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Lyapunov Matrix Equation *in* System Stability *and* Control

Zoran Gajić
Muhammad Tahir Javed Qureshi



Lyapunov Matrix Equation in System Stability and Control

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Lyapunov Matrix Equation in System Stability and Control

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Dedicated to my father Radivoj Gajić

Zoran Gajić

To my wife Shamaila Tahir

Muhammad Qureshi

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Preface

This book is intended for a wide readership including engineers, applied mathematicians, computer scientists, and graduate students who seek a comprehensive view of the main results on the Lyapunov matrix equation. The book presents different techniques for solving and analyzing the algebraic, differential, and difference Lyapunov matrix equations of continuous-time and discrete-time systems. The Lyapunov and Lyapunov-like equations arise in many different perspectives such as control theory, system theory, system identification, linear algebra, optimization, differential equations, boundary value problems and partial differential equations, mechanical engineering, power systems, signal processing, large space flexible structures, communications, and the like. Therefore, its solution is of great interest. The book provides easy and quick references for the solution of many engineering and mathematical problems related to the Lyapunov matrix equations. Because both the mathematical development and the applications are considered, this book is useful for solving problems as well as for research purposes.

In this book we are concerned with the “pure” Lyapunov equation, and only in rare cases the Lyapunov-like equations are discussed. The continuous and discrete Lyapunov matrix equations are considered in three categories: (1) explicit solutions, (2) bounds of the solutions main attributes (such as eigenvalues, determinant, trace), and (3) numerical solutions. The advancements made so far in all these categories are the topics of this book. Different approaches are compared, where possible, in order to demonstrate the efficiency of any particular method. In addition, the recent results on the stability robustness, sensitivity of the Lyapunov equation, parallel algorithms and iterative methods for

numerical solution of high dimensional algebraic Lyapunov equations. Also, the Lyapunov matrix equations corresponding to jump parameter linear systems, singularly perturbed and weakly coupled systems are included in this book. Several examples of real-world systems are given throughout of the book in order to demonstrate the effectiveness of the presented methods and algorithms. The book covers research work of more than 250 available journal papers on the Lyapunov matrix equation published in or before December of 1994, and the recent research work by the authors and their coworkers.

The authors are thankful for support and contributions from Professors T-Y. Li, P. Milojević, B. Petrović, and N. Puri, our colleagues Drs. X. Shen and M. Lim, graduate students I. Borno and V. Radisavljević. For technical support, we are indebted to J. Li, I. Seskar, and Dr. A. Kolarov.

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Chapter One

Introduction

The Lyapunov and Lyapunov-like matrix equations appear in many different engineering and mathematical perspectives such as control theory, system theory, optimization, power systems, signal processing, linear algebra, differential equations, boundary value problems, large space flexible structures, and communications, (Dou, 1966; Barnett and Storey, 1970; Kwakernaak and Sivan, 1972; Kreisselmeier, 1972; Balas, 1982; Wonham, 1985; Hodel and Poolla, 1992). It is named after the Russian mathematician Alexander Mikhailovitch Lyapunov (Shcherbakov, 1992; Axelby and Parks, 1992), who in 1892, in his doctoral dissertation, introduced the famous stability theory of linear and nonlinear systems (Lyapunov, 1892). A complete English translation of Lyapunov's 1892 doctoral dissertation is published in *International Journal of Control* in March of 1992. According to his definition of stability, so-called stability in the sense of Lyapunov, one can check the stability of a system by finding some functions, called the Lyapunov functions. There is no general procedure for finding a Lyapunov function for nonlinear systems, but for linear time invariant systems, the procedure comes down to the problem of solving the matrix Lyapunov equation. Since linear systems are mathematically very convenient and give fairly good approximations for nonlinear systems, mathematicians and engineers very often base their analysis on the linearized models. Therefore, the solu-