

# FUNDAMENTALS OF RESERVOIR ENGINEERING

L.P. DAKE





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### developments in petroleum science 8

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DEVELOPMENTS IN **PETROLEUM SCIENCE** 8

## FUNDAMENTALS OF RESERVOIR ENGINEERING

L.P. DAKE<sup>†</sup>



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#### To Grace

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#### PREFACE

This teaching textbook in Hydrocarbon Reservoir Engineering is based on various lecture courses given by the author while employed in the Training Division of Shell Internationale Petroleum Maatschappij B.V. (SIPM), in the Hague, between 1974 and 1977.

The primary aim of the book is to present the basic physics of reservoir engineering, using the simplest and most straightforward of mathematical techniques. It is only through having a complete understanding of the physics that the engineer can hope to appreciate and solve complex reservoir engineering problems in a practical manner.

Chapters 1 through 4 serve as an introduction to the subject and contain material presented on Shell's basic training courses. They should therefore be of interest to anyone even remotely connected with the business of developing and producing hydrocarbon reserves.

Chapters 5 through 8 are more specialised describing the theory and practice of well testing and pressure analysis techniques, which are probably the most important subjects in the whole of reservoir engineering. The approach is entirely general in recognising that the superposition of dimensionless pressure, or pseudo pressure functions, permits the analysis of any rate-pressure-time record retrieved from a well test, for any type of reservoir fluid. To appreciate this generality, the reader is advised to make a cursory inspection of section 8.13 (page 295), before embarking on a more thorough reading of these chapters. The author hopes that this will serve as a useful introduction to the recently published and, as usual, excellent SPE Monograph (Advances in Well Test Analysis; by Robert C. Earlougher, Jr.), in which a knowledge is assumed of much of the theory presented in these four chapters.

Chapter 9 describes the art of aquifer modelling, while Chapter 10, the final chapter, covers the subject of immiscible, incompressible displacement. The message here is – that there is but one displacement theory, that of Buckley and Leverett. Everything else is just a matter of "modifying" the relative permeability curves (known in the business as "scientific adjustment"), to account for the manner in which the fluid saturations are distributed in the dip-normal direction. These curves can then be used in conjunction with the one dimensional Buckley-Leverett equation to calculate the oil recovery. By stating the physics implicit in the generation of averaged (pseudo) relative permeabilities and illustrating their role in numerical simulation, it is hoped that this chapter will help to guide the hand of the scientific adjuster.

The book also contains numerous fully worked exercises which illustrate the theory. The most notable omission, amongst the subjects covered, is the lack of any serious discussion on the complexities of hydrocarbon phase behaviour. This has already been made the subject of several specialist text books, most notably that of Amyx, Bass and Whiting (reference 8, page 42), which is frequently referred to throughout this text. A difficult decision to make, at the time of writing, is which set of units to employ. Although the logical decision has been made that the industry should adopt the SI (Système Internationale) units, no agreement has yet been reached concerning the extent to which "allowable" units, expressed in terms of the basic units, will be tolerated. To avoid possible error the author has therefore elected to develop the important theoretical arguments in Darcy units, while equations required for application in the field are stated in Field units. Both these systems are defined in table 4.1, in Chapter 4, which appropriately is devoted to the description of Darcy's law. This chapter also contains a section, (4.4), which describes how to convert equations expressed in one set of units to the equivalent form in any other set of units. The choice of Darcy units is based largely on tradition. Equations expressed in these units have the same form as in absolute units except in their gravity terms. Field units have been used in practical equations to enable the reader to relate to the existing AIME literature.

#### ACKNOWLEDGEMENTS

The author wishes to express his thanks to SIPM for so readily granting permission to publish this work and, in particular, to H.L. Douwes Dekker, P.C. Kok and C.F.M. Heck for their sustained personal interest throughout the writing and publication, which has been a source of great encouragement.

Of those who have offered technical advice, I should like to acknowledge the assistance of G.J. Harmsen; L.A. Schipper; D. Leijnse; J. van der Burgh; L. Schenk; H. van Engen and H. Brummelkamp, all sometime members of Shell's reservoir engineering staff in the Hague. My thanks for technical assistance are also due to the following members of KSEPL (Koninklijke Shell Exploratie en Productie Laboratorium) in Rijswijk, Holland: J. Offeringa; H.L. van Domselaar; J.M. Dumoré; J. van Lookeren and A.S. Williamson. Further, I am grateful to all former lecturers in reservoir engineering in Shell Training, and also to my successor A.J. de la Mar for his many helpful suggestions. Sincere thanks also to S.H. Christiansen (P.D. Oman) for his dedicated attitude while correcting the text over a period of several months, and similarly to J.M. Willetts (Shell Expro, Aberdeen) and B.J.W. Woods (NAM, Assen) for their efforts.

For the preparation of the text I am indebted to G.J.W. Fransz for his co-ordinating work, and particularly to Vera A. Kuipers-Betke for her enthusiastic hard work while composing the final copy. For the drafting of the diagrams and the layout I am grateful to J.C. Janse; C.L. Slootweg; J.H. Bor and S.O. Fraser-Mackenzie.

Finally, my thanks are due to all those who suffered my lectures between 1974 and 1977 for their numerous suggestions which have helped to shape this textbook.

L.P. Dake, Shell Training, The Hague, October 1977.

#### **IN MEMORIAM: LAURENCE P. DAKE**

In the family of reservoir and petroleum engineers it was always so natural and rewarding to talk about "Laurie" (the name he preferred to his official one, Laurence Patrick Dake) about his point of view, and about his acceptance of, or opposition to, certain ideas or procedures. Today, sitting in front of a blank sheet of paper, I understand for the first time how difficult, how sad, and how impossible it is for any of his friends to talk about Laurie in *memoriam*. The only way to proceed is by remembering Laurie's life and his contribution to our petroleum engineering profession, and in evoking his exceptional creative spirit.

I remember the unforgettable conversations during the long winter nights of 1985 in my Norsk Hydro Oslo Office, when Laurie elaborated on the key objective of reservoir engineering: The capacity to turn the time-mirror around, so that a coherent image of the future prediction of an oil field can in return give us valuable insight into today's understanding of the same field, in order to ensure that every statement about the future behaviour of the reservoir is not accompanied by a long series of "ifs", "buts" and an avalanche of "maybes".

It was during this period that Laurie began using this approach to lay the foundations for the book "Practice of Reservoir Engineering".

Laurence Dake was born *11 March 1941* on the Isle of Man. He received his education at King Williams College and graduated in Natural Philosophy at the University of Glasgow in *1964*.

Recruited by Shell in *1964*, he joined Shell International as a Petroleum Engineer. Following a thorough training program at the Shell Training Center in The Hague, he participated as Petroleum Engineer in a variety of field operations in Australia, Brunei, Turkey and Australia until 1971, when he was once again called back to the Shell Training Center in The Hague. For seven years, from *1971 until 1978*, he taught the subject of Reservoir Engineering to Shell graduates.

In *1978* Laurie Dake left Shell after 14 years of service, at which time he made two significant steps which would determine his further professional career:

(1) He joined the newly established State Oil Company BNOC (British National Oil Cooperation) as Chief, Reservoir Engineering. In this function he participated in the discovery, development and deciphering of the secrets of the large North Sea reservoirs. His contribution during the early days of the UK offshore industry was so significant that in *1987* he received the OBE recognition for his Reservoir Engineering services to the UK industry. In these days this recognition not only honoured him for his exceptional work, but also indirectly honoured the reservoir engineering profession for its potential to influence the results of the oil and gas industry.

(2) In 1978 Laurie Dake published his first book with Elsevier on reservoir engineering under the title "The Fundamentals of Reservoir Engineering". In this work he introduced a modern vision on Reservoir Engineering based on the synthesis between rigorous physics and applied science, necessary in any field operative work. The exceptional success of this book with the entire petroleum world resulted from:

- its utility for Petroleum Engineers in applying simplified procedures to complex problems of hydrocarbon reservoirs;
- its utility as fundamental text for students at almost every University where the scientific basis of the reservoir discipline is combined with a large amount of field applications and examples.

In *1982* Laurie Dake left BNOC at the time of its privatisation and started as an independent consultant, based in Edinburgh. His comprehensive activities were divided among:

- a "direct consulting activity" with medium and large companies where Laurie made a substantial contribution to the appraisal and development of over 150 world wide oil and gas fields, *between 1982 and 1994*. He became one of the most appreciated international petroleum consultants, and was consulted by very large companies (BP, Agip, Norsk Hydro, Statoil, etc.) and banks (Bank of Scotland – Edinburgh, BankWest Perth, Australia, etc.);
- an important collaboration with the Petroleum Department of the Heriot—Watt University, where he started initially (*after 1978*) as an external examiner and where he later became a "Honorary Professor";
- the elaboration of his second book "The Practice of Reservoir Engineering", published by Elsevier in *1994.* In addition to many field operative concepts, the text included specific procedures and analyses developed by Laurie and proven successful in various fields studied by him.

In the middle of these exceptional activities, his real help to the entire petroleum engineering family through his books and courses, his consulting activities and his advice to the Financial World and Petroleum Companies, Laurie Dake's death on *July 19, 1999* left us disoriented. All of us who appreciated him, who admired his work and loved him for his exceptional qualities and distinction suddenly felt impoverished.

However, if we now look back to the horizon opened by Laurie, knowing that there exists an accepted horizon — visible but sterile, and another ... an imaginative and creative one, we may change our point of view. Knowing that the creative horizon in a sense defines the boundaries between spirit and matter, between resources and platitude, we start to understand the role played by Laurie Dake — who disregarded the customary procedure and fought to grasp the real meaning of reservoir behaviour.

He has been able with his intelligence to enlarge the opened horizon by combining the will of creativity with the knowledge of reality versus the size of possibility ..., all of which we find in the solutions proposed by him.

It is this enlarged horizon which Laurie left to all of us as a splendid heritage ...

Prof. Dr. T.D. van Golf Racht Petroleum Department, Trondheim University

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#### NOMENCLATURE

#### ENGLISH

- A area
- B Darcy coefficient in stabilized gas well inflow equations (ch. 8)
- B<sub>g</sub> gas formation volume factor
- B<sub>o</sub> oil formation volume factor
- B<sub>w</sub> water formation volume factor
- c isothermal compressibility
- ce effective compressibility (applied to hydrocarbon pore volume)
- cf pore compressibility
- ct total compressibility (applied to pore volume)
- c total aquifer compressibility (cw+cf)
- C arbitrary constant of integration
- C coefficient in gas well back-pressure equation (ch. 8)
- C pressure buildup correction factor (Russell afterflow analysis ch. 7)
- C<sub>A</sub> Dietz shape factor
- D non-Darcy flow constant appearing in rate dependent skin (ch. 8)
- D vertical depth
- e exponential
- ei exponential integral function
- E gas expansion factor
- $\mathsf{E}_{f,w}$  term in the material balance equation accounting for the expansion of the connate water and reduction in pore volume
- $\mathsf{E}_g$  term in the material balance equation accounting for the expansion of the gascap gas
- E<sub>o</sub> term in the material balance equation accounting for the expansion of the oil and its originally dissolved gas
- f fraction
- f fractional flow of any fluid in the reservoir
- f function; e.g. f(p) function of the pressure
- F cumulative relative gas volume in PVT differential liberation experiment (ch. 2)
- F non-Darcy coefficient in gas flow equations (ch. 8)
- F production term in the material balance equation (chs. 3, 9)

- F wellbore parameter in McKinley afterflow analysis (ch. 7)
- g acceleration due to gravity
- g function; e.g. g(p) function of pressure
- G gas initially in place GIIP
- G gravity number (ch. 10)
- G wellbore liquid gradient (McKinley afterflow analysis, ch. 7)
- G<sub>a</sub> apparent gas in place in a water drive gas reservoir (ch. 1)
- G<sub>p</sub> cumulative gas production
- h formation thickness
- hp thickness of the perforated interval
- H total height of the capillary transition zone
- J Bessel function (ch. 7)
- J Productivity index
- k absolute permeability (chs. 4,9,10)
- k effective permeability (chs. 5,6,7,8)
- k<sub>r</sub> relative permeability obtained by normalizing the effective permeability curves by dividing by the absolute permeability
- $\overline{k_r}$  thickness averaged relative permeability
- kr end point relative permeability
- k iteration counter
- Kr relative permeability obtained by normalizing the effective permeability curves by dividing by the end point permeability to oil (ch. 4)
- I length
- L length
- m ratio of the initial hydrocarbon pore volume of the gascap to that of the oil (material balance equation)
- m slope of the early, linear section of pressure analysis plots of pressure (pseudo pressure) vs. f(time), for pressure buildup, fall-off or multi-rate flow tests
- m(p) real gas pseudo pressure
- m'(p) pseudo pressure for two phases (gas-oil) flow
- M end point mobility ratio
- M molecular weight
- M<sub>s</sub> shock front mobility ratio
- n reciprocal of the slope of the gas well back pressure equation (ch. 8)
- n total number of moles

- N stock tank oil initially in place (STOIIP)
- Np cumulative oil production
- N<sub>pd</sub> dimensionless cumulative oil production (in pore volumes)
- N<sub>pD</sub> dimensionless cumulative oil production (in movable oil volumes)
- p pressure
- pa average pressure in the aquifer (ch. 9)
- pb bubble point pressure
- pc critical pressure
- pd dynamic grid block pressure
- p<sub>D</sub> dimensionless pressure
- pe pressure at the external boundary
- p<sub>i</sub> initial pressure
- ppc pseudo critical pressure
- ppr pseudo reduced pressure
- psc pressure at standard conditions
- pwf bottom hole flowing pressure
- Pwf(1 hr) bottom hole flowing pressure recorded one hour after the start of flow
- pws bottom hole static pressure
- pws(LIN) (hypothetical) static pressure on the extrapolation of the early linear trend of the Horner buildup plot
- p average pressure
- $p^{\star}$  specific value of  $p_{ws(\text{LIN})}$  at infinite closed-in time
- $\Delta p$  pressure drop

N.B. the same subscripts/superscripts, used to distinguish between the above pressures, are also used in conjunction with pseudo pressures, hence:  $m(p_i)$ ;  $m(p_{wf})$ ;  $m(p_{ws(LIN)})$ ; etc.

- Pc capillary pressure
- P<sub>c</sub><sup>o</sup> pseudo capillary pressure
- q production rate
- q<sub>i</sub> injection rate
- Q gas production rate
- r radiał distance
- re external boundary radius
- $r_D$  dimensionless radius =  $r/r_w$  (chs. 7,8) =  $r/r_o$  (ch. 9)
- $r_{eD}$  dimensionless radius =  $r_e/r_w$  (chs. 7,8) =  $r_e/r_o$  (ch. 9)
- rh radius of the heated zone around a steam soaked well
- ro reservoir radius
- rw wellbore radius
- $r'_{w}$  effective wellbore radius taking account of the mechanical skin ( $r'_{w} = r_{w}e^{-S}$ )
- R producing (or instantaneous) gas oil ratio
- R universal gas constant
- Rp cumulative gas oil ratio
- R<sub>s</sub> solution (or dissolved) gas oil ratio

- S mechanical skin factor
- S saturation (always expressed as a fraction of the pore volume)
- Sg gas saturation
- $S_{gr} \quad \mbox{residual gas saturation to water}$
- So oil saturation
- Sor residual oil saturation to water
- Sw water saturation
- Swc connate (or irreducible) water saturation
- Swf water saturation at the flood front
- $\overline{S}_{w}$  thickness averaged water saturation
- $\overline{S}_w$  volume averaged water saturation behind an advancing flood front
- t reciprocal pseudo reduced temperature  $(T_{pc}/T)$
- t time
- t<sub>D</sub> dimensionless time
- $t_{DA}$  dimensionless time (=  $t_D r_W^2/A$ )
- $\Delta t$  closed-in time during a pressure buildup
- $\Delta t_d$  closed in time during a buildup at which  $p_{ws(LIN)} = p_d$
- T absolute temperature
- T transmissibility (McKinley afterflow analysis, ch. 7)
- T<sub>c</sub> critical temperature
- T<sub>pc</sub> pseudo critical temperature
- Tpr pseudo reduced temperature
- u Darcy velocity (q/A)
- U aquifer constant
- v velocity
- $v_g$  relative gas volume, differential liberation experiment
- V volume
- V net bulk volume of reservoir
- V<sub>f</sub> pore volume (PV)
- V<sub>g</sub> cumulative relative gas volume (sc), differential liberation PVT experiment w width
- W<sub>D</sub> dimensionless cumulative water influx (ch. 9)
- We cumulative water influx
- $W_{ei}$  initial amount of encroachable water in an aquifer;  $W_{ei}=\bar{c}W_ip_i$  (ch. 9)
- W<sub>i</sub> initial volume of aquifer water (ch. 9)
- W<sub>i</sub> cumulative water injected (ch. 10)
- W<sub>id</sub> dimensionless cumulative water injected (pore volumes)
- W<sub>iD</sub> dimensionless cumulative water injected (movable oil volumes)
- Wp cumulative water produced
- y reduced density, (Hall-Yarborough equations, ch. 1)
- Z Z-factor

#### GREEK

- $\beta$  turbulent coefficient for non-Darcy flow (ch. 8)
- $\beta$  angle between the oil-water contact and the direction of flow, -stable segregated displacement (ch. 10)
- γ specific gravity (liquids,-relative to water
   =1 at standard conditions; gas,-relative to air=1 at standard conditions)
- $\gamma$  exponent of Euler's constant (=1.782)
- $\Delta \quad \text{difference (taken as a positive difference} \\ \text{e.g. } \Delta p = p_i p)$
- $\lambda$  mobility
- $\theta$  dip angle of the reservoir
- $\Theta$  contact angle
- μ viscosity
- ρ density
- σ surface tension
- $\phi$  porosity
- $\Phi$  fluid potential per unit mass
- $\Psi$  fluid potential per unit volume (datum pressure)

#### SUBSCRIPTS

- b bubble point
- bt breakthrough
- c capillary
- c critical
- d differential (PVT analysis)
- d dimensionless (expressed in pore volumes)

- d displacing phase
- D dimensionless (pressure, time, radius)
- D dimensionless (expressed in movable oil volumes)
- DA dimensionless (time)
- e effective
- e at the production end of a reservoir block (e.g. Swe)
- f flash separation (PVT)
- f flood front
- f pore (e.g. c<sub>f</sub> pore compressibility)
- g gas
- h heated zone
- i cumulative injection
- i initial conditions
- n number of flow period
- N " " " "
- n (superscript) time step number
- o oil
- p cumulative production
- r reduced
- r relative
- r residual
- s steam
- s solution gas
- sc standard conditions
- t total
- w water
- wf wellbore flowing
- ws wellbore static

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#### CHAPTER 1

#### SOME BASIC CONCEPTS IN RESERVOIR ENGINEERING

#### **1.1 INTRODUCTION**

In the process of illustrating the primary functions of a reservoir engineer, namely, the estimation of hydrocarbons in place, the calculation of a recovery factor and the attachment of a time scale to the recovery; this chapter introduces many of the fundamental concepts in reservoir engineering.

The description of the calculation of oil in place concentrates largely on the determination of fluid pressure regimes and the problem of locating fluid contacts in the reservoir. Primary recovery is described in general terms by considering the significance of the isothermal compressibilities of the reservoir fluids; while the determination of the recovery factor and attachment of a time scale are illustrated by describing volumetric gas reservoir engineering. The chapter finishes with a brief qualitative account of the phase behaviour of multi-component hydrocarbon systems.

#### **1.2 CALCULATION OF HYDROCARBON VOLUMES**

Consider a reservoir which is initially filled with liquid oil. The oil volume in the reservoir (oil in place) is

$$OIP = V\phi(1-S_{wc})$$
 (res.vol.) (1.1)

where V = the net bulk volume of the reservoir rock

 $\phi$  = the porosity, or volume fraction of the rock which is porous

and  $S_{wc}$  = the connate or irreducible water saturation and is expressed as a fraction of the pore volume.

The product  $V\phi$  is called the pore volume (PV) and is the total volume in the reservoir which can be occupied by fluids. Similarly, the product  $V\phi(1-S_{wc})$  is called the hydrocarbon pore volume (HCPV) and is the total reservoir volume which can be filled with hydrocarbons either oil, gas or both.

The existence of the connate water saturation, which is normally 10-25% (PV), is an example of a natural phenomenon which is fundamental to the flow of fluids in

porous media. That is, that when one fluid displaces another in a porous medium, the displaced fluid saturation can never be reduced to zero. This applies provided that the fluids are immiscible (do not mix) which implies that there is a finite surface tension at the interface between them.

Thus oil, which is generated in deep source rock, on migrating into a water filled reservoir trap displaces some, but not all, of the water, resulting in the presence of a connate water saturation. Since the water is immobile its only influence in reservoir engineering calculations is to reduce the reservoir volume which can be occupied by hydrocarbons.

The oil volume calculated using equ. (1.1) is expressed as a reservoir volume. Since all oils, at the high prevailing pressures and temperatures in reservoirs, contain different amounts of dissolved gas per unit volume, it is more meaningful to express oil volumes at stock tank (surface) conditions, at which the oil and gas will have separated. Thus the stock tank oil initially in place is

STOIIP = N = 
$$V\phi(1-S_{wc})/B_{oi}$$
 (stock tank volume) (1.2)

where  $B_{oi}$  is the oil formation volume factor, under initial conditions, and has the units reservoir volume/stock tank volume, usually, reservoir barrels/stock tank barrel (rb/stb). Thus a volume of  $B_{oi}$  rb of oil will produce one stb of oil at the surface together with the volume of gas which was originally dissolved in the oil in the reservoir. The determination of the oil formation volume factor and its general application in reservoir engineering will be described in detail in Chapter 2.

In equ. (1.2), the parameters  $\phi$  and  $S_{wc}$  are normally determined by petrophysical analysis and the manner of their evaluation will not be described in this text<sup>1</sup>. The net bulk volume, V, is obtained from geological and fluid pressure analysis.

The geologist provides contour maps of the top and base of the reservoir, as shown in fig. 1.1. Such maps have contour lines drawn for every 50 feet, or so, of elevation



Fig. 1.1 (a) Structural contour map of the top of the reservoir, and (b) cross section through the reservoir, along the line X-Y.

and the problem is to determine the level at which the oil water contact (OWC) is to be located. Measurement of the enclosed reservoir rock volume above this level will then give the net bulk volume V. For the situation depicted in fig. 1.1(b) it would not be possible to determine this contact by inspection of logs run in the well since only the oil zone has been penetrated. Such a technique could be applied, however, if the OWC were somewhat higher in the reservoir.

The manner in which the oil water contact, or fluid contacts in general, can be located requires a knowledge of fluid pressure regimes in the reservoir which will be described in the following section.

#### **1.3 FLUID PRESSURE REGIMES**

The total pressure at any depth, resulting from the combined weight of the formation rock and fluids, whether water, oil or gas, is known as the overburden pressure. In the majority of sedimentary basins the overburden pressure increases linearly with depth and typically has a pressure gradient of 1 psi/ft, fig. 1.2.



Fig. 1.2 Overburden and hydrostatic pressure regimes (FP = fluid pressure; GP = grain pressure).

At a given depth, the overburden pressure can be equated to the sum of the fluid pressure (FP) and the grain or matrix pressure (GP) acting between the individual rock particles, i.e.

$$OP = FP + GP$$

(1.3)

and, in particular, since the overburden pressure remains constant at any particular depth, then

$$d(FP) = -d(GP) \tag{1.4}$$

That is, a reduction in fluid pressure will lead to a corresponding increase in the grain pressure, and vice versa.

Fluid pressure regimes in hydrocarbon columns are dictated by the prevailing water pressure in the vicinity of the reservoir. In a perfectly normal case the water pressure at any depth can be calculated as

$$p_{w} = \left(\frac{dp}{dD}\right)_{water} \times D + 14.7 \qquad (psia) \tag{1.5}$$

in which dp/dD, the water pressure gradient, is dependent on the chemical composition (salinity), and for pure water has the value of 0.4335 psi/ft.

Addition of the surface pressure of one atmosphere (14.7 psia) results in the expression of the pressure in absolute rather than gauge units (psig), which are measured relative to atmospheric pressure. In many instances in reservoir engineering the main concern is with pressure differences, which are the same whether absolute or gauge pressures are employed, and are denoted simply as psi.

Equation (1.5) assumes that there is both continuity of water pressure to the surface and that the salinity does not vary with depth. The former assumption is valid, in the majority of cases, even though the water bearing sands are usually interspersed with impermeable shales, since any break in the areal continuity of such apparent seals will lead to the establishment of hydrostatic pressure continuity to the surface. The latter assumption, however, is rather naive since the salinity can vary markedly with depth. Nevertheless, for the moment, a constant hydrostatic pressure gradient will be assumed, for illustrative purposes. As will be shown presently, what really matters to the engineer is the definition of the hydrostatic pressure regime in the vicinity of the hydrocarbon bearing sands.

In contrast to this normal situation, abnormal hydrostatic pressure are encountered which can be defined by the equation

$$p_{w} = \left(\frac{dp}{dD}\right)_{water} \times D + 14.7 + C \qquad (psia)$$
(1.6)

where C is a constant which is positive if the water is overpressured and negative if underpressured.

For the water in any sand to be abormally pressured, the sand must be effectively sealed off from the surrounding strata so that hydrostatic pressure continuity to the

surface cannot be established. Bradley<sup>2</sup> has listed various conditions which can cause abnormal fluid pressures in enclosed water bearing sands, which include:

- temperature change; an increase in temperature of one degree Fahrenheit can cause an increase in pressure of 125 psi in a sealed fresh water system.
- geological changes such as the uplifting of the reservoir, or the equivalent, surface erosion, both of which result in the water pressure in the reservoir sand being too high for its depth of burial; the opposite effect occurs in a downthrown reservoir in which abnormally low fluid pressure can occur.
- osmosis between waters having different salinity, the sealing shale acting as the semi permeable membrane in this ionic exchange; if the water within the seal is more saline than the surrounding water the osmosis will cause an abormally high pressure and vice versa.

Some of these causes of abnormal pressuring are interactive, for instance, if a reservoir block is uplifted the resulting overpressure is partially alleviated by a decrease in reservoir temperature.

The geological textbook of Chapman<sup>3</sup> provides a comprehensive description of the mechanics of overpressuring. Reservoir engineers, however, tend to be more pragmatic about the subject of abnormal pressures than geologists, the main questions being; are the water bearing sands abormally pressured and if so, what effect does this have on the extent of any hydrocarbon accumulations?

So far only hydrostatic pressures have been considered. Hydrocarbon pressure regimes are different in that the densities of oil and gas are less than that of water and consequently, the pressure gradients are smaller, typical figures being

$$\left(\frac{dp}{dD}\right)_{water} = 0.45 \text{ psi/ft}$$

$$\left(\frac{dp}{dD}\right)_{oil} = 0.35 \text{ psi/ft}$$

$$\left(\frac{dp}{dD}\right)_{aas} = 0.08 \text{ psi/ft}$$

Thus for the reservoir containing both oil and a free gascap, shown in fig. 1.3; using the above gradients would give the pressure distribution shown on the left hand side of the diagram.

At the oil-water contact, at 5500 ft, the pressure in the oil and water must be equal otherwise a static interface would not exist. The pressure in the water can be determined using equ. (1.5), rounded off to the nearest psi, as

$$p_w = 0.45 D + 15$$
 (psia)

(1.7)