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Damage and Interfacial Debonding in Composites

edited by

G.Z. Voyiadjis
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Damage and Interfacial Debonding in Composites

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FOREWORD

This book represents thirteen papers that are based on the presentations made in the five session symposium on "Damage and Interfacial Debonding in Composites" on the occasion of the 32nd Society of Engineering Science Meeting; held in New Orleans, Louisiana, October 29 - November 1, 1995. The five sessions were mainly in the area of constitutive modeling of the micromechanics of damage of composites. It includes macromechanical/ micromechanical constitutive modeling, experimental procedures, and numerical modeling. Inelastic behavior, interfaces, damage, fracture, failure, and computational methods are included.

The book is divided into two parts. Part I deals with the study of damage of composites, and Part II is on the interfacial debonding of composites. The papers discuss topics ranging from theoretical treatments to experimental investigation. The papers investigate both micromechanics and continuum aspects of damage and interfacial debonding in composites.

We express our thanks to all the authors that contributed to this work. Their time and effort are greatly appreciated.

George Z. Voyiadjis
Baton Rouge, Louisiana

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March 1996

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PART I

DAMAGE IN COMPOSITES

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Damage and Interfacial Debonding in Composites
G.Z. Voyiadjis and D.H. Allen

The Stress Intensity Factors and Interaction Between Cylindrical Cracks in Fiber-Matrix Composites

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The elastic interaction between two cylindrical cracks in an infinite, homogeneous, isotropic, elastic medium is investigated. The cylindrical cracks represent a case of fiber-matrix debonding. We examine the effect of the cracks spacing and size on the stress intensity factors, K_I and K_{II}, which result from a pressure loading. Each crack is modeled as a pile-up of Somigliana ring dislocations. The solution is based on analytical expressions obtained earlier for the ring dislocation. Continuous distributions of dislocation densities, modeling the two cracks, are obtained numerically using a piecewise quadratic approximation and an iterative scheme to evaluate the interaction between the two cracks. The analysis provides estimates for the stress intensity factors and their relation to the cracks spacing and size. The analysis also reveals that each crack can be represented by a pair of superdislocations, which leads to the analytical solution. The interaction among the superdislocations also provides a closed form expression for the stress intensity factor.

1. INTRODUCTION

Recent advances in the field of material science have led to the development of a class of unconventional materials, such as fibrous composites. In general, composites are composed of strong, lightweight fibers embedded in a matrix. As the development of composites has progressed, the utilization of these materials in industry has become increasingly more common. As with any solid material, there are unavoidable stress raisers present, due to internal defects, which have important implications on the mechanical behavior of the material. Some of the most common defects which have been studied extensively include crystal defects and planar cracks. The development of composite materials has given rise to a series of internal defects which have not been thoroughly investigated. These defects arise from the characteristic, geometric structure of fiber-matrix composites, and include matrix cracking, broken fibers, fiber pull-out, and fiber-matrix debonding as shown in Figure 1. Stress singularities caused by cracks, voids or inclusions, may lead to structural failure at stress levels far below the limits estimated using a macromechanical analysis. Therefore, it is necessary to have a comprehensive understanding of fracture initiation and growth, the effects of voids and small inclusions, and their interaction with each other. In this instance, we examine the interaction between cylindrical cracks; a defect type which might occur in the case of fiber-matrix debonding, as shown in Figure 1d.

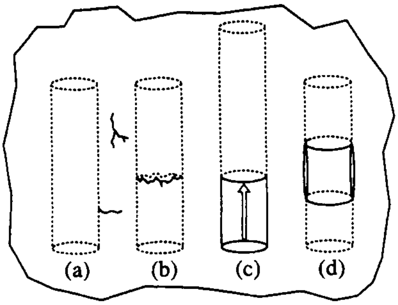

Figure 1. Defects in fibrous composite materials: (a) matrix cracking, (b) broken fibers, (c) fiber pull-out, (d) fiber-matrix debonding.

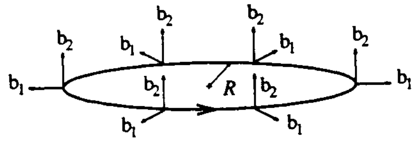

Figure 2. The Somigliana ring dislocation.

The fiber-matrix debonding problem has been previously addressed by a number of investigators. When debonding occurs, cylindrical cracks are formed at the fiber-matrix interface. The overall strength of the composite becomes dependent upon the sizes and geometries of these cracks. A review of the interface crack can be found in the recent work of Rice [1]. The most common method of modeling interfacial cracks is the dislocation approach, in which the crack is represented by a pile-up of dislocations. The work of Erdogan [2] gives a comprehensive review of fracture problems in composite materials with special emphasis on the linear elastic fracture mechanics models.

The theory of dislocations has become an increasingly useful tool for modeling many mechanical properties. Dislocation theory arose in an attempt to explain why the observed yield stresses of crystals are much lower than the theoretical yield stresses. Dislocations were first considered as singularities in continuous media and then later as crystal imperfections. A review of the early works and developments of the concepts of dislocations can be found in [3-5]. The original purpose for developing the theory of dislocations was to model singularities in a continuum, but later, the concepts and theories were adapted to a variety of problems in continuum mechanics. Today, the theory of dislocations is an important tool for modeling the continuum elastic-plastic description of deformable solids. By combining large numbers of dislocations in various ways, it is possible to model many different defects in both homogeneous and nonhomogeneous media. An introduction to the mathematical theory of dislocations can be found in [6].

The first dislocation models utilized straight dislocations of the edge and screw type. The Burgers vector of a straight dislocation remains constant and fixed at all positions along the dislocation line. Dislocations of this type are called Voltera dislocations. Later development led to the introduction of dislocations where the Burgers vector changed in magnitude and/or direction along the dislocation line. Dislocations of this type are called Somigliana dislocations [7]. We define a special type of Somigliana dislocation where the two ends of the dislocation are joined together to form a circular loop, as shown in Figure 2. This type of dislocation is called a Somigliana ring dislocation, and the stress and displacement fields associated with it are given in [8].

In continuum mechanics, dislocations are used to model internal defects. The defect which is most commonly modeled by dislocations is the planar crack. The planar crack is modeled as a pile-up of straight dislocations, and the macroscopic effects of the crack can be determined by summing the effects of the individual dislocations. The procedure for modeling planar cracks is thoroughly established [9], but this practice is not only limited to planar crack problems. The theory of dislocation pile-ups can also be applied to the cylindrical cracks which may occur in a fiber-matrix debonding problem. Cylindrical cracks may propagate along the fiber-matrix interface in composite materials. Since excessive crack propagation may ultimately lead to failure of the structure, one is very interested in the stress state in the neighboring region surrounding the crack. This problem has been investigated by a number of people who considered interfacial cracks between two isotropic materials [10,11], homogeneous transversely isotropic materials [12], and nonhomogeneous anisotropic materials [13]. These studies modeled the cylindrical crack as a pile-up of Somigliana ring dislocations. Approximate solutions for the stress fields near the crack tip were achieved by numerically solving a set of integral equations. This problem was recently re-examined by Demir *et al.* [14], who modeled the cylindrical crack as a pile-up of ring dislocations, but also utilized an earlier result they obtained for a single ring dislocation [8]. Demir *et al.* were able to achieve numerical solutions for the dislocation distributions, the extended stress field, and the stress intensity factors associated with a cylindrical crack. In addition, they were able to show that the pile-up of dislocations can be approximately represented by an equivalent pair of superdislocations, with magnitudes and positions determined to produce a similar stress field. Since the solution for the single dislocation was already given in [8], and the authors additionally provided an exact expression for the interaction between two Somigliana ring dislocations in [15], the superdislocation representation provided a closed form solution for the extended stress field of a cylindrical crack. The next logical step is to analyze a crack-crack interaction problem, establishing the framework for examining a multiple crack problem.

The two-crack problem shown in Figure 3 is proposed to investigate the macroscopic effects of the interaction between two collinear cylindrical cracks. The purpose of this study is to determine the total stress field and the stress intensity factors which arise from the interaction between the stress fields of the two cracks. Each crack is represented by a distribution of dislocation loops. From these distributions, we are able to numerically calculate the stress field and stress intensity factors resulting from applied stress in the presence of two cracks.

After the final dislocation distributions are determined, we replace the continuous distributions by sets of discrete superdislocations with magnitudes and positions calculated to produce similar extended stress fields. Based on these results, we then propose a simplified procedure to determine the extended stress field surrounding a pair of coupled cylindrical cracks. This procedure involves a series of graphs from which one can select the magnitudes and positions for the sets of superdislocations necessary to produce the extended stress field. Once the stress field has been established, the calculations for the stress intensity factors are performed by summing the interaction between all superdislocations representing the cracks. Furthermore, from the superdislocation representation, we then propose an approximate analytical model to calculate the magnitudes and positions for the sets of superdislocations. Once these expressions are established, they can be used to obtain an approximate expression for the stress intensity factors near the crack tips.

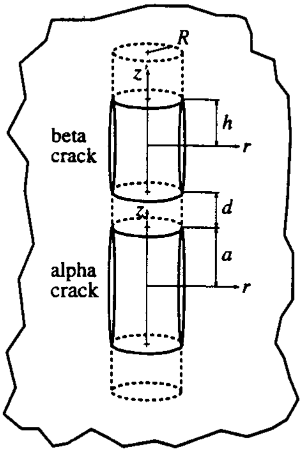

Figure 3. The dual, collinear cylindrical crack problem.

2. COLLINEAR CRACKS

We consider the case of two cylindrical cracks, both with radius R and collinear axes of symmetry as shown in Figure 3. The longer of the two cracks is designated the alpha crack and the remaining crack is designated the beta crack. The length of the alpha crack is $2a$ and the length of the beta crack is $2h$. The distance separating the two inner crack tips is d. We define two local cylindrical coordinate systems. The first coordinate system is defined with the origin at the center of the alpha crack, and the second is defined with the origin at the center of the beta crack. It is important to note that the z-axes of both coordinate systems are coincident with each other.

The same method that is used by Demir *et. al.* [14] to model the single cylindrical crack is utilized to model each of the cylindrical cracks in the two-crack problem. In an actual composite, the fiber and surrounding matrix are composed of two dissimilar materials. However, in this model, we consider the case of similar material because it can be treated analytically, which makes it possible to establish a framework for the treatment of the more complicated case of fiber-matrix problems where the solution must be carried out numerically. Therefore, although this case does