

SIGNAL PROCESSING IN ELECTRONIC COMMUNICATION



Michael J. Chapman, David P. Goodall *and* Nigel C. Steel

Horwood Series in



Engineering Science

Signal Processing in Electronic Communications

The direction in which education starts a man will determine his future life
Plato: *The Republic, Book* (427-347 BC)

“Talking of education, people have now a-days” (said he) “got a strange opinion that every thing should be taught by lectures. Now, I cannot see that lectures can do so much good as reading the books from which the lectures are taken. I know nothing that can be best taught by lectures, except where experiments are to be shewn. You may teach chymestry by lectures. — You might teach making of shoes by lectures!”

James Boswell: *Life of Samuel Johnson*, 1766 (1709-1784 AD)

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Preface

Communication is the process by which information is exchanged between human beings, between machines, or between human beings and machines. Communication Theory is the theory of the transmission process, and the language of this theory is mathematics. This book sets out to explain some of the mathematical concepts and techniques which form the elements and syntax of that language, and thus enable the reader to appreciate some of the results from the theory.

In some ways, the degree of evolution of a nation or state may be measured by the sophistication of its communication processes, particularly those based on electronic means. Within the lifetime of one of the authors, the telephone has become an everyday means of communication, and television has moved from a rarely seen novelty to the means of mass entertainment. Instantaneous communication over long distances, across continents or oceans, by voice or text has become an everyday requirement in many walks of life. More recently, the internet has provided a new dimension to the way many of us work. Electronic mail is not only an indispensable tool for collaborative research, it is a means by which colleagues, perhaps in different countries, communicate on a day-to-day basis. The wider resource of the so-called world-wide-web gives access to a mass of data. Perhaps the major problem which faces us at the time of writing is how these data can be turned into information efficiently, but that is a debate for a different forum!

All these forms of communication are conducted by the transmission of electronic signals using an appropriate method and this book concentrates on the description of signals and the systems which may be used to process them. Such processing is for the purpose of enabling the transmission of information by, and the extraction of information from, signals. Over the last few years, the unifying concepts of signals and linear systems have come to be recognised as a particularly convenient way of formulating and discussing those branches of applied mathematics concerned with the design and control of 'processes'. The process under discussion may be mechanical, electrical, biological, economic or sociological. In this text, we consider only a restricted subset of such processes, those related to communication by electronic means. There are several first class treatments of the general field of signals and linear systems and indeed, some specifically related to communication theory. However, in many cases, the haste to discuss the engineering applications of the material means that the mathematical development takes second place. Such a treatment may not appeal to, or even be readily accessible to those whose first subject, or interest, is mathematics, and who thus may not be able to supply the necessary engineering insight to follow the discussion easily. This book is aimed in

part at such readers who may wish to gain some understanding of this fascinating application area of mathematics.

We are also aware from our teaching experience that many of our engineering students (possibly more than is commonly acknowledged!) also appreciate such a development to complement and support their engineering studies. This book is also written for this readership. It is interesting to recall that, whilst the need for engineers to become competent applied mathematicians was widely recognised in the recent past, this need is not so often expressed, at least in some countries, today. It will be interesting to compare future performance in the field of design and innovation, and thus in economic performance, between countries which adopt different educational strategies.

The subject matter which is included within the book is largely self-contained, although we assume that the reader will have completed a first course in mathematical methods, as given for engineering and computer science students in most UK Universities. The style adopted is an attempt to capture that established for textbooks in other areas of applied mathematics, with an appropriate, but not overwhelming, level of mathematical rigour. We have derived results, but avoided theorems almost everywhere!

Many books on applied mathematics seem to concentrate almost exclusively on the analysis of 'given' systems or configurations. In producing this text, we have attempted to demonstrate the use of mathematics as a design or synthesis tool. Before such a task may be undertaken, it is necessary for the user or designer to achieve a considerable degree of experience of the field by the careful analysis of the relevant types of system or structure, and we have attempted to provide a suitable vehicle for this experience to be gained. Nevertheless, it has been at the forefront of our thinking as we have approached our task, that the aim of many readers will eventually be the production of network or system designs of their own. We cannot, in a book such as this, hope to give a sufficiently full treatment of any one topic area to satisfy this aim entirely. However, by focusing on the design task, we hope to demonstrate to the reader that an understanding of the underlying mathematics is an essential pre-requisite for such work.

Most of the material has been taught as a single course to students of Mathematics at Coventry University, where student reaction has been favourable. The material, to a suitable engineering interface, has been given to students of Engineering, again with encouraging results. Engineering students have generally acknowledged that the course provided an essential complement to their Engineering studies.

Many of the concepts considered within the book can be demonstrated on a PC using the MATLAB package, together with the various toolboxes. In Appendix B, we give some 'm' files, with application to the processing of speech signals.

OUTLINE OF THE BOOK.

Chapter 1 introduces the concepts of signals and linear systems. Mathematical models of some simple circuits are constructed, and the Laplace transform is introduced as a method of describing the input/output relationship. Simulation diagrams are also discussed, and a brief introduction to generalized functions is presented.

Chapters 2 and 3 provide much of the technique and analytical experience necessary for our later work. Chapter 2 is concerned with system responses. By examining the type of response which can be obtained from a linear, time invariant system, concepts of stability are developed. An introduction to signal decomposition and the convolution operation precedes a discussion of the frequency response. Chapter 3 is devoted to the harmonic decomposition of signals by use of the Fourier transform, leading to the idea of the amplitude and phase spectra of a signal. The effect of sampling a continuous-time signal on these spectra is first discussed here.

Chapter 4 deals with the design of analogue filters. Based on the analytical experience gained in the previous three chapters, the task of designing a low-pass filter is addressed first. Butterworth filters emerge as one solution to the design task, and the question of their realization using elementary circuits is considered. Transformations which produce band-pass, band-reject and high-pass filters are investigated, and a brief introduction is given to Chebyshev designs.

Chapters 5–7 provide a discussion of discrete-time signals and systems. Difference equations, the z -transform and the extension of Fourier techniques to discrete time are all discussed in some detail. The need for a fast, computationally efficient algorithm for Fourier analysis in discrete time rapidly emerges, and a Fast Fourier Transform algorithm is developed here. This chapter presents several opportunities for those readers with access to a personal computer to conduct their own investigations into the subject area.

Chapter 8 returns to the theme of design. Building on the material in the earlier chapters, it is now possible to see how digital filters can be designed either to emulate the analogue designs of Chapter 4, or from an *ab initio* basis. Infinite-impulse and finite-impulse response designs are developed, together with their realizations as difference equations. Here again, the reader with access to a personal computer, together with minimal coding skill, will find their study considerably enhanced.

Finally, in Chapter 9, we apply some of the work in the previous chapters to the processing of speech signals: speech processing, as well as image processing, being an important part of Communication Theory. There are many aspects of speech processing such as, for example, speech production, modelling of speech, speech analysis, speech recognition, speech enhancement, synthesis of speech, etc. All these aspects have numerous applications, including speech coding for communications, speech recognition systems in the robotic industry or world of finance (to name but a few), and speech synthesis, the technology for which has been incorporated in a number of modern educational toys for example. One important analysis technique is that of Linear Predictive Coding. Using this technique, a speech signal can be processed to extract salient features of the signal which can be used in various applications. For example, in the synthesis problem, these features can be used to synthesize electronically an approximation to the original speech sound. In this chapter, some specific topics considered are: a speech production model, linear predictive filters, lattice filters, and cepstral analysis, with application to recognition of non-nasal voiced speech and formant estimation.

PREREQUISITES

Chapter 1 assumes a knowledge of elementary calculus, and a familiarity with the Laplace transform would be helpful, but is not essential. Also in Chapter 1, we delve into a little analysis in connection with the discussion on generalized functions. The interested reader who wishes to develop understanding further should consult the excellent text by Hoskins, cited in the references, for a full and clearly presented account.

EXERCISES

These are designed principally to test the reader's understanding of the material but there are, in addition, some more testing exercises. In addition to stressing applications of the material, answers to Exercises are given in an Appendix.

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We have benefitted from discussions with a number of our colleagues at Coventry, in particular with Peter Lockett, Robert Low, and Ann Round. The book has been typeset by the authors using $\text{\LaTeX}$, the fact that this is the case is a tribute to the skills of its inventors. In undertaking a project of this nature, widespread encouragement is needed and, as in the earlier version of the work, there has been considerable feline support, albeit from a changed workforce! David Goodall acknowledges support given by his wife, Alison, his three daughters, Tanja, Emma and Fiona, and, in addition, helpful discussions with Mike Herring of Cheltenham and Gloucester College of Higher Education (U.K.) during initial exploratory excursions into Speech Processing. Others have helped too, and we would wish to express our thanks to Renate Fechter of Siemens (Austria), Les Balmer, Sandra Prince and Julie Clayton, all at Coventry.

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Notation	
$\in$	'is a member of'
$\subset$	'is contained in'
$\mathbb{Z}$	the integers: 0, ± 1 , ± 2 , ...
$\mathbb{N}$	the natural numbers: 1, 2, 3, ...
$\mathbb{R}$	the real numbers
$\mathbb{C}$	the complex numbers
$\mathcal{L}$	the Laplace transform operator
$\mathcal{Z}$	the z-transform operator
$\mathcal{F}$	the Fourier transform operator
$\Re$	'the real part of'
$\Im$	'the imaginary part of'
$\sim$	'is represented by'
$\leftrightarrow$	transform pairs
j	$\sqrt{-1}$
$\delta(t)$	the unit impulse (Dirac δ -) function
$\zeta(t)$	the Heaviside unit step function
$\{\delta_k\}$, $\{\delta(k)\}$	the unit impulse sequence
$\{\zeta_k\}$, $\{\zeta(k)\}$	the unit step sequence
δ_{ij}	the Kronecker delta, ($\delta_{ij} = \delta_{i-j} = \delta(i-j)$)
$\mathbf{A}^T$	the transpose of a matrix $\mathbf{A}$
$F(s)$	the Laplace transform of a signal, $f(t)$
$U(z)$	the z-transform of a sequence, $\{u_k\}$
$F(j\omega)$	the Fourier transform of a signal, $f(t)$
P	the period of a periodic signal
ω_0	the fundamental frequency in rads./sec. $\omega_0 = 2\pi/P$
T	sample period
$*$	convolution

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1

Signals and linear system fundamentals

1.1 INTRODUCTION

In this first chapter we concern ourselves with **signals** and operations on signals. The first task is to define what is meant by a signal and then to classify signals into different types according to their nature. When this has been achieved, the concept of a **system** is introduced by the consideration of some elementary ideas from the theory of electrical circuits. Useful ideas and insight can be obtained from **simulation diagrams** representing circuits (or systems) and these are discussed for the time domain. **Laplace transforms** are reviewed, with their rôle seen as that of system representation rather than as a method for the solution of differential equations.

1.2 SIGNALS AND SYSTEMS

A signal is a time-varying quantity, used to cause some effect or produce some action. Mathematically, we describe a signal as a function of time used to represent a variable of interest associated with a system, and we classify signals according to both the way in which they vary with time and the manner of that variation. The classification which we use discriminates between **continuous-time** and **discrete-time signals** and, thereafter, between **deterministic** and **stochastic signals**, although we concern ourselves only with deterministic signals. Deterministic signals can be modelled or represented using completely specified functions of time, for example:

1. $f_1(t) = a \sin(\omega t)$, with a and ω constant and $-\infty < t < \infty$,

2. $f_2(t) = ce^{-dt}$, with c and d constant and $t \geq 0$,

3. $f_3(t) = \begin{cases} 1, & |t| \leq A, \\ 0, & |t| > A, \end{cases}$ with A constant and $-\infty < t < \infty$.

Each signal $f_1(t)$, $f_2(t)$ and $f_3(t)$ above is a function of the continuous-time variable t , and thus are continuous-time signals. Notice that although $f_3(t)$ is a continuous-time signal, it is not a continuous function of time.

In some applications, notably in connection with digital computers, signals are represented at discrete (or separated) values of the time variable (or index). Between these discrete-time instants the signal may take the value zero, be undefined or be simply of no interest. Examples of discrete-time signals are

1. $f_4(nT) = a \sin(nT)$, with a and T constant and $n \in \mathbb{Z}$, i.e. n is an integer.
2. $f_5(k) = bk + c$, with b and c constants and $k = 0, 1, 2, \dots$, i.e. k is a non-negative integer.

The signals $f_4(nT)$ and $f_5(k)$ are functions of nT and k , respectively, where n and k may take only specified integer values. Thus, values of the signal are only defined at discrete points. The two notations have been used for a purpose: the origin of many discrete-time signals is in the **sampling** of a continuous-time signal $f(t)$ at (usually) equal intervals, T . If n represents a sampling index or counter, taking values from a set $I \subset \mathbb{Z}$, then this process generates the sequence of values $\{f(nT); n \in I\}$, with each term $f(nT)$ generated by a formula as in, for example, $f_4(nT)$ above. Using the notation as in $f_5(k) = bk + c$ merely suppresses the information on the sampling interval and uses instead the index of position, k , in the sequence $\{f(k); k \in I\}$ as the independent variable. Figures 1.1a–e exhibits the graphs of some of these signals. Stochastic signals, either continuous-time or discrete-time, cannot be

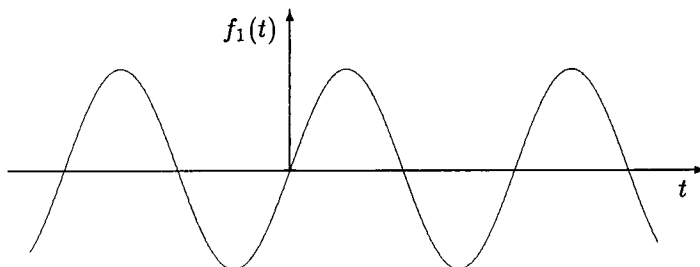


Figure 1.1a: Graph of $f_1(t)$.

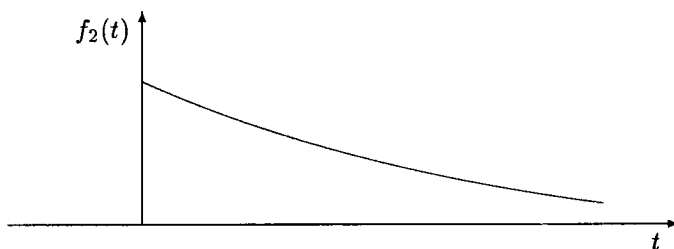


Figure 1.1b: Graph of $f_2(t)$.

so represented and their description has to be in terms of statistical properties. The analysis and processing of such signals is an important part of communication theory, but depends on an understanding of deterministic signal processing. This book concentrates on deterministic signals, and thus may serve as an introduction to the more advanced texts in this area.