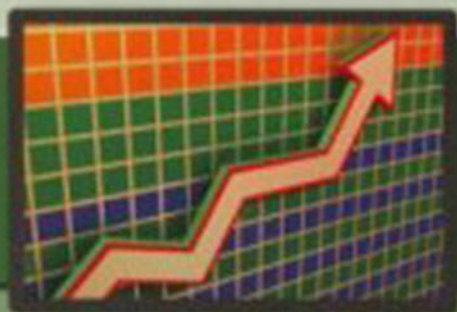


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Bhisham C. Gupta
& H. Fred Walker

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Bhisham C. Gupta
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∞ Printed on acid-free paper

In loving memory of my parents, Roshan Lal and Sodhan Devi.

—Bhisham

In loving memory of my father, Carl Ellsworth Walker.

—Fred

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Preface

Statistical *Quality Control for the Six Sigma Green Belt* was written as a desk reference and instructional aid for those individuals currently involved with, or preparing for involvement with, Six Sigma project teams. As Six Sigma team members, Green Belts help select, collect data for, and assist with the interpretation of a variety of statistical or quantitative tools within the context of the Six Sigma methodology.

Composed of steps or phases titled Define, Measure, Analyze, Improve, and Control (DMAIC), the Six Sigma methodology calls for the use of many more statistical tools than is reasonable to address in one book. Accordingly, the intent of this book is to provide for Green Belts and Six Sigma team members a thorough discussion of the statistical quality control tools addressing both the underlying statistical concepts and the application. More advanced topics of a statistical or quantitative nature will be discussed in two additional books that, together with the first book in this series, *Applied Statistics for the Six Sigma Green Belt*, and this book, will comprise a four-book series.

While it is beyond the scope of this book and series to cover the DMAIC methodology specifically, this book and series focus on concepts, applications, and interpretations of the statistical tools used during, and as part of, the DMAIC methodology. Of particular interest in the books in this series is an applied approach to the topics covered while providing a detailed discussion of the underlying concepts.

In fact, one very controversial aspect of Six Sigma training is that, in many cases, this training is targeted at the Six Sigma Black Belt and is all too commonly delivered to large groups of people with the assumption that all trainees have a fluent command of the statistically based tools and techniques. In practice this commonly leads to a good deal of concern and discomfort on behalf of trainees, as it quickly becomes difficult to keep up with and successfully complete Black Belt-level training without the benefit of truly understanding these tools and techniques.

So let us take a look together at *Statistical Quality Control for the Six Sigma Green Belt*. What you will learn is that these statistically based tools and techniques aren't mysterious, they aren't scary, and they aren't overly difficult to understand. As in learning any topic, once you learn the basics, it is easy to build on that knowledge—trying to start without a knowledge of the basics, however, is generally the beginning of a difficult situation.

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—Bhisham C. Gupta

—H. Fred Walker

1

Introduction to Statistical Quality Control

Statistical quality control (SQC) refers to a set of interrelated tools used to monitor and improve process performance.

Definition 1.1 A *process*, for the purposes of this book, is a set of tasks or activities that change the form, fit, or function of one or more input(s) by adding value as is required or requested by a customer.

Defined in this manner, a process is associated with production and service delivery operations. Because Six Sigma applies to both production and service delivery/transactional operations, understanding and mastering the topics related to SQC is important to the Six Sigma Green Belt.

In this book, SQC tools are introduced and discussed from the perspective of application rather than theoretical development. From this perspective, you can consider the SQC tools as statistical “alarm bells” that send signals when there are one or more problems with a particular process. As you learn more about the application of SQC tools, it will be helpful to understand that these tools have general guidelines and rules of thumb for both design and interpretation; however, these tools are intended to be tailored to each company for use in a specific application. This means that when preparing to use SQC tools, choices must be made that impact how certain parameters within the tools are calculated, as well as how individual stakeholders involved with these tools actually interpret statistical data and results. Accordingly, choices related to the types of tools used, sample size and frequency, rules of interpretation, and acceptable levels of risk have a substantial impact on what comes out of these tools as far as usable information.

Critical to your understanding of SQC as a Six Sigma Green Belt is that SQC and statistical process control (SPC) are different. As noted earlier, SQC refers to a set of interrelated tools. SPC is but one of the tools that make up SQC. Many quality professionals continue to use the term *SPC* incorrectly by implying that SPC is used for process monitoring as a stand-alone tool. Prior to using SPC, we need to ensure that our process is set up correctly and, as much as possible, is in a state of statistical control. Likewise, once the process is in a state of statistical control, we need valid SPC data to facilitate

our understanding of process capability and to enable the use of acceptance sampling.

1.1 Identifying the Tools of SQC

Figure 1.1 identifies the five basic tool types that make up SQC.

As can be seen in Figure 1.1, SQC consists of SPC (phase I and II), capability analysis (process and measurement systems), PRE-control, acceptance sampling (variables and attributes), and design of experiments (DOE). Within these five basic tool types are specific tools designed to provide information useful in a specific context or application. The remainder of this book will focus on the first four SQC tools, identified in Figure 1.1. DOE, as identified in Figure 1.1, is a component of SQC. However, DOE is also treated as a set of tools outside the context of SQC, and for this reason we will address DOE in the next two books in this series.

1.2 Relating SQC to Applied Statistics and to DOE

There is a distinct relationship among applied statistics, SQC, and DOE, as is seen in Figure 1.2.

Figure 1.2 shows that each of the SQC tools, as well as DOE, is based on applied statistics. The first book in this four-book series, *Applied Statistics for the Six Sigma Green Belt*, provides the foundational skills needed to learn the content presented here. Note that in Figure 1.2, with the possible exception of PRE-control, the level of statistical complexity increases with the use of the SQC tools moving from left to right. The level of statistical complexity is the greatest in DOE, and, in fact, there is an increasing level of statistical complexity within DOE, as you will see in the next two books in this series: *Introductory Design of Experiments for the Six Sigma Green Belt* and *Advanced Design of Experiments for the Six Sigma Green Belt*.

In understanding the relationship among applied statistics, SQC, and DOE, you should note the order in which they are presented—applied statistics, SQC, and then DOE. These topics are presented in this order in this

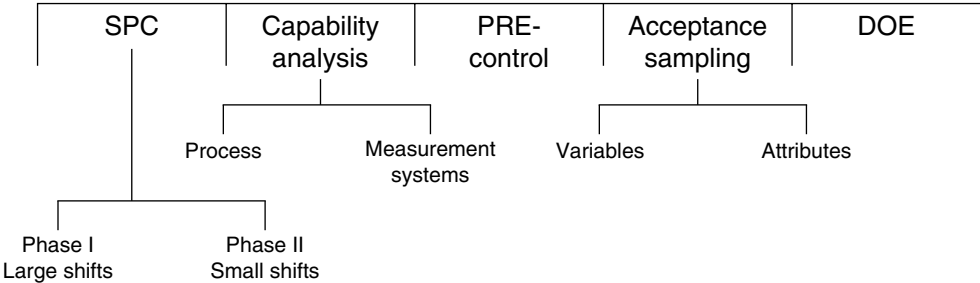


Figure 1.1 The five tool types of SQC.

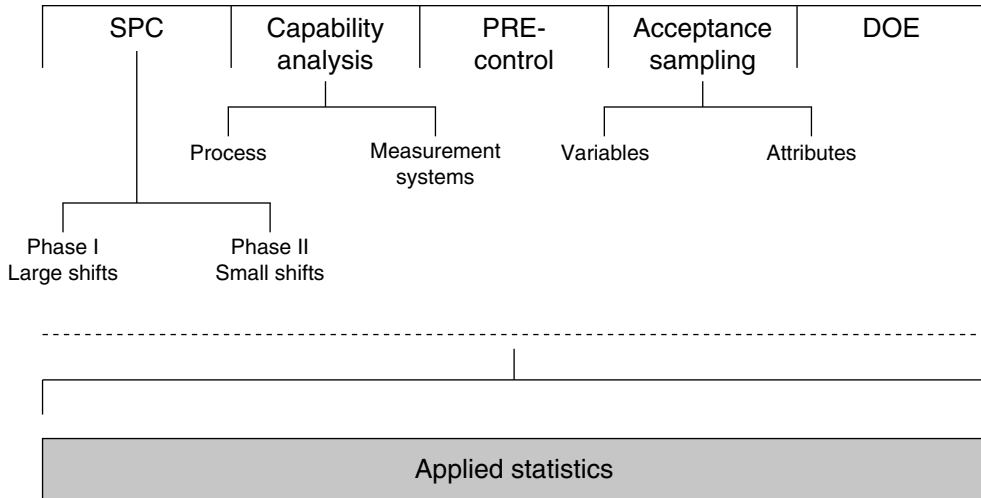


Figure 1.2 Relationship among applied statistics, SQC, and DOE.

book, and in most statistical and engineering texts and literature, to reflect the increasing level of computational difficulty. You should also know that in practice these tools would be used in a different order, which is applied statistics, DOE, and then SQC. There are three reasons, all quite logical, for changing the order of presentation: (1) moving from applied statistics to DOE is generally considered to be too rapid an increase in computational complexity for many people to easily grasp, (2) moving from applied statistics to DOE removes the opportunity for development of process knowledge, which provides the context for a study of experimental factors that come from product and process designs, and (3) it is generally necessary for us to determine which process parameters need to be monitored with DOE prior to using the process monitoring tools of SQC.

Figure 1.1 and Figure 1.2, then, represent maps of the topics addressed in SQC and provide the order of presentation of those topics in this book as well as in the greater statistical and engineering communities. Figure 1.3 represents an order in which those topics would be applied in process or transactional Six Sigma, assuming all the tools were to be applied.

The intent of Figure 1.3 is to illustrate that Six Sigma Green Belts would, where applicable, begin with DOE followed by the SQC tools. Further, Figure 1.3 illustrates that there is a cycle of iteration wherein DOE leads us to identify appropriate process parameters to monitor. We then use SQC tools to monitor those parameters, which may lead us to continue with additional experimentation and process monitoring as we refine our processes to better meet customer expectations.

Figure 1.3 also shows that once we identify with DOE the characteristics to monitor in our process, we then use SPC and capability analysis simultaneously to ensure that our process is in a state of statistical control and that our process variability and mean are consistent with our specifications. Another important point shown in Figure 1.3 is that we may or may not use a tool

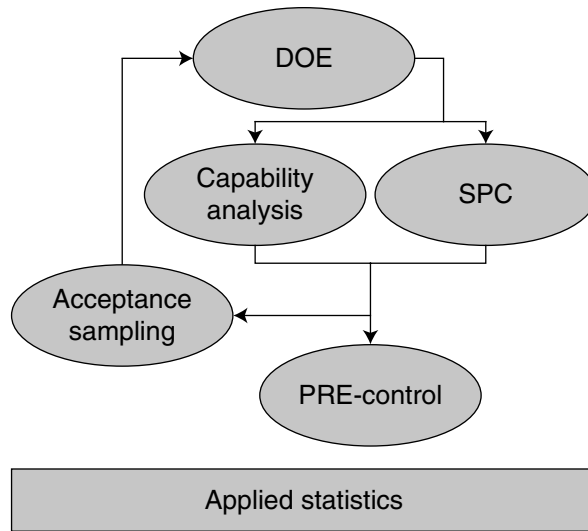


Figure 1.3 Order of SQC topics in process or transactional Six Sigma.

type called PRE-control. The very name *PRE-control* counterintuitively and incorrectly implies its use prior to SPC. If used at all, PRE-control is used after SPC, wherein processes are properly centered on target, are in a state of statistical control, are determined to be capable, and exhibit very low defect rates. Use of PRE-control as a means of reduced sampling and inspection continues to be controversial, and it is applicable only in a very small set of circumstances, as will be discussed more fully in Chapter 8. Whether or not PRE-control is used, the next tool type used, as identified in Figure 1.3, is acceptance sampling. What all these tools have in common is a statistical basis for analysis and decision making.

1.3 Understanding the Role of Statistics in SQC

As noted in section 1.2, the first book in this series focusing on the Six Sigma Green Belt is *Applied Statistics for the Six Sigma Green Belt*. Developing a working knowledge of basic statistics and how they apply to production and service delivery operations was an important step in enabling us to discuss SQC. Each SQC tool is based on statistical theory and application. The value and amount of information you are able to obtain from SQC tools are directly related to your level of understanding of basic statistical concepts. Because SQC is based on the application of statistics, much of what you read in this book assumes you have mastery of the prerequisite knowledge.

In practice, two groups of people use SQC tools:

1. Shop-floor operators and service delivery/transaction-focused people
2. Technicians, engineers, Six Sigma team members, and management team members

Each group using SQC tools has different roles and responsibilities relative to the use and implementation of the tools. For example, people in group 1 are commonly expected to collect data for, generate, and react to SQC tools. People in group 2 are commonly expected to design and implement SQC tools. They are also expected to critically analyze data from these tools and make decisions based on information gained from them.

1.4 Making Decisions Based on Quantitative Data

In practice, we are asked to make decisions based on quantitative and qualitative data on a regular basis.

Definition 1.2 *Quantitative data* are numerical data obtained from direct measurement or tally/count. Direct measurement uses a scale for measurement and reference, and tally/count uses direct observation as a basis for summarizing occurrences of some phenomenon.

Definition 1.3 *Qualitative data* are nonnumerical data obtained from direct observation, survey, personal experience, beliefs, perceptions, and perhaps historical records.

It is important to acknowledge that application of both quantitative and qualitative data has value and can be entirely appropriate in a professional work environment depending on the types of decisions we need to make. It is also important to acknowledge the difficulty in defending the use of qualitative data to make decisions in the design and process improvement efforts most commonly encountered by the Six Sigma Green Belt. Key, then, to obtaining the maximum value of information from SQC tools is realizing the power of quantitative data, because what can be directly measured can be validated and verified.

1.5 Practical versus Theoretical or Statistical Significance

As a Six Sigma Green Belt you will use applied statistics to make decisions. We emphasize the words *applied statistics* to note that applied statistics will be the basis for business decisions. When making decisions, we simply must temper our ability to detect statistical differences with our ability to act on designs and processes in a cost-effective manner. Figure 1.4 helps us visualize what we are trying to accomplish in detecting statistical differences.

In Figure 1.4 we see a normal distribution with a level of test significance (α) defined by the shaded regions in the tails of the distribution. The α identifies the region of the distribution wherein we would not expect to see evidence of process behavior if the process is behaving as intended. As a Six Sigma Green Belt you have the ability to set the level of α , which means you are actually making choices about the amount of area for the shaded region—the higher the level of α selected, the bigger the shaded region, the more discriminating the

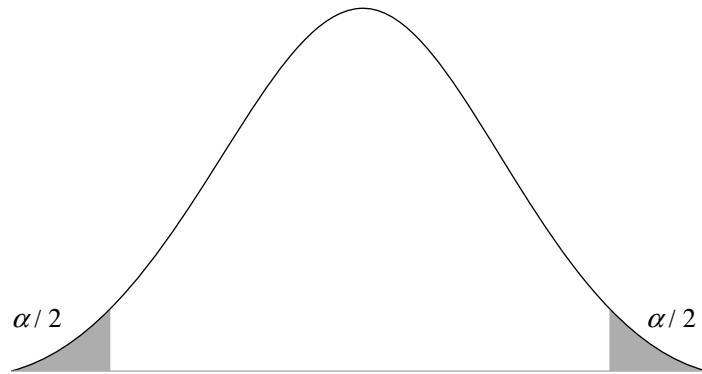


Figure 1.4 Detecting statistical differences.

test, and the more expensive it will be to make process improvement changes. While making any decisions related to α has financial implications, to understand practical differences we need to look at Figure 1.5.

In Figure 1.5 our comparison point changes from the shaded regions under the distribution tails of Figure 1.4 to the center of the distribution. Practical decisions then require that we consider how far off the intended target the observed process behavior is as compared with the statistical difference identified in Figure 1.4. It should be noted that differentiating between a practical and a statistical difference is a business or financial decision. When making a practical versus a statistical decision, we may very well be able to detect a statistical difference; however, it may not be cost effective or financially worth making the process improvements being considered.

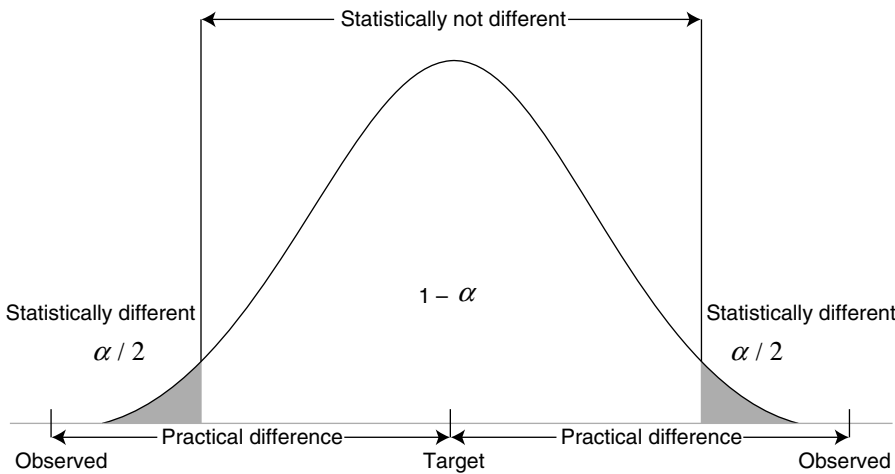


Figure 1.5 Detecting practical and statistical differences.

1.6 Why We Cannot Measure Everything

Whether in process-oriented industries such as manufacturing or in transactional-oriented industries, the volume of operations is sufficiently large to prohibit measurement of all the important characteristics in all units of production or all transactions. And even if we could measure some selected quality characteristic in all units of production or in all transactions, it is well known that we simply would not identify every discrepancy or mistake. So managing a balance between the volume of measurement and the probability of making errors during the measurement process requires us to rely on the power of statistics.

The power of statistics, in this case, refers to conclusions drawn from samples of data about a larger population, as shown in Figure 1.6.

Because we cannot afford the time or cost of measuring 100 percent of our products or transactions, sampling, along with appropriate descriptive or inferential statistics, is used to help us understand our processes. The important point contained in Figure 1.6 is that *samples*, by definition, are subsets of data drawn from a larger population. Because samples do not contain all the data from a population, there is a risk that we will draw incorrect conclusions about the larger population.

1.7 A Word on the Risks Associated with Making Bad Decisions

When relying on inferential or descriptive statistics based on samples of data, we risk making bad decisions. Bad decisions in practice lead to difficulties and problems for producers as well as consumers, and we refer to this as producer risk and consumer risk. The same bad decisions in statistical terms are referred to as Type I and Type II error, as well as alpha (α) and beta (β) risk, respectively. It is important for Six Sigma Green Belts to realize that these

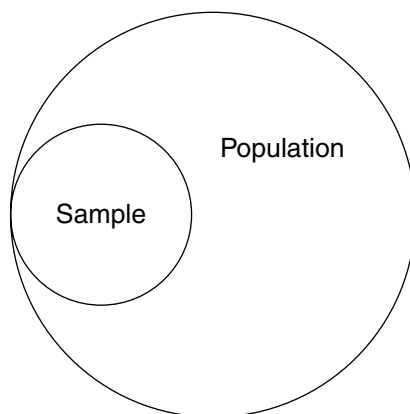


Figure 1.6 Sample versus population.

risks exist and that people from different functional areas within an organization may use different terms to describe the same thing. Lastly, we must realize that choices made during the design of SQC tools, choices related to selection of consumer and producer risk levels, quite dramatically impact performance of the tools and the subsequent information they produce for decision making.

It is not enough to simply identify the risks associated with making bad decisions; the Six Sigma Green Belt must also know the following key points:

- Sooner or later, a bad decision will be made
- The risks associated with making bad decisions are quantified in probabilistic terms
- α and β risks added together *do not* equal one
- Even though α and β go in opposite directions (that is, if α increases, β decreases), there is no direct relationship between α and β
- The values of α and β can be kept as low as we want by increasing the sample size

Definition 1.4 *Probability* is the chance that an event or outcome will or will not occur. Probability is quantified as a number between zero and one where the chance that an event or outcome will not occur in perfect certainty is zero and the chance that it will occur with perfect certainty is one. The chance that an event or outcome will not occur added to the chance that it will occur add up to one.

Definition 1.5 *Producer risk* is the risk of failing to pass a product or service delivery transaction on to a customer when, in fact, the product or service delivery transaction meets the customer quality expectations. The probability of making a producer risk error is quantified in terms of α .

Definition 1.6 *Consumer risk* is the risk of passing a product or service delivery transaction on to a customer under the assumption that the product or service delivery transaction meets customer quality expectations when, in fact, the product or service delivery is defective or unsatisfactory. The probability of making a consumer risk error is quantified in terms of β .

A critically important point, and a point that many people struggle to understand, is the difference between the probability that an event will or will not occur and the probabilities associated with consumer and producer risk—they *simply are not the same thing*. As noted earlier, probability is the percent chance that an event will or will not occur, wherein the percent chances of an event occurring or not occurring add up to one. The probability associated with making an error for the consumer, quantified as β , is a value ranging

between zero and one. The probability associated with making an error for the producer, quantified as α , is also a value between zero and one. The key here is that α and β *do not add up to one*. In practice, one sets an acceptable level of α and then applies some form of test procedure (some application of an SQC tool in this case) so that the probability of committing a β error is acceptably small. So defining a level of α does not automatically set the level of β .

In closing, the chapters that follow discuss the collection of data and the design, application, and interpretation of each of the various SQC tools. You should have the following two goals while learning about SQC: (1) to master these tools at a conceptual level, and (2) to keep in perspective that the use of these tools requires tailoring them to your specific application while balancing practical and statistical differences.

2

Elements of a Sample Survey

The science of sampling is as old as our civilization. When trying new cuisine, for example, we take only a small bite to decide whether the taste is to our liking—an idea that goes back to when civilization began. However, modern advances in sampling techniques have taken place only in the twentieth century. Now, sampling is a matter of routine, and the effects of the outcomes can be felt in our daily lives. Most of the decisions regarding government policies, marketing (including trade), and manufacturing are based on the outcomes of various samplings conducted in different fields. The particular type of sampling used for a given situation depends on factors such as the composition of the population and the objectives of the sampling as well as time and budget availability. Because sampling is an integral part of SQC, this chapter focuses on the various types of sampling, estimation problems, and sources of error.

2.1 Basic Concepts of Sampling

The primary objective of sampling is to make inferences about population parameters (population mean, population total, population proportion, and population variance) using information contained in a sample taken from the population. To make such inferences, which are usually in the form of estimates of parameters and which are otherwise unknown, we collect data from the population under investigation. The aggregate of these data constitutes a sample. Each data point in the sample provides us information about the population parameter. Collecting each data point costs time and money, so it is important that some balance is kept while taking a sample. Too small a sample may not provide enough information to obtain proper estimates, and too large a sample may result in a waste of resources. This is why it is very important to remember that in any sampling procedure an appropriate sampling scheme—normally known as the *sample design*—is put in place.

The sample size is usually determined by the degree of precision desired in the estimates and the budgetary restrictions. If θ is the population parameter of interest, $\hat{\theta}$ is an estimator of θ , and E is the desired margin of error of estimation (absolute value of the difference between θ and $\hat{\theta}$), then the sample

size is usually determined by specifying the value of E and the probability with which we will indeed achieve that value of E .

In this chapter we will briefly study four different sample designs: simple random sampling, stratified random sampling, systematic random sampling, and cluster random sampling from a finite population. But before we study these sample designs, some common terms used in sampling theory must be introduced.

Definition 2.1 A *population* is a collection of all conceivable individuals, elements, numbers, or entities that possess a characteristic of interest.

For example, if we are interested in the ability of employees with a specific job title or classification to perform specific job functions, the population may be defined as all employees with a specific job title working at the company of interest across all sites of the company. If, however, we are interested in the ability of employees with a specific job title or classification to perform specific job functions at a particular location, the population may be defined as all employees with the specific job title working only at the selected site or location. Populations, therefore, are shaped by the point or level of interest.

Populations can be finite or infinite. A population where all the elements are easily identifiable is considered *finite*, and a population where all the elements are not easily identifiable is considered *infinite*. For example, a batch or lot of production is normally considered a finite population, whereas all the production that may be produced from a certain manufacturing line would normally be considered infinite.

It is important to note that in statistical applications, the term *infinite* is used in the relative sense. For instance, if we are interested in studying the products produced or the service delivery iterations occurring over a given period of time, the population may be considered as finite or infinite, depending on one's frame of reference. It is important to note that the frame of reference (finite or infinite) directly impacts the selection of formulae used to calculate some statistics of interest.

In most statistical applications, studying each and every element of a population is not only time consuming and expensive but also potentially impossible. For example, if we want to study the average lifespan of a particular kind of electric bulb manufactured by a company, we cannot study the whole population without testing each and every bulb. Simply put, in almost all studies we end up studying only a small portion, called a sample, of the population.

Definition 2.2 A portion of a population selected for study is called a *sample*.

Definition 2.3 The *target population* is the population about which we want to make inferences based on the information contained in a sample.