# Lyapunov Matrix Equation *in* System Stability *and* Control

MATHEMATICS
*in*
SCIENCE
*and*
ENGINEERING

*Volume 195*


Zoran Gajić
Muhammad Tahir Javed Qureshi


# Lyapunov Matrix Equation in System Stability and Control

This is Volume 195 in
MATHEMATICS IN SCIENCE AND ENGINEERING
Edited by William F. Ames, *Georgia Institute of Technology*

A list of recent titles in this series appears at the end of this volume.

# Lyapunov Matrix Equation in System Stability and Control

Zoran Gajić
*Rutgers University*
*Piscataway, New Jersey*

Muhammad Tahir Javed Qureshi
*Water and Power Development Authority*
*Pakistan*

ACADEMIC PRESS
San Diego Boston New York London Sydney Tokyo Toronto

This book is printed on acid-free paper. ♾



Academic Press, Inc.
A Division of Harcourt Brace & Company
525 B Street, Suite 1900, San Diego, California 92101-4495

*United Kingdom Edition published by*
Academic Press Limited
24-28 Oval Road, London NW1 7DX

Library of Congress Cataloging-in-Publication Data

Gajic, Zoran.
The Lyapunov matrix equation in system stability and control / by Zoran Gajic, Muhammad Tahir Javed Qureshi.
p. cm. -- (Mathematics in sciences and engineering series : v. 195)
Includes index.
ISBN 0-12-273370-3 (alk. paper)
1. Control theory. 2. Lyapunov stability. I. Qureshi, Muhammad Tahir Javed. II. Title. III. Series: Mathematics in science and engineering : v. 195.
QA402.3.G3387 1995
003'.85'01135--dc20 95-16531
CIP

PRINTED IN THE UNITED STATES OF AMERICA
95 96 97 98 99 00 BB 9 8 7 6 5 4 3 2 1

*Dedicated to my father Radivoj Gajić*

Zoran Gajić

*To my wife Shamaila Tahir*

Muhammad Qureshi

This Page Intentionally Left Blank

# Contents

This Page Intentionally Left Blank

# Preface

This book is intended for a wide readership including engineers, applied mathematicians, computer scientists, and graduate students who seek a comprehensive view of the main results on the Lyapunov matrix equation. The book presents different techniques for solving and analyzing the algebraic, differential, and difference Lyapunov matrix equations of continuous-time and discrete-time systems. The Lyapunov and Lyapunov-like equations arise in many different prospectives such as control theory, system theory, system identification, linear algebra, optimization, differential equations, boundary value problems and partial differential equations, mechanical engineering, power systems, signal processing, large space flexible structures, communications, and the like. Therefore, its solution is of great interest. The book provides easy and quick references for the solution of many engineering and mathematical problems related to the Lyapunov matrix equations. Because both the mathematical development and the applications are considered, this book is useful for solving problems as well as for research purposes.

In this book we are concerned with the "pure" Lyapunov equation, and only in rare cases the Lyapunov-like equations are discussed. The continuous and discrete Lyapunov matrix equations are considered in three categories: (1) explicit solutions, (2) bounds of the solutions main attributes (such as eigenvalues, determinant, trace), and (3) numerical solutions. The advancements made so far in all these categories are the topics of this book. Different approaches are compared, where possible, in order to demonstrate the efficiency of any particular method. In addition, the recent results on the stability robustness, sensitivity of the Lyapunov equation, parallel algorithms and iterative methods for numerical solution of high dimensional algebraic Lyapunov equations. Also, the Lyapunov matrix equations corresponding to jump parameter linear systems, singularly perturbed and weakly coupled systems are included in this book. Several examples of real-world systems are given throughout of the book in order to demonstrate the effectiveness of the presented methods and algorithms. The book covers research work of more than 250 available journal papers on the Lyapunov matrix equation published in or before December of 1994, and the recent research work by the authors and their coworkers.

The authors are thankful for support and contributions from Professors T-Y. Li, P. Milojević, B. Petrović, and N. Puri, our colleagues Drs. X. Shen and M. Lim, graduate students I. Borno and V. Radisavljević. For technical support, we are indebted to J. Li, I. Seskar, and Dr. A. Kolarov.

Z. Gajić and M. Qureshi
Piscataway, NJ, USA
February 1995

# Chapter One

## Introduction

The Lyapunov and Lyapunov-like matrix equations appear in many different engineering and mathematical perspectives such as control theory, system theory, optimization, power systems, signal processing, linear algebra, differential equations, boundary value problems, large space flexible structures, and communications, (Dou, 1966; Barnett and Storey, 1970; Kwakernaak and Sivan, 1972; Kreisselmeier, 1972; Balas, 1982; Wonham, 1985; Hodel and Poolla, 1992). It is named after the Russian mathematician Alexander Mikhailovitch Lyapunov (Shcherbakov, 1992; Axelby and Parks, 1992), who in 1892, in his doctoral dissertation, introduced the famous stability theory of linear and nonlinear systems (Lyapunov, 1892). A complete English translation of Lyapunov's 1892 doctoral dissertation is published in *International Journal of Control* in March of 1992. According to his definition of stability, so-called stability in the sense of Lyapunov, one can check the stability of a system by finding some functions, called the Lyapunov functions. There is no general procedure for finding a Lyapunov function for nonlinear systems, but for linear time invariant systems, the procedure comes down to the problem of solving the matrix Lyapunov equation. Since linear systems are mathematically very convenient and give fairly good approximations for nonlinear systems, mathematicians and engineers very often base their analysis on the linearized models. Therefore, the solu-