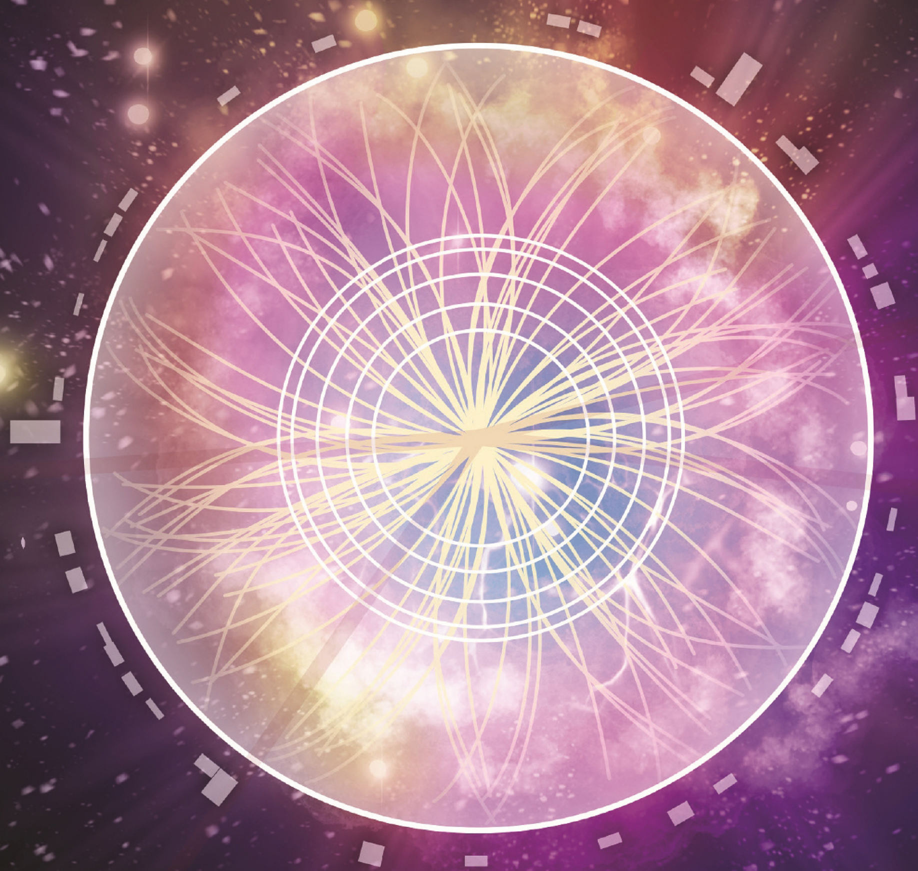
THE STANDARD MODEL

From Fundamental Symmetries to Experimental Tests

Yuval Grossman • Yossi Nir



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PREFACE

This book aims at teaching modern particle physics. The goal of particle physics is to understand what are the fundamental laws of nature. These are grand, yet true, words.

Particle physics is also known as high-energy physics. Particle physics experiments have probed energy scales as high as 10 TeV (which is 10,000 times the mass of the proton) and distance scales as small as 10^{-20} m. We find it amazing that by now, we have achieved a deep understanding of how nature works down to such distances. The framework that we use to describe the phenomena at this distance scale is based on quantum field theory (QFT), which is different from quantum mechanics (QM), which we use to deal with atomic physics; and which in turn is different from classical mechanics, which we apply to explain most of the macroscopic world. While these frameworks are very different from each other, the underlying principles of physics are surprisingly similar across all these scales. Specifically, in all of them, we use the principle of minimal action and symmetry arguments to construct the theory.

It is often stated that the aim of particle physics is to describe the elementary particles and the fundamental interactions among them. While this is true, particle physicists aim higher. In some sense, the particles and their interactions constitute a tool for us to obtain insights into deeper principles that describe nature. Our current understanding of the basic laws of nature is based on very elegant symmetry principles. Once we know the symmetries of the universe and how the fundamental fields respect them, much of nature is explained. In a way, the deeper we understand nature, the simpler and more abstract are the basic principles that we use to formulate this understanding.

The currently accepted theory of particle physics is called the Standard Model. The basic ingredients of the Standard Model were conceived in the late 1960s and early 1970s by Sheldon Lee Glashow, Abdus Salam, and Steven Weinberg. At that time, many of the particles that now constitute part of it were yet to be discovered. By 2012, the full list of the Standard Model particles have been directly produced and detected, and the full list of the Standard Model parameters have been measured with impressive accuracy. The Standard Model has been tested by numerous experimental measurements, and it has passed almost all of them with flying colors. The very few failures constitute the starting point for the road to an even deeper level of understanding nature.

It is customary to say that there are four forces in nature: gravitational, electromagnetic, weak, and strong forces. The Standard Model does not deal with gravity, but it does include the three other forces. As you read the book, it becomes clear that while these three forces seem very different from each other, they all arise at the fundamental level from a QFT incorporating gauge symmetries. We can think of this as a generalization of Quantum Electro Dynamics (QED) that leads to very different phenomenology. It is this unification of the underlying principle—gauge symmetries—that makes the theory so elegant.

Significant breakthroughs in physics were often achieved when realizing that phenomena that seem very different are, in fact, connected. One example is the understanding that the movement of the planets around the sun and the falling of an apple from a tree are both explained by the same law of gravity. Another example is the realization that electricity and magnetism are two manifestations of the unified electromagnetic theory. Particle physics keeps in the same path and provides a unified description of phenomena that seem very different from each other.

The way that the Standard Model has been developed was based on data that became available during the process. It took many years to reach the present status of the theory because it took time to develop new experiments and collect all the data. In this book, however, we do not follow a historical approach; rather, we describe things as we understand them today. We only briefly mention historical facts as we go. Given that all the particles of the Standard Model have been discovered and all the parameters measured, we can explain the Standard Model in a comprehensive and pedagogical way that captures the main points.

Most of the data that are relevant for particle physics came from collider experiments. In this book, we focus on the theory side and do not elaborate on the way that experiments are carried out. The various colliders differ in energy and luminosity, and in the particles that collide. It is the synergy of all of them that led to the construction of the Standard Model. At the time of writing this book, the highest energy accelerator is the Large Hadron Collider (LHC), which collides protons at a center-of-mass energy of 13 TeV. It is the energy of the LHC that sets the upper bound on the mass scale of particles that can be probed directly by being produced on-shell. High-luminosity machines help us search for quantum effects (i.e., loop effects that probe indirectly physics at even higher scales).

The data that are relevant to particle physics is collected by the Particle Data Group (PDG) in the Review of Particle Physics [1] (we loosely refer to this collection of data as the "PDG"). When we quote experimental data, they come from the PDG unless we explicitly state otherwise. For the untrained eye, the PDG may look like an (old-fashioned) phone book. There are numerous tables with data on decay rates and other properties of particles. Browsing through this data, you see large variations: some particles are stable, others have long lifetimes, and yet others have very short lifetimes before they decay. Some particles have masses that are almost a million times heavier than the electron, and others are more than a million times lighter. The decay products of various particles are very different, as are the corresponding branching ratios.

The task of particle physicists is to identify the patterns in these raw data and to organize them in such a way that the principles that explain this variety emerge. The language that we use to do this is QFT, and the main tool within this framework is the Lagrangian (which depends on fundamental fields and their couplings). Once organized in this way, we can extract the symmetry principles that dictate the Lagrangian and explain the rich phenomena observed.

In writing this book, we intended that it would serve as a textbook for a one-semester course. Thus, we omit many details and focus mainly on the phenomenological aspects. This means that we will be skipping a few interesting and relevant topics. We hope that our book will motivate you to explore these on your own.

The book, although very much self-contained, is written assuming preknowledge of basic QFT. It is aimed at students who have already taken a one-semester course in QFT and have an understanding of the concept, but may be unfamiliar with advanced topics. To gauge this statement, students familiar with the first part of Peskin and Schroeder [2] have the preknowledge necessary to follow our book. Some other books that we find useful and complementary to ours are Georgi [3], Quigg [4], Peskin [5], Burgess and Moore [6], Langacker [7], Ramond [8], Cottingham and Greenwood [9], Donoghue, Golowich, and Holstein [10], Goldberg [11], and Buras [12].

At the end of a course that follows this book, the student should gain knowledge and understanding in two areas:

- The Standard Model: the symmetry principles that define it, the fundamental interactions and elementary particles that it describes, the ways in which it has been tested, its many successes, and its very few failures.
- The principles of model building in particle physics: the tools that are used to interpret new experimental results and, in particular, to extend the Standard Model if future measurements cannot be explained by it.

Actually, in our minds, there is a third goal for this book as well. We think that the Standard Model is a scientific masterpiece, beautiful and elegant, and we hope to convey this sense of appreciation and intellectual joy to our readers.

1
Lagrangians

In this chapter, we review the basic tools that we will use in this book. In particular, we introduce the Lagrangian and present some simple Lagrangians involving scalar and fermion fields.

1.1 Introduction

Modern physics encodes the basic laws of nature in the action, S, and postulates the principle of minimal action in its quantum interpretation. In quantum field theory (QFT), the action is an integral over spacetime of the *Lagrangian density* or Lagrangian, $\mathcal{L}$, for short. For most of our purposes, it is enough to consider the Lagrangian rather than the action. In this chapter, we explain how Lagrangians are constructed. Later in the book, we discuss how the numerical values of the parameters that appear in the Lagrangian are determined and how to test if a Lagrangian provides a viable description of nature.

The QFT equivalent of the generalized coordinates of classical mechanics are *fields*. The action is given by

$$S = \int d^4x \, \mathcal{L}, \tag{1.1}$$

where $d^4x = dx^0 dx^1 dx^2 dx^3$ is the integration measure in four-dimensional Minkowski space. In general, we require the following properties for the Lagrangian:

1. It is a function of the fields and their derivatives only.
2. It depends on the fields taken at one spacetime point x^μ only, leading to a local field theory.
3. It is real, so the total probability is conserved.
4. It is invariant under the Poincaré group, which consists of spacetime translations and Lorentz transformations.
5. It is an analytic function in the fields. This is not a general requirement, but it is common to all field theories that are solved via perturbation theory. In these cases, we expand around a minimum, which means that we consider a Lagrangian that is a polynomial in the fields.

6. It is invariant under certain internal symmetry groups. The invariance of S (or $\mathcal{L}$) is in correspondence with conserved quantities and reflects basic symmetries of the physical system.
7. Every term in the Lagrangian that is not forbidden by a symmetry should appear.

We often impose an additional requirement as well:

8. Renormalizability. A renormalizable Lagrangian contains only terms that have a dimension less than or equal to four in the fields and their derivatives.

The renormalizability requirement ensures that the Lagrangian contains at most two ∂_μ operations, and it leads to classical equations of motion that are no higher than second-order derivatives. If the full theory of nature is described by a QFT, its Lagrangian should indeed be renormalizable. The theories that we consider, however, and, in particular, the Standard Model, are only low-energy-effective theories, that are valid up to some energy scale Λ. Therefore, we also must include nonrenormalizable terms, which have coefficients with inverse mass dimensions, $1/\Lambda^n$, $n = 1, 2, \ldots$ For most purposes, however, renormalizable terms constitute the leading terms in an expansion in E/Λ, where E is the energy scale of the physical processes under study. Therefore, the renormalizable part of the Lagrangian is a good starting point for our study. Thus, in chapters 1–10, we consider only renormalizable Lagrangians unless otherwise explicitly stated. In chapters 11–15, where we describe searches for physics beyond the Standard Model, we also consider nonrenormalizable Lagrangians.

Properties 1–5 are not the subject of this book. You should be familiar with them from your QFT course work. We do, however, deal intensively with the other requirements. Actually, the most important message that we would like to convey is the following: *(Almost) all experimental data for elementary particles and their interactions can be explained by the Standard Model of a spontaneously broken $SU(3) \times SU(2) \times U(1)$ gauge symmetry.*[1]

Writing down a specific Lagrangian is the end point of the process known as *model building*, and the starting point for a phenomenological interpretation and experimental testing. In this book, we explain both aspects of this modern way of understanding high-energy physics.

1.2 Examples of Simple Lagrangians

We next present a few examples of simple Lagrangians of scalar and fermion fields. They are simple in the sense that we are not yet imposing any internal symmetry. We use $\phi(x)$ for a scalar field and $\psi(x)$ for a fermion field. When we consider vector fields, as first done in section 2.2 of chapter 2, we use $A(x)$ for a vector field. We do not consider higher spin fields, as it is not simple to construct a QFT with them.

1. Actually, the great hope of the high-energy physics community is to prove this statement wrong and find an even more fundamental theory.

Two comments are in order:

- All fields that we consider here are functions of the spacetime coordinates $\phi(x)$, $\psi(x)$, and $A(x)$. We leave this spacetime dependence implicit except in cases where it is relevant.
- We use the notations ϕ, ψ, and A in the discussion of generic cases. When we refer to specific cases, we use different notation. For example, for the electron field, we use e instead of the generic ψ.

1.2.1 Scalars

The most general renormalizable Lagrangian for a single real scalar field ϕ is given by

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2 - \frac{\eta}{2\sqrt{2}}\phi^3 - \frac{\lambda}{4}\phi^4 . \tag{1.2}$$

We emphasize the following points:

- The term with derivatives is called the *kinetic term*. It is necessary if we want ϕ to be a dynamical field (namely, to be able to describe propagation in spacetime).
- The terms without derivatives are collectively denoted by $-V_\phi$. We then write $\mathcal{L}_S = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V_\phi$, and V_ϕ is called the *scalar potential.*
- We work in the canonically normalized basis where the coefficient of the kinetic term is 1/2. (This is true for a real scalar field. For a complex scalar field, the canonically normalized coefficient of the kinetic term is 1.)
- From here on, throughout the book, when we say "the most general Lagrangian," we are referring to a Lagrangian where the kinetic terms are canonically normalized, but the other terms are written in a general basis. (Question 2.8 in chapter 2 shows that there is no loss of generality in working in the canonically normalized basis.)
- We do not write a constant term since it does not enter the equation of motion for ϕ.
- We do not write a linear term in ϕ because when expanding around a minimum, the linear term vanishes.
- The quadratic term (ϕ^2) is a mass-squared term. (From here on we call it simply a *mass term.*)
- The trilinear (ϕ^3) and quartic (ϕ^4) terms describe interactions.
- Terms with five or more scalar fields (ϕ^n, $n \geq 5$) are nonrenormalizable.

1.2.2 Fermions

The basic fermion fields are two-component Weyl fermions, ψ_L and ψ_R, where L and R denote left-handed and right-handed chirality, respectively. Each of ψ_L and ψ_R has

2 degrees of freedom (DoF) and is a complex field. ψ_L and ψ_R are related to the four-component Dirac field ψ via

$$\psi_R = P_R\psi \equiv \frac{1+\gamma_5}{2}\psi, \qquad \psi_L = P_L\psi \equiv \frac{1-\gamma_5}{2}\psi. \tag{1.3}$$

It is useful to define the related left-handed Weyl fermion ψ_R^c and right-handed Weyl fermion ψ_L^c via

$$\psi_R^c = C\,\overline{\psi_R}^T, \qquad \psi_L^c = C\,\overline{\psi_L}^T, \tag{1.4}$$

where C is the *charge conjugation matrix*. (The reason for this name becomes clear once we define *charge* in chapter 2.)

The most general renormalizable Lagrangian for a single left-handed fermion field ψ_L and a single right-handed fermion field ψ_R is given by

$$\mathcal{L}_F = i\overline{\psi_L}\partial\!\!\!/\psi_L + i\overline{\psi_R}\partial\!\!\!/\psi_R - \left(\frac{m_{MR}}{2}\overline{\psi_R^c}\,\psi_R + \frac{m_{ML}}{2}\overline{\psi_L^c}\,\psi_L + m_D\overline{\psi_L}\psi_R + \text{h.c.}\right). \tag{1.5}$$

We emphasize the following points:

- The derivative terms are kinetic terms, and they are necessary if we want the field $\psi_{L,R}$ to be dynamical.
- We work in the canonically normalized basis, where the coefficient of the kinetic term is 1.
- Terms with an odd number of fermion fields violate Lorentz symmetry, and so they are forbidden.
- The quadratic terms are mass terms. The m_M terms are called *Majorana masses*, and the m_D terms are called *Dirac masses*.
- The relative factor of $1/2$ between Majorana and Dirac mass terms is the analog of the similar factor between the mass terms for real and complex scalar fields.
- Terms with four or more fermion fields are nonrenormalizable.
- Given the fact that Majorana mass terms are made of a pair of identical fields, we often write

$$\frac{m_{MR}}{2}\overline{\psi_R^c}\,\psi_R \to \frac{m_{MR}}{2}\psi_R^T\,\psi_R. \tag{1.6}$$

If the Majorana masses vanish, $m_{ML} = m_{MR} = 0$, $\mathcal{L}_F$ can be written in terms of the Dirac fermion field ψ:

$$\mathcal{L}_F(m_M = 0) = i\overline{\psi}\partial\!\!\!/\psi - m_D\overline{\psi}\psi. \tag{1.7}$$

Since ψ_L and ψ_R are different fields, there are 4 DoF with the same mass, m_D. In contrast, if the Majorana masses do not vanish, there are generally only 2 DoF that have the same mass. In section 2.1.5 in chapter 2, we discuss these issues in more detail and explain why often Majorana masses vanish.

1.2.3 Fermions and Scalars

Consider the case of a single left-handed fermion ψ_L, a single right-handed fermion ψ_R, and a single real scalar field ϕ. The Lagrangian includes, in addition to terms that involve only the scalar (equation (1.2)), and terms that involve only the fermions (equation (1.5)), terms that involve both the scalar and the fermions. They can be obtained by replacing the mass parameters for the fermions with a coupling multiplied by the scalar field:

$$-\mathcal{L}_{\text{Yuk}} = \frac{Y}{\sqrt{2}}\phi\overline{\psi_L}\psi_R + \frac{Y_{MR}}{2}\phi\overline{\psi_R^c}\,\psi_R + \frac{Y_{ML}}{2}\phi\overline{\psi_L^c}\,\psi_L + \text{h.c.} \tag{1.8}$$

These terms are called *Yukawa interactions*. The Y parameters are dimensionless and are called *Yukawa couplings*. Note that in equation (1.8), we use $-\mathcal{L}$, which is a common practice when we do not write the kinetic terms.

1.3 Symmetries

We always seek deeper reasons for the laws of nature that have been discovered. These reasons are often closely related to symmetries. The term *symmetry* refers to an invariance of the equations that describe a physical system. The fact that symmetry and invariance are related concepts is obvious enough—a smooth ball has a spherical symmetry, and its appearance is invariant under rotation.

Symmetries are built into physics as invariance properties of the Lagrangian. If we construct our theories to encode various empirical facts (and, in particular, the observed conservation laws), then the equations turn out to exhibit certain invariance properties. For example, if we want to incorporate energy conservation into the theory, then the Lagrangian must be invariant under time translations (and therefore cannot depend explicitly on time). From this point of view, the conservation law is the input and the symmetry is the output.

Conversely, if we take the symmetries to be the fundamental rules, then various observed features of particles and their interactions are a necessary consequence of the symmetry principle. In this sense, symmetries provide an explanation of these features. In modern particle physics (and in particular in this book), we often take the latter point of view, in which symmetries are the input and conservation laws are the output.

In the following, we discuss the consequences of *imposing* symmetry on a Lagrangian. This is the starting point of model building in particle physics: one defines the basic symmetries and the field content and then obtains the predictions that follow from these imposed symmetries.

There are symmetries that are not imposed, however, which are called *accidental symmetries*. They are outputs of the theory rather than external constraints. Accidental symmetries arise because we truncate our Lagrangian. In particular, the renormalizable terms in the Lagrangian often have accidental symmetries that are broken by non-renormalizable terms. Since we study mostly renormalizable Lagrangians, we will often encounter accidental symmetries.

There are various types of symmetries. First, we distinguish between *spacetime* and *internal* symmetries. Spacetime symmetries include the Poincaré group of translations, rotations, and boosts. They give the energy-momentum and angular momentum conservation laws. As mentioned previously, we always impose this symmetry. The list of possible spacetime symmetries includes, in addition, space inversion (also called *parity*) P, time-reversal T, and charge conjugation C. (While C is not truly a spacetime symmetry, the way that it acts on fermions and the CPT theorem make it simpler to include C in the same class of operators.) The discrete spacetime symmetries are usually covered in QFT courses, but for completeness, we discuss them briefly in Appendix 1.A.

Internal symmetries act on the fields, not directly on spacetime. In other words, they act in internal spaces that are mathematical spaces generated by the fields. These are the kind of symmetries that we discuss in detail. In chapter 2, we introduce Abelian symmetries; in chapter 4, we introduce non-Abelian symmetries.

1.4 Model Building

As stated already, writing a Lagrangian is the end point of model building. Our procedure of constructing Lagrangians goes as follows. We start by defining the following inputs:

1. The symmetry.
2. The transformation properties of the various scalar and fermion fields under the symmetry operation.

Then we write the most general Lagrangian that depends on the fields and is invariant under the symmetry.

A renormalizable Lagrangian (or a nonrenormalizable one truncated at a certain order) has a finite number of parameters. For a theory with N parameters, we need to perform N appropriate measurements such that additional measurements, from the $(N+1)$'th on, test the theory. In principle, we do not really need to determine the values of the parameters, we can just use experimental inputs to make predictions. In practice, however, it is usually convenient to use the N measurements to determine the values of the Lagrangian parameters and then use these parameters to make further predictions. It is important to remember that the values of the parameters are not inputs to model building.

At this point, this procedure may seem abstract, but it becomes clear and concrete as we work on examples. Throughout this book, we repeat the process of model building several times. We see how Quantum ElectroDynamics (QED), the theory of electromagnetic interactions, Quantum ChromoDynamics (QCD), the theory of strong interactions, the Leptonic Standard Model (LSM), the theory of electroweak interactions among leptons, and the Standard Model itself can be understood in this way of thinking, starting from a postulate of symmetry principles.

Appendix

1.A Discrete Spacetime Symmetries: *C*, *P*, and *T*

The discrete spacetime symmetries, *C*, *P*, and *T*, play an important role in our understanding of nature. Each of these three symmetries has been experimentally shown to be violated in nature, as discussed in detail next. The *CPT* combination seems, however, to be an exact symmetry of nature. On the experimental side, no sign of *CPT* violation has been observed. On the theoretical side, *CPT* must be conserved for any Lorentz-invariant local field theory. Since we only consider such theories, we assume that *CPT* holds. In this case, *CP* and *T* are equivalent. Thus, we usually refer to *CP*.

1.A.1 C and P

We consider *C* and *P* only in theories that involve fermions. Under *C*, particles and antiparticles are interchanged by conjugating all internal quantum numbers (e.g., reversing the sign of the electromagnetic charge, $Q \to -Q$). Under *P*, the handedness of space is reversed ($\vec{x} \to -\vec{x}$), and the chirality of fermion fields is reversed ($\psi_L \leftrightarrow \psi_R$). For example, a left-handed (LH) electron e_L^- transforms under *C* into an left-handed positron e_L^+, and under *P* into a right-handed (RH) electron e_R^-.

1.A.2 CP Violation and Complex Couplings

The *CP* transformation combines charge conjugation *C* with parity *P*. For example, a left-handed electron e_L^- transforms under *CP* into a right-handed positron, e_R^+. *CP* is a good symmetry if there is a basis where all the parameters of the Lagrangian are real. We do not prove it here, but we do provide a simple, intuitive explanation of this statement.

Consider a theory with a single complex scalar, ϕ, and two sets of *N* fermions, ψ_L^i and ψ_R^i $(i = 1, 2, \ldots, N)$ (we define a complex scalar in chapter 2). The Yukawa interactions are given by

$$-\mathcal{L}_{\text{Yuk}} = Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj} + Y_{ij}^*\overline{\psi_{Rj}}\phi^\dagger\psi_{Li}, \tag{1.9}$$

where we write the two Hermitian conjugate terms explicitly. The *CP* transformation of the fields is defined as follows:

$$\phi \to \phi^\dagger, \qquad \psi_{Li} \to \overline{\psi_{Li}}, \qquad \psi_{Ri} \to \overline{\psi_{Ri}}. \tag{1.10}$$

Therefore, a *CP* transformation exchanges the operators

$$\overline{\psi_{Li}}\phi\psi_{Rj} \longleftrightarrow \overline{\psi_{Rj}}\phi^\dagger\psi_{Li}, \tag{1.11}$$

but leaves their coefficients, Y_{ij} and Y_{ij}^*, unchanged. This means that *CP* is a symmetry of $\mathcal{L}$ if $Y_{ij} = Y_{ij}^*$.

In practice, things are more subtle since one can define the *CP* transformation in a more general way than equation (1.10), as follows

$$\phi \to e^{i\theta}\phi^{\dagger}, \qquad \psi_L^i \to e^{i\theta_{Li}}\overline{\psi_L^i}, \qquad \psi_R^i \to e^{i\theta_{Ri}}\overline{\psi_R^i}, \tag{1.12}$$

with $\theta, \theta_{Li}, \theta_{Ri}$ convention-dependent phases. Then, there can be complex couplings, and yet *CP* would be an exact symmetry. The correct statement is that *CP* is violated if, using the freedom to redefine the phases of the fields, one cannot find any basis where all couplings are real.

For Further Reading

There are many books that discuss in detail the QFT-related aspects relevant to this book. For example, some of the standard textbooks are by Peskin and Schroeder [2], Zee [13], Srednicki [14], and Schwartz [15]. Other textbooks that explain many of the relevant issues include Ramond [16], Dine [17], Nagashima [18, 19], and Petrov and Blechman [20].

With regard to some specific points, we mention the following sources:

- For a formal discussion of *C* and *P*, see section 3.6 of Peskin and Schroeder [2], or sections 11.4–11.6 of Schwartz [15].
- For a discussion of the issues about quantizing theories with higher-spin fields, see Peskin [21].
- For a discussion of Majorana fermions, see section 11.3 of Schwartz [15].
- For the *CPT* theorem, see Streater and Wightman [22].

Problems

Question 1.1: Algebra

1. Draw the Feynman diagrams for the interaction terms in the Lagrangian of equation (1.2).
2. Starting from equation (1.5) and using equation (1.3), derive equation (1.7).
3. Draw the Feynman diagrams for the Yukawa interaction terms in the Lagrangian of equation (1.8).

Question 1.2: Using natural units

In high-energy physics, since relativity and quantum mechanics are essential, it is convenient to use units where

$$\begin{aligned} \hbar &\approx 6.58 \times 10^{-22}\ \text{MeV s} = 1, \qquad c \approx 3 \times 10^{8}\ \text{m/s}^{-1} = 1\,, \\ \hbar c &\approx 2 \times 10^{-13}\ \text{MeV m} = 1\,. \end{aligned} \tag{1.13}$$

One can think of this convention as a choice of a unit system where the basis is $\{\hbar, c, \text{eV}\}$ instead of, for example, the {cm, g, sec} of the cgs system. In addition, it is common to make the factors of $\hbar$ and c implicit and measure everything in powers of eV. We reinstate the factors of $\hbar$ and c only when converting to a different unit system. The aim of this exercise is that you gain some practice in using these natural units.

1. The width of a particle is defined as the inverse of its lifetime. The mean lifetime for the B^+ meson is $\tau \approx 1.64 \times 10^{-12}$ s. What is its width in eV?
2. Consider a particle with a width of $\Gamma = 2.3$ eV. Recall that in the lab frame, $t = \gamma\tau$. What is the average distance that such a particle travels with $\gamma = 100$ before decaying (since $\gamma \gg 1$, you can use $\beta \approx 1$)?
3. Quantum gravity effects cannot be neglected at very short distances. This happens when the energy scale is of the order of the Planck mass:

$$M_{\rm Pl} \equiv \sqrt{\frac{\hbar c}{G_N}}, \tag{1.14}$$

   where G_N is the Newtonian gravitational constant. (The Planck scale constitutes an upper bound on the cutoff scale of all QFTs relevant to nature.) Express $M_{\rm Pl}$ in GeV, and the Planck length, $L_{\rm Pl} \equiv M_{\rm Pl}^{-1}$, in centimeters (cm).
4. In oscillation experiments for neutrinos, it is important to know the oscillation length, $L_{\rm osc} = 4\pi E/\Delta m^2$, where Δm^2 is the mass difference between the two neutrino states. For an experiment conducted with neutrinos of $E = 1.3$ GeV, find the value of Δm^2 in units of eV^2 that corresponds to $L_{\rm osc} = 140$ meters.

Question 1.3: Dimensions of terms

It is useful to understand what we refer to as the *dimension of operators* or the *dimension of Lagrangian terms*. The action has dimensions of angular momentum. Therefore, in the natural unit system, the action is dimensionless and the Lagrangian has a mass dimension of four (or, more generally, of the number of spacetime dimensions).

1. Based on the Lagrangians of equations (1.2) and (1.5), show that canonical scalar fields have dimension $d = 1$, and canonical fermion fields have dimension $d = 3/2$.
2. Find the dimensions of the m^2 parameter in equation (1.2) and of the m_{MR}, m_{ML}, and m_D parameters in equation (1.5).
3. What are the dimensions of η and λ in equation (1.2) and of Y in equation (1.8)?

Question 1.4: Accidental symmetries

In this question, we study a classical system to show examples of accidental symmetries. Consider a classical one-dimensional pendulum of length ℓ. The 1 DoF can be designated as θ, the angle of the pendulum. Then the Lagrangian is given by

$$L = \frac{m\ell^2\dot{\theta}^2}{2} - mg\ell(1 - \cos\theta). \tag{1.15}$$

Assuming small oscillations ($\theta \ll 1$), we can expand the potential. Keeping only terms up to the second order, we get

$$L = \frac{m\ell^2\dot{\theta}^2}{2} - \frac{mg\ell\theta^2}{2}, \tag{1.16}$$

which is the Lagrangian of a simple harmonic oscillator. It is well known that the frequency of a simple harmonic oscillator does not depend on its amplitude. Next, we aim to understand how this result is related to accidental symmetries.

1. Show that the equation of motion (EoM) derived from the Lagrangian of equation (1.16) is invariant under dilation, $\theta \to \lambda\theta$, for any finite λ. (We are then saying that L of equation (1.16) has dilation symmetry, despite the fact that it is only the EoM that is invariant.)
2. Does the Lagrangian of equation (1.15) also have dilation symmetry?
3. Expand the Lagrangian of equation (1.15) up to $O(\theta^4)$. Show explicitly that the θ^4 term breaks the dilation invariance. Explain why this implies that this symmetry is accidental.
4. Without a formal proof, argue that dilation symmetry implies that the frequency cannot depend on the amplitude.

What we have shown here is that the dilation symmetry is accidental and is broken by higher-order terms.

2

Abelian Symmetries

In section 1.3 of chapter 1, we explained the importance of symmetries in physics and presented the various types of symmetries that we encounter. In this chapter, we introduce various concepts and definitions concerning internal symmetries. In particular, we distinguish global from local symmetries, discrete from continuous symmetries, and chiral from vectorial symmetries. We further introduce the notion of charge and its relation to symmetries. In this chapter, we only discuss Abelian symmetries (i.e., symmetries that correspond to commuting symmetry groups). Non-Abelian symmetries are discussed in chapter 4.

2.1 Global Symmetries

The term *global symmetries* refers to symmetries under transformations that are constant in spacetime. This is in distinction to *local symmetries*, which are introduced in section 2.2, where the symmetry transformation is x_μ-dependent.

2.1.1 Global Discrete Symmetries

We start with a simple example of imposing an internal global discrete symmetry. Consider a real scalar field ϕ. The most general Lagrangian is given in equation (1.2) in chapter 1, which we rewrite here:

$$\mathcal{L}_S = \frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi) - \frac{m^2}{2}\phi^2 - \frac{\eta}{2\sqrt{2}}\phi^3 - \frac{\lambda}{4}\phi^4 \,. \tag{2.1}$$

We now impose a symmetry: we demand that $\mathcal{L}$ is invariant under $\phi \to -\phi$, namely,

$$\mathcal{L}(\phi) = \mathcal{L}(-\phi) \,. \tag{2.2}$$

$\mathcal{L}$ is invariant under this symmetry if $\eta = 0$. Thus, by imposing the symmetry, we force $\eta = 0$. The most general $\mathcal{L}(\phi)$ that is invariant under $\phi \to -\phi$, then, is

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi) - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4 \,. \tag{2.3}$$

What conservation law corresponds to this symmetry? We note that the number of ϕ-particles in a system that is described by the Lagrangian of equation (2.3) can change, but only by an even number. Therefore, if we define ϕ-parity as $(-1)^n$, where n is the number of ϕ-particles in the system, this ϕ-parity is conserved. If we do not impose the symmetry and $\eta \neq 0$, then the number of particles can change by any integer and ϕ-parity is not conserved.

It is a useful exercise to describe the symmetry in terms of group theory. Here, the relevant group is Z_2. It has two elements that we call *even* $(+)$ and *odd* $(-)$. The multiplication table is very simple:

$$\begin{array}{c|cc} Z_2 & (+) & (-) \\ \hline (+) & (+) & (-) \\ (-) & (-) & (+) \end{array} . \tag{2.4}$$

When we say that we impose a Z_2 symmetry on $\mathcal{L}$, with the Z_2 transformation law $\phi \to -\phi$, what we mean is that $\mathcal{L}$ belongs to the even representation of Z_2 and ϕ belongs to the odd representation. Clearly, for n being an integer, ϕ^{2n} belongs to the even representation and ϕ^{2n+1} belongs to the odd representation. For $\mathcal{L}$ to be Z_2-even, each term in $\mathcal{L}$ must be Z_2-even as well, and we must omit all terms with odd powers of ϕ. Then we can construct the most general $\mathcal{L}$, given by equation (2.3).

At this point, using the language of group theory may seem cumbersome and unnecessarily complicated. Later, however, we use this vocabulary to deal with more elaborate situations, where it has proved to be very useful.

2.1.2 Global Continuous Symmetries

We now extend our discussion to global continuous symmetries. The idea is that we demand that $\mathcal{L}$ is invariant under rotation in an internal space. While some of the fields are not invariant under rotation in that space, the combinations that appear in the Lagrangian are.

The fact that we can rotate between fields should not come as a surprise. This is nothing but the idea of generalized coordinates in action. Any linear combinations of the fields can be used as our coordinates, not just the one that we choose to start with.

Consider a complex scalar field, ϕ. A complex scalar field has 2 degrees of freedom (DoF). There are two useful ways to write the DoF explicitly. First, we can use a Cartesian form:

$$\phi \equiv \frac{1}{\sqrt{2}} \left(\phi_R + i\phi_I\right), \tag{2.5}$$

with ϕ_R and ϕ_I as real scalar fields. The most general renormalizable $\mathcal{L}(\phi_R, \phi_I)$ is given by

$$\mathcal{L}(\phi_R, \phi_I) = \frac{1}{2}\delta_{ij}\left(\partial^\mu \phi_i\right)\left(\partial_\mu \phi_j\right) - \frac{m_{ij}^2}{2}\phi_i\phi_j - \frac{\eta_{ijk}}{6}\phi_i\phi_j\phi_k - \frac{\lambda_{ijk\ell}}{24}\phi_i\phi_j\phi_k\phi_\ell\,,$$
$$i, j, k, \ell = R, I. \tag{2.6}$$

We now consider rotations in the complex plane:

$$\begin{pmatrix} \phi_R \\ \phi_I \end{pmatrix} \to O \begin{pmatrix} \phi_R \\ \phi_I \end{pmatrix}, \qquad O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{2.7}$$

When we say that we impose a *global* symmetry, we mean that θ is a number that does not depend on x_μ. Imposing that $\mathcal{L}(\phi_R, \phi_I)$ is invariant under the transformation of equation (2.7) forbids many terms and relates others, as follows:

$$\begin{aligned} \mathcal{L}(\phi_R, \phi_I) = {} & \frac{1}{2}\left(\partial^\mu \phi_R \partial_\mu \phi_R\right) + \frac{1}{2}\left(\partial^\mu \phi_I \partial_\mu \phi_I\right) - \frac{m^2}{2}\left(\phi_R\phi_R + \phi_I\phi_I\right) \\ & - \frac{\lambda}{4}\left(\phi_R^4 + \phi_I^4 + 2\phi_I^2\phi_R^2\right). \end{aligned} \tag{2.8}$$

In the language of group theory, the imposed symmetry—rotations in a two-dimensional real plane—is called $SO(2)$.

Second, we can formulate the transformation law directly in terms of the complex field ϕ:

$$\phi \to e^{i\theta}\phi, \qquad \phi^\dagger \to e^{-i\theta}\phi^\dagger. \tag{2.9}$$

Imposing that $\mathcal{L}(\phi, \phi^\dagger)$ is invariant under the transformation of equation (2.9) leads to

$$\mathcal{L}(\phi, \phi^\dagger) = \left(\partial^\mu \phi^\dagger\right)\left(\partial_\mu \phi\right) - m^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2. \tag{2.10}$$

In the language of group theory, the imposed symmetry—rotations in a one-dimensional complex plane—is called $U(1)$. (Mathematically, $SO(2)$ and $U(1)$ are equivalent. The different names represent the way that we think about the underlying space.) It is easy to check that the Lagrangians in equations (2.8) and (2.10) are equivalent. Equation (2.10), however, is more compact. We emphasize the following points regarding equation (2.10):

- The three terms that appear in this equation, and in particular the mass term, do not violate any internal symmetry. Thus, there is no way to forbid them by imposing an internal symmetry.
- We would obtain the same result if we scale θ by any nonzero number. Explicitly, we would obtain the same Lagrangian with a transformation law,

  $$\phi \to e^{iq\theta}\phi, \tag{2.11}$$

  for any finite q. Since q is arbitrary, we can choose the value to be 1, as we did. The situation is different when we have more than one complex field, as we discuss next.

In terms of group theory, the situation is explained as follows: we impose a $U(1)$ symmetry and assign the ϕ field to the $q = 1$ representation of that $U(1)$.

2.1.3 Charge

We are now ready to define charge. We are accustomed to the notion of charge from the specific case of electromagnetism. In electromagnetism, charge has two aspects: (1) It sets the strength of the interaction of the fermions with the photon; and (2) it is a conserved quantity. In this section, we deal with the latter point, while the aspect of interaction strength will emerge when we generalize our discussion to local symmetries in section 2.2. Charge conservation is related to symmetry. The general relation between internal global continuous symmetries and conserved charges is expressed by Noether's theorem. Here, we provide a simple example.

Consider a theory with two complex scalar fields, ϕ_1 and ϕ_2. To each field ϕ_i we assign a real number q_i. We impose a symmetry under the simultaneous phase rotation of both fields:

$$\phi_1 \to e^{iq_1\theta}\phi_1, \qquad \phi_2 \to e^{iq_2\theta}\phi_2. \tag{2.12}$$

The conjugate fields transform as follows:

$$\phi_1^\dagger \to e^{-iq_1\theta}\phi_1^\dagger, \qquad \phi_2^\dagger \to e^{-iq_2\theta}\phi_2^\dagger. \tag{2.13}$$

We say that q_i is the charge of the field ϕ_i. The charge of the conjugate field $\phi_i^\dagger$ is $-q_i$. While we can always set one of the charges, q_1 or q_2, to 1, we cannot do it for both. The ratio of charges, q_2/q_1, is a physical quantity.

The charge q_i is an input to model building: we assign charges to the fields and write the Lagrangian that is invariant under the rotations of equations (2.12) and (2.13). As a concrete example, consider a model with two complex scalar fields of charges: $q_1 = 1$ and $q_2 = 3$. Then the most general renormalizable Lagrangian that is invariant under equation (2.12) is

$$\begin{aligned}\mathcal{L} = {} & \left(\partial^\mu\phi_1^\dagger\right)\left(\partial_\mu\phi_1\right) + \left(\partial^\mu\phi_2^\dagger\right)\left(\partial_\mu\phi_2\right) - m_1^2\phi_1^\dagger\phi_1 - m_2^2\phi_2^\dagger\phi_2 \\ & - \lambda_{11}(\phi_1^\dagger\phi_1)^2 - \lambda_{22}(\phi_2^\dagger\phi_2)^2 - \lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\eta\phi_1^3\phi_2^\dagger + \text{h.c.}).\end{aligned} \tag{2.14}$$

A few comments are in order here:

- For a term to be allowed, the sum of the charges of the fields in this term must be zero.
- All the interactions that are allowed by the symmetry conserve the charge. This can be seen formally (and most generally) by Noether's theorem. It can also be seen for our specific example. Each term in the Lagrangian of equation (2.14) carries an overall charge of zero. Therefore, it corresponds to creation and annihilation of particles such that the sum of charges of the initial particles equals the sum of charges of the final particles.