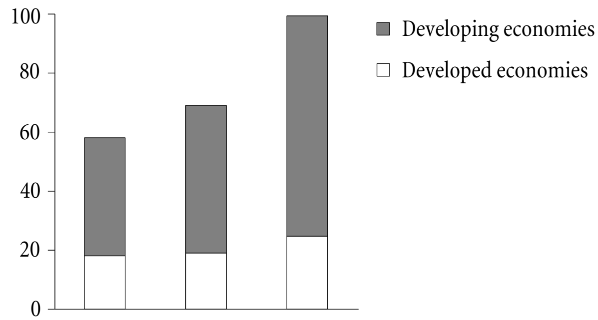

Figure 1.3: Number of stock exchanges around the world. Source: World Bank, OECD.

1.4 World bond markets

The origin of the spectacular increase in the size of global financial markets was the rise in oil prices in the early 1970s. Higher oil prices stimulated the development of a sophisticated international banking system, as they resulted in large capital inflows to developed country banks from the oil-producing countries. A significant proportion of these capital flows were placed in *Eurodollar* deposits in major banks. The growing trade deficit and level of public borrowing in the United States also contributed. The 1980s and 1990s saw tremendous growth in capital markets volumes and trading. As capital controls were eased and exchange rates moved from fixed to floating, domestic capital markets became internationalised. Growth was assisted by the rapid advance in information technology and the widespread use of financial engineering techniques. Today we would think nothing of dealing in virtually any liquid currency bond in financial centres around the world, often at the touch of a button. Global bond issues, underwritten by the subsidiaries of the same banks, are commonplace. The ease with which transactions can be undertaken has also contributed to a very competitive market in liquid currency assets.

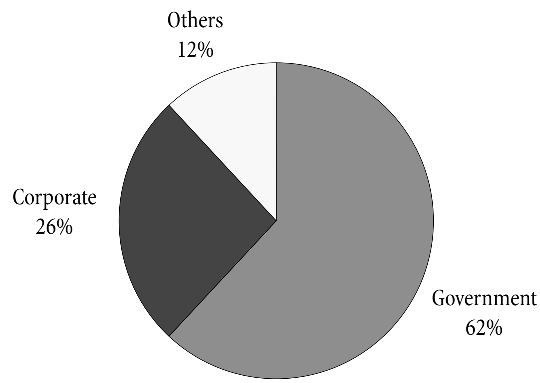

Figure 1.4: Global bond market issuers, December 1998. Source: IFC, 1998.

The world bond market has increased in size more than fifteen times since the 1970s. As at the end of 1998 outstanding volume stood at over $26 trillion. The majority of this debt is issued by governments, as shown in Figure 1.4.

The market in US Treasury securities is the largest bond market in the world. Like the government bond markets in the UK, Germany, France and other developed economies it also very liquid and transparent. Table 1.1 lists the major government bond markets in the world; the US market makes up nearly half of the total. The Japanese market is second in size, followed by the German market. A large part of the government bond market is concentrated therefore in just a few countries. Government bonds are traded on major exchanges as well as *over-the-counter* (OTC). Generally OTC refers to trades that are not carried out on an exchange but directly between the counterparties. Bonds are also listed on exchanges, for example the NYSE had over 600 government issues listed on it at the end of 1996, with a total par value of $2.6 billion.

Country	Nominal value ($ billion)	Percentage (rounded)
United States	5,490	48.5
Japan	2,980	26.3
Germany	1,236	10.9
France	513	4.5
Canada	335	3.0
United Kingdom	331	2.9
Netherlands	253	2.2
Australia	82	0.7
Denmark	72	0.6
Switzerland	37	0.3
Total	11,329	100

Table 1.1: Major government bond markets, December 1998. Source: IFC 1998.

The corporate bond market varies in liquidity, depending on the currency and type of issuer of any particular bond. Outstanding volume as at the end of 1998 was over $5.5 trillion. The global distribution of corporate bonds is shown at Figure 1.5, broken down by currency. The introduction of the euro across eleven member countries of the European Union in January 1999 now means that corporate bonds denominated in that currency form the second highest group.

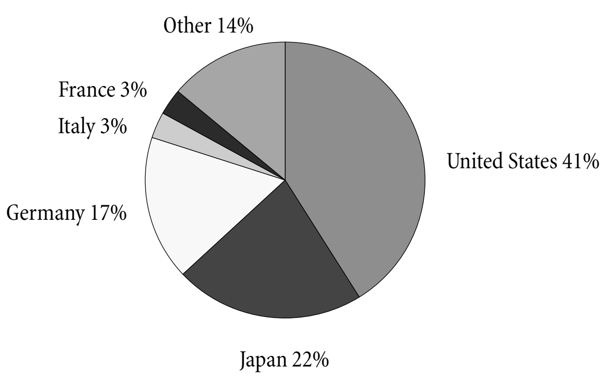

Figure 1.5: Global distribution of corporate bonds by currency, December 1998. Source: OECD.

Companies finance their operations in a number of ways, from equity to short term debt such as bank overdrafts. It is often advantageous for companies to fix longer term finance, which is why bonds are so popular. Bonds are also attractive as a means of raising finance because the interest payable on them to investors is tax deductible for the company. Dividends on equity are not tax deductible. A corporate needs to get a reasonable mix of debt versus equity in its funding however, as a high level of interest payments will be difficult to service in times of recession or general market downturn. For this reason the market views unfavourably companies that have a high level of debt. Corporate bonds are also traded on exchanges and OTC. One of the most liquid corporate bond types is the *Eurobond*, which is an international bond issued and traded across national boundaries. Sovereign governments have also issued Eurobonds.

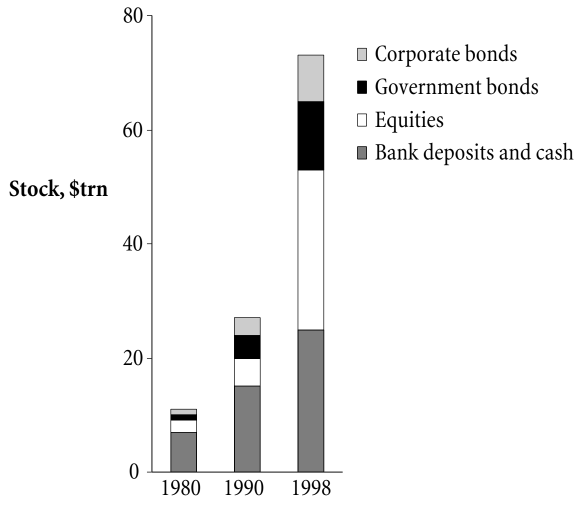

Figure 1.6 Global capital markets. Source: Strata Consulting.

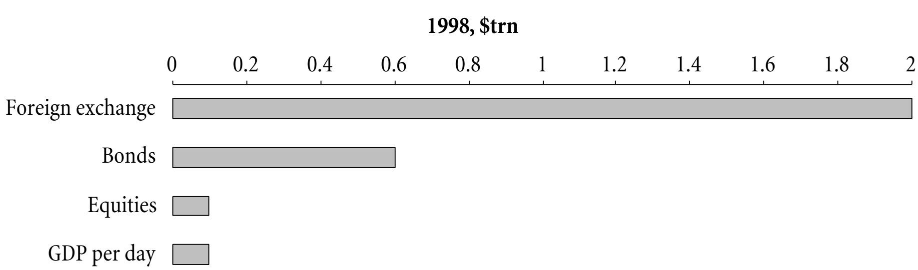

Figure 1.7: Global capital markets turnover. Source: Strata Consulting.

1.5 Overview of the main bond markets

So far we have established that bonds are debt capital market instruments, which means that they represent loans taken out by governments and corporations. The duration of any particular loan will vary from two years to thirty years or longer. In this chapter we introduce just a small proportion of the different bond instruments that trade in the market, together with a few words on different country markets. This will set the scene for later chapters, where we look at instruments and markets in greater detail.

1.5.1 Domestic and international bonds

In any market there is a primary distinction between *domestic* bonds and other bonds. Domestic bonds are issued by borrowers domiciled in the country of issue, and in the currency of the country of issue. Generally they trade only in their original market. A *Eurobond* is issued across national boundaries and can be in any currency, which is why they are also called *international* bonds. It is now more common for Eurobonds to be referred to as international bonds, to avoid confusion with "euro bonds", which are bonds denominated in *euros*, the currency of twelve countries of the European Union (EU). As an issue of Eurobonds is not restricted in terms of currency or country, the borrower is not restricted as to its nationality either. There are also *foreign* bonds, which are domestic bonds issued by foreign borrowers. An example of a foreign bond is a *Bulldog*, which is a sterling bond issued for trading in the United Kingdom (UK) market by a foreign borrower. The equivalent foreign bonds in other countries include *Yankee* bonds (United States), *Samurai* bonds (Japan), *Alpine* bonds (Switzerland) and *Matador* bonds (Spain).

There are detail differences between these bonds, for example in the frequency of interest payments that each one makes and the way the interest payment is calculated. Some bonds such as domestic bonds pay their interest *net*, which means net of a withholding tax such as income tax. Other bonds including Eurobonds make *gross* interest payments.

1.5.2 Government bonds

As their name suggests government bonds are issued by a government or *sovereign*. Government bonds in any country form the foundation for the entire domestic debt market. This is because the government market will be the largest in relation to the market as a whole. Government bonds also represent the best *credit risk* in any market as people do not expect the government to go bankrupt. As we see in a later chapter, professional institutions that analyse borrowers in terms of their credit risk always rate the government in any market as the highest credit available. While this may sometimes not be the case, it is usually a good rule of thumb.[3] The government bond market is usually also the most *liquid* in the domestic market due to its size and will form the benchmark against which other borrowers are rated. Generally, but not always, the yield offered on government debt will be the lowest in the market.

- **United States.** Government bonds in the US are known as *Treasuries*. Bonds issued with an original maturity of between two and ten years are known as *notes* (as in "Treasury note") while those issued with an original maturity of over ten years are known as *bonds*. In practice there is no real difference between notes and bonds and they trade the same way in the market. Treasuries pay semi-annual coupons. The US Treasury market is the largest single bond market anywhere and trades on a 24-hour basis all around the world. A large proportion of Treasuries are held by foreign governments and corporations. It is a very liquid and *transparent* market.
- **United Kingdom.** The UK government issues bonds known as *gilt-edged securities* or *gilts*.[4] The gilt market is another very liquid and transparent market, with prices being very competitive. Many of the more esoteric features of gilts such as "tick" pricing (where prices are quoted in 32nds and not decimals) and *special ex-dividend* trading have recently been removed in order to harmonise the market with euro government bonds. Gilts still pay coupon on a semi-annual basis though, unlike euro paper. The UK government also issues bonds known as *index-linked* gilts whose interest and redemption payments are linked to the rate of inflation. There are also older gilts with peculiar features such as no redemption date and quarterly-paid coupons.
- **Germany.** Government bonds in Germany are known as *bunds*, BOBLs or *Schatze*. These terms refer to the original maturity of the paper and has little effect on trading patterns. Bunds pay coupon on an annual basis and are of course, now denominated in euros.

  We will look at these markets and other government markets in greater detail in Chapter 13.

1.5.3 Non-conventional bonds

The definition of bonds given earlier in this chapter referred to conventional or *plain vanilla* bonds. There are many variations on vanilla bonds and we introduce a few of them here.

- **Floating Rate Notes.** The bond marked is often referred to as the *fixed income* market, or the *fixed interest* market in the UK. Floating rate notes (FRNs) do not have a fixed coupon at all but instead link their interest payments to an external reference, such as the three-month bank lending rate. Bank interest rates will fluctuate constantly during the life of the bond and so an FRNs cash flows are not known with certainty. Usually FRNs pay

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[3] Occasionally one may come across a corporate entity, such as Gazprom in Russia, that one may view as better rated in terms of credit risk compared to the government of the country in which the company is domiciled (in this case the Russian government).

[4] This is because early gilt issues are said to have been represented by certificates that were edged with gold leaf, hence the term gilt-edged. In fact the story is almost certainly apocryphal and it is unlikely that gilt certificates were ever edged with gold!

a fixed margin or *spread* over the specified reference rate; occasionally the spread is not fixed and such a bond is known as a *variable rate note*. Because FRNs pay coupons based on the three-month or six-month bank rate they are essentially money market instruments and are treated by bank dealing desks as such.

- **Index-linked bonds.** An index-linked bond as its coupon and redemption payment, or possibly just either one of these, linked to a specified index. When governments issue Index-linked bonds the cash flows are linked to a price index such as consumer or commodity prices. Corporates have issued index-linked bonds that are connected to inflation or a stock market index.
- **Zero-coupon bonds.** Certain bonds do not make any coupon payments at all and these are known as *zero-coupon bonds*. A zero-coupon bond or *strip* has only cash flow, the redemption payment on maturity. If we assume that the maturity payment is say, £100 per cent or *par* the issue price will be at a discount to par. Such bonds are also known therefore as *discount* bonds. The difference between the price paid on issue and the redemption payment is the interest realised by the bondholder. As we will discover when we look at strips this has certain advantages for investors, the main one being that there are no coupon payments to be invested during the bond's life. Both governments and corporates issue zero-coupon bonds. Conventional coupon-bearing bonds can be *stripped* into a series of individual cash flows, which would then trade as separate zero-coupon bonds. This is a common practice in government bond markets such as Treasuries or gilts where the borrowing authority does not actually issue strips, and they have to be created via the stripping process.
- **Amortised bonds.** A conventional bond will repay on maturity the entire nominal sum initially borrowed on issue. This is known as a *bullet* repayment (which is why vanilla bonds are sometimes known as bullet bonds). A bond that repays portions of the borrowing in stages during the its life is known as an *amortised* bond.
- **Bonds with embedded options**. Some bonds include a provision in their offer particulars that gives either the bondholder and/or the issuer an option to enforce early redemption of the bond. The most common type of option embedded in a bond is a *call feature.* A call provision grants the issuer the right to redeem all or part of the debt before the specified maturity date. An issuing company may wish to include such a feature as it allows it to replace an old bond issue with a lower coupon rate issue if interest rates in the market have declined. As a call feature allows the issuer to change the maturity date of a bond it is considered harmful to the bondholder's interests; therefore the market price of the bond at any time will reflect this. A call option is included in all asset-backed securities based on mortgages, for obvious reasons (asset-backed bonds are considered in a later chapter). A bond issue may also include a provision that allows the investor to change the maturity of the bond. This is known as a *put feature* and gives the bondholder the right to sell the bond back to the issuer at par on specified dates. The advantage to the bondholder is that if interest rates rise after the issue date, thus depressing the bond's value, the investor can realise par value by *putting* the bond back to the issuer. A *convertible* bond is an issue giving the bondholder the right to exchange the bond for a specified amount of shares (equity) in the issuing company. This feature allows the investor to take advantage of favourable movements in the price of the issuer's shares. The presence of embedded options in a bond makes valuation more complex compared to plain vanilla bonds, and will be considered separately.
- **Bond warrants.** A bond may be issued with a *warrant* attached to it, which entitles the bond holder to buy more of the bond (or a different bond issued by the same borrower) under specified terms and conditions at a later date. An issuer may include a warrant in order to make the bond more attractive to investors. Warrants are often detached from their host bond and traded separately.

Finally there is a large class of bonds known as *asset-backed securities.* These are bonds formed from pooling together a set of loans such as mortgages or car loans and issuing bonds against them. The interest payments on the original loans serve to back the interest payable on the asset-backed bond. We will look at these instruments in a later chapter.

1.6 Financial engineering in the bond markets

A quick glance through this book will show that we do not confine ourselves to the cash bond markets alone. The last twenty years has seen tremendous growth in the use of different and complex financial instruments, a result of *financial engineering*. These instruments have been introduced by banks to cater for customer demand, which includes the following:

- the demand for greater yield and diversification: investors in the capital markets are continually seeking opportunities to enhance yield and diversify across different markets, leading to products being introduced in new markets;
- the demand for lower borrowing costs: borrowers can issue bonds in virtually any currency where there is a demand, but still raise finance in the currency of their choice by means of a *swap* transaction;
- the demand to reduce risk exposure: market participants increasingly wish to transfer the risk that their operations expose them to, often by means of tailor-made *option* contracts.

Banks develop new instruments in response to customer demand, which will vary according to the current circumstances. This can include reaction to high levels of inflation, highly volatile interest or foreign exchange rates and other macroeconomic factors. In this book we will explore and analyse all these developments, including a detailed look at *derivatives* and risk management as well as bond analysis, trading and portfolio strategies.

Appendices

APPENDIX 1.1 List of World Stock Exchanges

Exchange	Country	Year established
Melbourne	SE Australia	1865
Dhaka SE	Bangladesh	1988
Brussels SE	Belgium	1801
Sao Paulo SE	Brazil	1850
Montreal SE	Canada	1874
Toronto SE	Canada	1852
Vancouver SE	Canada	1907
Copenhagen SE	Denmark	1690
Paris SE	France	1871
Frankfurt SE	Germany	1802
Hong Kong SE	Hong Kong	1891
Budapest SE	Hungary	1989
Milan SE	Italy	1808
Tokyo SE	Japan	1878
Amsterdam SE	Netherlands	1611
SE of Singapore	Singapore	1973
Madrid SE	Spain	1831
Zurich SE	Switzerland	1877
Taiwan SE	Taiwan	1961
London SE	United Kingdom	1805
New York SE	United States	1792

"SE" is a common abbreviation for Stock Exchange.

APPENDIX 1.2 Market-determined interest rates

This chapter has introduced bonds as a package of cash flows. As the cash flows in a plain vanilla bond are known with certainty we can use the principles of *discounting* and *present value* to calculate the price of a bond, and this is considered in Chapter 2. In this appendix we will look at how the discount rates used in present value calculations are determined by the market.

Market interest rates

An investor purchasing a bond security is sacrificing the consumption power of current funds in return for a future return. Whether the security is very short-dated or long-dated, the investor is locking away funds for a period of time, with the additional risk that the investment may not be returned because say, the borrower defaults on the loan. To compensate the investor on both these grounds the bond must pay an adequate return, sufficient to satisfy the investor that the expected return for locking funds away is worth the risk of so doing. The return from any investment will be in the form of *income*, by way of dividends or periodic interest payments, and *capital gain*, which is when the value of the original investment rises. It is reasonable to generalise that the return on equity investments is expected to be more in the form of capital gain while the return on a bond investment is expected to be more in the form of interest income. There are of course exceptions to this.

The *real interest* rate applicable to any investment is the interest rate minus the rate of *inflation*. In a zero inflation environment the real interest rate is the quoted interest rate for the instrument. The interest rate is determined by the time preference of individual investors, which is a function of investors' willingness to forgo current consumption in return for an increased consumption in the future. The investor will therefore in the first instance set the interest rate. If an investor will accept a rate of return of 5% over a period of one year, this means that she is indifferent to consuming £1 today or £1.05 in one year's time. In a world of just one borrower and one lender, the borrower will have to offer a rate of interest of at least 5% (in our example) in order to be able to sell his bond. In a world of many borrowers and lenders, if the interest rate was below the rate demanded, there would be an excess of borrowers over lenders, while if the rate was too high there would be an excess of lenders. The *market determined* rate of interest is therefore that rate which balances the supply of lent funds with the demand for borrowed funds. This is also known as the *equilibrium* rate of interest.

Real interest rates and inflation

The market determined interest rate is the rate that would apply in a market without inflation and no concern for *liquidity* (see below). Investors placing their funds with borrowers for any length of time over say, three months will require compensation for any inflation in the economy, otherwise the borrower would gain from paying back funds that had depreciated in real value. In an environment of high inflation investors will require a rate of return that is at least equal to the rate of inflation in order to be certain they are receiving back funds with the same level of purchasing power as originally lent. They will also add to this the rate of interest they require on the loan.

Consider an economy with an inflation rate of 10%. A lender will require a minimum of £1.10 on a loan of £1 at the end of one year simply to compensate for the effects of inflation. If the equilibrium rate of interest is 5%, the amount returned on an investment of £1 at the end of one year will be £1.1550. This is shown below.

$$\begin{aligned}\text{Return on loan} &= \pounds 1 \quad 1 + \text{real interest rate} \quad \frac{\text{inflation effect at end of year}}{\text{amount of loan}} \\ &= \pounds 1 \quad 1.05 \quad \frac{\pounds 1.10}{\pounds 1} = \pounds \; 1.05 \quad 1.10 \\ &= \pounds 1.1550.\end{aligned} \tag{1.1}$$

Therefore the *nominal* interest rate required is 15½%.

The "headline" quoted interest rate is therefore always taken to be the nominal interest rate that takes into account both the equilibrium interest rate and the rate of inflation. It is determined from equation (1.2) and is also known as the *Fisher equation* (1930):

$$\begin{aligned}1 + r &= \; 1 + \rho \quad 1 + i \\ &= 1 + \rho + i + \; \rho \quad i\end{aligned} \tag{1.2}$$

where

r	is the nominal interest rate
ρ	is the real interest rate
i	is the expected inflation rate.

In (1.2) the product of $\rho \quad i$ is often negligible and it is customary to ignore it, which simplifies the Fisher equation to (1.3) below.

$$r = \rho + i. \tag{1.3}$$

Note that the inflation rate is defined as the *expected* rate, the one expected to apply at the end of the investment period. The rate quoted in the media is always an historical one, for example the monthly retail price index quoted in the UK press is always the rate that applied in the previous month.

Liquidity premium

We saw earlier in the preface notes that it is common for investors to prefer to lend for short periods, as there is less risk involved and it is easier to convert investments back into cash if needed. Borrowers on the other hand prefer to fix their financing for as long as possible, as their cost of capital is known with certainty into the future. Short-term borrowing would also force them continually to refinance their activities, which exposes them to the risk that interest rates will rise and force up their financing costs. Because of this natural conflict between lenders and borrowers, the interest rate on longer term borrowing has to be higher than short-term borrowing, to compensate investors for the increased risk. In a conventional market environment therefore the interest rate for money increases steadily for investments as the maturity increases. The relationship between the maturity of the loan and the interest rate applicable to it is known as the *yield curve* or the *term structure of interest rates* (although strictly speaking this term should be reserved for the zero-coupon yield curve only) and this is examined in depth in later chapters. The yield curve usually slopes upwards as shown in Figure 1.8 and illustrates the *liquidity premium* associated with longer maturity loans.

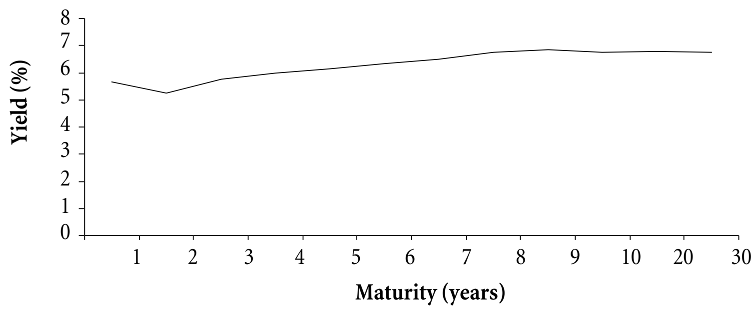

Figure 1.8: The yield curve

The liquidity premium will apply to all classes of investments, although it is greater for long-term instruments that are not as marketable as others. All buyers of long-term debt will require a liquidity premium to compensate them for locking their funds away. The liquidity premium will impact the required nominal interest rate, so we can adjust the Fisher equation as shown in (1.4) below.

$$r = \rho + i + l \tag{1.4}$$

where l is the liquidity premium.

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Questions and exercises

1. Identify and classify the main participants in the capital markets.
2. What are gilts? What are the main ways that gilts differ from Eurobonds?
3. What are the main ways that corporations raise finance?
4. Consider your local street market. Would you classify it as an efficient market?
5. Two acquaintances wish to borrow funds from you and each insists that you charge interest. One promises to repay the funds within a month, whereas the other is asking for a repayment window of between six and twelve months. Do you charge them the same interest rate? If not, why not?

2 Financial Markets Arithmetic

Since the early 1970s the mathematics required for the analysis of capital market instruments has become steadily more complex. Bonds have been no exception to this. In later chapters we will introduce and develop some of the analytical techniques important to an understanding of bonds, but to begin with in this chapter we introduce the basic building blocks of corporate finance. These include the principles of compounded interest, the time value of money, and future and present values. The concepts in this chapter are important in all aspects of finance and are a vital part of capital market mathematics. It is essential to have a firm understanding of the main principles before moving on to other areas. When reviewing the concept of the time value of money, assume that the interest rates used the market determined rates of interest which we discussed in Appendix 1.2 of Chapter 1.

2.1 Simple and compound interest

The principles of financial arithmetic have long been used to illustrate that £1 received today is not the same as £1 received at a point in the future. Faced with a choice between receiving £1 today or £1 in one year's time we would not be indifferent, given a rate of interest of say, 10% and provided that this rate is equal to our required nominal rate. Our choice would be between £1 today or £1 plus 10p – the interest on £1 for one year at 10% per annum. The notion that money has a time value is a basic concept in the analysis of financial instruments. Money has time value because of the opportunity to invest it at a rate of interest.

2.1.1 Simple interest

A loan that has one interest payment on maturity is accruing *simple interest*. On short-term instruments there is usually only the one interest payment on maturity, hence simple interest is received when the instrument expires. The terminal value of an investment with simple interest is given by (2.1):

$$FV = PV(1 + r) \tag{2.1}$$

where

FV	is the terminal value or *future value*
PV	is the initial investment or *present value*
r	is the interest rate.

So for example if PV is £100, r is 5% and the investment is one year then

$$FV = 100(1 + r) = 105.$$

The market convention is to quote interest rates as *annualised* interest rates, which is the interest that is earned if the investment term is one year. Consider a three-month deposit of £100 in a bank, placed at a rate of interest of 6%. In such an example the bank deposit will earn 6% interest for a period of 90 days. As the annual interest gain would be £6, the investor will expect to receive a proportion of this, which is calculated below:

$$£6.00 \times \frac{90}{365}.$$

So the investor will receive £1.479 interest at the end of the term. The total proceeds after the three months is therefore £100 plus £1.479. If we wish to calculate the terminal value of a short-term investment that is accruing simple interest we use the following expression:

$$FV = PV\left(1 + r \times \tfrac{\text{days}}{\text{year}}\right). \tag{2.2}$$

The fraction $\frac{\text{days}}{\text{year}}$ refers to the numerator, which is the number of days the investment runs, divided by the denominator which is the number of days in the year. In the sterling markets the number of days in the year is taken to be 365, however certain other markets (including the euro currency markets) have a 360-day year convention. For this reason we simply quote the expression as "days" divided by "year" to allow for either convention.

2.1.2 Compound interest

Let us now consider an investment of £100 made for three years, again at a rate of 6%, but this time fixed for three years. At the end of the first year the investor will be credited with interest of £6. Therefore for the second year the interest rate of 6% will be accruing on a principal sum of £106, which means that at the end of year 2 the interest credited will be £6.36. This illustrates how *compounding* works, which is the principle of earning interest on interest. What will the terminal value of our £100 three-year investment be?

In compounding we are seeking to find a *future value* given a *present value,* a *time period* and an *interest rate*. If £100 is invested today (at time t_0) at 6%, then one year later (t_1) the investor will have £100 $\times$ (1 + 0.06) = £106. In our example the capital is left in for another two years, so at the end of year 2 (t_2) we will have:

$$
\begin{aligned}
£100 \times (1 + 0.06) \times (1 + 0.06) &= £100 \times (1 + 0.06)^2 \\
&= £100 \times (1.06)^2 \\
&= £112.36.
\end{aligned}
$$

The outcome of the process of compounding is the *future value* of the initial amount. We don't have to calculate the terminal value long-hand as we can use the expression in (2.3).

$$FV = PV(1 + r)^n \tag{2.3}$$

where

r is the periodic rate of interest (expressed as a decimal)
n is the number of periods for which the sum is invested.

In our example the initial £100 investment becomes £100 $\times$ $(1 + 0.06)^3$ which is equal to £119.10.

When we compound interest we have to assume that the reinvestment of interest payments during the investment term is at the same rate as the first year's interest. That is why we stated that the 6% rate in our example was *fixed* for three years. We can see however that compounding increases our returns compared to investments that accrue only on a simple interest basis. If we had invested £100 for three years fixed at a rate of 6% but paying on a simple interest basis our terminal value would be £118, which is £1.10 less than our terminal value using a compound interest basis.

2.1.3 Compounding more than once a year

Now let us consider a deposit of £100 for one year, again at our rate of 6% but with quarterly interest payments. Such a deposit would accrue interest of £6 in the normal way but £1.50 would be credited to the account every quarter, and this would then benefit from compounding. Again assuming that we can reinvest at the same rate of 6%, the total return at the end of the year will be:

$$100 \times ((1 + 0.015) \times (1 + 0.015) \times (1 + 0.015) \times (1 + 0.015)) = 100 \times (1 + 0.015)^4$$

which gives us 100×1.06136, a terminal value of £106.136. This is some 13 pence more than the terminal value using annual compounded interest.

In general if compounding takes place m times per year, then at the end of n years mn interest payments will have been made and the future value of the principal is given by (2.4) below.

$$FV = PV\left(1 + \frac{r}{m}\right)^{mn}. \tag{2.4}$$

As we showed in our example the effect of more frequent compounding is to increase the value of the total return when compared to annual compounding. The effect of more frequent compounding is shown below, where we consider the annualised interest rate factors, for an annualised rate of 5%.

Compounding frequency		*Interest rate factor*
Annual	$(1+r)$	1.050000
Semi-annual	$\left(1+\frac{r}{2}\right)^2$	1.050625
Quarterly	$\left(1+\frac{r}{4}\right)^4$	1.050945
Monthly	$\left(1+\frac{r}{12}\right)^{12}$	1.051162
Daily	$\left(1+\frac{r}{365}\right)^{365}$	1.051267.

This shows us that the more frequent the compounding the higher the interest rate factor. The last case also illustrates how a limit occurs when interest is compounded continuously. Equation (2.4) can be rewritten as follows:

$$\begin{aligned} FV &= PV\left(1+\frac{r}{m}\right)^{m/r \quad rn} = PV \quad 1+\frac{1}{m/r}^{\quad m/r \quad rn} \\ &= PV\left(1+\frac{1}{n}\right)^{n \quad rn} \end{aligned} \tag{2.5}$$

where $n = m/r$. As compounding becomes continuous and m and hence n approach infinity, the expression in large brackets in (2.5) above approaches a value known as e, which is shown below.

$$e = \lim_{n} \left(1+\frac{1}{n}\right)^n = 2.718281\ldots$$

If we substitute this into (2.5) this gives us:

$$FV = PVe^{rn} \tag{2.6}$$

where we have continuous compounding. In (2.6) e^{rn} is known as the *exponential function* of rn and it tells us the continuously compounded interest rate factor. If $r = 5\%$ and $n = 1$ year then:

$$e^r = (2.718281)^{0.05} = 1.051271.$$

This is the limit reached with continuous compounding. From our initial example, to illustrate continuous compounding the future value of £100 at the end of three years when the interest rate is 6% is given by:

$$FV = 100e^{(0.06)\times 3} = £119.72.$$

2.1.4 Effective interest rates

The interest rate quoted on a deposit or loan is usually the *flat* rate. However we are often required to compare two interest rates which apply for a similar investment period but have different interest payment frequencies, for example a two-year interest rate with interest paid quarterly compared to a two-year rate with semi-annual interest payments. This is normally done by comparing equivalent *annualised* rates. The annualised rate is the interest rate with annual compounding that results in the same return at the end of the period as the rate we are comparing.

The concept of the effective interest rate allows us to state that:

$$PV \times \left(1+\frac{r}{n}\right)^n = PV \times (1 + aer) \tag{2.7}$$

where *aer* is the equivalent annual rate. Therefore if r is the interest rate quoted which pays n interest payments per year, the *aer* is given by (2.8):

$$aer = \left(1 + \frac{r}{n}\right)^n - 1 \,. \tag{2.8}$$

The equivalent annual interest rate *aer* is known as the *effective* interest rate. We have already referred to the quoted interest rate as the "nominal" interest rate. We can rearrange equation (2.8) above to give us (2.9) which allows us to calculate nominal rates.

$$r = \left((1 + aer)^{\frac{1}{n}} - 1\right) \times n. \tag{2.9}$$

We can see then that the effective rate will be greater than the flat rate if compounding takes place more than once a year. The effective rate is sometimes referred to as the *annualised percentage rate* or APR.

EXAMPLE 2.1

- Farhana has deposited funds in a building society 1-year fixed rate account with interest quoted at 5%, payable in semi-annual instalments. What is the effective rate that she earns at the end of the period?

$$\left(1 + \frac{0.05}{2}\right)^2 - 1 = 5.0625\%.$$

- Rupert is quoted a nominal interest rate of 6.40% for a 1-year time deposit where the interest is credited at maturity. What is the equivalent rate for the same building society's 1-year account that pays interest on a monthly basis?

$$\left((1 + 0.064)^{\frac{1}{12}} - 1\right) \times 12 = 6.2196\%.$$

2.1.5 Interest rate conventions

The convention in both wholesale or personal (retail) markets is to quote an annual interest rate. A lender who wishes to earn the interest at the rate quoted has to place their funds on deposit for one year. Annual rates are quoted irrespective of the maturity of a deposit, from overnight to ten years or longer. For example if one opens a bank account that pays interest at a rate of 3.5% but then closes it after six months, the actual interest earned will be equal to 1.75% of the sum deposited. The actual return on a three-year building society bond (fixed deposit) that pays 6.75% fixed for three years is 21.65% after three years. The quoted rate is the annual one-year equivalent. An overnight deposit in the wholesale or *interbank* market is still quoted as an annual rate, even though interest is earned for only one day.

The convention of quoting annualised rates is to allow deposits and loans of different maturities and different instruments to be compared on the basis of the interest rate applicable. We must also be careful when comparing interest rates for products that have different payment frequencies. As we have seen from the foregoing paragraphs the actual interest earned will be greater for a deposit earning 6% on a semi-annual basis compared to 6% on an annual basis. The convention in the money markets is to quote the equivalent interest rate applicable when taking into account an instrument's payment frequency.

2.2 The time value of money

2.2.1 Present values with single payments

Earlier in this chapter we saw how a *future value* could be calculated given a known *present value* and rate of interest. For example £100 invested today for one year at an interest rate of 6% will generate $100 \times (1 + 0.06) = £106$ at the end of the year. The future value of £100 in this case is £106. We can also say that £100 is the *present value* of £106 in our example.

In equation (2.3) we established the following future value relationship:

$$FV = PV(1 + r)^n \,.$$

By reversing this expression we arrive at the present value formula (2.10):

$$PV = \frac{FV}{(1+r)^n} \tag{2.10}$$

where terms are as before. Equation (2.10) applies in the case of annual interest payments and enables us to calculate the present value of a known future sum.

EXAMPLE 2.2

- Naseem is saving for a trip around the world after university and needs to have £1000 in three years' time. He can invest in a building society bond at 7% guaranteed fixed for three years. How much does he need to invest now?

  To solve this we require the PV of £1000 received in three years' time.

$$PV = \frac{1000}{(1+.07)^3} = \frac{1000}{1.225043} = 816.29787.$$

Naseem therefore needs to invest £816.30 today.

To calculate the present value for a short-term investment of less than one year we will need to adjust what would have been the interest earned for a whole year by the proportion of days of the investment period. Rearranging the basic equation, we can say that the present value of a known future value is

$$PV = \frac{FV}{\left(1 + r \times \frac{\text{days}}{\text{year}}\right)}. \tag{2.11}$$

Given a present value and a future value at the end of an investment period, what then is the interest rate earned? We can rearrange the basic equation again to solve for the *yield*.

$$yield = \left(\frac{FV}{PV} - 1\right) \times \frac{\text{year}}{\text{days}}. \tag{2.12}$$

Using equation (2.12) will give us the interest rate for the actual period. We can then convert this to an effective interest rate using (2.13).

$$r = 1 + yield \times \frac{\text{days}}{\text{year}}^{\frac{365}{days}} - 1. \tag{2.13}$$

When interest is compounded more than once a year, the formula for calculating present value is modified, as shown by (2.14):

$$PV = \frac{FV}{(1 + (r/m))^{mn}} \tag{2.14}$$

where as before FV is the cash flow at the end of year n, m is the number of times a year interest is compounded, and r is the rate of interest or discount rate. Illustrating this therefore, the present value of £100 that is received at the end of five years at a rate of interest rate of 5%, with quarterly compounding is:

$$PV = \frac{100}{(1 + (0.05/4))^{(4)(5)}} = £78.00.$$

2.2.2 Discount factors

The calculation of present values from future values is also known as *discounting*. The principles of present and future values demonstrate the concept of the *time value* of money which is that in an environment of positive interest rates a sum of money has greater value today than it does at some point in the future because we are able to invest the sum today and earn interest. We will only consider a sum in the future compared to a sum today if we are compensated by being paid interest at a sufficient rate. Discounting future values allows us to compare the value of a future sum with a present sum.

Another way to write the expression in example (2.14) is to say that we multiply £1000 by $1/(1.05)^5$, which is the *reciprocal* of $(1.05)^5$ and is denoted in this case as $(1 + 0.05)^{-5}$. The rate of interest r that we use in Example 2.2 is known as the *discount rate* and is the rate we use to *discount* a known future value in order to calculate a present value. We can rearrange equation (2.14) to give:

$$PV = FV(1 + r)^{-n}$$

and the term $(1 + r)^{-n}$ is known as the n-year discount factor.

$$df_n = (1 + r)^{-n} \tag{2.15}$$

where df_n is the n-year discount factor.

The three-year discount factor when the discount rate is 9% is:

$$df_3 = (1 + 0.09)^{-3} = 0.77218.$$

We can calculate discount factor for all possible interest rates and time periods to give us a *discount function.* Fortunately we don't need to calculate discount factors ourselves as this has been done for us and discount tables for a range of rates are provided in Appendix 2.2.

EXAMPLE 2.3 Formula summary

Discount factor with simple interest: $df = \dfrac{1}{\left(1 + r\frac{\text{days}}{\text{year}}\right)}.$

Discount factor with compound interest: $df_n = \left(\dfrac{1}{1 + r}\right)^n.$

Earlier we established the continuously compounded interest rate factor as e^{rn}. Using a continuously compounded interest rate therefore we can establish the discount factor to be:

$$df = \frac{1}{1 + \left(e^{r\times\frac{\text{days}}{\text{year}}} - 1\right)} = e^{-r\times\frac{\text{days}}{\text{year}}}$$
$$df_n = e^{-rn}. \tag{2.16}$$

The continuously compounded discount factor is part of the formula used in option pricing models, which are discussed in the chapter on options.

It is possible to calculate discount factors from the prices of government bonds. The traditional approach described in most textbooks requires that we first use the price of a bond that has only one remaining coupon, its last one, and calculate a discount factor from this bond's price. We then use this discount factor to calculate the discount factors of bonds with ever-increasing maturities, until we obtain the complete discount function. This method, which is illustrated in the box below, suffers from certain drawbacks and in practice more sophisticated techniques are used, which we will introduce later in the book.

EXAMPLE 2.4 Discount factors

- The following hypothetical government bonds pay coupon on a semi-annual basis. Consider the bond prices indicated, and assume that the first bond has precisely six months to maturity, so that it has only one more cash flow to pay, the redemption value and final coupon. Assume further that the remaining bonds mature at precise six-month intervals.

Bond	Price
8% June 2000	101.09
7% December 2000	101.03
7% June 2001	101.44
6.5% December 2001	101.21

The first bond has a redemption payment of 104.00, comprised of the redemption payment and the final coupon payment (remember that this is a semi-annual coupon bond). The present value of this bond is 101.09. This allows us to determine the discount factor of the bond as follows:

$$\begin{aligned} 101.09 &= 104.00 \times df_{6\text{-}month} \\ 0.97202 &= df_{6\text{-}month}. \end{aligned}$$

This shows that the six-month discount factor is 0.97202. We use the second bond in the table, which has cash flows of 3.50 and 103.50, to calculate the next period discount factor, using the following expression:

$$101.03 = 3.50 \times df_{6\text{-}month} + 103.5 \times df_{1\text{-}year}.$$

We have already calculated the six-month discount factor, and use this to calculate the one-year discount factor from the above expression, which solves to give 0.94327. We then carry on this procedure for the next bond, leaving us the following discount factors:

Bond	Price	Discount factor
8% June 2000	101.09	0.97202
7% December 2000	101.03	0.94327
7% June 2001	101.44	0.91533
6.5% December 2001	101.21	0.89114

Note how the discount factors progressively reduce in value over an increasing maturity period. Using one of a number of techniques (which we consider later in the book) we can graph the set of discount factors above to obtain the two-year discount function. In the same way, if we have government bond prices for all maturities from six months to 30 years, we can obtain the complete discount function for that currency.

2.2.3 Present values with multiple discounting

Present values for short-term investments of under one year maturity often involve a single interest payment. If there is more than one interest payment then any discounting needs to take this into account. If discounting takes place m times per then we can use equation (2.4) to derive the present value formula as follows.

$$PV = FV\left(1 + \frac{r}{m}\right)^{-mn}. \tag{2.17}$$

For example, what is the present value of the sum of £1000 that is to be received in five years where the discount rate is 5% and there is semi-annual discounting?

Using (2.17) above we see that

$$PV = 1000\left(1 + \frac{0.05}{2}\right)^{-2\times 5} = £781.20.$$

The effect of more frequent discounting is to lower the present value. As with continuous compounding, the limiting factor is reached with continuous discounting and we can use equation (2.6) to derive the present value formula for continuous discounting

$$PV = FVe^{-rn}. \tag{2.18}$$

Using this expression, if we consider the same example as before but now with continuous discounting we calculate the present value of £1000 to be received in five years' time as:

$$PV = 1000e^{-(0.05)\times 5} = £778.80.$$

EXAMPLE 2.5 Calculation summaries

- Grant invests £250 in a bank account for five years at a rate of 6.75%. What is the future value of this sum assuming annual compounding?

$$250 \times (1.0675)^5 = £346.56.$$

- After 180 days Grant decides to close the account and withdraw the cash. What is the terminal value?

$$250 \times (1 + 1.0675 \times 180/365) = £258.32.$$

- To pay off a personal loan Phil requires £500 in 30 days' time. What must he invest now if he can obtain 12% interest from a bank?

$$500/(1 + 0.12 \times 30/365) = £495.12.$$

- If Phil deposits £1000 today and receives a total of £1021 after 90 days, what yield has he earned on the investment?

$$((1021/1000) - 1) \times 365/90 = 8.52\%.$$

- What is the 180-day discount factor earned during this period is 6.15%? The ten-year discount factor?

$$1/(1 + 0.0615 \times 180/365) = 0.97056$$

$$1/(1 + 0.0615)^{10} = 0.55055.$$

- What is the present value of £100 in ten years' time at this discount rate?

$$100 \times 0.55055 = £55.06.$$

2.3 Multiple cash flows

2.3.1 Future values

In Chapter 1 we introduced bonds by describing them as a packages of cash flows. Up to now we have considered future values of a single cash flow. Of course the same principles of the time value of money can be applied to a bundle of cash flows. A series of cash flows can be regular or at irregular intervals. If we wish to calculate the total future value of a set of irregular payments made in the future we need to calculate each payment separately and then sum all the cash flows. The formula is represented with the equation given at (2.19):

$$FV = \sum_{n=1}^{N} C_n (1 + r)^{N-n} \tag{2.19}$$

where C_n is the payment in year n and the symbol Σ means "the sum of". We assume that payment is made and interest credited at the end of each year.

It is much more common to come across a regular stream of future payments. Such a cash flow is known as an *annuity*. In an annuity the payments are identical and so C_n as given in (2.19) simply becomes C. We can then rearrange (2.19) as shown below.

$$FV = C \sum_{n=1}^{N} (1 + r)^{N-n}. \tag{2.20}$$

This equation can be simplified to give us the expression at (2.21):[1]

$$FV = C \, \frac{(1 + r)^N - 1}{r} \, . \tag{2.21}$$

[1] If we multiply both sides of (2.20) by $1 + r$ and then subtract the result from (2.20) we obtain:

$$\begin{aligned} FV - (1 + r)FV &= C \sum_{n=1}^{N} (1 + r)^{N-n} - \sum_{n=1}^{N} (1 + r)^{N-n+1} \\ &= -C\big((1 + r)^N - 1\big). \end{aligned}$$

This formula can be used to calculate the future value of an annuity. For example, if we consider an annuity that pays £500 each year for ten years at a rate of 6%, its future value is given by:

$$FV = 500\,\frac{(1.06)^{10} - 1}{0.06} = £6{,}590.40.$$

EXAMPLE 2.6 Calculating pension contributions

- We can use the future value equation (2.21) to calculate the size of contributions required to establish a pension fund on retirement. If we rearrange (2.21) to obtain the size of the annuity C we obtain:

$$C = FV\,\frac{r}{(1+r)^{N} - 1}\,.$$

- Lita wishes to have a savings pool of £250,000 to fund her pension when she retires in 30 years' time. What annual pension contribution is required if the rate of interest is assumed to be a constant 7.9%?

$$C = 250{,}000\,\frac{0.079}{(1.079)^{30} - 1} = £2247.65.$$

The common definition of an annuity is a continuous stream of cash flows. In practice the pension represented by an annuity is usually paid in monthly instalments, similar to an employed person's annual salary. Certain regular payments compound interest on a more frequent basis than annually, so our formula at (2.20) needs to be adjusted slightly. If compounding occurs m times each year, then (2.20) needs to be altered to (2.22) to allow for this.

$$FV = C\sum_{n=1}^{N}\left(1 + \frac{r}{m}\right)^{m(N-n)}. \tag{2.22}$$

To make calculations simpler we can multiply both sides of (2.22) by $(1 + (r/m))$ and subtract the result from (2.22).[2] Simplifying this will then result in (2.23) below.

$$FV = C\,\frac{(1 + (r/m))^{mN} - 1}{(1 + (r/m))^{m} - 1}\,. \tag{2.23}$$

For example a ten year annuity that has annual payments of £5000 each year, but compounded on a quarterly basis at a rate of 5% will have a future value of £63,073 as shown below.

$$FV = 5000\,\frac{(1.025)^{20} - 1}{(1.025)^{2} - 1} = £63{,}073.$$

Where there is continuous compounding, as before the limiting factor will result in (2.23) becoming (2.24):

$$FV = C\left(\frac{e^{rN} - 1}{e^{r} - 1}\right). \tag{2.24}$$

Equations (2.23) and (2.24) can be adjusted yet again to allow for frequent payments together with frequent compounding, but such a stream of cash flows is rarely encountered in practice. For reference, in the case of continuous compounding of continuous payments, the limiting factor expression is as shown at (2.25):

$$FV = C\left(\frac{e^{rN} - 1}{r}\right). \tag{2.25}$$

[2] The process is:

$$\begin{aligned} FV - 1 + (r/m)^{m} FV &= C \sum_{n=1}^{N} 1 + (r/m)^{M(N-n)} - \sum_{n=1}^{N} 1 + (r/m)^{M(N-n)+m} \\ &= -C\left(1 + (r/m)^{mN} - 1\right). \end{aligned}$$

2.3.2 Present values

Using similar principles as we have employed for calculating future values, we can calculate present values for a stream of multiple of cash flows. The method employed is slightly different according to whether the cash flows are regular or irregular.

For irregular payments we calculate present value by applying the conventional present value formula to each separate cash flow and then summing the present values. This is represented by (2.26):

$$PV = \sum_{n=1}^{N} C_n (1+r)^{-n} \tag{2.26}$$

where C_n is the cash flow made in year n.

Consider a series of annual cash payments made up of £100 in the first year and then increasing by £100 each year until the fifth year. The present value of this cash flow stream is:

$$\begin{aligned} PV &= 100(1.05)^{-1} + 200(1.05)^{-2} + 300(1.05)^{-3} + 400(1.05)^{-4} + 500(1.05)^{-5} \\ &= 95.24 + 181.41 + 259.15 + 329.08 + 391.76 \\ &= £1256.64. \end{aligned}$$

The more frequently encountered type of cash flow stream is an *annuity*, regular annual payments with annual discounting. To calculate the present value of an annuity we can use a variation of (2.21) as shown at (2.27):

$$\begin{aligned} PV &= \frac{FV}{(1+r)^N} \\ &= C\,\frac{(1+r)^N - 1}{r}\;\frac{1}{(1+r)^N} \\ &= C\,\frac{1-(1+r)^{-N}}{r}\;. \end{aligned} \tag{2.27}$$

Consider now an annuity paying £5000 each year for twenty years at an interest rate 4.5%. The present value of this annuity is:

$$\begin{aligned} PV &= 5000\,\frac{1-(1.045)^{-20}}{0.045} \\ &= 65{,}039.68. \end{aligned}$$

We illustrated this principle using a 20-year annuity that employed annual discounting. If a cash flow stream employs more frequent discounting we need to adjust the formula again. If an annuity discounts its cash flows m times each year then the present value of its cash flow stream is found using the present value adjusted equation from (2.23). This becomes (2.28).

$$PV = \frac{FV}{\left(1+\frac{r}{m}\right)^{mN}} = c\,\frac{1-(1+(r/m))^{-mN}}{(1+(r/m))^m - 1}\;. \tag{2.28}$$

If continuous discounting is employed then this results again in the limiting factor for continuous discounting, so we adjust (2.28) and the new expression is given at (2.29):

$$PV = C\left(\frac{1-e^{-rN}}{e^r - 1}\right). \tag{2.29}$$

The last case to consider is that of the payments stream that has more frequent cash flows in addition to more frequent discounting. Such a payments stream will have m cash flows each year which are also discounted m times per year. To calculate the present value of the cash flows we use (2.30):

$$PV = \frac{FV}{(1 + (r/m))^{mN}} = C\frac{1 - (1 + (r/m))^{mN}}{r}. \tag{2.30}$$

The limiting factor for continuous discounting of continuous payments is given by (2.31):

$$PV = C\left(\frac{1 - e^{-rN}}{r}\right). \tag{2.31}$$

Payment streams that have cash flow frequencies greater than annually or semi-annually occur quite often in the markets. To illustrate how we might use (2.30), consider a mortgage-type loan taken out at the beginning of a period. If the borrower is able to fix the interest rate being charged to the whole life of the mortgage, she can calculate the size of the monthly payments that are required to pay off the loan at the end of the period.

For example consider a repayment mortgage of £76,000 taken out for 25 years at a fixed rate of interest of 6.99%. The monthly repayments that would be charged can be calculated using (2.30) as shown in (2.32):

$$C_i = \frac{C}{12} = \frac{PV}{12}\,\frac{r}{1 - (1 + (r/m))^{-12 \times N}} \tag{2.32}$$

where C_i is the size of the monthly payment. Substituting the terms of the mortgage payments in to the equation we obtain:

$$C_i = \frac{76{,}000}{12}\,\frac{0.0699}{1 - (1 + (0.0699/12))^{-12 \times 25}} = £536.67.$$

The monthly repayment is therefore £536.67 and includes the interest chargeable in addition to a repayment of some of the principal (hence the term *repayment* mortgage, as opposed to *endowment* mortgages which only pay off the monthly interest charge). A repayment mortgage is also known as an *amortised* mortgage. An amortised loan is one for which a proportion of the original loan capital is paid off each year. Loans that require the borrower to service the interest charge only each year are known as *straight* or *bullet* loans. It is for this reason that plain vanilla bonds are sometimes known as bullet bonds, since the capital element of a loan raised through a vanilla bond issue is repaid only on maturity.

2.3.3 Perpetual cash flows

The type of annuity that we as individuals are most familiar with is the *annuity pension*, purchased from a life assurance company using the proceeds of a pension fund at the time of retirement. Such an annuity pays a fixed annual cash amount for an undetermined period, usually up until the death of the beneficiary. An annuity with no set finish date is known as a *perpetuity*. As the end date of a perpetuity is unknown we are not able to calculate its present value with exact certainty, however a characteristic of the term $(1 + r)^{-N}$ is that it approaches zero as N tends to infinity. This fact reduces our present value expression to:

$$PV = \frac{C}{r} \tag{2.33}$$

and we can use this formula to approximate the present value of a perpetuity.

The UK gilt market includes four gilts that have no redemption date, so-called *undated* bonds. The largest issue amongst the undated gilts is the 3½% War Loan, a stock originally issued at the time of the 1914–18 war. This bond pays a coupon of £3½ per £100 nominal of stock. Since the cash flow structure of this bond matches a perpetual, its present value using (2.33) when long dated market interest rates are at say, 5% would be:

$$PV = \frac{3.5}{0.05} = £70.$$

The present value of the cash flow stream represented by War Loan when market rates are 5% would therefore be £70 per £100 nominal of stock. In fact because this bond pays coupon on a semi-annual basis we should adjust the calculation to account for the more frequent payment of coupons and discounting, so the present value (price) of the bond is more accurately described as:

$$PV = \frac{C/2}{r/2} = \frac{1.75}{0.025}$$

although as we would expect this still gives us a price of £70 per cent!

2.3.4 Varying discount rates

During our discussion we have assumed that a single discount factor is used whenever we wish to obtain the present value of a cash flow stream. In fact it is more realistic to expect that cash flows over a period of time are discounted at different rates. This is logical when one considers that over a certain period, different interest rates will apply to different time periods. If we wish to calculate the present value of a series of cash flows it is usually more realistic to do this by discounting each payment at its own appropriate rate and then taking the sum of all the individual present values.

Consider a financial instrument that pays £5 each year for four years, with a final payment of £105 in the fifth year (this sounds very similar to a 5% coupon five-year bond!). If the market required interest rate for five-year money is 5%, then all the cash flows will be discounted at 5%, as shown in Table 2.1.

Period	Cash flow	Present value
1	5	4.7619
2	5	4.5351
3	5	4.3192
4	5	4.1135
5	105	82.2702
	Total PV =	100.0000

Table 2.1: Present values using one discount rate.

Using this discount rate, the total present value of the cash flow stream is exactly £100. If however the cash flows are treated as individual payments in their own right we may opt to discount each cash flow at a discount rate that is more appropriate to it. On this basis it may be that the first payment is discounted at a lower rate because it occurs earlier, and that the payments are all discounted at lower than 5%, except for the final payment. If we assign discount rates that fit the following market requirements, we find that the total present value of the cash flow stream is now higher, at £100.2129. This is shown in Table 2.2.

Period	Cash flow	Required interest rate	Present value
1	5	4%	4.8077
2	5	4.25%	4.6006
3	5	4.50%	4.3815
4	5	4.75%	4.1529
5	105	5%	82.2702
		Total PV =	100.2129

Table 2.2: Present values using unique discount rates.

2.3.5 The discount function

Discount factors can be calculated for any discount rate that apply to any term to maturity, using the standard formulae. The complete range of discount factors for any particular rate is known as the *discount function*. Figure 2.1 illustrates the discount function when the discount rate selected is 5%. This is obtained by plotting continuous rather than discrete discount factors for a given rate. A discount factor table for selected rates and investment terms is given at Appendix 2.2.

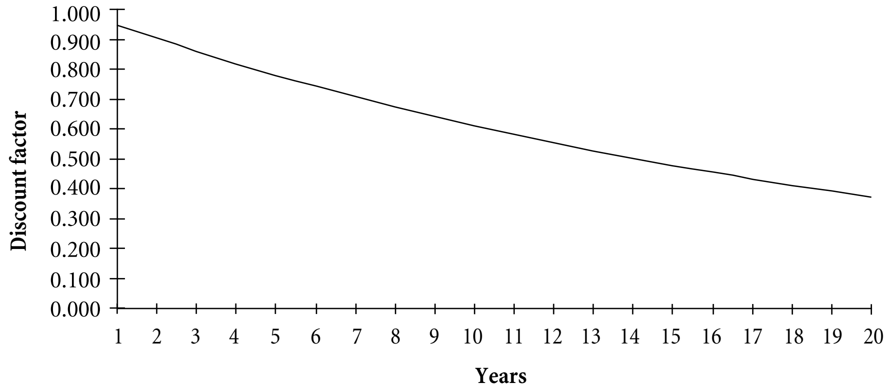

Figure 2.1: Discount function with the rate at 5%.

2.4 Corporate finance project appraisal

Two common techniques used by corporates and governments to evaluate whether a project is worth undertaking are *net present value* and *internal rate of return*. Both techniques evaluate the anticipated cash flows associated with a project, using the discounting and present value methods described so far in this chapter. Generally speaking it is a company's *cost of capital* that is used as the discount rate in project appraisal, and most companies attempt to ascertain the true cost of their capital as accurately as possible. As most corporate financing is usually a complex mixture of debt and equity this is sometimes problematic. A discussion of cost of capital is outside the scope of this book and we recommend the text in the reference section for readers wishing to know more about this subject.

2.4.1 Net present value

In the case of an investment of funds made as part of a project, we would have a series of cash flows of which some would be positive and others negative. Typically in the early stages of a project we would forecast negative cash flows as a result of investment outflows, followed by positive cash flows as the project began to show a return. Each cash flow can be present valued in the usual way. In project appraisal we would seek to find the present value of the entire stream of cash flows, and the sum of all positive and negative present values added together is the *net present value* (NPV). As the appraisal process takes place before the project is undertaken, the future cash flows that we are concerned with will be estimated forecasts and may not actually be received once the project is underway.

The present value equation is used to show that:

$$NPV = \sum_{n=1}^{N} \frac{C_n}{(1+r)^n} \tag{2.34}$$

where C_n is the cash flow in the project in period N. The rate r used to discount the cash flows can be the company's cost of capital or the rate of return required by the company to make the project viable.

Companies will apply NPV analysis to expected projected returns because funds invested in any undertaking has a time-related cost, the opportunity cost that is the corporate cost of capital. In effect NPV measures the present value of the gain achieved from investing in the project (provided that it is successful!). The general rule of thumb applied is that any project with a positive NPV is worthwhile, whereas those with a negative NPV, discounted at the required rate of return or the cost of capital, should be avoided.

EXAMPLE 2.7

- What is the NPV of the following set of expected cash flows, discounted at a rate of 15%?

Year 0: −£23,000
Year 1: +£8,000
Year 2: +£8,000
Year 3: +£8,000
Year 4: +11,000

$$NPV = 23{,}000 - \frac{8{,}000}{(1.15)} + \frac{8{,}000}{(1.15)^2} + \frac{8{,}000}{(1.15)^3} + \frac{11{,}000}{(1.15)^4} = £1{,}554.$$

2.4.2 The internal rate of return

The internal rate of return (IRR) for an investment is the discount rate that equates the present value of the expected cash flows (the NPV) to zero. Using the present value expression we can represent it by that rate r such that:

$$\sum_{n=0}^{N} \frac{C_n}{(1+r)^n} = 0 \tag{2.35}$$

where C_n is the cash flow for the period N, n is the last period in which a cash flow is expected, and Σ denotes the sum of discounted cash flows at the end of periods 0 through n. If the initial cash flow occurs at time 0, equation (2.35) can be expressed as follows:

$$C_0 = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_N}{(1+r)^N}. \tag{2.36}$$

In corporate finance project appraisal, C_0 is a cash outflow and C_1 to C_N are cash inflows. Thus r is the rate that discounts the stream of future cash flows (C_1 through C_n) to equal the initial outlay at time $0 - C_0$. We must therefore assume that the cash flows received subsequently are reinvested to realise the same rate of return as r. Solving for the internal rate of return r cannot be found analytically and has to be found through numerical iteration, or using a computer or programmable calculator. For the complete mathematical background to IRR consult Appendix 2.1.

To illustrate IRR consider the earlier project cash flows given in Example 2.7. If we wish to find the IRR long-hand then we would have to obtain the NPV using different discount rates until we found the rate that gave the NPV equal to zero. The quickest way to do this manually is to select two discount rates, one of which gives a negative NPV and the other a positive NPV, and then *interpolate* between these two rates. This method of solving for IRR is known as an *iterative* process, and involves converging on a solution through trial and error. This is in fact the only way to calculate the IRR for a set of cash flows and it is exactly an iterative process that a computer uses (the computer is just a touch quicker!). If we have two discount rates, say x and y that give positive and negative NPVs respectively for a set of cash flows, the IRR can be estimated using the equation at (2.37):

$$\text{IRR estimate} = x\% + (y\% - x\%) \times (+ve\text{ NPV value}/(+ve\text{ NPV value} - (-\text{NPV value}))). \tag{2.37}$$

EXAMPLE 2.8

- In Example 2.7 using a discount rate of 15% produced a positive NPV. Discounting the cash flows at 19% produces an NPV of $-£395$. Therefore the estimate for IRR is:

$$15\% + 4\% \times 1554/(1554 - (-395)) = 18.19\%.$$

The IRR is approximately 18.19%. This can be checked using a programmable calculator or spreadsheet programme, or may be checked manually by calculating the NPV of the original cash flows using a discount rate of 18.19%; it should come to $-£23{,}000$. Using an HP calculator we obtain an IRR of 18.14%.

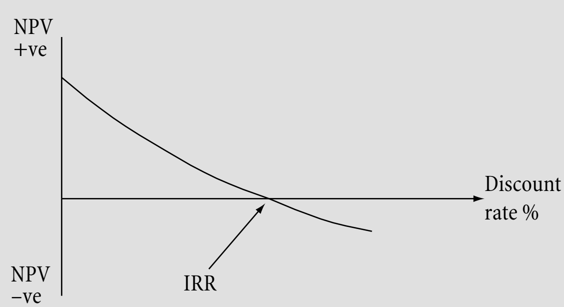

Figure 2.2: Relationship between NPV and IRR

The relationship between the IRR and the NPV of an investment is that while the NPV is the value of the projected returns from the investment using an appropriate discount rate (usually the company's cost of capital),