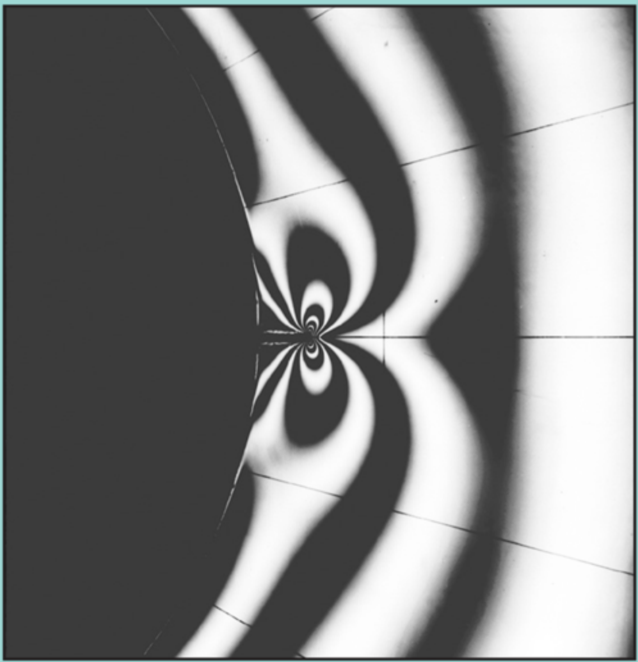
# Practical Fracture Mechanics in Design

## Second Edition, Revised and Expanded



Arun Shukla

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## Second Edition, Revised and Expanded

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# Preface to the Second Edition

This is the second edition of a book that was originally published by Alexander Blake. This edition contains revisions in each chapter, more design problems, and four additional new chapters. The underlying theme of the book "Practical Fracture Mechanics" has not been changed and the book is still intended for the design and practicing engineer. The new chapters deal with the fractographic analysis, fracture of composites, dynamic fracture, and experimental methods in fracture mechanics. Presented below is the preface from the first edition with some additions and deletions reflecting the new material included in the book.

For many years fracture mechanics has been developing from a purely theoretical basis of a new science to a more practical alternative of dealing with such matters as component design, control of brittle fracture in service, and development of material specifications. In particular, linear elastic fracture mechanics, generally referred to in the technical literature as LEFM, has now been established for many years as a design methodology that can be employed with confidence under elastic conditions. The pragmatic alternative, however, should be further refined because of so many areas of potential use that are developing in military applications and widespread private industry, particularly since the fundamental principles of fracture mechanics can cover both metallic and nonmetallic materials. Almost any type of brittle fracture—be that in a giant crane hook in heavy construction or a tiny layer of plaque inside a delicate human artery—can well be analyzed with the aid of this branch of engineering science.

Reliable fracture control plans are already required in such areas as offshore drilling rigs, nuclear power plants, space shuttles, ships, airplanes, bridges, pressure vessels, pipelines, lifting gear, and other mechanical and structural systems in which rapid fracturing cannot be tolerated.

These introductory comments are intended to set the tone for this text written for engineering practitioners, designers, and students interested in exploring the applications of fracture mechanics techniques to mechanical components, structures, and control of fractures. It appears that current researchers of fracture phenomena still outnumber practitioners at all levels of academic and industrial "hands-on" experience. And to a similar degree theoretical publications in the field vastly outnumber practice-oriented papers and books. It is therefore high time to emphasize the need for equal time and effort to balance the learning processes of theory and design practice. By the same token, this book is not directed specifically to researchers and experts in fracture mechanics.

One of the prime objectives of preparing this text is to convey a clear, practical message that the appropriate elements of fracture mechanics, materials science, and stress analysis must be closely interconnected in forming a modern approach to engineering design. These elements directly affect the process of assessing the causes of fracture and the development of the methodology for minimizing the frequency and extent of structural failures. It is, however, only proper to recognize that, in spite of the considerable progress in the area of fracture mechanics, our understanding of the basic fracture mechanism is still hampered by various uncertainties, so that, even with the best analytical tools, certain approximations and simplifying assumptions can hardly be avoided.

The calculations and experimental aspects of fracture mechanics discussed in this volume have naturally been considered supplements to conventional stress analysis and materials technology. For this reason, this book is a compilation of fundamentals, definitions, basic formulas, elementary worked examples, and references with a general emphasis on linear elastic fracture mechanics, supported by several case studies and a survey of stress calculations and material selection used for developing fracture control decisions. Pertinent numerical results included in several chapters are given in English and SI units. Furthermore, in the spirit of recognizing the need for simplicity of presentation, the academic rigor of derivations and mathematical details has been reduced to a minimum during the preliminaries of dealing with toughness parameters, defect characterization, and design constraints.

That a practical book such as this is heavily derived from the contributions of others is evident. I have taken utmost care to acknowledge all the sources of information consulted. At the same time no statement is advanced that all up-to-date standards, codes, specifications, and regulatory guides were utilized in the preparation of the book material, since the process of updating standards and improving fracture mechanics technology continues unabated.

The first two chapters provide a short historical sketch outlining in simple terms the development of the concepts of stress, energy, and material behavior and their relationship to the fundamental parameters of fracture mechanics.

Chapters 3-5 are intended to serve as a general primer for calculation, involving fundamental definitions and symbols describing fracture mechanics and materials input. These chapters contain the majority of design formulas and the numerical illustrations consistent with the typical line of questions encountered in design.

Chapter 6 includes a brief review of fracture modes and design methodologies that should be of direct interest to design engineers. It deals with the elements of structural integrity in relation to materials behavior under brittle or ductile conditions and describes specific design approaches and criteria.

Chapter 7 presents a brief discussion on experimental techniques currently used by practitioners to determine stress intensity factors for different loadings in a given geometry.

Chapter 8 deals with different aspects of dynamic fracture mechanics including crack initiation, crack arrest and crack propagation.

Chapter 9 presents a discussion on fracture of fiber reinforced composite materials.

Chapter 10 covers the important topic of fractographic analysis. The fracture surface features have a wealth of information and this chapter attempts in brief to elucidate this.

Many aspects of the book are based on the direct experiences of Alexander Blake, that he accumulated over a long period working on programs of national defense. Chapter 11 is based on his documented experience, and it highlights a number of field problems involving a mix of materials issues, fracture mechanics parameters, and stress analysis techniques at work.

Chapters 12 and 13 deal with the practical aspects of fracture control, selected design formulas, and definitions in fracture mechanics. This material is supplemented with a number of “design comments” on the application of special formulas to various design situations requiring a typical mix of knowledge of elementary fracture mechanics, materials, and stress analysis.

The invaluable contributions of Alexander Blake to this book are gratefully acknowledged. On his behalf, I want to acknowledge the contributions of all the individuals who helped him in the preparation of the first edition. I also wish to acknowledge the help of my graduate students V. Parameswaran, V. Chalivendra, N. Jain, A. Tekalur and V. Srivastava in the preparation of this revised edition. I also thank Anish Shukla for proofreading the original manuscript.

*Arun Shukla*

# Preface to the First Edition

For many years fracture mechanics has been developing from a purely theoretical basis of a new science to a more practical alternative of dealing with such matters as component design, control of brittle fracture in service, and development of material specifications. In particular, linear elastic fracture mechanics, generally referred to in the technical literature as LEFM, is now established as a design methodology that can be employed with confidence under elastic conditions. The pragmatic alternative, however, should be further refined because of so many areas of potential use that are developing in military applications and widespread private industry, particularly since the fundamental principles of fracture mechanics can cover both metallic and nonmetallic materials. Almost any type of brittle fracture, be that in a giant crane hook in heavy construction or a tiny layer of plaque inside a delicate human artery, can well be analyzed with the aid of this branch of engineering science.

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The calculational and experimental aspects of fracture mechanics discussed in this volume have naturally been considered supplements to conventional stress analysis and materials technology. For this reason, this book is a compilation of fundamentals, definitions, basic formulas, elementary worked examples, and references with a general emphasis on linear elastic fracture mechanics, supported by several case studies and a survey of stress calculations and material selection used for developing fracture control decisions. Pertinent numerical results included in several chapters are given in English and SI units. Furthermore, in the spirit of recognizing the need for simplicity of presentation, the academic rigor of derivations and mathematical details has been reduced to a minimum during the preliminaries of dealing with toughness parameters, defect characterization, and design constraints.

That a practical book such as this is heavily derived from the contributions of others is evident. I have taken utmost care to acknowledge all the sources of information consulted. At the same time no statement is advanced that all up-to-date standards, codes, specifications, and regulatory guides were utilized in the preparation of the book material, since the process of updating standards and improving fracture mechanics technology continues unabated.

Several recent textbooks were found to be of unusual value in the author's search for practical design solutions and graphical illustrations of fracture mechanics parameters to assist the creation of this volume. The authors of the selected titles [References 2, 14, 20, 21, 36, 121, 122 and 140] deserve a special credit for giving the engineering profession a pool of scientific and technical wisdom derived from countless papers and years of research in fracture mechanics and related issues.

The opening two chapters provide a short historical sketch outlining in simple terms the development of the concepts of stress, energy, and material behavior and their relationship to the fundamental parameters of fracture mechanics.

Chapters 3–5 are intended to serve as a general primer for calculation, involving fundamental definitions and symbols describing fracture mechanics and materials input. These chapters contain the majority of design formulas and the numerical illustrations consistent with the typical line of questions encountered in design.

Chapter 6 includes a brief review of fracture modes and design methodologies that should be of direct interest to design engineers. It deals with the elements of structural integrity in relation to materials behavior under brittle or ductile conditions and describes specific approaches and criteria.

The task of preparing the book material based on direct experience with fracture control would not have been possible without cooperation of the research laboratories and industrial organizations supporting the programs of national defense over the past 30 years. During that period the mechanical and structural design philosophy was shaped in terms of the principles of practical fracture mechanics at all stages of procurement and fielding of critical hardware components and systems. One of the specific programs was concerned with the engineering aspects of fielding underground experiments, reflected briefly in the case studies and the elements of fracture control in Chapter 7. The entire chapter is based on documented experience, and it highlights a number of field problems involving a mix of materials issues, fracture mechanics parameters, and stress analysis techniques at work.

Chapters 8 and 9 deal with the practical aspects of fracture control, selected design formulas, and definitions. This material is supplemented with a number of "design comments" on the application of special formulas to various design situations requiring a typical mix of knowledge of elementary fracture mechanics, materials, and stress analysis.

I wish to acknowledge the individual contributions and reviews that greatly influenced the intent, scope, and technical presentation of the material.

Mr. Philip R. Landon offered his extensive knowledge of metallurgy, practical use of fracture mechanics, and history of special case studies related to structural failures in support of the presentation of Chapter 7 and other portions of the book. His insight into the process of blending materials science, the fundamentals of fracture control, and basic stress analysis was of special value during the planning and development of the book's concept.

Mr. Anthony M. Davito undertook the painstaking task of a detailed technical review of the entire manuscript with special emphasis on fracture mechanics methodology, selection of working formulas, and the numerical illustrations. His long-standing experience in advanced engineering design and his superior technical knowledge of the structural and mechanical aspects of modern technology have provided a high level of confidence in presentation of

the design arguments and the numerical accuracy of the calculations. Last but not least, the reviewer has performed a valuable service to the reader by confirming the relevance of the book's material to design.

Dr. Donald W. Moon conducted a critical review of the manuscript from the point of view of a specialist in materials science and the engineering aspects of fracture mechanics. His varied professional background and interest in this book project have been of special help in clarifying the presentation of scientific concepts, definitions, and pragmatic elements of the two disciplines in relation to a modern approach to fracture control technology. The emphasis of the review was on the scientific and educational aspects of the book and reader-friendly characteristics.

I am open to comments and constructive suggestions for future improvements of this book's substance and style. This work is offered here as a plea for equal time in dealing with the theory and practice of fracture mechanics.

*Alexander Blake*

# Contents

# 1

# Historical Developments in Fracture Mechanics and Overview

Several structural failures can be associated with the fracture of one or more of the materials making that structure. When such events occur, they are mostly unexpected, sudden, and unfortunate, and it is natural for us to focus attention on minimizing the undesired consequences when designing and analyzing modern-day structures. The study of crack behavior, prevention and analysis of fracture of materials is known as *fracture mechanics*.

In every discipline, including fracture mechanics, it is of critical importance to examine the historical antecedents. Progress not only depends on revolutionary ideas, but a significant part of it depends on retentiveness as well. People who tend to ignore the past are more prone to repeat mistakes. Although developments in fracture mechanics concepts are quite new, designing structures to avoid fracture is not a new idea. The fact that many ancient structures are still standing is a testimony to this. The stability of some of the ancient structures is quite amazing when we consider the fact that the choice of construction material was limited at that time. Brick and mortar, which were relatively brittle and unreliable for carrying tensile loads, were the primary construction materials. Even though the concept of brittle fracture did not exist, the structures were inadvertently designed against fracture by ensuring that the weaker components of the structure were always in compression. An arch-shaped Roman bridge design, as shown in Fig. 1.1, is an excellent example of a structure where fracture was avoided by virtue of design. The possibility of fracture in the bridge design

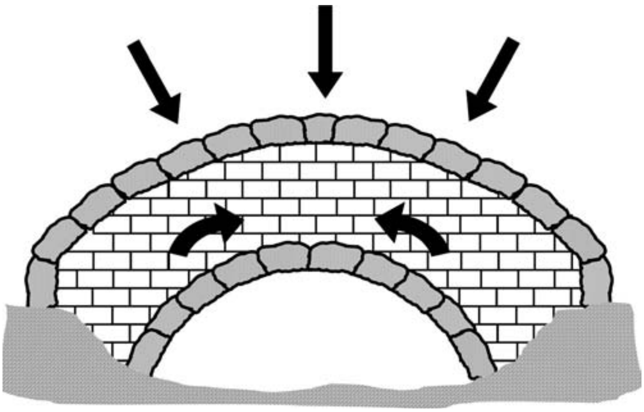

**FIGURE 1.1** Schematic of an ancient Roman bridge design.

was avoided as the arch shape of the bridge results in compressive rather than tensile stresses being transmitted through the structure.

## EARLY FOUNDATIONS OF STRENGTH OF MATERIAL CONCEPTS

The notes of Leonardo da Vinci (1452–1519) are the earliest records indicating a concept for evaluating the strength of materials. He suggested an experiment (da Vinci's sketch shown in Fig. 1.2), which is believed, was intended to establish a "law" for the influence of length on the strength of all types of materials. Even though it is unknown if the experiments were performed at that time, this was an early indication of the size effects on the strength of material. A longer wire corresponds to a larger sample volume and provides a higher probability of sampling a region containing a flaw.

The science and early evolution of the strength of materials concepts can be attributed to Galileo. In the early 17th century, Galileo turned his attention to structural mechanics while he was under house arrest and was banned from celestial mechanics. In his book *Due Nuove Scienze*, which was published in 1638,[1] Galileo introduced the concept of tensile strength in simple tension (Fig. 1.3), which he referred to as "*absolute resistance to fracture*." His observation that the strength of the bar is proportional to the cross-sectional area and is independent of the length produced early strength of material concepts. Figure 1.3 illustrates Galileo's method of evaluating tensile strength in a column. It is an interesting fact that even Galileo noted an indication of size effect while he was visiting the Venetian Arsenal. He noticed a greater attention used by workers

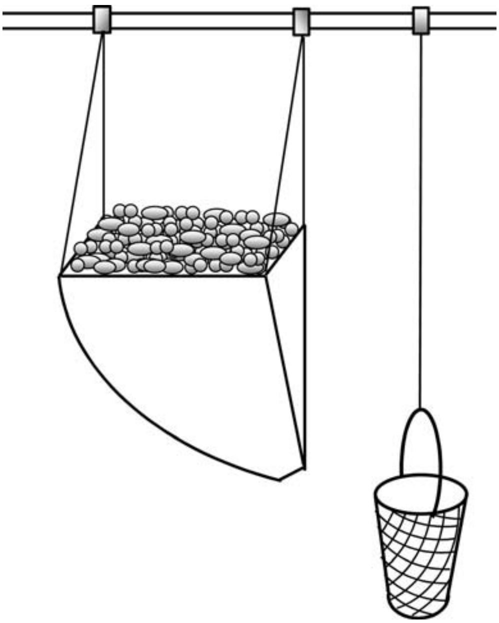

**FIGURE 1.2** Da Vinci's concept for measuring the strength of wires.

in the construction of big ships than in small ships. At that time one of the master builders explained to him that the large ships were assumed to be more brittle than the smaller vessels.

## DEVELOPMENT OF CONCEPT OF STRESSES

It was Robert Hooke (1635–1702) who broke away from the traditional thinking of his era and introduced the concept of the *true theory of elasticity or springiness* in 1678. Hooke tested wire strings of 20 to 40 ft in length by adding weight and measuring displacements. He made an important observation: that the wire always returned to its original length after several tests on the same wire. Hooke published his research in 1679,[2] outlining the principle that has since been known as Hooke's law. His far-reaching statement *ut tensio sic vis (Latin)* implied that when a mechanical force is applied to a solid object, change in shape (by extension or compression) must take place, and accordingly the solid produces a reaction.

The nature of a relationship between forces and deflections in a solid, under normal engineering conditions, is of course macroscopic. However, Hooke reasoned that when a structure is deflected, the structural material is also

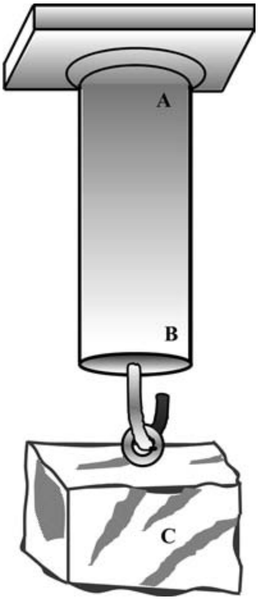


**FIGURE 1.3** Galileo's illustration of tensile strength in a column.

deformed internally. This was a remarkable observation, because we know nowadays that the atoms and molecules can move under external forces. The chemical bonds joining the atoms can therefore be stretched or compressed, although on a nanoscopic scale. Hooke continued his work (in spite of experimental difficulties) to prove this point, and he also showed that the deflection of a structure was proportional to the load. Essentially, Hooke arrived at the conclusion that all solids and objects can behave like springs. Gordon provides an excellent assessment of Hooke's mental effort by saying that it is perhaps one of the great intellectual achievements of history.[3] Hooke's law advanced two very important principles:

1. Recovery from elastic deformation.
2. Linear relationship between applied load and elastic deformation.

Although rather simplistic in mathematical terms, this principle has been a significant help to engineering practitioners for more than 300 years. It certainly represents the early history of a conventional stress analysis that deals with relationships between the deflection, geometry, and material parameters of a given structure. It further denotes the science of elasticity concerned with the interactions between forces and displacements.

From the principles of Hooke's law, all subsequent contributions were based on the theory of elasticity. In 1807, Thomas Young published[4] the definition of modulus of elasticity, which is also known as Young's modulus. Young related stress ($\sigma$) and strain ($\varepsilon$) by using the modulus of elasticity ($E$) with a very simple equation

$$\sigma = E\varepsilon \tag{1.1}$$

From this point on in structural mechanics, quantitative methods could be used to design structures without having to constantly resort to testing.

## DEVELOPMENT OF MODERN FRACTURE MECHANICS

By the end of the 19th century, the influence of crack on the structural strength was widely appreciated, but its nature and influence was still unknown. In 1913, Inglis published the first significant work in the development of fracture mechanics.[5] The work was an analytical formulation of stresses in a plate in the vicinity of a two-dimensional elliptical hole. The plate was pulled at both ends perpendicular to the ellipse as shown in Fig. 1.4. Inglis observed that the corner of the ellipse (point A) was feeling the most pressure and as the ellipse gets longer and thinner the stresses at A become larger. He examined local stresses at the tip of the ellipse and estimated that the stress concentration was

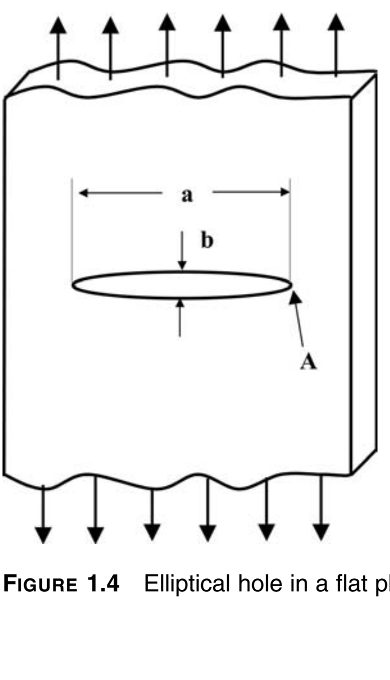


**FIGURE 1.4** Elliptical hole in a flat plate.

approximately

$$2\left(\frac{a}{b}\right) \quad \text{or} \quad 2\sqrt{\frac{a}{\rho}} \tag{1.2}$$

where $a$ and $b$ are the semimajor and semiminor axes respectively and $\rho$ is the root radius at the tip of the ellipse. Inglis evaluated various hole geometries and realized it is not really the shape of the hole that matters but the length of hole perpendicular to the load and the curvature at the end of the hole that matters in cracking. He also noticed that pulling in a direction parallel to the hole does not produce a great effect.

The basic ideas leading to the start of modern fracture mechanics can be attributed to a theory of fracture strength of glass, which was published by A.A. Griffith in 1920.[6] Using Inglis' work as a foundation, Griffith proposed an energy balance approach to study the fracture phenomenon in cracked bodies. A great contribution to the ideas about breaking strength of materials emerged when Griffith suggested that the weakening of material by a crack could be treated as an equilibrium problem. He proposed that the reduction in strain energy of a body when the crack propagates could be equated to the increase in surface energy due to the increase in the surface area. The Griffith theory assumed that the fracture strength was limited by the existence of initial cracks and that brittle materials contain elliptical microcracks, which introduce high stress concentrations near their tips. He developed a relationship between crack length ($a$), surface energy connected with traction-free crack surfaces ($2\gamma$), and applied stress, which is given by

$$\sigma^2 = \frac{2\gamma E}{\pi a} \tag{1.3}$$

Plasticity effects in metals limited the theorem and it was not until Irwin's work in 1948, that a modification was made to Griffith's model to make it applicable to metals. Irwin's first major contribution was to extend the Griffith approach to metals by including the energy dissipated by local plastic flow.[7] Orowan independently proposed a similar modification[8] to Griffith's theory in 1949. Orowan limited practical use to brittle materials while Irwin made no such restrictions.

It is an interesting fact and perhaps relevant to point out that the scientific curiosity towards fracture mechanics became a significantly important engineering discipline after the unfortunate failures of Liberty ships during World War II. The Liberty ships were built by the United States to support Britain's war effort and used a new construction method for mass production in which the hull was welded instead of riveted. The Liberty ship program was an astounding success until 1943, when a Liberty ship broke completely in two while sailing in the

North Pacific. Later, hundreds of other vessels sustained fractures. An investigation into Liberty ship failures pointed out poor toughness of steel and transition from ductile to brittle behavior at the service temperatures that ships experienced. It was noticed that the fractures initiated at the square hatched corners on the deck where there was a local stress concentration and the sharp corners acted like starter cracks. Research into this problem was led by George Rankine Irwin at the Naval Research Laboratory in Washington, DC. It was the research during this period that resulted in the development and definition of what we now refer to as linear-elastic fracture mechanics (LEFM). A major breakthrough occurred in the early 1950s when Irwin and Kies[9,10] and Irwin[11] provided the extension of Griffith theory for an arbitrary crack and proposed the criteria for the growth of this crack. The criterion was that the *strain energy release rate* ($G$) must be larger than the critical work ($G_c$), which is required to create a new unit crack area. Irwin also related strain energy release rate to the stress field at the crack tip using Westergaard's work.[12] Westergaard had developed a semi-inverse technique for analyzing stresses and displacements ahead of a crack tip. Using Westergaards' method, Irwin showed that the stress field in the area of the crack tip is completely determined by a quantity $K$ called the *stress intensity factor*. Using the method of virtual work, Irwin presented a relationship between the energy release rate and the stress intensity factor as

$$\sigma_{ij} = \frac{K f_{ij}(\theta)}{\sqrt{2\pi r}} \tag{1.4}$$

$$K^2 = EG \tag{1.5}$$

where $E$ is Young's modulus.

Other serious failures that were experienced during that period were those of the de Havilland "Comet" commercial aircraft. The Comet was first manufactured in 1952, and was the first two-jet-engine aircraft to fly at 40,000 ft with a pressurized cabin. After about a year in service, three aircraft failed, resulting in the tragic loss of several lives. In 1955, Wells[13] used fracture mechanics to show that the fuselage failures in several Comet jet aircraft resulted from fatigue cracks reaching a critical size. These cracks were initiated at windows and were caused by insufficient local reinforcement in combination with square corners, which produced higher stress concentrations. It was noticed that the fracture of welded Liberty ships, the pressurized cabin fractures of de Havilland Comet jet airplanes, bursts of several large petroleum storage tanks, and several other unpredicted failures, all seemed understandable in terms of the new fracture-strength points of view. The evaluation method was straightforward, a value of $G_c$ was established from laboratory tests on precracked specimens and the value of the driving force $G$ that tended to extend the starting crack was computed using appropriate stress analysis methods. The comparison showed that the fracture toughness had not been large enough to prevent crack propagation in the failure cases mentioned above.

In 1957, Williams[14] developed an infinite series that defined stress around a crack for any geometry. The use of the optical method "photoelasticity" to examine the stress fields around the tip of a running crack was published by Wells and Post in 1958,[15] and Irwin[16] observed that the photoelastic fringes not only formed closed loops at the crack tip as predicted by singular stress field equations but also showed a tilt as a result of the near specimen boundaries. In 1960, a significant contribution to the development of LEFM was put forth when Paris and his coworkers advanced an idea to apply fracture mechanics principles to fatigue crack growth. Although they provided convincing experimental and theoretical arguments for their approach, the initial resistance to their work was intense and they could not find a peer-reviewed technical journal to publish their manuscript. They finally opted to publish their work in a University of Washington periodical entitled *The Trend in Engineering*.[17] The work by Paris and colleagues was a landmark in the fatigue aspects of fracture mechanics, and yielded the equation

$$\frac{da}{dN} = c(\Delta K)^n \tag{1.6}$$

where $c$ and $n$ are the curve-fitting parameters of experimentally obtained fatigue data.

Linear elastic fracture mechanics is not valid when significant plastic deformation precedes failure. Although earlier theoretical developments were aimed at understanding brittle crack behavior, it became apparent from experiments that except for a few, most materials are ductile and therefore linear elastic analysis should be modified accordingly. Dugdale[18] in 1960 and Barenbelt[19] in 1962 made the first attempts to include cohesive forces in the crack tip region by developing an elaborate model within the limits of elasticity. Later, in 1968, Rice conducted a simplified analysis of complete plastic zone formation, approximated by a circular region ahead of the crack tip.[20,21] The results derived from the energy–momentum tensor concept and applied to elastic cracks were extended to include plastic cracks by defining a path-independent integral termed the $J$ integral. The plastic zone size and the crack opening displacement were found to correlate with the elastic stress intensity factor criterion. The successful experiments in 1971 by Begley and Landes,[22] who were the research engineers at Westinghouse, led to the publication of a standard procedure for $J$ testing of metals in 1981.[23] In 1976, Sih[24] introduced the strain–energy density concept, which was a departure from classical fracture mechanics. He was able to characterize mixed-mode extension problems with this method, which also provided the direction of the crack propagation in addition to the amplitude of the stress field.

Contemporary research and development in fracture mechanics focuses on several interesting areas, such as dynamic fracture mechanics, interface fracture mechanics, shear ruptures in earthquakes, stress corrosion cracking, environmental effects on fatigue crack propagation, fracture of novel materials such as nanocomposites and graded materials. Extremely powerful numerical codes that are able to investigate fracture due to separation of atoms are being developed. Also, experimental techniques have progressed enough to investigate fracture in materials at nanometer length scales and nanosecond time resolution. However, experimental techniques that could provide spatial and temporal resolution simultaneously at the nanolevel are still not available. At this point it is pertinent to point out that in the present age of unprecedented technological growth, we are inclined to believe that technology and knowledge are accelerating in an exponential fashion. However, we must recognize that we are observing an exceedingly tiny period of human development on a historic time scale. Table 1.1 represents a compression of the elapsed time from the Big Bang event to the present day and puts in perspective that on a cosmic scale, our knowledge is still in its infancy. Although the field of fracture mechanics has matured in recent years, there will be a lot more to learn in the future.

## BASIC CONCEPTS OF STRESS AND STRAIN

It is a rather strange twist of history that no real progress was made in solving practical problems in stress analysis and elasticity until 120 years after Hooke's

**TABLE 1.1** Historical Events on a Cosmic Time Scale.

| Events | Time: Compressed and scaled to 1 year |
|---|---|
| Big Bang | Jan 1st |
| Origin of Milky Way galaxy | May 1st |
| Origin of Solar System | Sept 9th |
| Formation of Earth | Sept 14th |
| First Humans | Dec 31st, 10:30 p.m. |
| Euclidian Geometry, Archimedean Physics, Roman Empire, Birth of Christ | Dec 31st, 11:59:56 p.m. |
| Renaissance in Europe, experimental methods in science | Dec 31st, 11:59:59 p.m. |
| Major developments in science and technology, power, flight, space, computers and strength of material concepts | Last second of the year (present) |

death. In terms of our modern rush to saturate every scientific field with advanced numerical techniques, theoretical solutions, and software tools, 18th century experience appears to be unreal. There were, of course, valid reasons for the lack of progress, some of which have persisted until today. Theoretically inclined engineers and scientists, as well as most philosophers, appear to have limited interest in the problems of design and manufacture of industrial products, and in the numerous technical decisions affecting the integrity and economics of structures and machines. Hooke's accomplishments in the field of mechanics and his long-standing inventions were more than enough for the majority of interests of a scientific and technical nature during the 18th century. Finally, after many years of outstanding intellectual effort on the part of Leonhard Euler,[25] Thomas Young,[4] and an applied mathematician, Augustin Cauchy,[26] the concept of stress and strain was becoming a practical engineering tool, with Cauchy securing the major part of credit for this development in 1822.

Although they are rudimentary in nature, it is well at this point of our trek through history to sum up the concepts of stress and strain without which even the current, sophisticated science could not survive. While these concepts were few and, in modern terms, so obvious, they took centuries to develop. It appears that Galileo himself (1564–1642) almost stumbled upon the idea of stress, but the world needed another 180 years for this concept to mature. The simplicity is bewildering when we say that

$$\sigma = \frac{load}{area} = \frac{W}{A} \tag{1.7}$$

This can be tension or compression at a given point in a material that is acting in the direction of the applied load. In English units the stress ($\sigma$) is usually given in pounds (force) per square inch, or psi for short. Scientists prefer using the SI (Système Internationale) units while the Continental countries (generally speaking, Europe and Asia) employ metric units such as kilogram (force) per square centimeter, or $\mathrm{kg/cm^2}$ for short. The basis for the SI, applicable to conventional stress analysis and fracture mechanics, involves the *newton* as a unit of force or weight while the unit of stress is known as the *pascal*, with the relevant symbols of (N) and (Pa). Hence pressure, stress, material strength, or elastic constants in the SI world are denoted by pascals or their more convenient multiples. Unfortunately, in engineering work, the unit of the pascal is far too small for all practical purposes. After struggling with the pascal unit for years it is hard not to promote the use of $\mathrm{N/mm^2}$. The dimensions of countless machine components and structural elements are still expressed in millimeters worldwide. Also, 1 atmosphere, for practical reasons, can be taken as $0.1\ \mathrm{N/mm^2}$, the strength of typical mild steel as $250\ \mathrm{N/mm^2}$, or the elastic modulus for steel in general as

$2 \times 10^5$ N/mm$^2$. Of course, the proposed unit relates to the pascal as follows

$$\frac{\mathrm{N}}{\mathrm{mm}^2} = \frac{\mathrm{N}}{\mathrm{m}^2 \times 10^{-6}} = \frac{10^6\,\mathrm{N}}{\mathrm{m}^2} = 10^6\,\mathrm{Pa} = 1\,\mathrm{MPa}$$

Here MPa denotes 1 megapascal. Unfortunately, the literature lacks uniformity because of the open choice of pascal, kilopascal, megapascal, and other potential multiples of the small and cumbersome unit of stress. The traditional unit of stress (psi) is still used by many engineers in English-speaking countries in spite of the efforts to convert industries and the public at large to the SI standard of weights and measures.

In dealing with practical issues of engineering formulas and the meaning of numerical results in various portions of this book, it may be helpful to the reader to have the following brief summary of the basic conversions at hand.

1 lb = 4.4482 N
1 psi = lb/in$^2$ = 4.4482 N/(0.0254 m)$^2$ = 6895 Pa
1 ksi = 6.895 MPa
1 in. = 25.4 mm
1 psi = 0.006895 N/mm$^2$
1 lb-in. = 4.4482 N $\times$ 25.4 mm = 112.9842 N-mm
1 lb/in. = 4.4482 N/25.4 mm = 0.1751 N/mm
1 MPa = 1 N/mm$^2$ = 145 psi
1 kg = 9.8066 N
1 MN = 1 meganewton $\cong$ 100 long tons force

The second elementary but very important formula defines the concept of strain that enters considerations of stress and fracture in engineering materials. For the case of a bar in uniaxial tension or compression, the strain is

$$\varepsilon = \frac{\Delta L}{L} \tag{1.8}$$

where $\Delta L$ denotes the increase or decrease of the original length of the bar and $L$ is the bar length. Engineering strain given by Eq. (1.8) is relatively small under elastic conditions and it is convenient to express strains as percentages in order to minimize typographical errors with zeros and decimal points.

It should be added that the original efforts to define and verify the concept of strain in Hooke's days were rather irksome because of experimental difficulties and a certain amount of confusion about whether to deal with the structure as a whole or at any given point within the material. Today, of course, we take a "test piece" from the structure under consideration and the stress–strain diagram obtained from the test is essentially unaffected by the size of the test piece. However, the shape of the diagram is characteristic of any given material, as shown in Fig. 1.5.

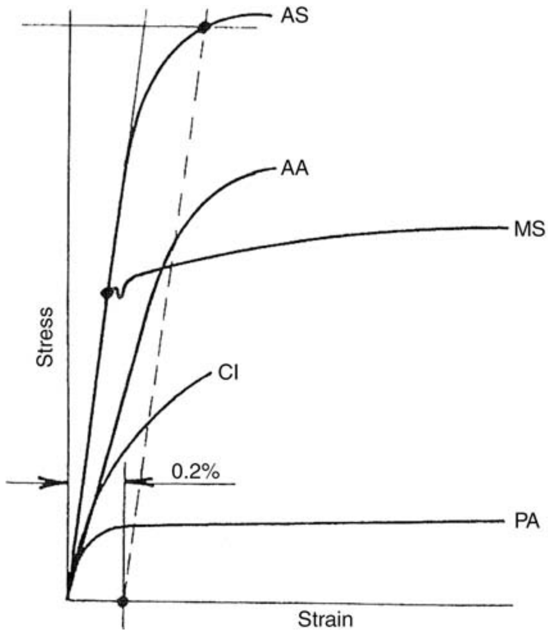


**FIGURE 1.5** Typical shapes of stress–strain curves (AS, alloy steel; AA, aluminum alloy; MS, mild steel; CI, cast iron; PA, pure aluminum) (from Ref. 27).

When the major portion of the stress–strain curve is a straight line, it is customary to say that we are dealing with a Hookean material. The slope of the curve indicates the degree of stiffness of a given solid, which leads to the definition of a material constant $E$ known as Young's modulus, also known as the modulus of elasticity or the elastic modulus:

$$E = \frac{\sigma}{\varepsilon} \tag{1.9}$$

The elastic modulus has the same dimensions as stress. This physical property is now regarded as a fundamental concept in materials science and engineering, and it has made some inroads in other science disciplines such as biology. For instance, in the cardiovascular field Young's moduli are measured for plaque and artery materials in order to better understand plaque failure strength and fracture characteristics. The importance of the Young's modulus concept can hardly be disputed, and yet it took the entire first half of the 19th century for scientists and engineers to accept it. The stress analysis and fracture mechanics principles cannot be understood and applied without the full acceptance of this key concept. Table 1.2 shows approximate values of Young's modulus for various materials.

**TABLE 1.2** Approximate Young's Moduli.

| Material | Young's modulus ($E$) psi | Young's modulus ($E$) (MPa) or ($N/mm^2$) |
|---|---|---|
| Artery | 14.5 | 0.1 |
| Plaque | 145 | 1 |
| Rubber | 1,000 | 6.9 |
| Plastics | 200,000 | 1,380 |
| Plywood | 1,000,000 | 6,897 |
| Birch | 2,070,000 | 14,280 |
| Fresh bone | 3,000,000 | 20,690 |
| Brick (hard)[a] | 3,500,000 | 24,140 |
| Magnesium | 6,000,000 | 41,380 |
| Granite | 7,000,000 | 48,280 |
| Marble | 8,000,000 | 55,170 |
| Glass | 10,000,000 | 68,970 |
| Aluminum | 10,000,000 | 68,970 |
| Brass (naval) | 15,000,000 | 103,450 |
| Cast iron | 15,000,000 | 103,450 |
| Titanium (alloy) | 17,000,000 | 117,240 |
| Cast iron (malleable) | 26,000,000 | 179,310 |
| Steel | 30,000,000 | 206,900 |
| Chromium | 42,000,000 | 289,700 |
| Tungsten | 58,000,000 | 400,000 |
| Aluminum oxide (sapphire) | 60,000,000 | 413,800 |
| Tungsten carbide | 102,000,000 | 703,400 |
| Diamond | 170,000,000 | 1,172,000 |

[a]Concrete, not shown here, has an average modulus of 3,000,000 psi. However, its value depends upon the ultimate strength according to the formula $E = 1000 \times$ compressive strength.[28]

Combining Eqs. (1.7) through (1.9) leads to a simple formula for estimating the amount of tension or compression in uniaxial loading:

$$\Delta L = \frac{WL}{AE} \tag{1.10}$$

To clarify some of the basic relationships in uniaxial loading we can look at the test piece in Fig. 1.6, where $L$ denotes the original length of the specimen and $\Delta r$ is the amount of lateral contraction. The corresponding lateral strain can be defined as

$$u = \frac{\Delta r}{r} \tag{1.11}$$

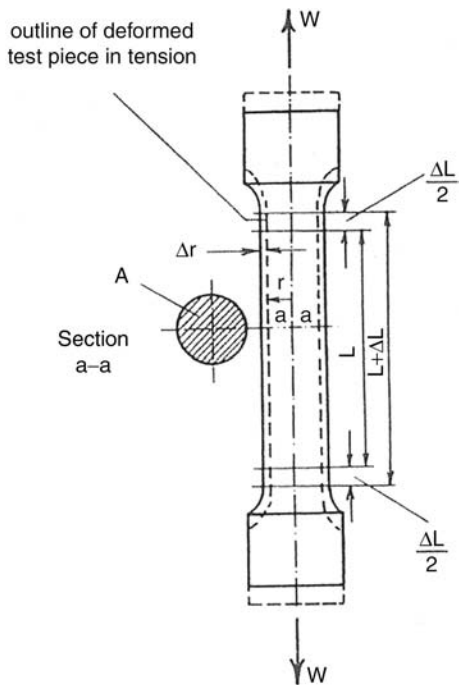


**FIGURE 1.6** Test specimen symbols (from Ref. 27).

Since the test piece in a standard case is cylindrical, $r$ is also the original radius before the tensile loads are applied.

The relationship given by Eq. (1.11) is identical in form and meaning to Eq. (1.8). Both equations represent strain but in two directions, and the relevant absolute values of strain are applicable to tension and compression. The outline shown in Fig. 1.6 gives elongation as $\Delta L$ and contraction as $\Delta r$.

At this point it is quite easy to get into an argument as to what relation should exist between $\varepsilon$, of Eq. (1.8), and $u$, of Eq. (1.11), because it is difficult to measure small changes in axial and radial displacements, even with modern technology. This limitation must have been very acute at the close of the 18th century and required mathematical insight such as that of S.D. Poisson (1781–1840). Poisson determined the ratio $(u/\varepsilon)$ analytically by employing the molecular theory of structure of the material. For elastic, isotropic materials Poisson calculated the values of this ratio, which were confirmed experimentally, and, to this day, the ratio

$$\nu = \frac{\text{unit lateral contraction}}{\text{unit axial elongation}} = \frac{u}{\varepsilon} \tag{1.12}$$

within the elastic limit is known as *Poisson's ratio*. This is constant for a given material with the theoretical limits of 0.50 and zero for ductile and brittle materials, respectively.[29]

It follows directly from Eqs. (1.9) and (1.12) that

$$u = \frac{\nu\sigma}{E} \tag{1.13}$$

Hence for a given Poisson's ratio, $\nu$, and Young's modulus, $E$, one can calculate the axial and lateral strains using Eqs. (1.9) and (1.13). It should be noted that in these calculations we have ignored any sign convention, although strictly speaking one strain such as $u$ is always of opposite sign to $\varepsilon$. Poisson's ratio is never shown in materials properties tables as negative. Several typical values of this ratio are given in Table 1.3.

Theoretically, Poisson's ratio applies to elastic conditions, although we normally use the same definition when the ratio increases with the increase, say, of the stress when the stress–strain curve is no longer a straight line. This characteristic has been proven experimentally,[30] justifying the use of the Poisson ratio term for both elastic and plastic strains. The effects of Poisson's ratio are especially significant in biomechanics, and the theoretical limit of 0.5, so well defended by the elasticians, is likely to be stretched to about 1.0, as shown by a fascinating discussion of biological materials by Gordon.[3] It seems that no matter what twists and turns modern scientific disciplines can take, the archaic but essential concepts of stress and strain survive.

Another view of Poisson's ratio can be acquired by calculating the change in volume due to strain for a bar of uniform circular cross-section subjected to tension,[27] which is given by

$$\frac{\Delta V}{V} = \frac{\Delta L}{L}(1 - 2\nu) \tag{1.14}$$

The practical values of $\nu$ vary within a relatively narrow range, say between 0.25 and 0.35. For a material subjected to stresses in the plastic range and with the Poisson ratio reaching the theoretical limit of 0.5, Eq. (1.14) gives $(\Delta V/V) = 0$, so that the volume of the material remains essentially unchanged. For the ideally brittle material behavior, the volume change becomes equal to the linear change in a dimension. It should be noted here that Eq. (1.14), derived for a bar of circular cross-section, also applies to bars with other uniform cross-sections.

The basic relations discussed so far can be extended to the case of volumetric strain of an elemental cube of a material subjected to hydrostatic

**TABLE 1.3** Poisson's Ratio for Various Materials.

| | |
|---|---|
| Upper theoretical limit | 0.50 |
| Calcified plaque | 0.48 |
| Lead | 0.43 |
| Gold | 0.42 |
| Platinum | 0.39 |
| Silver | 0.37 |
| Aluminum (pure) | 0.36 |
| Phosphor bronze | 0.35 |
| Tantalum | 0.35 |
| Copper | 0.34 |
| Titanium (pure) | 0.34 |
| Aluminum (wrought) | 0.33 |
| Titanium (alloy) | 0.33 |
| Brass | 0.33 |
| Molybdenum | 0.32 |
| Stainless steel | 0.31 |
| Structural steel | 0.30 |
| Fiberglass | 0.30 |
| Magnesium (alloy) | 0.28 |
| Tungsten | 0.28 |
| Granite | 0.28 |
| Sandstone | 0.28 |
| Plaque | 0.27 |
| Artery | 0.27 |
| Cast iron (gray) | 0.26 |
| Marble | 0.26 |
| Glass | 0.24 |
| Limestone | 0.21 |
| Uranium (D-38) | 0.21 |
| Plutonium (alpha phase) | 0.18 |
| Concrete (average water content) | 0.12 |
| Beryllium (vacuum-pressed powder) | 0.027 |
| Lower theoretical limit | 0.000 |

pressure.[31] This gives

$$\frac{\Delta V}{V} = \frac{3\sigma}{E}(1 - 2\nu) \tag{1.15}$$

In Eq. (1.15) $\sigma$ represents hydrostatic pressure acting on all sides of the cube and $(\Delta V/V)$ is the change in volume, which for a fully plastic condition of the material is a negligible quantity. The ratio of the hydrostatic pressure ($\sigma$) to the volumetric strain $(\Delta V/V)$ from Eq. (1.15) is called the bulk modulus of the material, denoted here by $E_b$. This yields

$$E_b = \frac{E}{3(1 - 2\nu)} \tag{1.16}$$

Another useful formula that employs Young's modulus and Poisson's ratio concepts, discussed in this chapter, defines the modulus of rigidity or the shearing modulus of elasticity denoted by $E_S$

$$E_S = \frac{E}{2(1 + \nu)} \tag{1.17}$$

Note that the bulk and shear moduli in engineering literature are usually denoted by $K$ and $G$, respectively. These symbols, however, are not used in this book as it may become a little confusing later when dealing with stress intensity factors and energy release rates in fracture analysis.

## THEORY OF ELASTICITY

This section introduces the concept of stress and strain in three dimensions and briefly discusses governing equations in the theory of elasticity from which linear elastic fracture mechanics is derived. In concept, stresses at a point are defined with respect to a plane or area passing through that point and can be obtained by shrinking that area to an infinitesimally small size. Conventionally, the stresses at a point are defined in terms of normal stresses ($\sigma$), which act perpendicular to the plane passing through the point and in terms of shear stresses ($\tau$), which act along that plane. In a homogeneous body, the stresses at a point will depend on the orientation of the plane passing through the point and will vary from one point to another.

Stresses in a three-dimensional system can be defined by constructing a Cartesian coordinate system at a point and considering the average forces acting on the faces of an infinitesimal cube surrounding that point. The stresses on each face of an infinitesimal cube around a point in a Cartesian coordinate system are shown in Fig. 1.7. As a general convention, the tensile stresses are considered positive normal stresses and they are presented as arrows pointing outwards and along the surface normal for that face and are denoted with the corresponding coordinate as subscript, that is, $\sigma_x$, $\sigma_y$, $\sigma_z$. Shear forces require two subscripts. The first subscript denotes the face on which the shear forces act and the second subscript indicates the direction in which the resultant shear is resolved. All the stresses shown in Fig. 1.7 are positive stresses.

Since the cube is infinitesimally small and the stresses are slowly varying across the cube, the moment equilibrium about the centroid of the cube gives

$$\tau_{xy} = \tau_{yx} \qquad \tau_{xz} = \tau_{zx} \qquad \tau_{yz} = \tau_{zy} \tag{1.18}$$

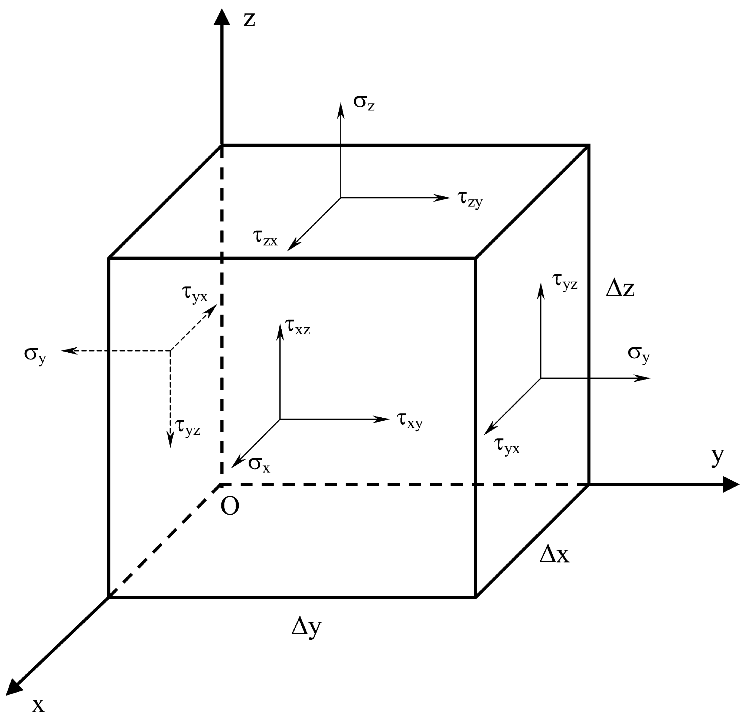


**Figure 1.7** Stress components in a three-dimensional Cartesian coordinate system.

Equation (1.18) reduces nine stress components to six independent components. The stress components can be represented in an array form as

$$\sigma_{ij} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix} \tag{1.19}$$

The above array has the transformation properties of a symmetric second-order tensor and is called the *stress tensor*.[32] If we rotate the infinitesimal element, all the shear stresses vanish at one particular orientation of that element. In essence, in a three-dimensional stress system one can find three mutually perpendicular directions in which only normal stresses $\sigma_1$, $\sigma_2$, and $\sigma_3$ are acting. Under these conditions where no shearing stresses are present, $\sigma_1$, $\sigma_2$, and $\sigma_3$ are defined as principal stresses. The corresponding principal strain in one direction can be stated as

$$\varepsilon = \frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3) \tag{1.20}$$

This equation indicates the extension of Hooke's law to the triaxial state of stress and it helps to define the degree of a constraint at the corner of a notch or a similar discontinuity. If $\sigma_2$ and $\sigma_3$ act in the same plane, then the direction of $\sigma_1$ must be perpendicular to the $\sigma_2$–$\sigma_3$ plane. Further ramifications of Eq. (1.20) are relegated to Chapter 2. The stress tensor when the coordinate system is oriented in principal stress directions will be

$$\sigma_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \tag{1.21}$$

where by convention, $\sigma_1 > \sigma_2 > \sigma_3$.

The strains are defined in terms of displacements of a point from its undistorted position and its derivatives. In the Cartesian system we can define the displacements in the $x$, $y$, and $z$ directions as $u$, $v$, and $w$, respectively. For small displacements in a continuous body, the normal strains (also known as dilatational strain) in terms of displacement are given by

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} \qquad \varepsilon_z = \frac{\partial w}{\partial z} \tag{1.22}$$

The engineering shear strains (measure of angular distortion) are given by

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \qquad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \qquad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \tag{1.23}$$

An array that defines a complete state of strain and turns out to be symmetric is

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_x & \dfrac{\gamma_{xy}}{2} & \dfrac{\gamma_{xz}}{2} \\ \dfrac{\gamma_{xy}}{2} & \varepsilon_y & \dfrac{\gamma_{yz}}{2} \\ \dfrac{\gamma_{xz}}{2} & \dfrac{\gamma_{yz}}{2} & \varepsilon_z \end{pmatrix} \tag{1.24}$$

When defining stresses, it is assumed that a material medium exists for the stress to act against some resistance and the medium is continuous so that the derivatives defining strain in Eqs. (1.22) and (1.23) are meaningful. The equations relating the state of stress with the state of strain are called the *constitutive equations*. While introducing the basic theory of elasticity, we will assume isotropic, homogeneous, and elastic material medium. In addition, strains are assumed to be sufficiently small and it is also assumed that the normal and shear modes of deformation are uncoupled. A set of constitutive equations that can be applied to linear, elastic, and isotropic materials are referred to as “generalized Hooke’s law.” In a three-dimensional Cartesian coordinate system, the strain is related to stress by the following relations:

$$\begin{aligned}
\varepsilon_x &= \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \\
\varepsilon_y &= \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \\
\varepsilon_z &= \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] \\
\gamma_{xy} &= \frac{1}{E_\mathrm{S}}\tau_{xy} \\
\gamma_{xz} &= \frac{1}{E_\mathrm{S}}\tau_{xz} \\
\gamma_{yz} &= \frac{1}{E_\mathrm{S}}\tau_{yz}
\end{aligned} \tag{1.25}$$

where $E$ is the elastic modulus, $\nu$ is Poisson’s ratio, and $E_\mathrm{S}$ is the shear modulus.

If $\nu \neq \frac{1}{2}$ then Eqs. (1.25) can be inverted to express stress in terms of strain components:

$$\begin{aligned}
\sigma_x &= \frac{E}{1+\nu}\varepsilon_x + \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_x + \varepsilon_y + \varepsilon_z) \\
\sigma_y &= \frac{E}{1+\nu}\varepsilon_y + \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_x + \varepsilon_y + \varepsilon_z) \\
\sigma_z &= \frac{E}{1+\nu}\varepsilon_z + \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_x + \varepsilon_y + \varepsilon_z) \\
\tau_{xy} &= E_\mathrm{S}\gamma_{xy} \\
\tau_{xz} &= E_\mathrm{S}\gamma_{xz} \\
\tau_{yz} &= E_\mathrm{S}\gamma_{yz}
\end{aligned} \tag{1.26}$$

The governing equations of elasticity can be simplified and formulated in two dimensions for the two special cases of plain strain and plain stress. If a body is in a state of plain strain such that all its strain components in the $z$-direction are zero, then

$$\varepsilon_z = 0 \qquad \gamma_{xz} = 0 \qquad \gamma_{yz} = 0 \tag{1.27}$$

and we obtain the plane-strain form of Hooke's law as

$$
\begin{aligned}
\varepsilon_x &= \frac{1+\nu}{E}[(1-\nu)\sigma_x - \nu\sigma_y] \\
\varepsilon_y &= \frac{1+\nu}{E}[(1-\nu)\sigma_y - \nu\sigma_x] \\
\sigma_x &= \frac{\nu E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_x + \nu\varepsilon_y] \\
\sigma_y &= \frac{\nu E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_y - \nu\varepsilon_x] \\
\sigma_z &= \nu(\sigma_x + \sigma_y) \\
\tau_{xy} &= E_{\mathrm{S}}\gamma_{xy}
\end{aligned}
\tag{1.28}
$$

If a body is in a state of plane stress such that all the stresses in the $z$-direction are zero, then

$$\sigma_z = 0 \qquad \tau_{xz} = 0 \qquad \tau_{yz} = 0 \tag{1.29}$$

and

$$
\begin{aligned}
\varepsilon_x &= \frac{1}{E}(\sigma_x - \nu\sigma_y) \\
\varepsilon_y &= \frac{1}{E}(\sigma_y - \nu\sigma_x) \\
\varepsilon_z &= -\frac{\nu}{E}(\sigma_x + \sigma_y) \\
\sigma_x &= \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \\
\sigma_y &= \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) \\
\tau_{xy} &= E_{\mathrm{S}}\gamma_{xy}
\end{aligned}
\tag{1.30}
$$

The above section summarizes some of the basic equations that are required in the theory of linear elastic fracture mechanics. A textbook on the theory of elasticity should be referred to for a detailed study of this section.

## STRENGTH THEORIES AND DESIGN

The first half of this century exhibited considerable interest in developing practical techniques for dealing with more classical behavior of ductile and brittle materials and the justification of the various strength theories in design.[33] Ductile material, where the plastic deformation region on the stress–strain curve is well defined, was considered to have failed when the last point on the elastic portion of the curve was reached, that is, plastic deformation began. A brittle material, on the other hand, was not considered completely failed until it had broken through a tensile fracture at ultimate strength. In compression the failure of a brittle material appears to be a shear fracture. The elongation of 5% was used as the arbitrary dividing line between ductile and brittle materials. However, under special circumstances involving low temperature, high strain rate, combined loading, residual stress, stress raisers, large size, or hydrogen absorption, ductile steel may show a brittle response.

The following four theories of elastic failure received probably the widest acceptance:

Maximum stress theory: Elastic failure occurs when the maximum working stress equals the yield value $\sigma_y$.

Maximum strain theory: Elastic failure occurs when the maximum tensile strain reaches $(\sigma_y/E)$.

Maximum shear theory: Elastic failure occurs when the maximum shear stress becomes equal to $(0.5\sigma_y)$.

Distortion energy theory: Elastic failure occurs when the principal stresses $\sigma_1$, $\sigma_2$, $\sigma_3$ satisfy the following relation:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2 \tag{1.31}$$

For the case of a two-dimensional stress the foregoing equation simplifies to

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_y^2 \tag{1.32}$$

Experiments indicate that brittle materials such as glass and Bakelite®[34] fracture in general agreement with the maximum stress and strain theories in tension. The conditions of yielding given by Eqs. (1.31) and (1.32) are usually accepted as valid for ductile materials. Similar comment can be made about the maximum shear theory, which is in good agreement with experiments on ductile materials and is rather simple to apply:

$$\sigma_y = \sigma_1 - \sigma_3 \tag{1.33}$$

This brief chapter on the issues of stress and strain shows how the basic elements of practical stress analysis have developed prior to more modern techniques of design reliability and safety. In the second half of the 19th century, British and American engineers in particular relied on calculated tensile stresses in structures, with factors of safety between 3 and 7, as certification of the tensile strength of a material. There was no real pressure to trim the weight and cost of the structures and, for all practical purposes, the discrepancies between the theoretical and the actual strengths of the materials used in construction projects were not alarming. Certainly the “factor of safety” or “ignorance” in vogue at the time was an order of magnitude greater than a natural few percent variation in strength.

The design of ships, boilers, bridges, support beams, parts of locomotives, and various structural members was based essentially on tensile stresses using relatively safe materials such as wrought iron or mild steel. However, in spite of large factors of safety, some accidents continued to occur. The demand for speed and lower weight, particularly in the shipbuilding industry, has gradually eroded the level of design confidence and opened new areas of experimental and analytical scrutiny. The problems of structural failure — whether from simple overload, insidious stress concentration, or crack propagation — were equally distressing.

## SYMBOLS

| | |
|---|---|
| $A$ | Cross-sectional area, $\text{in}^2$ ($\text{mm}^2$) |
| $E$ | Modulus of elasticity, psi ($\text{N/mm}^2$) |
| $E_b$ | Bulk modulus, psi ($\text{N/mm}^2$) |
| $E_s$ | Shearing modulus of elasticity, psi ($\text{N/mm}^2$) |
| $L$ | Length, in. (mm) |
| MPa | Megapascal, $10^6$ Pa ($\text{N/mm}^2$) |
| N | Newton |
| Pa | Pascal |
| $r$ | Radius of solid bar, in. (mm) |
| $u$ | Lateral strain, in./in. (mm/mm) |
| $V$ | Volume of stressed material, $\text{in.}^3$ ($\text{mm}^3$) |
| $W$ | External load, lb (N) |
| $\Delta L$ | Change in length, in. (mm) |
| $\Delta r$ | Change in radius, in. (mm) |
| $\Delta V$ | Change in volume, $\text{in.}^3$ ($\text{mm}^3$) |
| $\varepsilon$ | Engineering strain, in./in. (mm/mm) |
| $\nu$ | Poisson’s ratio |
| $\sigma$ | General symbol for normal stress, psi ($\text{N/mm}^2$) |
| $\sigma_y$ | Yield strength, psi ($\text{N/mm}^2$) |
| $\sigma_1, \sigma_2, \sigma_3$ | Principal stresses, psi ($\text{N/mm}^2$) |
| $\tau$ | General symbol for shear stress, psi ($\text{N/mm}^2$) |

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# 2

# Stress and Energy Criteria and Fracture

## EFFECTS OF GEOMETRY

For many years the assumption of uniform distribution of normal stresses over a cross-section of a nonprismatic bar gave satisfactory results as long as no abrupt changes in cross-section along the bar axis were involved. The general design reliability was even better when, over many years, engineers utilized high factors of safety. Although the presence of higher factors obscured some of the more questionable design details, the notion that smooth structural surfaces and limited changes of shape provided a certain amount of reliability and safety was hard to dispute.

In most early designs geometrical features such as holes, cracks, and sharp corners had been known in advance, and some of them were utilized for a specific purpose such as, for instance, grooves in slabs of chocolate or perforations in postage stamps. Although these particular geometric discontinuities were convenient, they were not engineered properly by calculating the stress concentration effects to, at least, indicate the ratio of the elevation of the local stress to the nominal stress. This practice existed in spite of the fact that there was some understanding of the general problem of perturbation in the stress field due to the presence of a hole or groove in a continuous solid. An example of a sharp groove effect is illustrated in Fig. 2.1.

The “trajectories” (Fig. 2.1) are simply “pathways” of stress that go around a particular irregularity such as a groove because the tensile forces applied to the

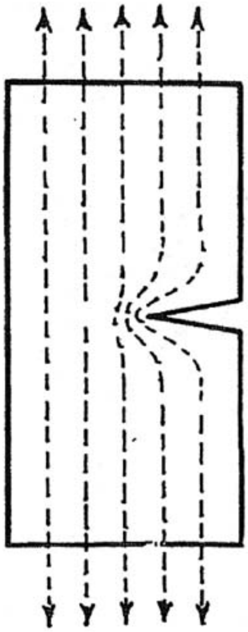

**FIGURE 2.1** Typical trajectories of stress in a grooved solid.

solid, as in Fig. 2.1, must be balanced in some way.[1] The stress trajectories are crowded together near the bottom of the groove where the force per unit area is higher than at any other location within the boundaries of the solid. The degree of "crowding together" of the trajectories depends upon the shape of the discontinuity, and, indeed, around a sharp corner this crowding can be rather severe. Where there is no groove, as in the left side of the solid in Fig. 2.1, our imaginary flow lines are straight and equally spaced, indicating a uniform stress field. Another important feature of this stress concentration mechanism is that the crowding effect is very local. In a practical way this feature can be verified by pushing a solid wedge against a rubber hose. These flow lines (trajectories) show the direction of the local tensile stress. Furthermore, in the vicinity of the groove the local stress can have vertical and horizontal components that, in the language of the theoretical elasticity, constitute a biaxial stress field. Hence the groove causes stress concentration and lateral stresses.[2]

## INGLIS THEORY OF STRESS

At the turn of this century the practice of using high factors of safety (up to 18 in locomotive design) was not always successful in preventing structural failures, particularly in the areas of large and complex systems such as in the shipbuilding industry. In 1901 the fastest ship in the world (*H.M.S. Cobra*) suddenly broke in two and sank with loss of life in the North Sea during ordinary weather.[1] Subsequent experiments on full-scale structures and verification of engineering calculations using a factor of safety higher than 5 did not provide sufficient explanation of the fracture mechanism responsible for the North Sea disaster.

One of the first investigations into the general area of modeling geometric irregularities and defects was conducted several years later by Professor Inglis of Cambridge University.[3] His theoretical analysis resulted in a design formula for an elliptical hole that also applied to openings such as portholes, doors, and hatchways with reasonable accuracy.

$$\sigma_{\max} = \sigma[1 + 2(h/\rho)^{1/2}] \tag{2.1}$$

where $\sigma_{\max}$ = maximum elastic stress at the tip of hole, $\sigma$ = nominal stress away from the stress concentration, $h$ = major semiaxis of the ellipse, and $\rho$ = root radius of an ellipse, and

$$\rho = b^2/h \tag{2.2}$$

where $b$ is the minor semiaxis of the ellipse. The dimension $\rho$ can also be described as the local radius of curvature at the tip of the ellipse (Krummungsradius), which can be derived from the general parametric equations of the ellipse.[4] The notation for the elliptical hole in the Inglis formula is given in Fig. 2.2.

An alternative form of the Inglis equation can be obtained by substituting the tip radius, $\rho$, in Eq. (2.1). This should yield

$$\sigma_{\max} = \sigma\left(1 + 2\frac{h}{b}\right) \tag{2.3}$$

When $h = b$, Eq. (2.3) gives $(\sigma_{\max}/\sigma) = 3$, which is the conventional stress concentration factor for a small circular hole or a semicircular notch. This is indeed a remarkable result considering that Eq. (2.1) was proposed many years ago. At about the same time, Kirsch in Germany (1898) and Kolosoff in Russia (1910) derived similar equations, and it was generally disappointing that little notice was taken of this development in shipbuilding and other industries.

When $b$ tends to a small value in comparison with the dimension $h$, the stress concentration factor increases markedly, as illustrated in Fig. 2.3. This suggests that a rather narrow opening perpendicular to the direction of nominal tension can produce a very high stress concentration, which may account for unexpected fractures even under moderate applied stresses. Under these conditions, however, dominated by high $(h/b)$ ratios, we are entering a rather

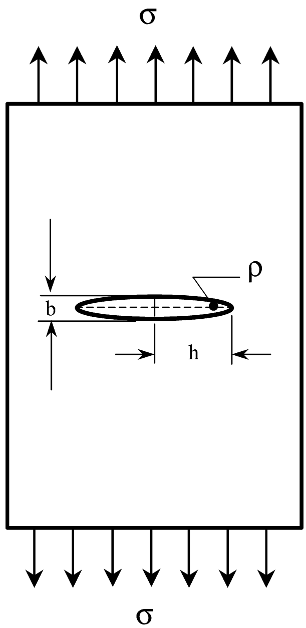


**FIGURE 2.2** Notation for elliptical hole.

different approach to judging the degree of severity of a particular discontinuity. Here we no longer deal with a conventional notch geometry but with a deep and sharp crack, with the tip-of-the-crack radius having, perhaps, molecular dimensions. Hence the conventional definition of stress concentration factor cannot be applied.

One should not take the results based on the Inglis formula entirely at their face value. This would only lead to a conclusion that it is impossible to design any structure to carry tensile loads because all structures and materials are scarcely free of discontinuities and cracks. In real-life situations bridges, machinery members, ships, and airplanes may well be infested with stress concentrations caused by holes, notches, and cracks, and yet such irregularities are seldom dangerous. Certainly, since the appearance of Inglis's paper,[3] a wealth of information on classical stress concentration methodology has developed,[5–7] so that almost any geometrical transition can be handled by calculation to enhance product safety. However, we should be careful with designing around a particular geometrical weakness by adding extra material, such as gussets or webs, so that "strengthening" does not produce some other form of weakness. It may not be easy to assure a proper design balance, because only nature is really good in mitigating undue stress effects.

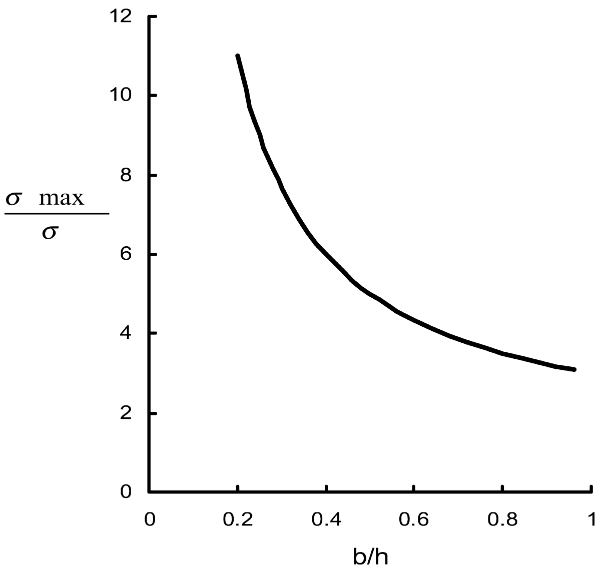


**FIGURE 2.3** Variation of stress factor.

## ADVENT OF THE ENERGY CONCEPT

While the Inglis formula planted certain questions in the minds of practical engineers and some startling results could be predicted, the design profession as a whole was, for a long time, eager to dismiss Inglis's implications by invoking the ductility of metals and plastic flow around the tip of a crack or a geometrical discontinuity. In effect, local plastic action was regarded as a "rounding off" mechanism for blunting the sharp tip.

In the meantime, additional structural failures continued to crop up and persisted until modern times, with some spectacular incidents involving ships, bridges, and oil rigs. It has become painfully obvious that the classical concepts of stress and strain — developed by Hooke, Young, Cauchy, and others — were not really enough, by themselves, to predict structural failures. After all, until quite recently, elasticity was taught in terms of forces and distances, and even now we seldom think of the stress–strain curve as a symbol of energy and the measure of conservation of energy. Yet the quantity of energy required to break a given material or structure defines the toughness, sometimes called fracture energy or work of fracture.

Although energy can exist in different forms — such as, for instance, electrical, chemical, heat, and potential energy — in the field of mechanical engineering and biomechanics, the concept of strain energy is more widespread because every elastic material under stress contains strain energy. In its simplest definition, the area under the stress–strain curve, shown in Fig. 2.4, represents strain energy, where the stress can be either tensile or compressive. Hence the strain energy per unit volume of the material, in line with Fig. 2.4, is

$$U = \frac{\sigma\varepsilon}{2} \tag{2.4}$$

or using Eq. (1.3), we can directly obtain

$$U = \frac{\sigma^2}{2E} \tag{2.5}$$

The basic physical rule is that in any manner of transformation of energy, we cannot get something for nothing. Also, energy can neither be created nor destroyed, which is known as the principle of conservation of energy. However, this principle was not generally accepted until quite late in the 19th century.

As far as the units of energy are concerned, there is little uniformity. In mechanical engineering the tradition is still to use foot-pounds, while the SI unit of energy is the joule. It represents the work done when 1 newton (N) acts through 1 meter (m), or in short

$$1 \text{ joule (J)} = \text{N} \times \text{m}$$

Other equivalents are

$$1 \text{ joule} = 10^7 \text{ ergs} = 0.734 \text{ ft-lb} = 0.239 \text{ calories}$$

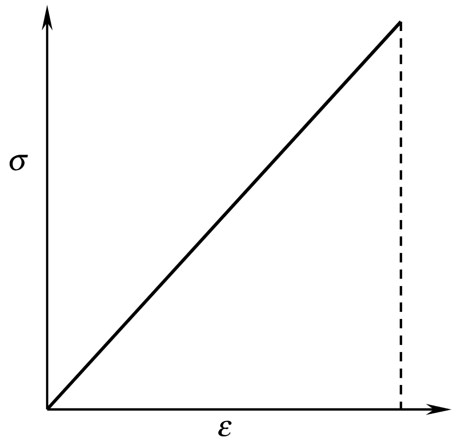


**FIGURE 2.4** Strain energy diagram.